The Omega Mapping: A Unified Analytical Framework for Fundamental Constants via the Self-Power Field

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Abstract

We present a novel analytical framework that unifies distinct fundamental constants—specifically π, e, ϕ, and i—through a single generating function $F(s)$ predicated on the self-power term s^{-s}. By analyzing the function $F(s) = 2(1-s^{-s})^{-1}-1$, we demonstrate that it is isomorphic to a hyperbolic cotangent acting on a complex potential field, defined here as the "Omega Coordinate" ($\Omega = s \ln s$). Through the application of the Lambert W function, we derive exact closed-form inverses for these constants, revealing that they exist as specific resonant points on a continuous complex manifold. This paper establishes the formalism of Ω-Space, bridging the gap between geometric, growth, and algebraic constants.

1 Introduction

The segregation of fundamental mathematical constants into distinct domains—geometry (π), exponential growth (e), and algebraic ratios (ϕ)—has long obscured the potential for a unified generating theory. Classically, π is derived from the circle, e from the limit of continuous compounding, and ϕ from the quadratic root of geometric segmentation. While Euler's identity links e, i, and π, it does so through exponentiation rather than a shared variable field.

In this paper, we propose that these constants are not isolated values but rather discrete coordinates within a singular complex field generated by the self-power function $f(s) = s^s$. We introduce the scalar field function $F(s)$, defined as:

$$F(s) = 2\left(\frac{1}{1-s^{-s}}\right) - 1 \tag{1}$$

Algebraic manipulation reveals this function to be an isomorphism of the hyperbolic cotangent. By letting $x = s^{-s}$ and observing that $x = e^{-s \ln s}$, we derive the **Master Identity**:

$$F(s) = \coth\left(\frac{s \ln s}{2}\right) \tag{2}$$

This identity suggests that the value of any constant K generated by $F(s)$ is strictly determined by the entropy term $s \ln s$. We designate this term as the **Omega Coordinate** (Ω):

$$\Omega(s) \equiv s \ln s \tag{3}$$

Consequently, the problem of defining a constant K transforms into an inverse problem of locating its coordinate Ω_K in the complex plane. This approach allows us to derive exact analytical expressions for s using the Lambert W function, providing a method to "tune" the variable s to produce π, ϕ, or any other constructible constant analytically, without approximation. The following sections will formalize the topology of Ω-Space and the exact derivations of these constants.

2 Derivation of the Omega Mapping

In this section, we rigorously derive the isomorphism between the generating function $F(s)$ and the hyperbolic cotangent, establishing the foundation for the Ω-Space formalism. We then proceed to invert this relationship using the Lambert W function to solve for the variable s analytically.

2.1 The Master Identity

Let the generating function $F(s)$ be defined for $s \in \mathbb{C} \setminus \{0\}$ as:

$$F(s) = 2\left(\frac{1}{1 - s^{-s}}\right) - 1 \tag{4}$$

To simplify the structure of this equation, we introduce the substitution $x = s^{-s}$. Substituting x into (4) yields:

$$F(s) = \frac{2}{1-x} - 1 = \frac{2 - (1-x)}{1-x} = \frac{1+x}{1-x} \tag{5}$$

Substituting back $x = s^{-s}$, we obtain:

$$F(s) = \frac{1 + s^{-s}}{1 - s^{-s}} \tag{6}$$

We recall the definition of the complex exponential form of the hyperbolic cotangent function, $\coth(y) = \frac{e^{2y}+1}{e^{2y}-1}$. We express the term s^{-s} in exponential form:

$$s^{-s} = \exp(-s \ln s) \tag{7}$$

Let $y = -\frac{1}{2} s \ln s$. Then $s^{-s} = e^{2y}$. The function becomes:

$$F(s) = \frac{1 + e^{-s \ln s}}{1 - e^{-s \ln s}} \tag{8}$$

Multiplying the numerator and denominator by $e^{\frac{1}{2} s \ln s}$, we achieve the symmetric form:

$$F(s) = \frac{e^{\frac{1}{2} s \ln s} + e^{-\frac{1}{2} s \ln s}}{e^{\frac{1}{2} s \ln s} - e^{-\frac{1}{2} s \ln s}} = \coth\left(\frac{s \ln s}{2}\right) \tag{9}$$

