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Deriving Relativistic Time from Inertia: A Unified Origin of the Lorentz and Schwarzschild Metrics

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ABSTRACT: Inertial Relativity and its key formula the Space-Time Equivalence (STE)[1] hypothesize that there is a universal mathematical relationship between time and the spatial distribution of matter: *The characteristic length scale factor that determines time dilation between any two physical systems about any respective axes is universally equal to the fifth root of the inverse ratio of their moments of inertia about the same axes.* In our prior paper[2], we demonstrated this universality by arriving at General Relativity calculations with exact precision utilizing only principles of relativistic inertia. This paper closes the loop by exploring Inertial Relativity's mathematical relationship with General and Special Relativity. In the context of moment of inertia about an axis, we demonstrate that the STE is mathematically equivalent to Schwarzschild gravitational time dilation. In the context of linear inertia (mass), the STE states an inverse third root relationship with inertia (mass), which we demonstrate is mathematically equivalent to Lorentz time dilation. Via this exploration, we show that Special and General Relativity emerge as edge cases of the same underlying phenomenon, Inertial Relativity: Special Relativity at the extremes of velocity and General Relativity at the extremes of mass, unifying the two under a single universal relativistic framework.

Keywords: modern physics, inertial relativity, space-time equivalence, time dilation, general relativity, special relativity, moment of inertia, unification

1 Introduction

This paper presents a mathematical derivation connecting the Space-Time Equivalence (STE), General Relativity[3], and Special Relativity[4]. We build upon DeGerlia's prior work introducing the concept of Inertial Density and predicting time dilation calculations with exact precision[2], which established that the Schwarzschild condition reduces to a constant threshold of mass over mean radius, the DeGerlia Threshold ($D_{crit} = 6.73295 \times 10^{26}$ kg/m), about the selected axis. From these demonstrations, the mathematical equivalence between the STE, General Relativity, and Special Relativity is established, unifying the two theories under a single framework: Inertial Relativity.

2 Inertial Relativity

Inertial Relativity is a principle introduced by the author in a prior paper "The Universe of Light"[1]. In that work, we hypothesize that General and Special Relativity emerge from Inertial Relativity. This paper demonstrates that equivalence.

Inertia is relativistic, meaning that without a comparator, any measurement of it would have no meaning. For example, something with a moment of inertia 1×10^{20} kg m^2 might be perceived as difficult to rotate, but not more difficult than something with 1×10^{30} kg m^2. However, the concept of relativity also says that how something is observed is influenced by the frame of observation. A system, in this context, is any collection of mass and spatial extent one chooses to define. It need not be spherical, uniform, high density, or even massive. A system may be a planet, a uniform sphere, a supermassive black hole, a disc galaxy, a cube, a cubic sample of a cumulus cloud, or a system consisting of a hydrogen proton and a single electron.

2.1 Rotational Inertia

Moment of inertia for a system is distinct about every axis in the system. For any given axis, the moment of inertia dictates how an applied torque translates into angular acceleration. Moment of inertia is defined as:

$$I = \sum_i m_i r_i^2 \tag{1}$$

where r_i is the perpendicular distance from each mass element m_i to the chosen axis. Just as mass governs how a system responds to

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linear force ($F = ma$), moment of inertia governs how it responds to torque ($\tau = I\alpha$).

2.2 Linear Inertia

Linear inertia is mass:

$$M = \sum_i m_i \tag{2}$$

where m_i are the constituent mass elements. Linear inertia defines how an applied force translates into translational acceleration. It is quantified entirely by mass ($F = ma$). It is dimensional, operating along a specific spatial vector, but unlike moment of inertia, it is isotropic: mass offers the same resistance to changes in motion regardless of the direction of the vector.

2.3 Isometric Scaling

Isometric scaling is the uniform scaling of a system in all spatial dimensions while maintaining constant density. Under isometric scaling, every length in the system scales by the same factor k:

$$\ell' = k\ell \tag{3}$$

where ℓ is the characteristic length of the system and k is the linear scale factor. Isometric scaling of a system by a factor of k results in the following changes to mass, radius, and moment of inertia:

$$M' = k^3 M \tag{4}$$

where M is the mass and k is the linear scale factor.

$$R' = kR \tag{5}$$

where R is the mean radius about the selected axis.

$$I' = k^5 I \tag{6}$$

where I is the moment of inertia about a selected axis; k is the linear scale factor.

