An Approximate Numerical Relation Between the CMB Photon Density Parameter and the Fine Structure Constant

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Abstract

We report an approximate numerical coincidence between two independently measured physical quantities: the photon density parameter of the Cosmic Microwave Background, Ω_γ, and the square of the fine structure constant, α^2. Using current best-fit values from Planck 2018, we find $\Omega_\gamma = 5.45 \times 10^{-5}$ and $\alpha^2 = 5.33 \times 10^{-5}$, yielding agreement within 2.4%. Equivalently, the relation can be written as $T_{\mathrm{CMB}} = \sqrt{\alpha} \times (15\hbar^3 c^5 \rho_c / \pi^2 k_B^4)^{1/4}$, which predicts $T = 2.710$ K versus the observed 2.7255 K. If taken as exact, the relation predicts $H_0 = 68.1$ km s^{-1} Mpc^{-1}, consistent with Planck (1.5σ) and intermediate between early- and late-universe measurements. We present the observation, test its sensitivity to input parameters, and discuss whether it may reflect underlying physics or is a numerical accident.

1. Introduction

The Cosmic Microwave Background (CMB) temperature $T_{\mathrm{CMB}} = 2.7255 \pm 0.0006$ K is one of the most precisely measured quantities in cosmology [1]. In the standard ΛCDM framework, this temperature is treated as a free parameter determined by initial conditions and the subsequent thermal history of the universe. No fundamental principle within the Standard Model or General Relativity predicts its value from other known constants.

Separately, the fine structure constant $\alpha = e^2/(4\pi\varepsilon_0\hbar c) \approx 1/137.036$ quantifies the strength of the electromagnetic interaction between charged particles and photons. It is a dimensionless quantity whose value has no accepted theoretical derivation.

In this note, we point out an approximate numerical relationship between these two quantities. Specifically, the CMB photon density parameter Ω_γ is approximately equal to α^2. We present the arithmetic, test sensitivity to cosmological parameters, and discuss possible interpretations.

2. Statement of the Relation

The energy density of CMB radiation at temperature T is given by the Stefan–Boltzmann law:

$$u_{\mathrm{CMB}} = (\pi^2 k_B^4 / 15\hbar^3 c^3)\, T_{\mathrm{CMB}}^4 \tag{1}$$

The critical density of the universe is:

$$\rho_c = 3H_0^2 / 8\pi G \tag{2}$$

The photon density parameter is defined as $\Omega_\gamma \equiv u_{\mathrm{CMB}} / (\rho_c c^2)$. We observe that:

$$\boldsymbol{\Omega_\gamma \approx \alpha^2} \tag{3}$$

Equivalently, this can be rewritten as a prediction for the CMB temperature:

$$T_{\mathrm{CMB}} = \sqrt{\alpha} \times (15\hbar^3 c^5 \rho_c / \pi^2 k_B^4)^{1/4} \tag{4}$$

where the term in parentheses is the temperature T_{pot} whose blackbody radiation energy density equals $\rho_c c^2$.

3. Numerical Verification

3.1 Forward Calculation

Using Planck 2018 cosmological parameters [2] ($H_0 = 67.36$ km s^{-1} Mpc^{-1}) and CODATA 2018 fundamental constants:

Quantity	Value
α^2	5.325×10^{-5}
Ω_γ (Planck H_0)	5.450×10^{-5}
Ω_γ / α^2	1.024
Discrepancy	2.4%
$T_{\mathrm{predicted}}$	2.710 K
T_{observed} (FIRAS)	2.7255 ± 0.0006 K

Table 1. *Comparison of Ω_γ and α^2* using Planck 2018 parameters.

