Singularity Regularization and the Emergent Planck Scale in the Logarithmic Superfluid Vacuum

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Abstract

General relativity predicts spacetime singularities where curvature invariants diverge. We show that the logarithmic superfluid vacuum framework provides a natural regularization mechanism through the quantum potential (Bohm potential) $Q = \Box\sqrt{\rho}/\sqrt{\rho}$, which was neglected at macroscopic scales in the companion paper but dominates at the soliton coherence scale. The healing length $\xi = \hbar_{\rm eff}/(m_{\rm eff}c)$ of the condensate sets a minimum spatial resolution below which the hydrodynamic description breaks down and quantum pressure prevents gravitational collapse to a point. We derive ξ from the dimensional structure of the framework's gravitational coupling and show that self-consistency—combining $G \sim c^2/(\xi^2\rho_0)$ with the healing-length identification $\xi = \hbar_{\rm eff}/(m_{\rm eff}c)$ and the natural packing density $n_0 \sim 1/\xi^3$—uniquely determines $\xi \sim \sqrt{\hbar G/c^3} = \ell_{\rm P}$, the Planck length, and identifies the condensate particle mass as $m_{\rm eff} \sim m_{\rm P}$ and the vacuum mass density as $\rho_0 \sim m_{\rm P}/\ell_{\rm P}^3$, the Planck density. The Planck scale thus emerges as the healing length of the vacuum condensate, not as a fundamental discreteness of spacetime. The temporal singularity is simultaneously resolved: the chemical potential $\mu(x) = m_{\rm eff}c_s^2/\hbar_{\rm eff}$, which governs the local clock rate, is bounded by the same mechanism, ensuring that gravitational time dilation is always finite and the phase of the condensate remains well-defined everywhere. The corresponding cutoffs $\Delta x_{\rm min} \sim \ell_{\rm P}$ and $\Delta t_{\rm min} \sim t_{\rm P}$ are related by c, respecting the emergent Lorentz invariance they regularize. These identifications unify $\hbar$, c, and G as properties of a single physical substrate: the speed of sound, the quantum coherence scale, and the emergent coupling of the vacuum condensate.

1 Introduction

The Penrose-Hawking singularity theorems [7, 8] establish that geodesic incompleteness is a generic feature of general relativity under physically reasonable energy conditions. At a singularity, the curvature invariants diverge, the metric ceases to be defined, and the predictive power of the theory breaks down. This is widely regarded as a signal that GR must be replaced by a more fundamental theory at sufficiently small scales [9].

In quantum gravity approaches, the Planck length $\ell_{\rm P} = \sqrt{\hbar G/c^3} \approx 1.616 \times 10^{-35}$ m is typically introduced as a fundamental discreteness scale or minimum length, either through dimensional analysis or through specific mechanisms such as loop quantization [5, 10] or string theory [11]. However, the physical origin of $\ell_{\rm P}$ as a minimum length remains unclear: why should spacetime have a minimum resolution, and why at precisely this scale [6]?

In a companion paper [12], we developed a superfluid vacuum model based on the relativistic logarithmic Klein-Gordon equation, deriving emergent gravitational light bending with PPN parameter $\gamma = 1$. The framework treats the vacuum as a complex order parameter $\psi(x,t)$

whose Madelung decomposition $\psi = \sqrt{\rho}\, e^{iS/\hbar_{\rm eff}}$ produces hydrodynamic equations with a density-dependent sound speed and a macroscopic flow velocity. Throughout that paper, the quantum potential (Bohm potential) [3] $Q = \Box\sqrt{\rho}/\sqrt{\rho}$ was neglected in the macroscopic limit, valid at length scales much larger than the condensate's coherence scale.

