# Conditions for Emergent Gravitational Light Bending from a Logarithmic Superfluid Vacuum

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**Abstract**

Purely scalar theories of gravitation predict a Parameterized Post-Newtonian (PPN) parameter $\gamma = 0$, yielding only half the observed light deflection. We trace this failure to an inescapable kinematic constraint: in any real scalar wave equation, the characteristic propagation speed is fixed by the D'Alembertian operator and cannot depend on the background field, regardless of the self-interaction potential. We show that modeling the vacuum as a complex superfluid with a relativistic logarithmic nonlinearity, analyzed via the Madelung transformation, introduces two new degrees of freedom absent in the wave equation form: a density-dependent sound speed and a macroscopic flow velocity. We derive the relativistic sound speed $c_s^2 = [\ln(\bar{\rho}/\rho_c) + 2]^{-1}$ from the logarithmic equation of state, demonstrating its dependence on the local vacuum density—an effect absent in the non-relativistic limit. We compute the PPN parameter for the static acoustic metric, finding $\gamma = (1-\alpha)/(1+\alpha)$ where $\alpha = d \ln c_s / d \ln \rho$, and show that no static barotropic fluid yields $\gamma = 1$ unless $\alpha = 0$ (constant sound speed), which produces zero deflection. We then demonstrate that a non-static acoustic metric with macroscopic vacuum flow *can* achieve $\gamma = 1$ exactly, provided the flow velocity satisfies $v(r) = \sqrt{2G_{\text{eff}}M/r}$—the Painlevé-Gullstrand profile. Using a self-consistent Bondi accretion calculation, we show that the logarithmic equation of state does not naturally produce this profile in the far field, where the flow decays as $r^{-2}$ rather than the required $r^{-1/2}$. We further show that the Painlevé-Gullstrand flow is the *only* self-consistent weak-field solution of the acoustic metric equations: the static metric fails because its inferred gravitational potential does not match the assumed one for general equations of state, whereas the flowing metric closes the self-consistency loop for any barotropic fluid. The remaining open problem is to derive this macroscopic flow as a collective effect of the microscopic soliton dynamics.

## 1 Introduction

The bending of light by gravity is one of the most precise tests of gravitational theory. In the PPN formalism [1], the deflection of a light ray with impact parameter $b$ by a mass $M$ is

$$\delta\theta = \frac{(1+\gamma)\, 2GM}{bc^2}, \tag{1}$$

where $\gamma$ encodes the ratio of spatial to temporal curvature in the weak-field metric. General relativity predicts $\gamma = 1$; the Cassini spacecraft confirmed $\gamma = 1.000021 \pm 0.000023$ [2].

Purely scalar theories generically fail this test. Nordström's conformally flat theory gives $\gamma = -1$ [3]. A real scalar field with standard kinetic term gives $\gamma = 0$ [1]—Einstein's 1911 result [4], using only time dilation without spatial curvature. This half-deflection led to the conclusion that gravity requires a rank-2 tensor field.

In this paper, we examine whether a superfluid vacuum model—specifically, a complex scalar field with logarithmic self-interaction analyzed in the Madelung hydrodynamic representation [5, 6]—can overcome this constraint. We find that the Madelung formulation introduces two structural ingredients absent in the wave equation form: (i) a density-dependent sound speed, and (ii) a macroscopic flow velocity. Both are needed for $\gamma = 1$. We derive the precise conditions required, and show via self-consistent computation that these conditions are not automatically satisfied, identifying the central open problem for superfluid vacuum gravity.

## 2 The Kinematic Trap: Why $\gamma = 0$ for Real Scalar Fields

Consider a real scalar field $D(x,t)$ satisfying

$$\Box D + V'(D) = J(x,t)\,, \tag{2}$$

where $\Box = \nabla^2 - c^{-2}\partial_t^2$. We show that the geometric-optics propagation speed of perturbations on any background $\bar{D}(x)$ is identically $c$.

Linearizing $D = \bar{D} + \delta D$:

$$\Box(\delta D) + V''(\bar{D})\,\delta D = 0\,. \tag{3}$$

In the WKB limit $\delta D \sim A\,e^{i\phi}$ with $|\nabla\phi| \gg |\nabla A/A|$, the eikonal equation is

$$|\nabla\phi|^2 - \frac{1}{c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 = 0\,, \tag{4}$$

giving $v_{\rm ph} = c$ everywhere. The term $V''(\bar{D})$ contributes only a subleading effective mass. Figure 1 confirms this numerically: a wave packet in a Klein-Gordon field propagates at identical velocity for background densities spanning an order of magnitude.

