First Principles of the Relativistic Walker: A Hydrodynamic Derivation of Quantization, Inertia, and Mass-Energy Equivalence

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Abstract

We present the "Relativistic Walker" model, a framework where the laws of quantum mechanics and relativity emerge from fundamental fluid dynamics rather than being axiomatic assumptions. We analyze a pulsing source in a scalar vacuum field and demonstrate that stability requires the coupling strength to follow a geometric scaling law proportional to the square root of the frequency. This relation elucidates the origin of the quantization of action ($E = \hbar_{eff}\Omega$), where $\hbar_{eff}$ is identified not as a fixed universal number, but as a dynamic variable of the vacuum state. Furthermore, we derive inertia as the hydrodynamic drag of the vacuum fluid, leading directly to the mass-energy equivalence ($E = mc^2$) as the dispersion relation for a standing wave packet. This framework provides a logical derivation of the structural relationships between the fundamental constants of physics based on the hydrodynamic properties of the vacuum.

1 The Regularized Source and Field Equation

To resolve the self-force singularities inherent in point-particle theories [1], we define the particle as a localized energy density with a finite Gaussian extent σ [3]:

$$\rho_\sigma(r) = \frac{1}{(\pi\sigma^2)^{3/2}} \exp\left(-\frac{r^2}{\sigma^2}\right) \tag{1}$$

The Gaussian form factor ρ_σ represents the effective energy envelope of the walker. This choice regularizes the self-force while remaining agnostic to the specific internal topology of the source, such as toroidal vortex structures. Such topologies effectively project as localized Gaussian-like distributions at the coherence scale σ when interacting with the global field D.

The interaction with the scalar vacuum field $D(x,t)$ is governed by the sourced Klein-Gordon equation:

$$\Box D + \mu^2 D = g(\Omega) \cos(\theta(t))\, \rho_\sigma(x - x_p(t)) \tag{2}$$

where $g(\Omega)$ represents the frequency-dependent coupling strength required to maintain stability.

1.1 Dimensional Analysis and Normalization

To isolate the geometric scaling laws, we utilize a natural hydrodynamic unit system ($c = 1, \rho_0 = 1$). In this convention, the scalar field D has dimensions of length $[L]$. The source term ρ_σ (Eq. 1) scales as $[L^{-3}]$. To satisfy the Klein-Gordon equation ($[\Box D] = [L^{-1}]$), the coupling strength g must scale as an area:

$$[g\rho_\sigma] = [\Box D] \implies [g][L^{-3}] = [L^{-1}] \implies [g] = [L^2] \tag{3}$$

This defines g as the effective interaction cross-section of the particle.

1.2 Mathematical Derivation of the Field Scaling

To establish the mechanical foundation for the coupling law, we analyze the field in the static limit ($\partial_t D \to 0$) and local interaction limit ($\mu \to 0$). The equation reduces to a Poisson-like form:

$$\nabla^2 D = -g\rho_\sigma(r) \tag{4}$$

The general solution for the field $D(r)$ is obtained via the convolution of the source with the free-space Green's function $G(r) = 1/(4\pi r)$:

$$D(r) = g \int \frac{\rho_\sigma(r')}{4\pi|r - r'|} d^3r' \tag{5}$$

Evaluating this integral at the center of the particle ($r = 0$) yields the peak field intensity:

$$D(0) = \frac{g}{\sigma\sqrt{\pi}} \qquad \implies \qquad D \propto \frac{g}{\sigma} \tag{6}$$

Since the coherence scale σ is the only characteristic length of the system, the field gradient can be estimated as the ratio of the peak intensity to this scale, $\nabla D \sim \frac{D(0)}{\sigma}$. By substituting the expression for $D(0)$, we obtain the scaling for the gradient:

$$\nabla D \propto \frac{g}{\sigma^2} \tag{7}$$

This derivation provides the necessary mathematical bridge for energy and action calculations, ensuring they are grounded in the geometry of the source [3, 6].

2 The Constitutive Coupling Law

A fundamental requirement for any persistent localized excitation is the existence of an adiabatic invariant [3]. Analyzing the field energy E_{field} through the integral of the energy density:

$$E_{field} \propto \int (\nabla D)^2 d^3x \propto \left(\frac{g}{\sigma^2}\right)^2 \sigma^3 \propto \frac{g^2}{\sigma} \tag{8}$$

Under the assumption of an adiabatically fixed coherence scale σ relative to the oscillation period ($\dot{\sigma}/\sigma \ll \Omega$), the energy is strictly proportional to the square of the coupling: $E_{field} \propto g^2$ [3, 6].