In this paper, we restore Q and show that it provides a natural singularity regularization mechanism. The healing length $\xi = \hbar_{\rm eff}/(m_{\rm eff}c)$ of the condensate—the scale at which Q balances the logarithmic interaction—sets a minimum spatial resolution. We then show that self-consistency of the framework, requiring the dimensionally-correct gravitational coupling $G \sim c^2/(\xi^2\rho_0)$ to match the observed Newton's constant when combined with the healing-length structure of the soliton and the natural condensate packing density $n_0 \sim 1/\xi^3$, uniquely fixes $\xi = \ell_{\rm P}$, the Planck length.

2 The Quantum Potential at Short Distances

The Madelung decomposition [2] of the logarithmic Klein-Gordon equation [12] yields the Hamilton-Jacobi equation (in energy units, so that both sides have dimensions of energy):

$$\frac{1}{2m_{\rm eff}}\,\partial_\mu S\,\partial^\mu S = m_{\rm eff}c^2 - b\,\ln\!\left(\frac{\rho}{\rho_0}\right) + Q[\rho]\,, \tag{1}$$

where b is the logarithmic coupling with units of energy, and the quantum potential (Bohm potential) is

$$Q[\rho] = -\frac{\hbar_{\rm eff}^2}{2m_{\rm eff}}\,\frac{\Box\sqrt{\rho}}{\sqrt{\rho}}\,. \tag{2}$$

Q has units of energy, consistent with the other terms. In the companion paper, Q was neglected at macroscopic scales ($r \gg \xi$), where the density varies slowly. We now examine the regime $r \sim \xi$ where Q becomes dominant.

2.1 Scaling analysis

For a density profile varying on a length scale L, the quantum potential (Eq. 2) scales as:

$$Q \sim \frac{\hbar_{\rm eff}^2}{m_{\rm eff}\,L^2}\,. \tag{3}$$

The gravitational interaction term (from the logarithmic nonlinearity) scales as:

$$b\,\ln\!\left(\frac{\rho}{\rho_0}\right) \sim b\,\frac{\delta\rho}{\rho_0}\,, \tag{4}$$

which for a weak-field gravitating source becomes $b\,\delta\rho/\rho_0 \sim b\,GM/(c^2 r)$, using the Bernoulli relation [12].

The quantum potential dominates over the logarithmic interaction when $Q \sim b$, i.e., when $\hbar_{\rm eff}^2/(m_{\rm eff}\,L^2) \sim b$. This defines a crossover length scale:

$$L_* = \sqrt{\frac{\hbar_{\rm eff}^2}{m_{\rm eff}\,b}}\,. \tag{5}$$

Identifying $b = m_{\rm eff}c^2$ at the relativistic background (so that the logarithmic term has magnitude comparable to the rest energy), we obtain the healing length:

$$\boxed{\xi \equiv \frac{\hbar_{\rm eff}}{m_{\rm eff}\,c}} \tag{6}$$

consistent with [12]. At $r \lesssim \xi$, the quantum pressure from Q overwhelms the interaction term, preventing the density from diverging.

2.2 The repulsive core

The physical mechanism is identical to that which stabilizes vortex cores in laboratory superfluids [13]: the quantum pressure creates a repulsive force that increases faster than the attractive force as the core is compressed. Quantitatively, for a radially symmetric density profile $\rho(r)$:

$$Q = \frac{1}{\sqrt{\rho}} \left(\frac{d^2\sqrt{\rho}}{dr^2} + \frac{2}{r} \frac{d\sqrt{\rho}}{dr} \right) . \tag{7}$$

If $\rho \to \rho_{\max}$ as $r \to 0$ (a regularized core), then $Q \to Q_0 = \text{const}$ at the center. If instead $\rho \to \infty$ (a singularity), then $Q \to -\infty$, providing an arbitrarily strong repulsive force. The singularity is self-regulating: any attempt to compress the density beyond $\rho_{\max}$ generates a quantum pressure that exceeds the gravitational compression.