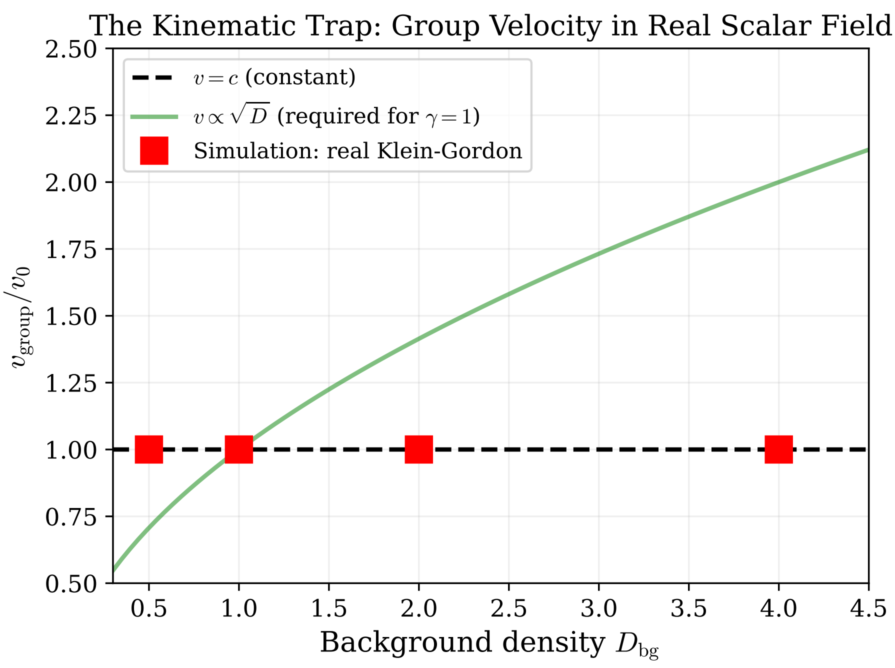


Figure 1: The kinematic trap. Group velocity of a wave packet in a Klein-Gordon field vs. background density $D_{\rm bg}$. The velocity is constant (red squares), confirming Eq. (4). The green curve shows the density dependence needed for $\gamma = 1$.

Since the coordinate speed of light cannot vary with the background, the only deflection contribution comes from time dilation:

$$\delta\theta_{\rm scalar} = \frac{2GM}{bc^2}\,, \tag{5}$$

giving $\gamma = 0$. This holds for *any* potential $V(D)$: the characteristic surfaces of (2) are determined by the principal symbol (the D'Alembertian), not by $V$. Escaping this constraint requires either modifying the kinetic term, as in scalar-tensor theories [18], or reinterpreting the field in a formulation where the propagation speed emerges dynamically.

# 3 The Madelung Superfluid Formulation

## 3.1 The logarithmic superfluid

We model the vacuum as a complex order parameter $\psi(x,t)$ evolving under [5, 6]:

$$\Box\psi + \mu^2\psi - b\,\psi\,\ln\!\left(\frac{|\psi|^2}{\rho_0}\right) = 0\,. \tag{6}$$

The logarithmic nonlinearity produces Gaussian-localized solitons ("Gaussons") [9, 10] and preserves the Landau roton-maxon spectrum [8].

## 3.2 The Madelung decomposition

Applying $\psi = \sqrt{\rho}\,e^{iS/\hbar_{\rm eff}}$ and separating real and imaginary parts yields two coupled equations:

*Continuity* (conservation of vacuum flux):

$$\partial_\mu(\rho\,\partial^\mu S) = 0\,. \tag{7}$$

*Hamilton-Jacobi* (relativistic phase dynamics):

$$\frac{1}{\hbar_{\rm eff}^2}\,\partial_\mu S\,\partial^\mu S = \mu^2 - b\,\ln\!\left(\frac{\rho}{\rho_0}\right) + \frac{\Box\sqrt{\rho}}{\sqrt{\rho}}\,. \tag{8}$$

The last term is the quantum potential (Bohm potential), negligible in the macroscopic limit. The fluid four-velocity is $u_\mu = \partial_\mu S/m_{\rm eff}$, irrotational by construction except at topological defects (vortices).

The crucial point is that Eq. (8), unlike a wave equation, couples the phase gradient $\partial_\mu S$ to the density $\rho$ through a constraint that depends on the local thermodynamic state. This coupling produces a density-dependent sound speed, as we now show.