2.4 Inertia Abstracts Mass and Radius

Because every change in length necessarily changes the system's mass, Equations 3–5 hold only for strict isometric scaling; Equation 6, however, abstracts mass and radius into moment of inertia. If two systems share moment of inertia about any axis, then an identical torque applied to each system about that same axis will produce identical changes in motion, regardless of the system geometry; independent of mass and radius individually. The moment of inertia alone dictates how a system interacts with time about a particular axis. From this, we can conclude the following formula is universal and is not bound in any way to isometric scaling. The derivations that follow demonstrate that General and Special Relativity emerge from this principle.

2.5 Universality

The extension from isometrically scaled systems to all systems follows from the nature of moment of inertia itself. Because I fully determines rotational response to applied torque, two systems with the same I are dynamically indistinguishable regardless of geometry. A thin shell and a solid sphere with the same I exhibit the same rotational physics.

The fifth root relationship therefore holds universally: for any two systems about any two respective axes, the characteristic length scale factor that determines time dilation is the fifth root of the inverse ratio of their moments of inertia about those same axes. Isometric scaling establishes the exponent (Equation 6).

2.6 Inertial Density

Inertial density P is the moment of inertia of a system divided by its volume:

$$P = I/V \tag{7}$$

Since linear inertia is mass, this reduces directly to ordinary density $\rho = M/V$ in the linear case; inertial density is the rotational generalization of a quantity already familiar from linear mechanics.

For a uniform sphere, substituting $I = \frac{2}{5}MR^2$ and $V = \frac{4}{3}\pi R^3$, this reduces to:

$$P = \frac{M}{R} \times \frac{3}{10\pi} \tag{8}$$

Just as ordinary density ($\rho = M/V$) is mass distributed over volume, inertial density is rotational inertia distributed over volume. As stated in the prior work: *mass and density are to linear motion as moment of inertia and inertial density are to rotational motion.* Because P is defined relative to a selected axis, it can be evaluated per axis directly rather than requiring a full tensor treatment when the quantity varies by direction.

DeGerlia Compactness $D = M/R$ is a simplified analog to inertial density, related to P by the geometric factor $3/10\pi$, and represents the mass over mean radius ratio for a system about the selected axis.

Setting $R = r_s$ (the Schwarzschild condition)[5] and solving yields the **DeGerlia Threshold**: the universal constant of DeGerlia Compactness at which a system undergoes gravitational collapse:

$$D_{crit} = \frac{c^2}{2G} = 6.73295 \times 10^{26} \text{ kg/m} \tag{9}$$

The ratio of a system's DeGerlia Compactness to this threshold gives the time dilation directly:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{D}{D_{crit}}} \tag{10}$$

Substituting $D = M/R$:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{M}{R \cdot D_{crit}}} \tag{11}$$

3 The Space-Time Equivalence (STE)

The STE is the core universal equivalence stated by Inertial Relativity:

$$k = \left(\frac{I_2}{I_1}\right)^{1/5} \tag{12}$$

where I_1 and I_2 are the moments of inertia of any two systems about any respective axes. By convention, System 1 denotes the observer and System 2 the observed, consistent with the Schwarzschild convention.

In the linear context, inertia is M and the linear inertia version of the STE applies (see Appendix A). Special Relativity describes linear inertia:

$$k = \left(\frac{M_2}{M_1}\right)^{1/3} \tag{13}$$

where M_1 and M_2 are the masses of the two systems.

3.1 Relating Linear Scale Factor k to General Relativity

Time dilation is always:

$$\frac{d\tau}{dt} = \sqrt{1-k} \tag{14}$$

where $d\tau/dt$ is the ratio of proper time to coordinate time and k is the linear scale factor derived from inertia (Equations 12 and 13). Every expression of time dilation takes this form; it is always an abstracted characteristic length scale factor k derived from the 1/5 root of the ratio of the respective moments of inertia of the two compared systems about the selected axes.