3.2 Inverse Prediction: Determining H_0

If Eq. (3) is taken as exact, it can be inverted to predict the Hubble constant from the measured CMB temperature and the fine structure constant:

$$H_0 = T_{\mathrm{CMB}}^2 / \alpha \times \sqrt{(8\pi^3 G k_B^4 / 45\hbar^3 c^5)} \tag{5}$$

Substituting T_{CMB} = 2.7255 K and α = 1/137.036 yields:

H_0^{pred} = 68.1 km s^{-1} Mpc^{-1}

This value lies between the two major determinations of the Hubble constant:

Measurement	H_0 (km s^{-1} Mpc^{-1})	Tension
Planck 2018 (CMB)	67.36 ± 0.54	1.4σ
ACT DR4	67.6 ± 1.1	0.5σ
This relation	**68.1**	—
TRGB (Freedman)	69.8 ± 1.7	1.0σ
SH0ES (Cepheids)	73.04 ± 1.04	4.7σ

Table 2. Predicted H_0 from Eq. (5) compared with current measurements. Tension column shows distance from the predicted value.

3.3 Sensitivity to H_0

Since $\Omega_\gamma \propto H_0^{-2}$ (at fixed T_{CMB}), the relation is sensitive to the assumed Hubble constant. The match improves for H_0 values near 68 km s^{-1} Mpc^{-1}:

H_0 (km/s/Mpc)	Ω_γ / α^2	Deviation
65.0	1.099	+9.9%
67.0	1.035	+3.5%
67.36	1.024	+2.4%
68.0	1.004	+0.4%
68.15	**1.000**	**0.0%**
69.0	0.975	−2.5%
70.0	0.948	−5.2%
73.0	0.871	−13.0%

Table 3. *Sensitivity of* Ω_γ/α^2 to the assumed Hubble constant. Exact equality requires H_0 = 68.15 km s^{-1} Mpc^{-1}.

3.4 Optimality of the Exponent

The relation $\Omega_\gamma = \alpha^2$ is equivalent to $T_{CMB} = T_{pot} \times \alpha^n$ with $n = 1/2$. To test whether this exponent is optimal, we solve for $n = \ln(T_{CMB}/T_{pot}) / \ln(\alpha)$. Using Planck H_0, we find $n = 0.4988$, deviating from 1/2 by only 0.24%. No other simple rational exponent (1/3, 2/5, 3/7, etc.) produces a match better than 30%.

4. Discussion

4.1 Nature of the Relation

The relation Eq. (3) connects a cosmological quantity (Ω_γ, determined by T_{CMB} and H_0) with an atomic quantity (α, determined by the elementary charge, Planck's constant, and the speed of light). In standard ΛCDM cosmology, there is no known reason for these quantities to be related. T_{CMB} is set by the baryon-to-photon ratio η and the thermal history of the universe, while α is a fundamental coupling constant of quantum electrodynamics.

The relation is *not* a Dirac large-number coincidence. Those typically arise from ratios of Planck-scale and Hubble-scale quantities, producing numbers of order 10^{60}. Here, $\alpha \approx 1/137$ is of order 10^{-2}, and the coincidence involves $\alpha^2 \approx 5.3 \times 10^{-5}$ matching $\Omega_\gamma \approx 5.4 \times 10^{-5}$. Both quantities are small but not astronomically so, and neither involves a ratio of cosmological to Planck scales.

4.2 Arguments for Physical Significance

Several features suggest this may not be accidental. First, the match holds to 2.4% despite the quantities having completely independent observational origins. Second, the optimal exponent $n = 0.4988$ is extremely close to the clean fraction 1/2. Third, the predicted $H_0 = 68.1$ km s^{-1} Mpc^{-1} falls naturally between the two competing measurements, suggesting it may capture the "true" value more accurately than either.

One possible physical pathway deserves mention. The photon number density in the universe is determined by electron–positron annihilation during the radiation epoch, a process whose cross-section is proportional to α^2. The baryon-to-photon ratio $\eta \approx 6 \times 10^{-10}$ encodes the efficiency of baryogenesis, and the final photon density depends on the interplay between η, the number of relativistic degrees of freedom g_*, and α. Whether this chain of dependencies could produce $\Omega_\gamma = \alpha^2$ as an exact result within standard Big Bang nucleosynthesis (BBN) remains an open question.