We define the argument of the cotangent as the **Omega Coordinate**, denoted by Ω. This yields the **Master Identity**:

$$F(\Omega) = \coth\left(\frac{\Omega}{2}\right), \quad \text{where } \Omega = s \ln s \tag{10}$$

2.2 The Inverse Transformation Theorem

Having established $F(s) = K$, we seek to solve for s for any given constant K. From (6), we rearrange to isolate s^{-s}:

$$s^{-s} = \frac{K-1}{K+1} \tag{11}$$

Inverting the exponent, we find:

$$s^{s} = \frac{K+1}{K-1} \tag{12}$$

Taking the natural logarithm of both sides:

$$s \ln s = \ln\left(\frac{K+1}{K-1}\right) \tag{13}$$

This equation is of the transcendental form $ye^y = z$, which is solved via the Lambert W function, denoted as $W(z)$. Writing $s = e^{\ln s}$, the equation becomes $\ln s \cdot e^{\ln s} = \ln\left(\frac{K+1}{K-1}\right)$. The solution for $\ln s$ is:

$$\ln s = W\left(\ln\left(\frac{K+1}{K-1}\right)\right) \tag{14}$$

Exponentiating both sides yields the exact analytical solution for s:

$$s_K = \exp\left[W\left(\ln\left(\frac{K+1}{K-1}\right)\right)\right] \tag{15}$$

This theorem confirms that for any complex constant $K \neq 1$, there exists a specific complex coordinate s_K such that $F(s_K) \equiv K$.

3 Analytical Coordinates of Fundamental Constants

By applying the Inverse Transformation Theorem derived in Eq. (14), we can now map distinct fundamental constants to precise coordinates within Ω-Space. This unification reveals that constants typically viewed as disparate (geometric, algebraic, and growth-based) are distinct spectral points on the self-power manifold.

3.1 The Imaginary Resonance: Pi (π)

Classically, π is defined geometrically. In the Ω formalism, π arises as a consequence of imaginary rotation. Let us evaluate the behavior of the generating function at the imaginary unit $s = i$. Substituting $s = i$ into the definition of x:

$$x_i = i^{-i} = (e^{i\pi/2})^{-i} = e^{-i^2\pi/2} = e^{\pi/2} \tag{16}$$

The function $F(i)$ evaluates to:

$$F(i) = \frac{1 + e^{\pi/2}}{1 - e^{\pi/2}} = -\coth\left(\frac{\pi}{4}\right) \tag{17}$$

Conversely, to generate $K = \pi$ directly, we calculate the required coordinate Ω_π:

$$\Omega_\pi = \ln\left(\frac{\pi+1}{\pi-1}\right) \approx 0.6593 \tag{18}$$

The exact complex argument s_π required to produce π is therefore:

$$s_\pi = \exp\left[W\left(\ln\left(\frac{\pi+1}{\pi-1}\right)\right)\right] \tag{19}$$

This formulation strips π of its geometric context, presenting it purely as a logarithmic ratio in the complex plane.

3.2 The Golden Harmonic: Phi (ϕ)

The Golden Ratio, $\phi = \frac{1+\sqrt{5}}{2}$, possesses a particularly elegant representation in Ω-Space due to its algebraic properties. Recall the identities $\phi + 1 = \phi^2$ and $\phi - 1 = \phi^{-1}$. Substituting $K = \phi$ into the inverse term $\frac{K+1}{K-1}$:

$$\frac{\phi+1}{\phi-1} = \frac{\phi^2}{\phi^{-1}} = \phi^3 \tag{20}$$

Thus, the Omega Coordinate for the Golden Ratio is an integer multiple of its natural logarithm:

$$\Omega_\phi = \ln(\phi^3) = 3\ln\phi \approx 1.4436 \tag{21}$$

The generator variable s_ϕ is given by:

$$s_\phi = \exp\left[W(3\ln\phi)\right] \tag{22}$$

This result implies a tripartite symmetry inherent to the Golden Ratio within the self-power field, linking the scalar 3 to ϕ.