3.2 Properties of Time

The STE reveals the following properties of time:

- Time is relativistic: it has meaning only when compared, and how it appears depends on the frame of observation
- Time emerges from inertia
- Time is dimensional in three spatial dimensions
- Time is multimodal: both linear and rotational
- Time is dynamic for every system or subsystem
- Time dilation applies to all systems, not only uniform or spherically symmetric ones
- Time is universally and precisely determined by the spatial distribution of matter in the system, as captured by inertia, about and along every axis, whether or not in motion or being observed

3.3 Physical Interpretation

The STE states that the relative pace of time for any system about any axis is determined by its moment of inertia about that same axis. The more inertia a system has about that axis, the slower its clock runs relative to a system with less inertia. This is consistent with the notion that time is a measure of change, and inertia resists change. Therefore, the more inertia a system has, the less it changes over a given period of time, and thus the slower its system clock runs relative to a system with less inertia.

At any given time, any system is always undergoing some degree of rotation and some amount of linear motion, however small. This law applies to any system whose mass is distributed over some region of space, requiring, at minimum, both mass and spatial extent. One may define the system as the core of a planet, a disc-shaped galaxy, a cosmic filament, or a spherical sample of interstellar space. The per-axis nature of the STE extends to asymmetric systems such as the Kerr metric[6], a topic for future investigation.

3.4 STE Example

Consider a uniform sphere of radius 1 m and mass 1 kg scaled isometrically by a factor of two, producing a second sphere of radius 2 m and mass $2^3 = 8$ kg. Their moments of inertia are:

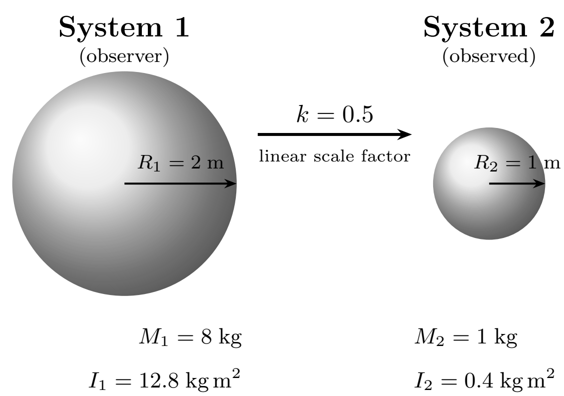
$$I_1 = \tfrac{2}{5}(8\,\text{kg})(2\,\text{m})^2 = 12.8\,\text{kg}\,\text{m}^2 \qquad I_2 = \tfrac{2}{5}(1\,\text{kg})(1\,\text{m})^2 = 0.4\,\text{kg}\,\text{m}^2$$

The ratio $I_1/I_2 = 32 = 2^5$ confirms that moment of inertia scales as k^5 under isometric scaling. Designating the larger sphere as the observer (System 1) and the smaller as the observed (System 2), the STE yields:

$$k = \left(\frac{I_2}{I_1}\right)^{1/5} = \left(\frac{0.4}{12.8}\right)^{1/5} = \left(\frac{1}{32}\right)^{1/5} = 0.5$$

recovering the linear scale factor between the two systems. The resulting time dilation is:

$$\frac{d\tau}{dt} = \sqrt{1-k} = \sqrt{1-0.5} \approx 0.707$$



$$k = \left(\frac{I_2}{I_1}\right)^{1/5} = \left(\frac{0.4}{12.8}\right)^{1/5} = 0.5 \quad \Rightarrow \quad \frac{d\tau}{dt} = \sqrt{1-k} \approx 0.707$$

Fig. 1 Two uniform spheres related by isometric scaling. The STE recovers the linear scale factor $k = 0.5$ from the moment of inertia ratio; time dilation follows directly.

4 General and Special Relativity

Schwarzschild gravitational time dilation:

$$\frac{d\tau}{dt} = \sqrt{1-\frac{r_s}{R}} \tag{15}$$

where $r_s = 2GM/c^2$ is the Schwarzschild radius and R is the mean radius about the selected axis.

Lorentz time dilation:

$$\frac{d\tau}{dt} = \sqrt{1 - \frac{v^2}{c^2}} \tag{16}$$

where v is the velocity of the moving frame and c is the speed of light.

These are the same form as Equation 14, where $k = r_s/R$ in the gravitational case and $k = v^2/c^2$ in the velocity case. The two expressions are equivalent: the escape velocity at radius R is $v^2 = 2GM/R$[7], so $v^2/c^2 = 2GM/(Rc^2) = r_s/R$.