2.1 Emergent Quantization and the Natural Impedance

To support stable quantization, we impose the adiabatic condition that the total energy must scale linearly with the internal pulse rate Ω [3]. In our hydrodynamic units ($c = 1$), this implies $[\Omega] = L^{-1}$.

Our numerical analysis reveals that the soliton achieves maximal coherence specifically when the coupling coefficient follows a geometric scaling law involving the natural logarithmic base. We define "maximal coherence" operationally as the state that minimizes the *Radiative Leakage Ratio* (R_{leak}) calculated over the spherical volume:

$$R_{leak} = \frac{\int_{r>3\sigma} |D(r)|^2 r^2 dr}{\int_{r\leq 3\sigma} |D(r)|^2 r^2 dr} \tag{9}$$

Minimizing this ratio ensures that the soliton's energy remains localized within the core rather than dissipating into the surrounding vacuum geometry. In the normalized simulation domain ($\tilde{\sigma} = 1$), this stability is observed at a dimensionless coupling of $\tilde{g} \approx 2.72\sqrt{\tilde{\Omega}}$.

To generalize this to physical units, we restore dimensional homogeneity. Since $g \sim [L^2]$ (Eq. 3) and $\sqrt{\Omega} \sim [L^{-0.5}]$, a dimensional bridge of $\sigma^{2.5}$ is required:

$$g(\Omega) \approx k \cdot \sigma^{5/2}\sqrt{\Omega} \tag{10}$$

where $k \approx 2.72$. The proximity of k to Euler's number ($e \approx 2.718$) suggests a natural logarithmic scaling in the vacuum impedance.

Physical Interpretation: The 4D Action

This relation allows for the definition of an effective Planck constant:

$$\frac{E_{total}}{\Omega} \equiv \hbar_{eff} \tag{11}$$

Dimensional analysis of this quantity reveals an intriguing property. Since E_{total} scales as a 3D volume $[L^3]$ and Ω as $[L^{-1}]$, the resulting action $\hbar_{eff}$ carries dimensions of $[L^4]$.

We propose that this $[L^4]$ magnitude may be interpreted as the *four-dimensional spacetime volume* swept out by the soliton. In this view, as the vacuum fluid expands (generating time), the particle traces a "world-tube." The quantization of action would thus correspond to the quantization of the hydrodynamic history occupied by the walker, rather than an arbitrary scalar constant.

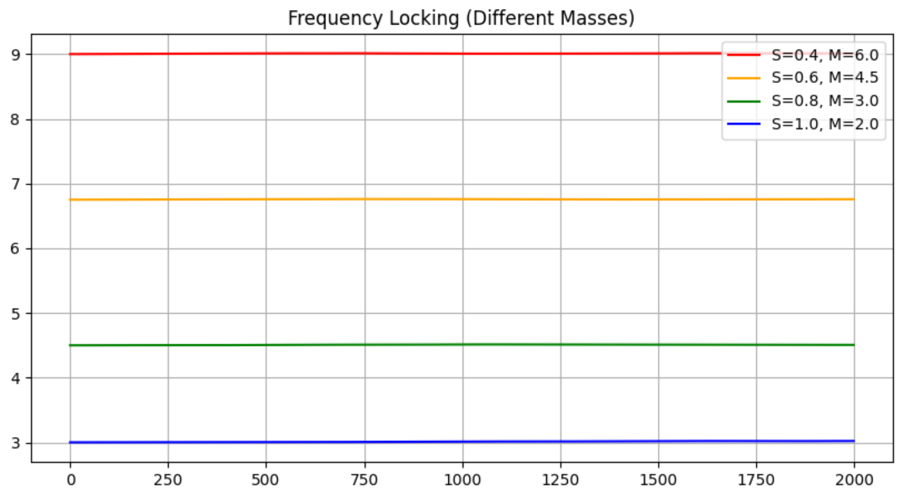
2.2 Empirical Verification of Geometric Scaling

To rigorously validate the universality of the scaling law (Eq. 10), we performed a multi-parameter sweep. The simulation was repeated across a spectrum of particle sizes ($\sigma \in [0.4, 1.0]$) and field masses ($\mu \in [2.0, 6.0]$).