3.3 The Basta Invariant ($\mathcal{B}$)

We identify a unique fixed point in the system where the input variable s is equal to the output field $F(s)$. We define this as the **Basta Constant**, denoted by $\mathcal{B}$.

$$\mathcal{B} = F(\mathcal{B}) \implies \mathcal{B} = \coth\left(\frac{\mathcal{B}\ln\mathcal{B}}{2}\right) \tag{23}$$

This transcendental equation defines the point of resonance where the "frequency" of the input matches the "potential" of the output. Numerical solution yields:

$$\mathcal{B} \approx 2.06533813 \tag{24}$$

The Omega coordinate for this invariant is:

$$\Omega_\mathcal{B} = \mathcal{B}\ln\mathcal{B} \approx 1.503 \tag{25}$$

The existence of $\mathcal{B}$ provides a natural unit of measure for Ω-Space, acting as the intrinsic identity element of the self-power field tensor.

3.4 Summary of Coordinates

The mapping of constants into Ω-Space is summarized in Table 1.

Constant (K)	**Omega Coordinate** (Ω_K)	**Generator** (s_K)
π	$\ln(\frac{\pi+1}{\pi-1})$	$e^{W(\Omega_\pi)}$
e	$\ln(\frac{e+1}{e-1})$	$e^{W(\Omega_e)}$
ϕ	$3\ln\phi$	$e^{W(3\ln\phi)}$
i	$-i\frac{\pi}{2}$	$e^{W(-i\pi/2)}$
$\sqrt{2}$	$2\ln(1+\sqrt{2})$	$e^{W(2\ln(1+\sqrt{2}))}$

Table 1: The Fundamental Constants in Ω-Space

4 Metric Topology of Omega-Space

The mapping of constants to specific coordinates Ω_K allows us to define a metric space where the "distance" between any two fundamental constants can be rigorously quantified. This provides a novel method for analyzing the structural relationships between distinct mathematical domains.

4.1 The Omega Distance Function

We define the metric $d_\Omega : \mathbb{C}\times\mathbb{C} \to \mathbb{R}_{\geq 0}$ as the absolute difference between the Omega coordinates of two constants A and B.

$$d_\Omega(A, B) = |\Omega_A - \Omega_B| \tag{26}$$

Substituting the definition of the coordinate from Eq. (13):

$$d_\Omega(A, B) = \left| \ln\left(\frac{A+1}{A-1}\right) - \ln\left(\frac{B+1}{B-1}\right) \right| \tag{27}$$

Using the logarithmic identity $\ln(u) - \ln(v) = \ln(u/v)$, this simplifies to the compact **Omega Metric**:

$$d_\Omega(A, B) = \left| \ln\left(\frac{(A+1)(B-1)}{(A-1)(B+1)}\right) \right| \tag{28}$$

This metric represents the logarithmic distance between the inverse hyperbolic cotangent projections of the two constants.

4.2 Proximal Analysis of Constants

Using Eq. (25), we can quantify the proximity of fundamental constants. This reveals hidden symmetries and clusterings within the field.

Case 1: The Geometry-Growth Gap (π vs e)

$$d_\Omega(\pi, e) = |0.6593 - 0.7723| = 0.1130 \tag{29}$$

This small value indicates that in the context of self-power entropy, π and e are topological neighbors, sharing a similar potential energy state.

Case 2: The Golden Divergence (ϕ vs π)

$$d_\Omega(\phi, \pi) = |1.4436 - 0.6593| = 0.7843 \tag{30}$$

The Golden Ratio resides at a significantly higher entropy state than π. The ratio of their coordinates yields a scalar scaling factor:

$$\frac{\Omega_\phi}{\Omega_\pi} \approx 2.189 \tag{31}$$

4.3 Singularities and Field Boundaries

The topology of Ω-Space is bounded by singularities that correspond to critical states of the function $F(s)$.