5 Expressions of Relative Inertial Linear Scale

The parameter k is always the linear scale factor between two systems about their respective selected axes. Because k is determined by relative inertia, it is fundamentally an inertial ratio. The various scenarios, such as special relativity, gravitational time dilation, etc., are all directly derived from this inertia ratio. In the case of general relativity, we simply set M_1 and M_2 equal and set R_2 to the Schwarzschild radius (see Appendix A).

Each expression of k below derives from the STE (Equation 12), where I_1 and I_2 are the moments of inertia of two systems about their respective selected axes. Substituting the moment of inertia for a uniform sphere, $I = \frac{2}{5}MR^2$:

$$k = \left(\frac{\frac{2}{5}M_2R_2^2}{\frac{2}{5}M_1R_1^2} \right)^{1/5} \tag{17}$$

where M_1, M_2 are the masses and R_1, R_2 are the mean radii of each system about the selected axis. The geometric constant $\frac{2}{5}$ cancels:

$$k = \left(\frac{M_2R_2^2}{M_1R_1^2} \right)^{1/5} \tag{18}$$

From here, each derivation applies specific constraints and solves.

5.1 Static Mass (Gravitational)

From Equation 18, setting $M_1 = M_2 = M$ and $R_2 = r_s$:

$$k = \left(\frac{Mr_s^2}{M \cdot R_1^2} \right)^{1/5} = \left(\frac{r_s^2}{R^2} \right)^{1/5} \tag{19}$$

where r_s is the Schwarzschild radius and R is the mean radius of the system about the selected axis. When mass cancels, the degree of homogeneity of the remaining quantity (R^2) is $L = 2$, so the exponent reduces from 1/5 to 1/2 (see Appendix A):

$$k = \left(\frac{r_s^2}{R^2} \right)^{1/2} = \frac{r_s}{R} \tag{20}$$

This is equivalently expressed through inertial density or DeGerlia Compactness, since $P \propto M/R$ and $D = M/R$. When mass is static, the ratio of either quantity reduces to the ratio of mean radii about the selected axis:

$$k = \frac{P_1}{P_2} = \frac{D_1}{D_2} = \frac{R_2}{R_1} = \frac{r_s}{R} \tag{21}$$

When compared against the DeGerlia Threshold $D_{crit} = 6.73295 \times 10^{26}$ kg/m:

$$k = \frac{D}{D_{crit}} \tag{22}$$

5.2 Static Radius (Linear)

From Equation 18, when $R_1 = R_2$, the STE reduces to (see Appendix A):

$$k = \left(\frac{M_2}{M_1} \right)^{1/3} \tag{23}$$

This is Equation 13. The Lorentz factor (Equation 16) has the same form as Equation 14, with $k = v^2/c^2$. Since velocity is distance per unit time, v^2/c^2 is the square of a ratio of two lengths, with the shared time unit cancelling. Gravitational time dilation confirms this directly: $v^2/c^2 = r_s/R$ via the escape velocity identity, so the velocity-based k of special relativity and the radius-based k of general relativity are the same length ratio.

5.3 Constant Density (Isometric)

When density is held constant, $M \propto R^3$, and the STE reduces to a pure linear scale ratio (see Appendix A):

$$k = \left(\frac{R_2^3R_2^2}{R_1^3R_1^2} \right)^{1/5} = \left(\frac{R_2^5}{R_1^5} \right)^{1/5} = \frac{R_2}{R_1} \tag{24}$$

5.4 General Case

When both mass and radius differ between two systems, no component cancels:

$$k = \left(\frac{I_2}{I_1} \right)^{1/5} = \left(\frac{M_2R_2^2}{M_1R_1^2} \right)^{1/5} \tag{25}$$

This is the full STE (Equation 12). The static-mass and static-radius cases derived above, producing Schwarzschild and Lorentz time dilation respectively, are edge cases of this formula, each arising when one component of inertia is held constant. The general case applies to any two systems without constraint.

6 Unification of GR and SR

From Equation 14, time dilation is always $d\tau/dt = \sqrt{1-k}$, and k is always the linear scale factor between two systems. In all cases, k reduces to a ratio of characteristic lengths. The Schwarzschild and Lorentz results are not independent discoveries; they are the same expression evaluated under different physical constraints on the same underlying parameter k.