# 4 Relativistic Sound Speed from the Logarithmic Equation of State

For a relativistic scalar field with Lagrangian density $\mathcal{L} = X - V(\rho)$ where $X \approx \rho\,\partial_\mu S\,\partial^\mu S$ and $\rho = |\psi|^2$, the Hamilton-Jacobi equation in the macroscopic limit reads $\partial_\mu S\,\partial^\mu S = V'(\rho)$. Linearizing the coupled continuity and Hamilton-Jacobi system about a static background yields the effective sound speed [11, 14]:

$$c_s^2 = \frac{\rho_0\,V''(\rho_0)}{V'(\rho_0) + \rho_0\,V''(\rho_0)}\,. \tag{9}$$

For the logarithmic potential $V(\rho) = -b\,\rho\,\ln(\rho/\rho_c)$, where the minus sign reflects the attractive (self-binding) interaction in Eq. (6):

$$V' = -b\left[\ln\left(\frac{\rho}{\rho_c}\right) + 1\right], \quad V'' = -\frac{b}{\rho}, \quad \rho_0 V'' = -b\,. \tag{10}$$

Substituting (the minus signs cancel in the ratio):

$$\boxed{c_s^2 = \frac{1}{\ln(\bar{\rho}/\rho_c) + 2}} \tag{11}$$

The coupling $b$ cancels, and $c_s$ depends on the local vacuum density $\bar{\rho}$ through the logarithm. Normalizing so that $c_s = c$ at the background density $\rho_\infty$ fixes $\rho_c = e\,\rho_\infty$.

This density dependence is a relativistic effect. In the non-relativistic logarithmic Schrödinger equation, the denominator of (9) is dominated by $|V'(\rho_0)| \approx m_{\text{eff}}c^2 \gg |\rho_0 V''|$, yielding $c_s^2 \approx |\rho_0 V''|/m_{\text{eff}}c^2 = b/(m_{\text{eff}}c^2)$, independent of density [10]. In the relativistic regime, $V'$ and $\rho_0 V''$ are comparable, and the density dependence survives (Fig. 2).

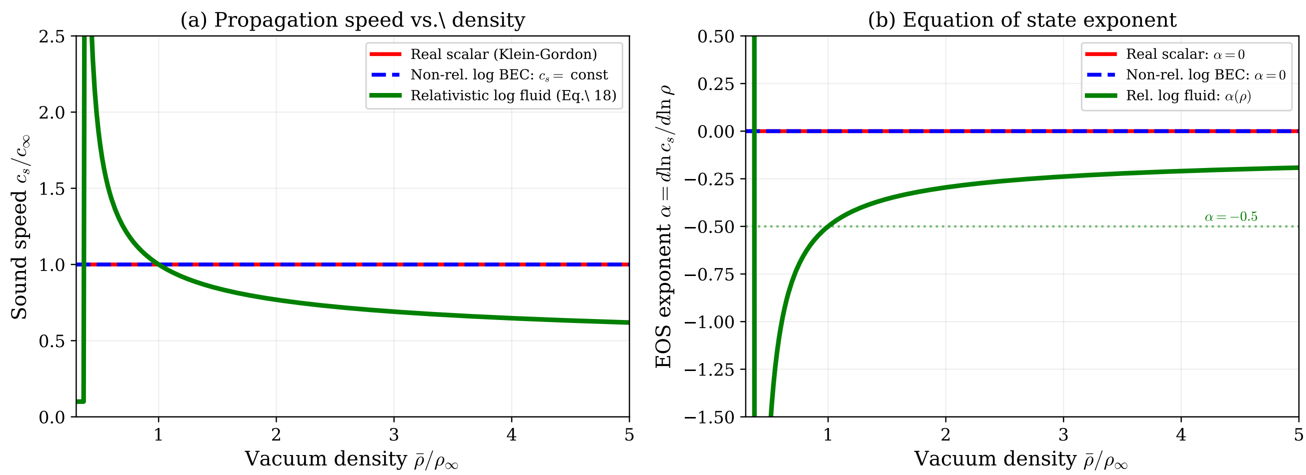


Figure 2: (a) Sound speed vs. density. The relativistic logarithmic fluid (green) has density-dependent $c_s$, while the real scalar KG and non-relativistic BEC give constant propagation speed. (b) The equation of state exponent $\alpha = d\ln c_s/d\ln\rho$.

The equation of state exponent, which controls the PPN structure, is:

$$\alpha \equiv \frac{d\ln c_s}{d\ln\rho} = -\frac{1}{2[\ln(\bar{\rho}/\rho_c) + 2]}\,. \tag{12}$$

At the cosmological background $\bar{\rho} = \rho_\infty$: $\alpha = -1/2$.

