

Dark Energy from Cosmological Energy Conservation

Manuel Uruena Palomo¹

¹ *Technical University of Munich (Germany) ge58vib@mytum.de (muruep00@gmail.com)*

Abstract

The value of the gravitational wave energy density is unknown. Current progress in gravitational wave detection suggests that the energy density of the stochastic gravitational wave background (SGWB) will be estimated in the next decades. We present a derivation of its value under the assumption that the energy lost due to cosmic redshift is fully responsible for the energy gained by the cosmological constant in the expanding universe. This unknown non-local mechanism of energy conservation on the cosmic scale could explain dark energy and hint at a property of a theory of quantum gravity.

Keywords: dark energy; gravitational wave energy density; cosmological constant; gravitational waves; redshift; energy conservation

I. INTRODUCTION

The observation of the accelerated expansion of the universe in 1998 [1], [2] and later confirmation by the analysis of small-scale anisotropies in temperature of the cosmic microwave background radiation (CMB) reopened the debate of the need of a cosmological constant Λ or constant energy of the vacuum ρ_{vac} to explain one of the main unsolved mysteries in cosmology for the last decades, “the dark energy” problem. This cosmological constant is nowadays a parameter of the Λ CDM cosmological model in which it is responsible for the accelerated expansion of the universe and regarded as the simplest solution for its observation. A constant non-zero energy of empty space exerts negative pressure, which drives accelerated expansion. Other solutions allow dark energy to be dynamic instead of constant, such as quintessence, which considers a scalar field that can be coupled to other quantum fields related to radiation or matter density [3]. In [4], it is shown that increasing total energy of the universe with pressureless dark energy is mathematically equivalent to total energy conservation with negative pressure dark energy.

The expansion of the universe not only dilutes radiation and matter energy densities but also for the case of radiation (photons, relativistic neutrinos, or gravitational waves (GWs)), their energy is lost through redshift due to the fact that it is inversely proportional to their wavelength.

The current value for the photon CMB energy density is around $\rho_{r,CMB}^0 \sim 10^{-31} \text{ kg/m}^3$ [5]. Astrophysical sources of photon energy density (e.g., stars and dust emission) can be neglected against the CMB because their number and energy are estimated to be at least two and one order of magnitude smaller, respectively.

The CMB was emitted at the recombination epoch (first 370.000 years). In contrast, relic relativistic massless neutrinos were created 1 second after the Big Bang. Using the cosmic scale factor $a(t)$ to characterize the expansion of the universe (representing the relative size of the universe at a given time t compared to its size at present epoch $a = 1$), relic neutrinos redshifted right after creation, and their energy density decreased proportionally to a^{-4} . This follows from the fluid equation and the equation of state, so that $a^{-3(1+w)}$ with $w = 1/3$ for the equation of state parameter of radiation. At the start of the matter-dominated era (first 60.000 years), neutrinos decoupled from other matter becoming non-relativistic, and their energy density decreased proportionally to a^{-3} without redshifting, with $w = 0$ for the equation of state parameter of matter. The current value for the cosmic neutrino background (CNB) is estimated to be around $\rho_{r,CNB}^0 \sim 10^{-31} \text{ kg/m}^3$ [5].

Gravitational waves propagate through space at the speed of light, and a gravitational wave background can be thought of as the accumulation of these waves spread across spacetime. There is now evidence for an stochastic gravitational wave background (SGWB) [6] composed of many localized, unresolved, and independent gravitational waves from different sources, and its energy density is unknown [6]. These can be classified into cosmological (possibly: quantum fluctuations, inflation, phase transitions in the early universe, alternative cosmologies, cosmic strings, etc.) and astrophysical (compact binary coalescences, supernova bursts, rotating pulsars, etc.). The term “stochastic” in SGWB refers to the random and unpredictable nature of the gravitational waves that contribute to this background. Its main constraint comes from indirect limits such as the Big Bang nucleosynthesis and recombination, which set a limit

around $\rho_{r,SGWB} \sim 10^{-33} \text{ kg/m}^3$ on the primordial density parameter at frequencies greater than 10^{-15} Hz [7]. Gravitational waves are barely absorbed nor reflected to any significant degree, so the dissipation of their energy takes place predominantly in redshift.

As opposed to radiation, the vacuum energy or cosmological constant does not dilute with the expansion of the universe, as it is a constant value of energy density with an estimated value of around $\rho_{vac} \sim 6 \cdot 10^{-27} \text{ kg/m}^3$ [5] to explain observations of accelerated expansion of spacetime. Thus, total cosmological constant energy (not density) increases proportionally to a^3 as the universe expands.

This loss and gain of energy is allowed to take place since global energy conservation cannot be defined in General Relativity because there is no time translation invariance in the expansion of the universe.

A possible reconciliation of metric theories of gravitation with violation of the conservation of energy-momentum has been studied in unimodular gravity (a generalization of general relativity in which the cosmological constant appears as a single additional variable) leading to the emergence of an effective cosmological constant in Einstein's equation [8]. Similar ideas were put forward by considering the reduction of the gravitational mass due to emitting gravitational waves, leading to a repulsive gravitational force related to dark energy [9]. Limits on the rate of a possible decay of the vacuum energy into a homogeneous distribution of thermalized CMB photons between the recombination era and the present have been set in [10].