As derived in Sections 5.1–5.4, each physical constraint yields a specific expression for k: static mass produces r_s/R, static radius produces $(M_2/M_1)^{1/3}$, constant density produces R_2/R_1, and the unconstrained general case retains the full $(I_2/I_1)^{1/5}$. In every case, k reduces to a ratio of characteristic lengths. The velocity-based k of special relativity and the radius-based k of general relativity are connected directly through the escape velocity identity ($v^2/c^2 = r_s/R$). Both classical relativistic results emerge from this single expression without additional parameters, geometric similarity, or domain restrictions.

7 Analysis and Conclusion

The Space-Time Equivalence (Equation 14) states that time dilation is:

$$\frac{d\tau}{dt} = \sqrt{1-k}$$

where k is the linear scale factor derived from inertia. Substituting the gravitational case ($k = r_s/R$) directly produces Schwarzschild time dilation (Equation 15):

$$\frac{d\tau}{dt} = \sqrt{1-\frac{r_s}{R}}$$

Substituting the linear case ($k = v^2/c^2$) directly produces Lorentz time dilation (Equation 16):

$$\frac{d\tau}{dt} = \sqrt{1-\frac{v^2}{c^2}}$$

Equivalently, $k = D/D_{crit}$ gives the fraction of the DeGerlia Threshold consumed by the system, and is algebraically identical to r_s/R.

The STE produces both Schwarzschild and Lorentz time dilation exactly, with no free parameters. Gravitational and kinematic time dilation are not distinct phenomena requiring separate theoretical frameworks. They are the same phenomenon, expressed under different physical constraints, unified by a single inertial parameter whose physical meaning is always and only linear scale. Both are edge cases of the general formula (Appendix A), which is required whenever both mass and radius differ between systems. Because the STE operates on moment of inertia, a quantity defined for any mass distribution about any axis, its applicability is not limited to the symmetric cases treated here. Exploration of asymmetric and higher-order systems is deferred to future work. The framework further suggests that the relationship between time and inertia may hold without bound in both scale and precision, with implications for the unification of phenomena conventionally treated as distinct, subjects to be developed in subsequent work.

Appendix A Special-Case Derivations

The main text presents four expressions of k under different physical conditions (Sections 5.1–5.4). When a component of inertia is held constant between two systems, the exponent of the STE changes from $1/5$ to reflect the degree of homogeneity of the remaining quantity. This appendix derives each case from the general form using Euler's theorem of homogeneous functions [8], then verifies the general case through decomposition and numerical example.

The fundamental form of the STE is $k = (I_2/I_1)^{1/L}$, where L is the degree of homogeneity of the compared quantity in the characteristic length. Under isometric scaling by factor k, a quantity with degree of homogeneity L scales as k^L. The specific exponents used throughout this paper ($1/5$ for moment of inertia, $1/3$ for mass, $1/2$ for R^2) each emerge from the value of L for that quantity.

$$k = \left(\frac{I_2}{I_1}\right)^{1/L} = \left(\frac{M_2 R_2^2}{M_1 R_1^2}\right)^{1/L} \tag{26}$$

Table 1 Isometric Scaling and Static Density: Scale factors for various properties under isometric scaling with constant density

Ratio	Metric	L
$M_a R_a^2/M_b R_b^2$	Moment of inertia ($= MR^2$)	5
$I_{r,a}/I_{r,b}$	Moment of inertia	5
$I_{l,a}/I_{l,b}$	Linear inertia ($= M$)	3
M_a/M_b	Mass	3
V_a/V_b	Volume	3
R_a^2/R_b^2	Radius squared	2
A_a/A_b	Area	2
R_a/R_b	Radius	1
ℓ_a/ℓ_b	Length	1

$I \propto MR^2$ where R is the mean radius about the selected axis.