3 The Gausson Ground State

The ground-state soliton of the logarithmic wave equation is the Gausson—a Gaussian-localized solution discovered by Białynicki-Birula and Mycielski [14]. For the three-dimensional stationary equation with the logarithmic potential, the density profile is:

$$\rho(r) = \rho_{\max} \exp\left(-\frac{r^2}{2\sigma^2} \right) , \tag{8}$$

where σ is the Gausson width. Substituting into the stationary Hamilton-Jacobi equation with Q retained yields the width:

$$\sigma = \frac{\hbar_{\text{eff}}}{\sqrt{2\, m_{\text{eff}}\, b}} . \tag{9}$$

With the relativistic identification $b \sim m_{\text{eff}} c^2$ from the balance between the logarithmic potential and the rest energy in Eq. (1):

$$\sigma \sim \frac{\hbar_{\text{eff}}}{m_{\text{eff}}\, c} = \xi . \tag{10}$$

The Gausson width equals the healing length: the soliton core size IS the minimum spatial resolution of the condensate.

The quantum potential at the Gausson center evaluates to:

$$Q(0) = \frac{3\, \hbar_{\text{eff}}^2}{2\, m_{\text{eff}}\, \sigma^2} = \frac{3\, m_{\text{eff}}\, c^2}{2} , \tag{11}$$

which is of order the rest energy $m_{\text{eff}} c^2$. The associated curvature scale is $Q(0)/\hbar_{\text{eff}}^2 \sim 1/\xi^2$. This is the maximum curvature the acoustic metric can sustain: the Kretschmann scalar [1] of the acoustic metric is bounded by $K \lesssim 1/\xi^4$.

4 Derivation of the Healing Length from Known Parameters

We now show that ξ is determined by the parameters derived in this paper from dimensional analysis. The chain of constraints is:

4.1 Dimensional structure of the gravitational coupling

For an emergent gravity sourced by perturbations of a condensate of mass density ρ_0, dimensional analysis fixes the form of Newton's constant. The available parameters are the sound speed c, the condensate mass density ρ_0, and a length scale ξ characterizing the response of the condensate to a localized perturbation. The unique combination with the dimensions of $G = [\mathrm{m}^3/(\mathrm{kg\,s}^2)]$ is:

$$G \sim \frac{c^2}{\xi^2 \rho_0}, \tag{12}$$

up to a dimensionless prefactor of order unity. The length scale ξ in the denominator must enter squared by inversion of the Laplacian in the linearized condensate response; we will identify this with the healing length below.

4.2 The Gausson energy

The Gausson rest energy is given by the Białynicki-Birula–Mycielski energy functional [14] of the logarithmic Schrödinger equation. For a Gaussian density profile $\rho(r) = n_0\, e^{-r^2/(2\sigma^2)}$ of width $\sigma = \xi = \hbar_{\mathrm{eff}}/(m_{\mathrm{eff}} c)$, the quantum-pressure contribution to the energy is, at the order of magnitude,

$$E_v \sim \int Q\, \rho\, d^3x \sim \frac{\hbar_{\mathrm{eff}}^2}{m_{\mathrm{eff}}\, \xi^2} \cdot \xi^3\, n_0 = \frac{\hbar_{\mathrm{eff}}^2\, n_0\, \xi}{m_{\mathrm{eff}}}, \tag{13}$$

where n_0 is the central condensate number density (units m^{-3}) and $Q \sim \hbar_{\mathrm{eff}}^2/(m_{\mathrm{eff}}\, \xi^2)$ is the quantum potential evaluated on the Gausson (Eq. 11). Substituting $\xi = \hbar_{\mathrm{eff}}/(m_{\mathrm{eff}} c)$:

$$E_v \sim \frac{\hbar_{\mathrm{eff}}^3\, n_0}{m_{\mathrm{eff}}^2\, c}. \tag{14}$$

Dimensional check: $[\hbar_{\mathrm{eff}}^3 n_0/(m_{\mathrm{eff}}^2 c)] = (\mathrm{J\cdot s})^3 \cdot \mathrm{m}^{-3}/(\mathrm{kg}^2 \cdot \mathrm{m/s}) = \mathrm{J}$ ✓.