Finally, the predicted $H_0 = 68.15$ km s^{-1} Mpc^{-1} may be naturally reconciled with higher local measurements through the observed KBC void. Galaxy number counts across multiple wavelengths indicate that the Local Group resides near the centre of a Gpc-scale underdensity with a density contrast of ~46% [6, 7]. Gravitationally driven outflows from this void would add peculiar velocities to the Hubble flow, inflating locally measured H_0 values relative to the true background rate. Recent modelling by Haslbauer, Banik & Kroupa [8] shows that a ~20% local underdensity, evolved with enhanced structure growth, can boost the apparent local H_0 from a Planck-consistent background value of ~67–68 to approximately 72–73 km s^{-1} Mpc^{-1}. Mazurenko et al. [9] further demonstrate that the observed redshift-dependent decline of $H_0(z)$ toward the Planck value at high redshift is consistent with void outflow models. In this picture, the relation $\Omega_\gamma = \alpha^2$ would yield the true *global* expansion rate, while the SH0ES measurement reflects a local environmental bias. This interpretation does not require new physics beyond the void itself, although the existence of such a large void is in ~6σ tension with structure formation predictions in standard ΛCDM [8].

4.3 Arguments Against Physical Significance

The most obvious objection is the look-elsewhere effect. Given the many dimensionless constants in physics (α, α_s, $\sin^2\theta_W$, η, etc.) and cosmological density parameters (Ω_b, Ω_c, Ω_γ, Ω_Λ), the number of possible pair combinations with various powers is large. A 2% coincidence among ~100 possible trials has a non-negligible probability of occurring by chance.

Furthermore, the relation is H_0-dependent: it works well for Planck $H_0 \approx 67.4$ but fails at 13% for SH0ES $H_0 \approx 73$. If the higher local measurement reflects the true global expansion rate (rather than a local environmental bias as discussed in Section 4.2), the relation would be falsified.

Finally, we note that in standard cosmology, $\Omega_\gamma h^2 = 2.47 \times 10^{-5}$ is fully determined by T_{CMB} alone (independent of H_0). The quantity $\alpha^2 h^2 = 5.33 \times 10^{-5} \times 0.4537 = 2.42 \times 10^{-5}$, which also matches to ~2%. This formulation removes the H_0 dependence entirely and may be the more fundamental statement, though the physical content is identical.

4.4 Falsifiability and Future Tests

The relation makes a sharp, testable prediction: $H_0 = 68.15 \pm 0.15$ km s^{-1} Mpc^{-1} (where the uncertainty comes from the FIRAS temperature measurement). Upcoming independent measurements of H_0 from gravitational wave standard sirens (LIGO/Virgo/KAGRA), strong lensing time delays (TDCOSMO), and tip-of-the-red-giant-branch calibrations will either confirm or rule out this value within the next several years.

Additionally, a non-trivial consistency check is available. The relation $\Omega_\gamma = \alpha^2$ should hold under changes to the effective number of neutrino species N_{eff}, since Ω_γ accounts only for photons while N_{eff} adds additional relativistic energy density. Any future determination that the *total* relativistic density parameter Ω_r (including neutrinos) satisfies a cleaner relation with α would strengthen the case for underlying physics.

5. Conclusion

We have identified an approximate numerical relation $\Omega_\gamma \approx \alpha^2$ connecting the CMB photon density parameter to the square of the fine structure constant. The relation holds to ~2% with current best-fit Planck parameters and predicts $H_0 = 68.1$ km s^{-1} Mpc^{-1}. We present this as an empirical observation without claiming a derivation from first principles. Whether it reflects an unrecognized constraint from primordial thermodynamics or is a numerical accident awaits both theoretical investigation and improved measurements of the Hubble constant.

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