Case 1: Static Mass

When $M_1 = M_2$ and $R_1 > R_2$:

$$k = \underbrace{\left(\frac{I_2}{I_1}\right)^{1/L}}_{L=5} = \underbrace{\left(\frac{MR_2^2}{MR_1^2}\right)^{1/L}}_{L=5} = \underbrace{\left(\frac{R_2^2}{R_1^2}\right)^{1/L}}_{L=2}$$

$$k = \left(\frac{R_2^2}{R_1^2}\right)^{1/2} = \frac{R_2}{R_1} \tag{27}$$

Example (Case 1): Gravitational time dilation compares a system at a given mass and radius to that same mass at the Schwarzschild radius, where time dilation is zero. This is a static-mass comparison between two uniform spheres. S_1: $M = 1.9885 \times 10^{30}$ kg, $R_1 = 6.9570 \times 10^8$ m. S_2: same mass at $R_2 = r_s = 2953.38$ m.

$$k = \frac{r_s}{R_1} = \frac{2953.38}{6.9570 \times 10^8} = 4.24520 \times 10^{-6}$$

$$\frac{d\tau}{dt} = \sqrt{1-k} = \sqrt{1 - 4.24520 \times 10^{-6}} = 0.99999787740$$

Confirmation: Schwarzschild gravitational time dilation at R_1:

$$\frac{d\tau}{dt} = \sqrt{1-\frac{2GM}{R_1 c^2}} = \sqrt{1-\frac{2.65437 \times 10^{20}}{6.25264 \times 10^{25}}}$$

$$= \sqrt{1 - 4.24520 \times 10^{-6}} = 0.99999787740$$

Case 2: Static Radius

When $R_1 = R_2$ and $M_1 > M_2$:

$$k = \underbrace{\left(\frac{I_2}{I_1}\right)^{1/L}}_{L=5} = \underbrace{\left(\frac{M_2 R^2}{M_1 R^2}\right)^{1/L}}_{L=5} = \underbrace{\left(\frac{M_2}{M_1}\right)^{1/L}}_{L=3}$$

$$k = \left(\frac{M_2}{M_1}\right)^{1/3} \tag{28}$$

Example (Case 2): S_1: $M_1 = 1\times10^{30}$ kg, S_2: $M_2 = 1\times10^{24}$ kg, both at $R = 1\times10^{8}$ m. Starting from the STE with static radius ($R_1 = R_2$, $L = 3$):

$$k = \left(\frac{M_2}{M_1}\right)^{1/3} = \left(\frac{1\times10^{24}}{1\times10^{30}}\right)^{1/3} = (1\times10^{-6})^{1/3} = 1\times10^{-2}$$

$$= 1.00000\times10^{-2}$$

Since $k = (k_2/k_1)^{1/3}$ where $k_i = D_i/D_{crit}$ and $D = M/R$:

$$k_1 = \frac{M_1/R}{D_{crit}} = \frac{1\times10^{30}/1\times10^{8}}{6.73295\times10^{26}} = 1.48523\times10^{-5}$$

$$k_2 = k_1\cdot k^3 = 1.48523\times10^{-5}\times(1.00000\times10^{-2})^3$$

$$= 1.48523\times10^{-5}\times1.00000\times10^{-6} = 1.48523\times10^{-11}$$

$$\frac{d\tau}{dt} = \frac{\sqrt{1-k_1}}{\sqrt{1-k_2}} = \frac{\sqrt{1-1.48523\times10^{-5}}}{\sqrt{1-1.48523\times10^{-11}}}$$

$$= 9.9999257381\times10^{-1}$$

Confirmation: Schwarzschild gravitational time dilation:

$$\frac{d\tau}{dt} = \frac{d\tau_1/dt}{d\tau_2/dt} = \frac{\sqrt{1-\frac{2GM_1}{Rc^2}}}{\sqrt{1-\frac{2GM_2}{Rc^2}}}$$

$$= \frac{\sqrt{1-\frac{2(6.67430\times10^{-11})(1\times10^{30})}{(1\times10^{8})(8.98755\times10^{16})}}}{\sqrt{1-\frac{2(6.67430\times10^{-11})(1\times10^{24})}{(1\times10^{8})(8.98755\times10^{16})}}}$$

$$= \frac{\sqrt{1-1.48523\times10^{-5}}}{\sqrt{1-1.48523\times10^{-11}}} = 9.9999257381\times10^{-1}$$