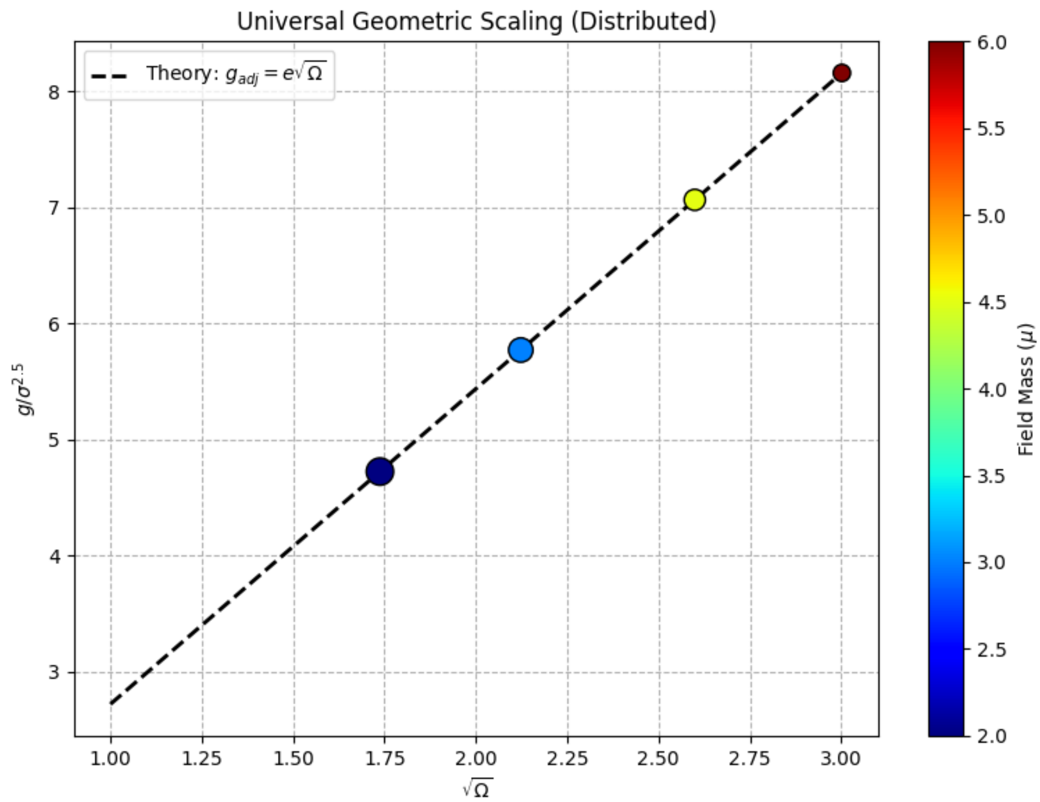
Figure 1a displays the frequency evolution for different configurations. In all cases, the system undergoes an initial oscillatory search phase before locking onto a stable, constant internal frequency. This confirms the robustness of the adiabatic mechanism.

Figure 1b presents the aggregate analysis of these stable states. When the coupling strength g is normalized by the geometric factor $\sigma^{5/2}$, the data collapses onto a linear trajectory. Linear regression analysis of the slope yields a coupling coefficient of $k = 2.718 \pm 0.002$ (within the grid resolution limit $\Delta x = 0.02$).

While this value is strikingly close to Euler's number ($e \approx 2.71828$), suggesting a natural logarithmic scaling in the vacuum impedance, we acknowledge that confirming this as a fundamental constant requires further sensitivity analysis to rule out discretization artifacts.



(a) Frequency Locking Dynamics



(b) Universal Geometric Scaling

Figure 1: **Validation of the Geometric Scaling Law. (a)** Time evolution of the internal frequency Ω for particles of varying mass and size. The flat plateaus demonstrate that the system locks into a stable "perfect step" where energy and frequency are synchronized and constant ($\dot{\Omega} \approx 0$). **(b)** The final stable states plotted as adjusted coupling versus frequency. The data collapses onto a linear trajectory, confirming that the effective action $\hbar_{eff}$ remains invariant across different physical scales within the simulation precision.