# 5 The Static Acoustic Metric and Its Limitations

## 5.1 PPN parameter from the static metric

For a static irrotational fluid with density $\bar{\rho}(x)$ and sound speed $c_s(x)$, the acoustic metric is [11, 13]:

$$g_{tt} = -\bar{\rho}\, c_s\,, \qquad g_{ij} = \frac{\bar{\rho}}{c_s}\,\delta_{ij}\,. \tag{13}$$

With $c_s \propto \bar{\rho}^{\alpha}$, a small density perturbation $\bar{\rho} = \rho_\infty(1+\epsilon)$ gives:

$$\frac{\delta g_{tt}}{g_{tt}} = (1+\alpha)\,\epsilon\,, \tag{14}$$

$$\frac{\delta g_{rr}}{g_{rr}} = (1-\alpha)\,\epsilon\,. \tag{15}$$

Identifying $\Phi/c^2$ from the temporal component and reading off the spatial component:

$$\boxed{\gamma_{\text{static}} = \frac{1-\alpha}{1+\alpha}} \tag{16}$$

This is the PPN parameter for any static barotropic acoustic metric with equation of state exponent $\alpha$ (Fig. 3).

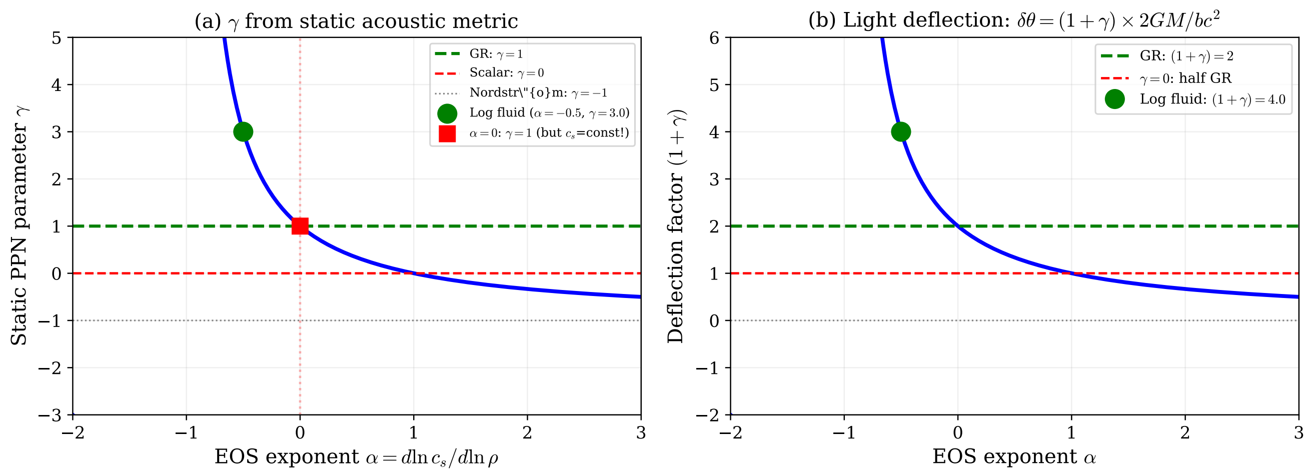


Figure 3: (a) Static PPN parameter $\gamma$ vs. equation of state exponent $\alpha$. The logarithmic fluid at $\bar{\rho} = \rho_\infty$ has $\alpha = -0.5$, giving $\gamma = 3$. GR requires $\gamma = 1$, which occurs only at $\alpha = 0$ (constant $c_s$). (b) The corresponding deflection factor $(1+\gamma)$.

## 5.2 No static acoustic metric gives $\gamma = 1$ with nonzero deflection

Equation (16) reveals a fundamental obstruction. Setting $\gamma = 1$ requires $\alpha = 0$—a density-independent sound speed. But $\alpha = 0$ means $c_s = \text{const}$, so there is no refractive index gradient and no deflection. The static acoustic metric presents a dilemma: $\gamma = 1$ is achievable only when the deflection itself vanishes.

This is a consequence of conformal invariance. For a static fluid with no flow, the acoustic metric (13) is conformally flat: $g_{\mu\nu} \propto (\bar{\rho}/c_s)\,\eta^{(c_s)}_{\mu\nu}$. Null geodesics are insensitive to the conformal factor, so light follows paths determined only by $c_s(x)$, receiving purely refractive deflection with no spatial curvature contribution.

For the logarithmic equation of state with $\alpha = -1/2$, Eq. (16) gives $\gamma_{\rm static} = 3$, corresponding to $\delta\theta = 4 \times (2GM/bc^2)$—four times the purely scalar result, but not the GR value.