Case 3: Constant Density

When $M \propto R^3$ and $R_1 > R_2$:

$$k = \underbrace{\left(\frac{I_2}{I_1}\right)^{1/L}}_{L=5} = \underbrace{\left(\frac{M_2R_2^2}{M_1R_1^2}\right)^{1/L}}_{L=5} = \underbrace{\left(\frac{R_2^3R_2^2}{R_1^3R_1^2}\right)^{1/L}}_{L=5} = \underbrace{\left(\frac{R_2^5}{R_1^5}\right)^{1/L}}_{L=5}$$

$$k = \left(\frac{R_2^5}{R_1^5}\right)^{1/5} = \frac{R_2}{R_1} \tag{29}$$

Example (Case 3): S_1: $M_1 = 1\times10^{30}$ kg, $R_1 = 1\times10^{8}$ m. S_2: $M_2 = 1\times10^{27}$ kg, $R_2 = 1\times10^{7}$ m. Both at $\rho = 2.38732\times10^{5}$ kg/m^3. Starting from the STE with constant density ($M \propto R^3$, $L = 5$):

$$k = \frac{R_2}{R_1} = \frac{1\times10^{7}}{1\times10^{8}} = 1\times10^{-1} = 1.00000\times10^{-1}$$

Since $D = M/R \propto R^2$ at constant density, $k_2/k_1 = (R_2/R_1)^2 = k^2$ where $k_i = D_i/D_{crit}$:

$$k_1 = \frac{M_1/R_1}{D_{crit}} = \frac{1\times10^{30}/1\times10^{8}}{6.73295\times10^{26}} = 1.48523\times10^{-5}$$

$$k_2 = k_1\cdot k^2 = 1.48523\times10^{-5}\times(1.00000\times10^{-1})^2$$

$$= 1.48523\times10^{-5}\times1.00000\times10^{-2} = 1.48523\times10^{-7}$$

$$\frac{d\tau}{dt} = \frac{\sqrt{1-k_1}}{\sqrt{1-k_2}} = \frac{\sqrt{1-1.48523\times10^{-5}}}{\sqrt{1-1.48523\times10^{-7}}} = 9.9999264807\times10^{-1}$$

Confirmation: Schwarzschild gravitational time dilation:

$$\frac{d\tau}{dt} = \frac{d\tau_1/dt}{d\tau_2/dt} = \frac{\sqrt{1-\frac{2GM_1}{R_1c^2}}}{\sqrt{1-\frac{2GM_2}{R_2c^2}}}$$

$$= \frac{\sqrt{1-\frac{2(6.67430\times10^{-11})(1\times10^{30})}{(1\times10^{8})(8.98755\times10^{16})}}}{\sqrt{1-\frac{2(6.67430\times10^{-11})(1\times10^{27})}{(1\times10^{7})(8.98755\times10^{16})}}}$$

$$= \frac{\sqrt{1-1.48523\times10^{-5}}}{\sqrt{1-1.48523\times10^{-7}}} = 9.9999264807\times10^{-1}$$

Case 4: Constant Inertia

When $M_1R_1^2 = M_2R_2^2$:

$$k = \underbrace{\left(\frac{I_2}{I_1}\right)^{1/L}}_{L=5} = \underbrace{\left(\frac{M_2R_2^2}{M_1R_1^2}\right)^{1/L}}_{L=5}$$

$$k = \left(\frac{M_2R_2^2}{M_1R_1^2}\right)^{1/5} = (1)^{1/5} = 1 \tag{30}$$

Example (Case 4): S_1: $M_1 = 1\times10^{30}$ kg, $R_1 = 1\times10^{8}$ m. S_2: $M_2 = 1\times10^{32}$ kg, $R_2 = 1\times10^{7}$ m. Both have $I = MR^2 = 1\times10^{46}$ kg·m^2. Starting from the STE:

$$I_1 = M_1R_1^2 = (1\times10^{30})(1\times10^{8})^2 = 1\times10^{46}$$

$$I_2 = M_2R_2^2 = (1\times10^{32})(1\times10^{7})^2 = 1\times10^{46}$$

$$k = \left(\frac{M_2R_2^2}{M_1R_1^2}\right)^{1/5} = \left(\frac{1\times10^{46}}{1\times10^{46}}\right)^{1/5} = 1$$

Since $k = (k_2/k_1)^{1/5}$ where $k_i = D_i/D_{crit}$ and $D = M/R$:

$$k_1 = \frac{M_1/R_1}{D_{crit}} = \frac{1\times10^{30}/1\times10^{8}}{6.73295\times10^{26}} = 1.48523\times10^{-5}$$

$$k_2 = k_1\cdot k^5 = 1.48523\times10^{-5}\times(1)^5$$

$$= 1.48523\times10^{-5}\times1 = 1.48523\times10^{-5}$$

$$\frac{d\tau}{dt} = \frac{\sqrt{1-k_2}}{\sqrt{1-k_1}} = \frac{\sqrt{1-1.48523\times10^{-5}}}{\sqrt{1-1.48523\times10^{-5}}} = 1 = 1.00000\times10^{0}$$