4.3 Self-consistency

We combine three relations:

- the gravitational coupling Eq. (12): $G \sim c^2/(\xi^2 \rho_0)$,
- the healing-length structure of the soliton Eq. (6): $\xi = \hbar_{\mathrm{eff}}/(m_{\mathrm{eff}} c)$,
- the relation between mass density and number density: $\rho_0 = m_{\mathrm{eff}}\, n_0$.

Substituting ξ and ρ_0 into Eq. (12):

$$G \sim \frac{c^2}{\xi^2\, m_{\mathrm{eff}}\, n_0} = \frac{c^2 \cdot m_{\mathrm{eff}}^2 c^2}{\hbar_{\mathrm{eff}}^2\, m_{\mathrm{eff}}\, n_0} = \frac{m_{\mathrm{eff}}\, c^4}{\hbar_{\mathrm{eff}}^2\, n_0}, \tag{15}$$

which can be solved for m_{eff} in terms of n_0:

$$m_{\mathrm{eff}} \sim \frac{G\, \hbar_{\mathrm{eff}}^2\, n_0}{c^4}. \tag{16}$$

Dimensional check: $[G\hbar_{\mathrm{eff}}^2 n_0/c^4] = (\mathrm{m}^3/(\mathrm{kg\,s}^2))(\mathrm{J\,s})^2(\mathrm{m}^{-3})/(\mathrm{m/s})^4 = \mathrm{kg}$ ✓.

4.4 Closure: $\hbar_{\rm eff} = \hbar$ and $n_0 \sim 1/\xi^3$

Equation (16) contains three unknowns: $\hbar_{\rm eff}$, $m_{\rm eff}$, and n_0. We fix them by two natural identifications.

First, $\hbar_{\rm eff} = \hbar$: the condensate's quantum mechanics reduces to standard quantum mechanics for its excitations (phonons identified with photons, solitons with particles).

Second, $n_0 \sim 1/\xi^3$: the condensate has one particle per healing volume. This is the natural packing density for a self-interacting relativistic condensate whose healing length equals the minimum spatial resolution. With $\xi = \hbar/(m_{\rm eff}c)$:

$$n_0 \;\sim\; \frac{1}{\xi^3} \;=\; \frac{m_{\rm eff}^3 c^3}{\hbar^3}\,. \tag{17}$$

Substituting (17) into (16):

$$m_{\rm eff} \;\sim\; \frac{G\hbar^2}{c^4}\cdot\frac{m_{\rm eff}^3 c^3}{\hbar^3} \;=\; \frac{G\,m_{\rm eff}^3}{\hbar\,c}\,, \tag{18}$$

which simplifies to:

$$m_{\rm eff}^2 \;\sim\; \frac{\hbar\,c}{G}\,. \tag{19}$$

The right-hand side is precisely $m_{\rm P}^2$, where $m_{\rm P} = \sqrt{\hbar c/G}$ is the Planck mass. Therefore:

$$\boxed{m_{\rm eff} \sim m_{\rm P}\,, \qquad \xi \sim \ell_{\rm P}\,, \qquad n_0 \sim 1/\ell_{\rm P}^3} \tag{20}$$

The condensate particle mass is the Planck mass, the healing length is the Planck length, and the vacuum number density is one particle per Planck volume. The $O(1)$ prefactors depend on the detailed Gausson energy integral; the essential result is that all three scales are fixed at the Planck scale by self-consistency.

4.5 The healing length IS the Planck length

Substituting the Planck-mass identification (20) into $\xi = \hbar/(m_{\rm eff}c)$:

$$\boxed{\xi \sim \frac{\hbar}{m_{\rm P}\,c} = \ell_{\rm P}} \tag{21}$$

where $\ell_{\rm P} = \sqrt{\hbar G/c^3} \approx 1.616 \times 10^{-35}$ m. The Planck length emerges as the healing length of the vacuum condensate, determined by self-consistency of the framework—not assumed. An equivalent statement is that the vacuum condensate has one particle per Planck volume, giving a Planck-density reservoir.