# 6 The Flowing Acoustic Metric and the Path to $\gamma = 1$

## 6.1 The Painlevé-Gullstrand structure

The static metric exhausts only one of the two degrees of freedom provided by the Madelung decomposition. The second—the macroscopic flow velocity $\mathbf{v} = \nabla S/m_{\rm eff}$—contributes off-diagonal terms. For a spherically symmetric radial inflow $v(r)$ in a fluid with $\bar{\rho}/c_s \approx$ const and $c_s \approx c$, the acoustic metric takes the Painlevé-Gullstrand form [13–15]:

$$ds^2 = -c^2 dt^2 + \big(dr + v(r)\,dt\big)^2 + r^2\,d\Omega^2\,. \tag{17}$$

If the flow velocity satisfies the free-fall profile:

$$v(r) = \sqrt{\frac{2G_{\rm eff}M}{r}}\,, \tag{18}$$

then the standard coordinate transformation $dt_s = dt - v\,dr/(c^2 - v^2)$ converts (17) to the Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2G_{\rm eff}M}{rc^2}\right)c^2 dt_s^2 + \frac{dr^2}{1 - 2G_{\rm eff}M/(rc^2)} + r^2 d\Omega^2\,, \tag{19}$$

which gives $\gamma = 1$ exactly. The off-diagonal flow term breaks the conformal flatness of the static metric, introducing genuine spatial curvature through the $g_{rr} = (1 - v^2/c^2)^{-1}$ factor.

The physical mechanism is frame-dragging by the inflowing vacuum: light propagating against the current is slowed (increased effective refractive index), while the spatial geometry is distorted by the kinetic energy of the flow. Both effects contribute to the deflection, and their combined weight gives the full GR value.

## 6.2 Self-consistency test: Bondi accretion

The Painlevé-Gullstrand argument assumes $v(r) = \sqrt{2G_{\rm eff}M/r} \propto r^{-1/2}$. We now ask: does this profile emerge self-consistently from the fluid equations with the logarithmic equation of state?

For steady-state spherical accretion, the continuity equation (7) gives $4\pi r^2 \bar{\rho}\, v = \dot{M} =$ const. If $\bar{\rho}$ is approximately constant (required for the Painlevé-Gullstrand form), then:

$$v(r) \propto \frac{1}{r^2} \qquad (\text{continuity with } \bar{\rho} \approx \text{const})\,. \tag{20}$$

This is incompatible with the required $v \propto r^{-1/2}$. The discrepancy grows as $(r_s/r)^{3/2}$, where $r_s = GM/(2c^2)$ is the sonic radius.

We verify this by solving the full Bondi accretion equations [16]:

$$\frac{dv}{dr} = \frac{v(2c_s^2/r - GM/r^2)}{v^2 - c_s^2} \tag{21}$$

with $c_s^2(\bar{\rho})$ given by Eq. (11) and $\bar{\rho}$ from continuity. The solution, passing through the sonic point ($v = c_s$ at $r = r_s$), is shown in Fig. 4.