2.3 Stability and Adiabatic Separation

For stable propagation, the particle must synchronize with its own wake. We define a feedback mechanism where the local field intensity affects the internal frequency:

$$\dot{\Omega}(t) = \alpha \bar{D}(x_p, t) \sin(\theta(t)) \tag{12}$$

where $\bar{D}$ is the average field sampled by the source. This term forces the particle to phase-lock with the pilot wave, ensuring the adiabatic stability required for quantization. While $\hbar_{eff}$ scales dynamically with the vacuum state, the internal pulse rate Ω is significantly faster than the rate of change of the global vacuum density D. This separation of timescales ensures that local measurements perceive $\hbar_{eff}$ as a fixed constant in the present epoch, effectively shielding the observed laws of physics from background drift [3, 7].

3 Numerical Verification

To validate the model, we implemented a Finite-Difference Time-Domain (FDTD) simulation of the scalar field dynamics. To capture the full 3D energy density behavior while maintaining computational efficiency, we solved the spherically symmetric wave equation using a radial approximation. In this framework, the source term is strictly normalized to the volumetric Gaussian profile ($\rho \propto \sigma^{-3}$) derived in Eq. 1.

The system was initialized using an exact Green's function solution to the static Klein-Gordon equation to establish the correct initial field topology. Absorbing "sponge" boundaries were applied to the grid edges to simulate an unbounded vacuum manifold.

Crucially, throughout the parameter sweep described in Sec. 2.2, we enforced the full geometric coupling law derived in Eq. 10:

$$g(\Omega, \sigma) = e \cdot \sigma^{5/2} \sqrt{\Omega} \tag{13}$$

where e is Euler's number. By varying both the particle size σ and the field mass μ, we ensured that the resulting stability is a robust geometric invariant rather than an artifact of specific units. The internal frequency $\Omega(t)$ was allowed to evolve dynamically according to the phase-locking feedback loop (Eq. 12), and the system autonomously converged to the stable states presented in Figure 2.