5 The Spacetime UV Cutoff

5.1 Minimum spatial resolution

Below the healing length $\xi \approx \ell_{\rm P}$, the hydrodynamic (Madelung) description breaks down: the quantum potential Q dominates all other terms, and the system is governed by the full wave equation rather than the emergent acoustic metric. The acoustic metric—and with it, the emergent spacetime geometry—ceases to be well-defined at scales $\Delta x \lesssim \xi$.

This is not a discretization of spacetime. The condensate wave function $\psi(x,t)$ remains continuous and well-defined at all scales. What has a minimum resolution is the *emergent* spacetime geometry experienced by collective excitations (phonons/photons). The underlying substrate is continuous; the emergent geometry is coarse-grained.

5.2 Minimum temporal resolution

The phase of the condensate oscillates as $S(t) = -m_{\rm eff}c^2 t/\hbar_{\rm eff}$ in the rest frame [12]. The minimum temporal interval resolvable by the acoustic metric is the time for a phase oscillation to cross the healing length:

$$\Delta t_{\rm min} = \frac{\xi}{c} = \frac{\hbar}{m_{\rm eff}\, c^2} \sim \frac{\ell_{\rm P}}{c} = t_{\rm P}\,, \tag{22}$$

where $t_{\rm P} = \sqrt{\hbar G/c^5} \approx 5.39 \times 10^{-44}$ s is the Planck time.

5.3 Unified UV cutoff

The spatial and temporal cutoffs are related by the sound speed:

$$\frac{\Delta x_{\rm min}}{\Delta t_{\rm min}} = c\,. \tag{23}$$

This is a consequence of the Lorentz invariance of the acoustic metric at scales $\gg \xi$. The UV cutoff respects the emergent symmetry that it regularizes.

5.4 Maximum curvature

The Kretschmann scalar of the acoustic metric is bounded:

$$K = R_{\mu\nu\alpha\beta}\, R^{\mu\nu\alpha\beta} \lesssim \frac{1}{\xi^4} \approx \frac{1}{\ell_{\rm P}^4}\,. \tag{24}$$

This is the same bound obtained in loop quantum gravity [10], but here it arises from the condensate microphysics rather than from a quantization of geometry.

6 Temporal Regularization: The Phase Counter

6.1 Time as phase accumulation

In the Madelung representation, the condensate phase $S(x,t)$ satisfies the Hamilton-Jacobi equation (1). In the rest frame of the condensate, $S(t) = -\mu\, t$, where the chemical potential

$$\mu(x) = \frac{m_{\rm eff}\, c_s^2(x)}{\hbar_{\rm eff}} \tag{25}$$

sets the local oscillation rate of the order parameter $\psi \propto e^{-i\mu t}$. This phase accumulation IS the local clock: two events separated by Δt accumulate a phase difference $\Delta\phi = \mu\, \Delta t$. A clock measures time by counting phase cycles of a physical oscillator; here, the oscillator is the vacuum itself.

Gravitational time dilation is the spatial variation of $\mu(x)$. In the weak-field limit, the acoustic metric gives $g_{00} \propto -c_s^2(x) \propto -\rho(x)$, so:

$$\frac{d\tau}{dt} = \sqrt{-g_{00}} \propto c_s(x) \propto \sqrt{\rho(x)}\,. \tag{26}$$

Deeper in a gravitational well (lower ρ), the sound speed is lower, the phase accumulates more slowly, and clocks run slower—exactly the gravitational redshift.

6.2 The temporal singularity in GR

In general relativity, the singularity at $r = 0$ inside a black hole is not merely a point of infinite density; it is a boundary where time itself ceases to exist. The Kretschmann scalar diverges, the metric is undefined, and no further time evolution is possible. At the event horizon ($r = r_H$), an external observer sees clocks approach infinite redshift: $g_{00} \to 0$, so $d\tau/dt \to 0$.

Both pathologies involve the metric component g_{00} reaching extreme values: zero at the horizon (for external coordinates) and undefined at the singularity.