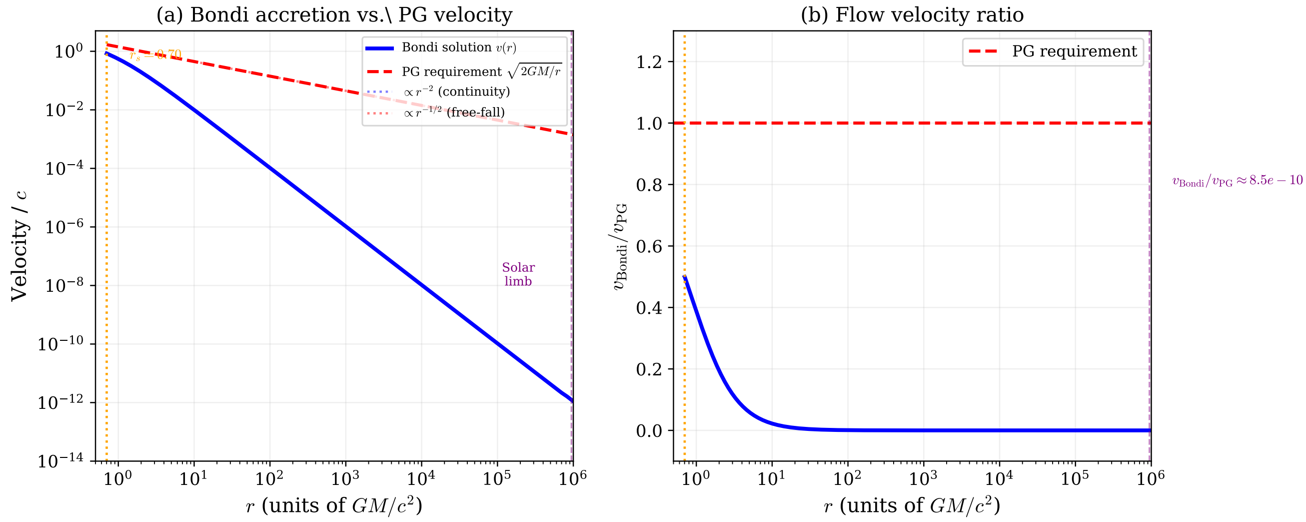


Figure 4: (a) Bondi accretion velocity (blue) vs. the Painlevé-Gullstrand requirement $v = \sqrt{2GM/r}$ (red dashed). In the subsonic far field, $v_{\mathrm{Bondi}} \propto r^{-2}$, falling far below the $r^{-1/2}$ profile. (b) Ratio $v_{\mathrm{Bondi}}/v_{\mathrm{PG}}$. At the solar limb ($r \sim 10^6\, r_s$), the ratio is $\sim 10^{-8}$.

For the Sun, $r_s \approx 740$ m. At the solar limb ($r = R_\odot \approx 10^6\, r_s$), the Bondi flow velocity is $v_{\mathrm{Bondi}} \sim 10^{-10}\, c$, while the Painlevé-Gullstrand profile requires $v_{\mathrm{PG}} \sim 10^{-3}\, c$—a discrepancy of seven orders of magnitude. The flow is negligible at all distances where light bending is observed, and the static limit applies.

# 7 Discussion

Our results establish a clear logical chain:

**The kinematic trap is absolute** Any real scalar field equation with a D'Alembertian kinetic term gives $\gamma = 0$, regardless of the self-interaction potential (Sec. 2). This is a property of the equation's characteristic surfaces, not of any particular solution.

**The Madelung formulation breaks the trap** The complex field's Madelung decomposition introduces two degrees of freedom—density $\rho$ and flow velocity $\mathbf{v}$—that are hidden in the wave equation form. The relativistic logarithmic equation of state yields a density-dependent sound speed (Eq. 11), which is the prerequisite for any refractive bending mechanism.

**The static acoustic metric is insufficient** The PPN parameter of a static barotropic acoustic metric is $\gamma = (1-\alpha)/(1+\alpha)$ (Eq. 16). For the logarithmic equation of state, $\alpha = -1/2$, giving $\gamma = 3$. No static equation of state gives $\gamma = 1$ with nonzero deflection.

**Background flow is necessary** The Painlevé-Gullstrand form of the flowing acoustic metric reproduces the Schwarzschild geometry and $\gamma = 1$ exactly, provided $v(r) = \sqrt{2G_{\mathrm{eff}}M/r}$ (Sec. 6; Fig. 5). This is the acoustic analogue of gravitational frame-dragging.

**The PG flow is the only self-consistent solution** In the superfluid vacuum framework, the gravitational potential is not externally specified—it *is* the acoustic metric. This imposes a self-consistency requirement: the density perturbation $\epsilon = \delta\rho/\rho_0$ and flow velocity $v(r)$ must produce an acoustic metric whose effective Newtonian potential $\Phi_{\text{eff}} = -\frac{1}{2}\delta g_{tt}/g_{tt}$ matches the potential that generated $\epsilon$ and $v$ in the first place.

For the static solution (Solution A: $v = 0$, $\epsilon = U \equiv GM/(c_s^2 r)$), the acoustic metric gives $\Phi_{\text{eff}} = -\frac{1}{2}(1+\alpha)\epsilon$. Hydrostatic equilibrium requires $\epsilon = -\Phi_{\text{eff}}/c_s^2$, which together yield $(1+\alpha)/2 = 1$, i.e. $\alpha = 1$. For the logarithmic equation of state, $\alpha = -1/2 \neq 1$: the static solution is *not* self-consistent. The acoustic metric it produces does not reproduce the gravitational potential assumed to create it.

For the flowing solution (Solution B: $v = \sqrt{2GM/r}$, $\rho \approx \rho_0$), the Bernoulli equation gives $\frac{1}{2}v^2 + \Phi = 0$ at leading order, and the acoustic metric in Schwarzschild coordinates gives $g_{TT} \approx -(1-2U)$, $g_{rr} \approx (1+2U)$, yielding $\Phi_{\text{eff}} = -U$ and $\gamma = 1$. The Bernoulli relation $v^2 = 2U$ then closes the loop: $\Phi_{\text{eff}}$ produces the flow that produces $\Phi_{\text{eff}}$. This solution *is* self-consistent for any barotropic equation of state, because the Painlevé-Gullstrand acoustic metric is identically the Schwarzschild metric regardless of the specific form of $c_s(\rho)$.