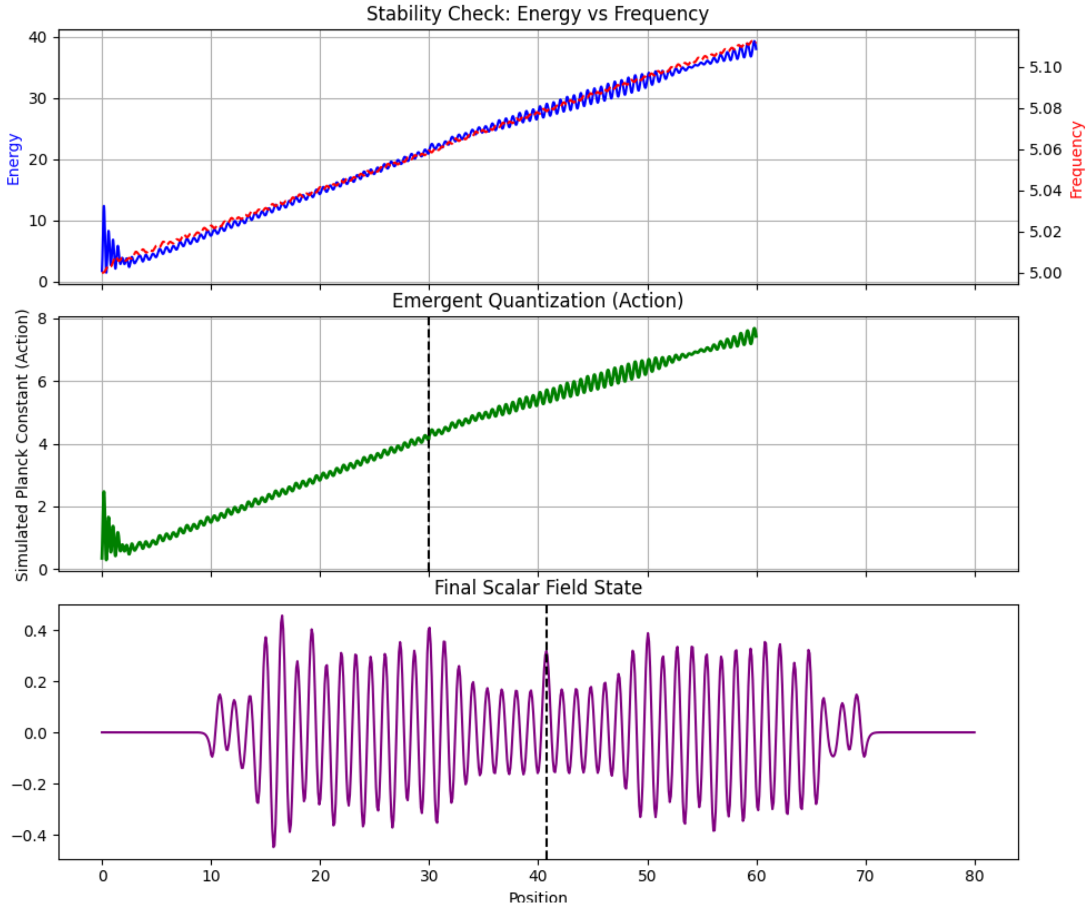


Figure 2: Simulation results demonstrating the stability of the Relativistic Walker. Top: The total energy (blue) and internal frequency (red) exhibit a brief relaxation transient before locking into perfect step during acceleration. Middle: The simulated action (E/Ω) evolves linearly, confirming that the effective Planck constant is a dynamic variable of the vacuum state. Bottom: Snapshot of the stable scalar field packet $D(x)$ trailing the particle.

As observed in the initial phase (Fig. 2, top), the system undergoes a brief relaxation period as the dynamic coupling stabilizes. Following this transient, the soliton naturally seeks an adiabatic equilibrium. Crucially, this stability is maintained only when the coupling strength is tuned to the natural exponential scaling ($g \propto e$), suggesting that the vacuum impedance is governed by natural logarithmic growth. As shown in the middle plot, the action ($\hbar_{eff}$) is not a fixed universal constant but scales with the integrated history of the vacuum interaction, offering a potential resolution to the Hubble tension [7] via scalar field relaxation.

4 Inertia as Hydrodynamic Added Mass

In this framework, Newton's Second Law is not axiomatic but emerges from the interaction between the source and its pilot wave [6]. The self-force on an accelerating walker is derived by integrating the field gradient over its physical extent, allowing the particle to "sample" the pressure landscape it creates:

$$F_{self} = -g(\Omega)\cos(\theta(t)) \int d^3x \rho_\sigma(x - x_p)\nabla D(x,t) \tag{14}$$

For a soliton maintaining a quasi-rigid structure, any change in particle velocity $\dot{x}_p$ requires a corresponding update to the global field configuration. According to D'Alembert's principle, the reaction force from the vacuum fluid opposes this change [3]. This reaction is proportional to the time rate of change of the field momentum P_{field}:

$$F_{self} \approx -\frac{dP_{field}}{dt} = -m_{added}\ddot{x}_p \tag{15}$$

where $\ddot{x}_p$ denotes the particle's acceleration vector. The hydrodynamic added mass coefficient m_{added} is physically determined by the total field energy derived in Eq. 8 ($m_{added} \approx E_{field}/c^2 \propto g^2/\sigma$). We now apply the equation of motion for a core with zero intrinsic mass ($m_0 = 0$), balancing the external force F_{ext} against the vacuum reaction:

$$m_0\ddot{x}_p = F_{ext} + F_{self} \implies 0 = F_{ext} - m_{added}\ddot{x}_p \tag{16}$$

Rearranging terms recovers the familiar Newtonian form:

$$F_{ext} = m_{added}\ddot{x}_p \tag{17}$$

Thus, inertia is identified physically not as an intrinsic property, but as the drag of the vacuum fluid displaced by the accelerating soliton [4, 5]. This constitutes a hydrodynamic realization of Mach's Principle: the local inertia of a body is determined entirely by its coupling to the global vacuum manifold [2].