6.3 Regularization by the bounded chemical potential

In the superfluid framework, $g_{00} \propto -\rho\, c_s \propto -\rho^{3/2}$. Since the Bohm potential bounds the density:

$$0 < \rho_{\min} \leq \rho(x) \leq \rho_{\max} \sim e\, \rho_0\,, \tag{27}$$

the chemical potential is bounded from both sides:

$$0 < \mu_{\min} \leq \mu(x) \leq \mu_{\max}\,. \tag{28}$$

The clock never stops ($\mu > 0$ everywhere) and never runs infinitely fast ($\mu < \mu_{\max}$).

The period of the fastest possible phase oscillation of the condensate is set by the maximum chemical potential:

$$T_{\text{phase}} = \frac{2\pi}{\mu_{\max}} = \frac{2\pi\, \hbar_{\text{eff}}}{m_{\text{eff}}\, c^2} \;\sim\; 2\pi\, t_{\text{P}}\,. \tag{29}$$

This is consistent with the resolution limit $\Delta t_{\min} \sim t_{\text{P}}$ derived from the sound-crossing time of the healing length (Eq. 22); the two definitions differ by a factor of 2π corresponding to one full phase cycle versus a half-period crossing. No physical process within the emergent spacetime can resolve time intervals shorter than $\sim t_{\text{P}}$.

The maximum resolvable time dilation ratio (the gravitational redshift between the deepest well and the cosmological background) is:

$$\frac{\mu_{\max}}{\mu_{\min}} = \frac{c_{s,\max}^2}{c_{s,\min}^2} = \frac{\rho_{\max}}{\rho_{\min}} \leq \frac{e\, \rho_0}{\rho_{\min}}\,. \tag{30}$$

This is finite. There is no infinite redshift surface—the horizon is replaced by a region of large but finite time dilation, across which the condensate phase remains well-defined and continuous.

6.4 The arrow of time

The phase of the condensate advances monotonically: $S(t + \Delta t) = S(t) - \mu\, \Delta t$ with $\mu > 0$. This provides a microscopic arrow of time that is absent in the classical wave equation (which is time-reversal symmetric). The phase winds monotonically in one direction because μ is positive-definite throughout the condensate, and this directional phase accumulation defines the forward direction of time.

This connects the thermodynamic arrow (entropy increases toward equilibrium with the Bulk) to the cosmological arrow (the Universe expands as the condensate evolves) and to the quantum-mechanical arrow (the phase accumulates monotonically).

7 Singularity Regularization

7.1 Black holes as phase boundaries

In the acoustic metric framework, the event horizon occurs where the inflow velocity equals the sound speed: $v(r_H) = c$. The Schwarzschild radius is:

$$r_H = \frac{2GM}{c^2}\,. \tag{31}$$

For a solar-mass object, $r_H \approx 3$ km $\gg \xi$. The acoustic metric description is valid at and outside the horizon.

Inside the horizon, the inflow velocity exceeds c, and the acoustic metric describes a trapped region. However, as $r \to 0$, the density profile of the infalling condensate steepens until the gradient reaches the healing-length scale. At $r \sim \xi$, the quantum potential (11) becomes comparable to the gravitational term, and the density is regularized to a finite maximum:

$$\rho_{\max} \sim \rho_0 \, \exp\!\left(\frac{m_{\rm eff}c^2}{\hbar_{\rm eff}^2/(m_{\rm eff}\xi^2)}\right) = \rho_0\, e^{\,m_{\rm eff}^2 c^2 \xi^2/\hbar_{\rm eff}^2} = \rho_0\, e\,, \tag{32}$$

where the last equality uses $\xi = \hbar_{\rm eff}/(m_{\rm eff}c)$ so that the exponent equals 1. The singularity is replaced by a Planck-scale core of radius $\sim \xi \sim \ell_{\rm P}$ operating at the maximum vacuum density $e\,\rho_0$, where the hydrodynamic description gives way to the full quantum dynamics of the condensate.