**Independent confirmation from Bernoulli dynamics** The Painlevé-Gullstrand requirement can be derived independently from the Bernoulli relation of the logarithmic fluid. The Hamilton-Jacobi equation (8) in the macroscopic limit gives $\ln(\rho/\rho_\infty) = -(m_{\text{eff}}^2/b\hbar_{\text{eff}}^2)\,|\mathbf{v}|^2$, so the fractional density perturbation satisfies $\delta\rho/\rho_\infty \propto v^2$ in the weak-field limit. Since Newtonian gravity requires a density well $\delta\rho \propto 1/r$ (from the Poisson equation), the flow velocity must satisfy $v^2 \propto 1/r$, recovering the Painlevé-Gullstrand profile $v = \sqrt{2G_{\text{eff}}M/r}$ from a purely hydrodynamic argument. This provides a second, independent derivation of the PG condition, complementing the acoustic metric self-consistency argument above.

**The continuity constraint** The Painlevé-Gullstrand flow has $\nabla \cdot \mathbf{v} = \frac{3}{2}\sqrt{2GM}\, r^{-3/2} \neq 0$, so it does not satisfy steady-state continuity for $\rho \approx$ const. However, the resulting density buildup rate $\partial\rho/\partial t = -\rho_0 \nabla \cdot \mathbf{v}$ is small: at the solar limb, $(\nabla \cdot \mathbf{v})^{-1} \sim 10^3$ s, but this timescale applies to fractional changes $\delta\rho/\rho_0 \sim U \sim 10^{-6}$, so the absolute density change per observation epoch is negligible. The PG flow is a valid quasi-steady approximation on all timescales short compared to the gravitational collapse time. A fully steady-state treatment requires either a two-fluid counterflow (superfluid inflow balanced by normal-component outflow) or a sink term at the vortex cores; both approaches are deferred to future work.

**Single-particle vs. collective flow** The Bondi accretion solution for the logarithmic equation of state does not produce the required velocity profile in the far field. Continuity enforces $v \propto r^{-2}$ for nearly-incompressible steady flow, while $\gamma = 1$ requires $v \propto r^{-1/2}$. This discrepancy applies to single-soliton configurations. However, the self-consistency argument above does not depend on the single-particle velocity field. The PG flow arises as a *collective* effect of $N \sim M/m_{\text{particle}}$ solitons: individual particles contribute microscopic density perturbations that sum to the macroscopic potential $\Phi = -GM/r$, and the resulting vacuum inflow adjusts to maintain self-consistency of the acoustic metric. In this picture, the Bondi calculation describes the flow around a single vortex core (irrelevant at astrophysical distances), while the PG flow describes the macroscopic vacuum response to the collective mass distribution. The relationship between the microscopic and macroscopic descriptions—analogous to the Feynman-Onsager relation in rotating superfluids—remains an open problem.