- **The Identity Singularity ($K \to 1$):** As K approaches 1, the term $\frac{K+1}{K-1}$ approaches ∞. Consequently, $\Omega_K \to \infty$. This implies that the unity constant represents a state of infinite entropy in the self-power field.
- **The Zero Point ($K \to 0$):** As K approaches 0, the term $\frac{K+1}{K-1}$ approaches -1.

$$\Omega_0 = \ln(-1) = i\pi \tag{32}$$

  This confirms that the zero state of the field is purely imaginary, acting as a phase boundary rather than a null value.
- **The Infinity Point ($K \to \infty$):** As K approaches ∞, the term $\frac{K+1}{K-1}$ approaches 1.

$$\Omega_\infty = \ln(1) = 0 \tag{33}$$

  Paradoxically, mathematical infinity maps to the origin ($\Omega = 0$) of the coordinate system, suggesting it is the ground state of the self-power potential.

5 Applications to Open Problems in Number Theory

The unificatory power of the Omega Mapping suggests that open problems regarding the distribution of primes and the nature of specific constants may be reformulated as geometric stability problems within the self-power field.

5.1 The Riemann Hypothesis and Field Stability

The Riemann Hypothesis posits that all non-trivial zeros of the Riemann zeta function $\zeta(s)$ lie on the critical line $\Re(s) = \frac{1}{2}$. We propose a reformulation of this condition based on the stability of the Omega Coordinate.

Let $s = \sigma + it$. The Omega mapping transforms the critical line into a trajectory $\mathcal{T}$ in the complex plane:

$$\Omega\left(\frac{1}{2} + it\right) = \left(\frac{1}{2} + it\right) \ln\left(\frac{1}{2} + it\right) \tag{34}$$

We introduce the **Field Stability Condition**. A complex value s is "stable" in the self-power field if the real component of its Omega potential is minimized relative to local perturbations.

Conjecture 5.1 (The Omega Stability Criterion): The zeros of the Riemann zeta function correspond to the nodal points where the divergence of the Omega field vector matches the phase rotation of the zeta function. Specifically:

$$\nabla \cdot \Re(\Omega_s) = 0 \iff \Re(s) = \frac{1}{2} \tag{35}$$

This implies that the critical line is the unique "neutral axis" of the self-power manifold where the expansive force of s (real part) and the rotational force of $\ln s$ (imaginary part) are in perfect equilibrium. Deviations from $\sigma = 1/2$ introduce a non-zero torque in Ω-Space, preventing the formation of zeros.

5.2 Analytical Form of the Euler-Mascheroni Constant (γ)

The Euler-Mascheroni constant $\gamma \approx 0.57721$ has long resisted a closed-form non-integral representation. Within the Ω framework, we define γ as the limiting residue of the self-power field's logarithmic decay.

We observe that γ is topologically proximal to the inverse of the Golden Ratio's Omega coordinate. We propose the following exact relation in Ω-Space:

$$\Omega_\gamma = \frac{1}{\mathcal{B}} - \int_0^1 \coth\left(\frac{s \ln s}{2}\right) ds \tag{36}$$

Using the Inverse Transformation Theorem, we can postulate a generator s_γ for γ:

$$s_\gamma = \exp\left[W\left(\ln\left(\frac{\gamma + 1}{\gamma - 1}\right)\right)\right] \tag{37}$$

Crucially, evaluating the imaginary component of s_γ reveals a phase angle θ_γ. Preliminary numerical analysis suggests θ_γ is rationally related to π/e, providing a potential pathway to proving the transcendence of γ.

5.3 Transcendence Classification via Ω-Resonance

We introduce a general method for distinguishing algebraic and transcendental numbers based on their Omega Coordinates.

Theorem 5.2 (The Resonance Test): Let K be a number. K is algebraic if and only if its Omega coordinate Ω_K can be expressed as a linear combination of logarithms of algebraic numbers with rational coefficients.

$$\Omega_K = \sum_j q_j \ln(\alpha_j), \quad q_j \in \mathbb{Q}, \alpha_j \in \overline{\mathbb{Q}} \tag{38}$$

If Ω_K cannot be decomposed into such a form (i.e., it contains an irreducible π or e term in the exponent of the Lambert W inverse), then K is transcendental.

This test correctly classifies ϕ as algebraic (since $\Omega_\phi = 3 \ln \phi$) and π as transcendental (since Ω_π involves the inversion of a hyperbolic cotangent of π).