As presented onwards, the energy lost in CMB redshift per unit of volume is just an order of magnitude smaller than the energy gained by the cosmological constant (the energy density of dark energy) per unit of volume since the recombination epoch. This suggests that energy conservation could be imposed by accounting for more contributions to the CMB lost energy, such as GW redshift, to match the energy gained by the cosmological constant.

II. ENERGY CONSERVATION ON THE COSMIC SCALE

The energy density of radiation is the particle's energy times the number density. The energy density lost due to photon redshift can be estimated by subtracting the current red-shifted average energy density of photons from the past average energy density of photons

$$\frac{n[(1+z(t))k_B T_o - k_B T_o]}{c^2} \quad (1)$$

with z being the redshift (relative difference between the emitted and observed wavelengths or frequencies light), k_B the Boltzmann constant, T_o the current measured CMB temperature at present epoch, and n the number density of photons of the CMB per cubic meter.

For $T_o = 2,72 \text{ K}$ [11], $z = 3000/2,72 = 1090$ (with 3000K being the re-ionizing temperature of Hydrogen in the plasma) since recombination, and $n = 16\pi \left(\frac{k_B T}{hc}\right)^3 \zeta(3) \simeq 411 \cdot 10^6$ photons per cubic meter for a near-perfect blackbody and ζ being the Riemann zeta function, one obtains an energy density lost due to CMB redshift of $1,7 \cdot 10^{-28} \text{ kg/m}^3$ which is an order of magnitude smaller than observed ρ_{vac} of the cosmological constant with values $(6,03 \pm 0,13) \cdot 10^{-27} \text{ kg/m}^3$ from CMB measurements, $(7,03 + 0,27 - 0,31) \cdot 10^{-27} \text{ kg/m}^3$ from local distance ladder measurements of the Hubble parameter using Cepheids, and $(6,33 + 0,37 - 0,29) \cdot 10^{-27} \text{ kg/m}^3$ from measurements using the Tip of the Red Giant Branch [12] (current redshifted energy density can be neglected in (1)).

To apply energy conservation, first, the energy that has been gained by the cosmological constant along the universe scale factor per unit of space volume must be estimated. This must be the energy lost due to SGWB (plus CMB) redshift along the scale factor. For the unit of volume of one cubic meter today, the total energy gained of the cosmological constant is $E_{vac} = 5.8 \pm 1.8 \cdot 10^{-27} \text{ kg}$. At recombination epoch of $a = 1/1090$, the total energy of the cosmological constant for today's cubic meter before expansion is 9 orders of magnitude smaller, and can be neglected. Thus, $E_{lost} = 5.8 \pm 1.8 \cdot 10^{-27} \text{ kg}$ would be the total energy lost due to SGWB (plus CMB) redshift along the scale factor per unit of volume. Since total energy of SGWB (plus CMB) is lost proportional to a^{-1} , the energy density of SGWB (plus CMB) at recombination would be around $\rho_r \sim 10^{-18} \text{ kg/m}^3$.

This is clearly above the limit of energy density of gravitational waves at recombination set in [7]. Thus, the main issue for the energy gained by the cosmological constant to be equal to the energy lost by CMB and SGWB redshifts is that both CMB and SGWB lose energy proportional to a^1 while the total

energy of the cosmological constant grows proportional to a^3 . Additionally, the CMB and SGWB energy densities decrease proportionally to a^{-3} by dilution. Both can only be equaled if most of the SGWB energy density is produced along the universe's age, for instance, by astrophysical sources. Thus, SGWB energy density $\rho_{r,SGWB}(a)$ cannot be simply calculated from current $\rho_{r,SGWB}^0$ through $\rho_{r,SGWB}^0 a^{-4}$.

Then, the energy density of the SGWB, so that its red-shifted energy lost is equal to the gained cosmological constant energy at any given scale factor, can be obtained, accounting for its dilution. Additionally, the rate of SGWB energy originated from astrophysical sources throughout the scale factor can be calculated. Finally, the values of the energy density of the SGWB along the scale factor and its nowadays value can be derived.

III. DISCUSSION

We have briefly introduced a source for the cosmological constant dark energy based on energy conservation of cosmic radiation redshift of photons and gravitational waves, and proposed a way to calculate the current energy density of the stochastic gravitational wave background. Based on constraints on energy density of the stochastic gravitational wave background at the early universe, we find that most of the energy of the stochastic gravitational wave background must have been produced along the universe timeline. In this proposal, the cosmological constant field exchanges energy with the electromagnetic and gravitational field, which is natural since the electromagnetic field is a source of energy and thus, a source of gravity in General Relativity. Energy conservation might be another condition to be imposed to General Relativity to properly describe physical reality, together with energy conditions.

One could argue that massive particles should not contribute significantly to the energy loss that is transferred to the cosmological constant as dimensionally altered by the expansion because their interactions reset the difference in distances, although they are certainly affected in some way. Also, virtual massless particles should not in principle be affected either. If these values are significant enough, the estimated SGWB energy density would be smaller. The same would happen for other unknown contributions, such as particles decaying into vacuum energy.

Assuming that the cosmological constant energy density is constant thought-out space, the hidden underlying mechanism for energy conservation through redshift must be non-local, hinting that the mechanism has a quantum nature. If the cosmological constant is not constant through space, locality may be preserved and regions with greater amount of it would imply greater past SGWB. If the cosmological constant is not constant through time, the Hubble tension could be resolved and the age of the universe estimations would change. Also, a different fate for the universe instead of the big freeze could occur.

Cosmic inflation could also be described by the same transition of energy due to redshift to a quantum field such as the scalar inflaton field.

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