Confirmation: Schwarzschild gravitational time dilation:

$$\frac{d\tau}{dt} = \frac{d\tau_2/dt}{d\tau_1/dt} = \frac{\sqrt{1-k_2}}{\sqrt{1-k_1}}$$

$$= \frac{\sqrt{1-1.48523\times10^{-5}}}{\sqrt{1-1.48523\times10^{-5}}} = 1 = 1.00000\times10^{0}$$

General Case

When $M_1 \neq M_2$, $R_1 \neq R_2$, $M_1 \neq R_1^3$, and $M_2 \neq R_2^3$:

$$k = \underbrace{\left(\frac{I_2}{I_1}\right)^{1/L}}_{L=5} = \underbrace{\left(\frac{M_2R_2^2}{M_1R_1^2}\right)^{1/L}}_{L=5} = \underbrace{\left(\frac{M_2}{M_1}\right)^{1/L}}_{L=5} \cdot \underbrace{\left(\frac{R_2}{R_1}\right)^{2/L}}_{L=5}$$

$$k = \left(\frac{M_2R_2^2}{M_1R_1^2}\right)^{1/5} = \left(\frac{M_2}{M_1}\right)^{1/5} \cdot \left(\frac{R_2}{R_1}\right)^{2/5} \tag{31}$$

Example (General Case): S_1: $M_1 = 1\times10^{30}$ kg, $R_1 = 1\times10^{7}$ m. S_2: $M_2 = 1\times10^{20}$ kg, $R_2 = \frac{4}{3}r_s = 1.98031\times10^{-7}$ m. Starting from the STE:

$$k = \left(\frac{M_2R_2^2}{M_1R_1^2}\right)^{1/5} = \left(\frac{(1\times10^{20})\cdot(1.98031\times10^{-7})^2}{(1\times10^{30})\cdot(1\times10^{7})^2}\right)^{1/5}$$

$$= \left(\frac{3.92163\times10^{6}}{1\times10^{44}}\right)^{1/5} = (3.92163\times10^{-38})^{1/5}$$

$$= 3.30136\times10^{-8}$$

Since $k_i = D_i/D_{crit}$ and $D = M/R$:

$$k_1 = \frac{M_1/R_1}{D_{crit}} = \frac{1\times10^{30}/1\times10^{7}}{6.73295\times10^{26}} = 1.48523\times10^{-4}$$

$$k_2 = \frac{M_2/R_2}{D_{crit}} = \frac{1\times10^{20}/1.98031\times10^{-7}}{6.73295\times10^{26}} = 7.50000\times10^{-1}$$

$$\frac{d\tau}{dt} = \frac{\sqrt{1-k_2}}{\sqrt{1-k_1}} = \frac{\sqrt{1-7.50000\times10^{-1}}}{\sqrt{1-1.48523\times10^{-4}}}$$

$$= 5.00037\times10^{-1}$$

Confirmation: Schwarzschild gravitational time dilation:

$$\frac{d\tau}{dt} = \frac{d\tau_2/dt}{d\tau_1/dt} = \frac{\sqrt{1-\frac{2GM_2}{R_2c^2}}}{\sqrt{1-\frac{2GM_1}{R_1c^2}}}$$

$$= \frac{\sqrt{1-\frac{2(6.67430\times10^{-11})(1\times10^{20})}{(1.98031\times10^{-7})(8.98755\times10^{16})}}}{\sqrt{1-\frac{2(6.67430\times10^{-11})(1\times10^{30})}{(1\times10^{7})(8.98755\times10^{16})}}}$$

$$= \frac{\sqrt{1-7.50000\times10^{-1}}}{\sqrt{1-1.48523\times10^{-4}}}$$

$$= 5.00037\times10^{-1}$$

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Conflicts of Interest

The author declares no competing interests.

Data Availability

The complete source materials for this paper, including all LaTeX source files, derivation documents, and revision history, are publicly available at: `https://github.com/denverdata/academic_InertialRelativity-deriving-gr-sr-from-ste`.

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