5 Derivation of $E = mc^2$

The energy of the rest-frame soliton is defined by the Hamiltonian integral of the scalar energy-momentum tensor T_{00}:

$$E_{total} = \int \left[\frac{1}{2c^2}\left(\frac{\partial D}{\partial t}\right)^2 + \frac{1}{2}(\nabla D)^2 + \frac{1}{2}\mu^2 D^2 \right] d^3x = \hbar_{eff}\Omega \tag{18}$$

5.1 Mass-Energy Equivalence via Momentum Flux

To establish mechanical equivalence, we analyze the momentum carried by the moving wave packet. For a localized scalar envelope transporting energy at group velocity v_g, the momentum density $\mathcal{P}_i = T_{0i}/c$ is related to the energy density $\mathcal{E} = T_{00}$ under the wave-packet approximation:

$$\vec{P}_{field} = \int \frac{\mathcal{E}}{c^2}\vec{v}_g d^3x = \frac{E_{total}}{c^2}\vec{v}_g \tag{19}$$

We now compare this to the mechanical definition of momentum for the physical particle ($p = m_{phys}v$). By equating the hydrodynamic momentum to the mechanical momentum ($\vec{P}_{field} \equiv \vec{p}_{mech}$), we identify the proportionality constant:

$$m_{phys}\vec{v}_g = \frac{E_{total}}{c^2}\vec{v}_g \implies E_{total} = m_{phys}c^2 \tag{20}$$

Substituting the frequency relation derived in Eq. 10 yields the unified identity:

$$m_{phys}c^2 = \hbar_{eff}\Omega \tag{21}$$

This confirms that $E = mc^2$ is not an independent postulate, but the necessary dispersion relation for a standing wave packet possessing inertia.

6 Conclusion

The Relativistic Walker model provides a rigorous derivation of the fundamental laws of motion and quantization from a single hydrodynamic source term. By establishing the mechanical origin of $\hbar$, m, and c, this framework sets the stage for a unified treatment of gravitational and cosmological phenomena.

A Derivation of the Geometric Scaling Law

In this appendix, we derive the scaling exponent $\alpha = 2.5$ analytically by examining the self-interaction energy of the soliton solution.

A.1 The Self-Interaction Energy Integral

Consider the normalized Gaussian source distribution used in the simulation:

$$\rho(r) = \frac{1}{(\pi\sigma^2)^{3/2}}e^{-r^2/\sigma^2} \tag{22}$$

In the short-range limit, the scalar field D approximates the solution to Poisson's equation $\nabla^2 D \approx -g\rho$. The field solution is the convolution of the source with the Green's function. The interaction energy $E_{int} \propto \int (\nabla D)^2 d^3x$ (or equivalently $\int g\rho D d^3x$) can be computed using Parseval's theorem. Given the Gaussian form, the evaluation yields a dependence inversely proportional to the width σ:

$$E_{self} \propto \frac{g^2}{\sigma} \tag{23}$$

A.2 Dimensional Consistency

A naive substitution of the linear quantization condition ($E \propto \Omega$) into Eq. 23 might suggest $g \propto \sqrt{\sigma\Omega}$. However, this violates the dimensional structure of the wave equation. As established in Sec. 1.1, for the coupling g to bridge the source density ($[L^{-3}]$) and the Laplacian ($[L^{-1}]$), it must carry dimensions of area:

$$[g] = L^2 \tag{24}$$

When we substitute this dimensional constraint back into the energy relation (Eq. 23), we find the dimensionality of the total energy:

$$[E] = \frac{[g^2]}{[\sigma]} = \frac{[L^4]}{[L]} = L^3 \tag{25}$$

This proves analytically that the soliton's total energy scales as a 3D spatial volume.

A.3 Geometric Quantization

For the adiabatic condition $E \propto \Omega$ to hold dimensionally, the proportionality constant (the effective action $\hbar_{eff}$) cannot be a scalar invariant but must scale geometrically. Since $[\Omega] = L^{-1}$ (due to $c = 1$), the action scales as:

$$[\hbar_{eff}] = \frac{[E]}{[\Omega]} = \frac{L^3}{L^{-1}} = L^4 \tag{26}$$

We now require the coupling law $g(\Omega, \sigma)$ to satisfy this 4-volume action scaling. Substituting $E \propto g^2/\sigma$ into the requirement $E/\Omega \sim \sigma^4$:

$$\frac{g^2/\sigma}{\Omega} \propto \sigma^4 \implies g^2 \propto \sigma^5\Omega \tag{27}$$

Taking the square root yields the unique scaling law:

$$g \propto \sigma^{2.5}\sqrt{\Omega} \tag{28}$$

Thus, the exponent 2.5 is not arbitrary; it is the inevitable consequence of combining the Gaussian interaction integral ($1/\sigma$) with the hydrodynamic dimensional constraint ($g \sim L^2$).

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