7.2 Comparison with other approaches

The regularization mechanism is physically distinct from other quantum gravity proposals:

In *loop quantum gravity* [10], the singularity is resolved by quantum geometry: area and volume operators have discrete spectra with minimum eigenvalues of order $\ell_{\rm P}^2$ and $\ell_{\rm P}^3$.

In *string theory* [11], the finite size of the string ($\ell_s \sim \ell_{\rm P}$) prevents localization below the string scale.

In the *superfluid vacuum*, the singularity is resolved by the Bohm potential of the condensate: the quantum pressure of the vacuum fluid prevents gravitational collapse below the healing length. The mechanism is hydrodynamic, not geometric or string-theoretic, but produces the same minimum length $\xi \sim \ell_{\rm P}$.

The key distinction is that $\ell_{\rm P}$ is *derived* from the condensate parameters, not introduced as a fundamental scale. The "quantization of geometry" is an emergent consequence of the quantum coherence of the vacuum condensate.

8 Unification of Fundamental Constants

The identification $\xi = \ell_{\rm P}$ provides a unified interpretation of the three fundamental constants $\hbar$, c, and G as properties of a single physical substrate—the vacuum condensate:

- $c = c_s$: the speed of light is the sound speed of the condensate at the cosmological background density [12]. It is determined by the logarithmic equation of state.
- $G \sim c^2/(\xi^2\rho_0)$: Newton's constant is the emergent gravitational coupling, fixed by the healing length $\xi = \ell_{\rm P}$ and the vacuum mass density $\rho_0 \sim m_{\rm P}/\ell_{\rm P}^3$ through the dimensional structure of the linearized condensate response (Sec. 4).
- $\hbar = \hbar_{\rm eff}$: Planck's constant governs the quantum coherence of the condensate. It sets the healing length $\xi = \hbar/(m_{\rm eff}c)$ and the Gausson width. The de Broglie relation [4] $p = \hbar k$ is the dispersion relation of condensate excitations.

The Planck units are then:

$$\ell_{\rm P} = \xi = \text{healing length of the vacuum}\,, \tag{33}$$
$$t_{\rm P} = \xi/c = \text{phase crossing time}\,, \tag{34}$$
$$m_{\rm P} \sim m_{\rm eff} = \text{constituent mass of the vacuum}\,, \tag{35}$$
$$E_{\rm P} = m_{\rm P}c^2 = \text{rest energy of one constituent}\,. \tag{36}$$

The Planck scale is not a fundamental discreteness of spacetime but the scale at which the emergent spacetime description (acoustic metric) gives way to the underlying quantum dynamics of the vacuum condensate.

9 Discussion

9.1 What is derived and what is assumed

The derivation of $\xi = \ell_{\rm P}$ rests on four inputs:

(i) The dimensional form of the gravitational coupling, $G \sim c^2/(\xi^2\rho_0)$ (Eq. 12), which follows from dimensional analysis once the available parameters of the emergent-gravity sector are fixed (c, ρ_0, and a length scale).

(ii) The healing-length structure of the soliton, $\xi = \hbar_{\rm eff}/(m_{\rm eff}c)$ (Eq. 6), which arises from balancing the quantum potential against the logarithmic interaction at the soliton core.

(iii) The natural packing density $n_0 \sim 1/\xi^3$ (Eq. 17), reflecting one condensate particle per healing volume.

(iv) The identification $\hbar_{\rm eff} = \hbar$, the simplest assumption consistent with the condensate's excitations being identified with standard quantum particles.

Of these, (i) and (ii) follow from dimensional analysis once the framework's structure is fixed, while (iii) and (iv) are physical assumptions. The result $m_{\rm eff} \sim m_{\rm P}$, $\xi \sim \ell_{\rm P}$, $\rho_0 \sim m_{\rm P}/\ell_{\rm P}^3$ is robust to $O(1)$ factors throughout the chain.

9.2 The vacuum density

From the self-consistent identifications (20), the condensate has one particle of mass $m_{\rm eff} \sim m_{\rm P}$ per Planck volume, giving a mass density at the Planck scale:

$$\rho_0 \;=\; m_{\rm eff}\, n_0 \;\sim\; \frac{m_{\rm P}}{\ell_{\rm P}^3} \;\approx\; 5.2 \times 10^{96}\ \text{kg/m}^3\,. \tag{37}$$

This is the Planck density—the natural maximum density for any relativistic quantum field. It is, however, the density of the *vacuum condensate* itself, not of ordinary matter. A useful intuition: the condensate has bulk modulus $K \sim \rho_0 c^2 \sim 10^{113}$ Pa, larger than that of diamond by $\sim 10^{101}$ orders of magnitude. Gravity is weak because the vacuum is stiff: a massive source produces only a tiny fractional density perturbation in the condensate, and this small perturbation is what drives gravitational acceleration. The energy density of the background itself, $\rho_0 c^2 \sim 10^{113}$ J/m^3, is comparable to the QFT zero-point estimate of the cosmological constant; in the present framework this is not directly observable, since gravitational effects are sourced by perturbations $\delta\rho$ rather than by the background density ρ_0.

9.3 Testable consequences

The healing-length regularization makes two predictions that distinguish it from classical GR:

(i) *No singularity at $r = 0$*: The density inside a black hole is bounded by $\rho_{\max} \sim e\,\rho_0$. This affects the information paradox: if no singularity forms, the interior is a finite-density condensate core, and unitarity may be preserved.

(ii) *Modified dispersion at high energies*: At energies approaching $E_{\rm P} = m_{\rm P}c^2$, the phonon dispersion relation deviates from $\omega = ck$ due to the Bohm potential corrections. This is analogous to the roton-maxon spectrum in superfluid helium [15] and is in principle testable through gamma-ray burst time-of-flight measurements [16].

9.4 Limitations

The derivation assumes a single species of condensate particle with mass $m_{\rm eff} \sim m_{\rm P}$. If the condensate has a more complex structure (e.g., multiple species or a composite fundamental unit), the healing length could differ from $\ell_{\rm P}$ by numerical factors. The $O(1)$ uncertainty in the Gausson energy integral propagates to an $O(1)$ factor in $\xi/\ell_{\rm P}$, which is within the expected precision of the order-of-magnitude analysis.

The interior structure of black holes in the superfluid framework requires solving the full nonlinear wave equation (1) with Q retained, which is beyond the scope of this paper. The regularization argument establishes the existence of a maximum density and minimum length, but not the detailed density profile inside the horizon.

10 Conclusion

We have shown that the logarithmic superfluid vacuum framework provides a complete UV regularization of the two singular limits of general relativity:

(1) *Spatial singularity.* The quantum potential $Q = \Box\sqrt{\rho}/\sqrt{\rho}$ prevents gravitational collapse below the healing length $\xi \sim \ell_{\rm P}$. The density is bounded by $\rho_{\max} \sim e\,\rho_0$, and the Kretschmann scalar is bounded by $K \lesssim 1/\ell_{\rm P}^4$.

(2) *Temporal singularity.* The chemical potential $\mu(x) = m_{\rm eff}c_s^2/\hbar_{\rm eff}$, which sets the local clock rate, is bounded from both sides by the bounded density. The clock never stops ($\mu > 0$), time dilation is always finite, and the minimum resolvable time interval is $\Delta t_{\min} \sim t_{\rm P}$.

Both regularizations arise from the same physical mechanism—the Bohm potential of the vacuum condensate—and produce the same characteristic scale—the Planck scale—which emerges as the healing length $\xi = \hbar/(m_{\rm eff}c) \sim \ell_{\rm P}$, not as an assumed discreteness.

The framework unifies $\hbar$, c, and G as properties of a single substrate: the speed of sound, the quantum coherence scale, and the emergent gravitational coupling of the vacuum condensate. The Planck mass $m_{\rm P} \sim m_{\rm eff}$ is the constituent mass, and the Planck length is the coherence length at which the emergent spacetime gives way to the underlying quantum dynamics.

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