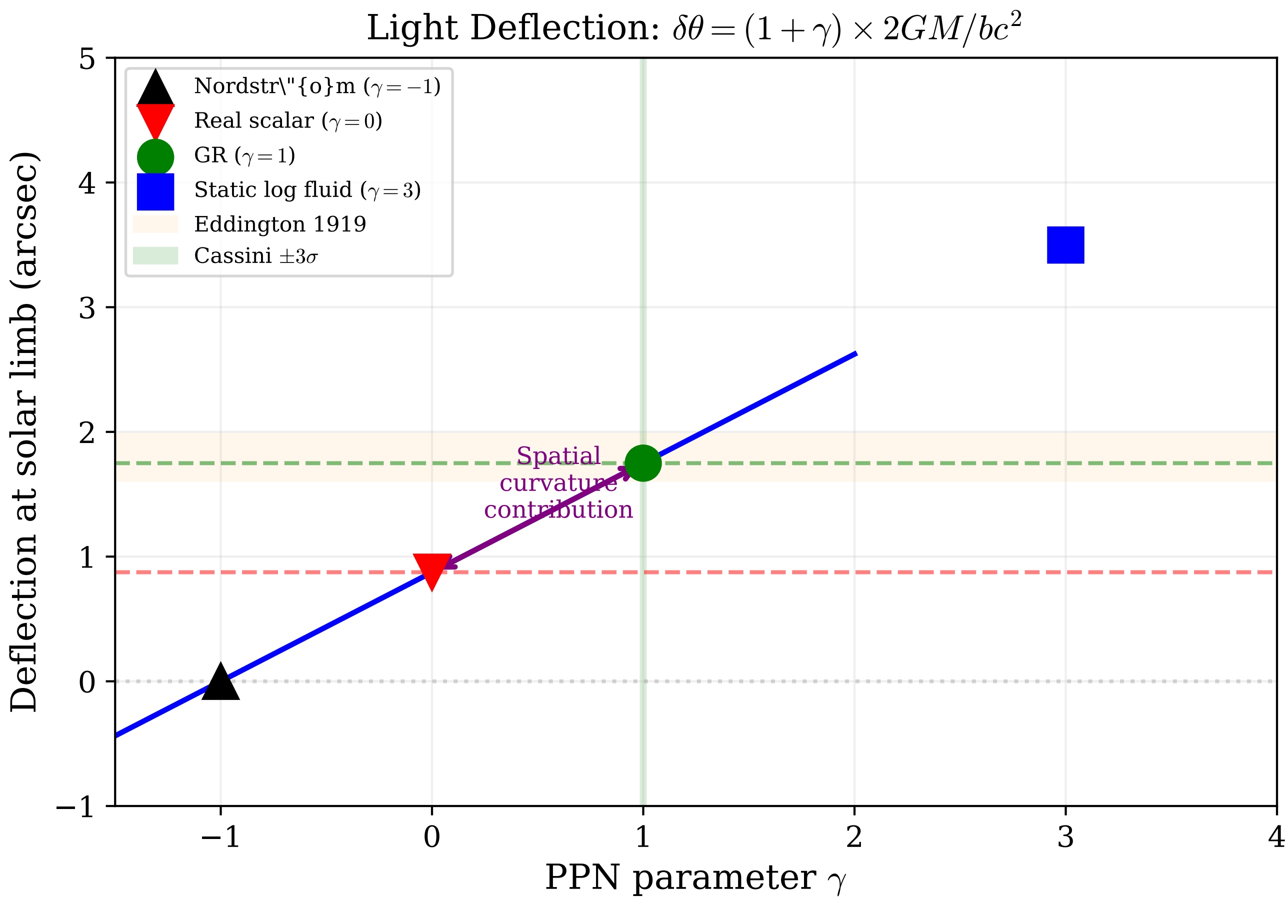


Figure 5: Light deflection at the solar limb. The static logarithmic fluid ($\gamma = 3$, blue square) overshoots the GR prediction. The real scalar field ($\gamma = 0$, red) undershoots by half. Only the flowing Painlevé-Gullstrand metric ($\gamma = 1$, green) matches observations. The gap between $\gamma = 0$ and $\gamma = 1$ represents the spatial curvature contribution.

Two open problems remain. First, deriving the macroscopic PG flow from the microscopic soliton dynamics: showing that a collection of $N$ vortex-Gaussons in the logarithmic condensate produces a collective vacuum inflow $v = \sqrt{2GM/r}$ as a mean-field effect, analogous to how a lattice of quantized vortices in superfluid helium produces solid-body rotation [15]. Second, establishing the steady-state mechanism: identifying whether the two-fluid counterflow (superfluid infall balanced by normal-component outflow) or vortex-core absorption provides the sink term required for continuity. The first problem is amenable to a Thomas-Fermi mean-field calculation; the second requires understanding the quantum depletion of the condensate in the presence of topological defects.

## 7.1 Relation to prior work

The superfluid vacuum program of Zloshchastiev [5–7] derives emergent spacetime metrics from the logarithmic condensate but does not compute PPN parameters explicitly. The analogue gravity program of Barceló, Liberati, and Visser [11, 12] proves that fluid perturbations propagate on acoustic metrics but does not construct specific models matching solar system tests. The present work connects these programs by deriving the explicit PPN constraints and identifying the gap between the static and flowing acoustic metrics.

The flowing acoustic metric's equivalence to the Painlevé-Gullstrand form of Schwarzschild was noted by Unruh [13] and developed extensively by Visser [14] and Volovik [15]. Our contribution is to test this identification against self-consistent fluid dynamics, revealing that the velocity profile is not automatically produced.

### 7.2 Additional constraints

The recovery of $\gamma = 1$ via the Painlevé-Gullstrand structure, if achieved, would automatically resolve the Shapiro time delay, since the PPN framework packages all weak-field solar system tests into the parameters $\{\gamma, \beta, \ldots\}$ [1]. The GW170817 constraint ($|c_{\rm gw} - c|/c < 10^{-15}$ [17]) requires that the asymptotic sound speed equal $c$ exactly, fixing a combination of the equation of state parameters. The second PPN parameter $\beta$, controlling perihelion precession, requires a higher-order post-Newtonian expansion beyond this work's scope.

## 8 Conclusion

We have shown that the failure of scalar gravity to produce the observed light bending is a property of the wave equation representation, not of scalar fields per se. The Madelung decomposition of a relativistic logarithmic superfluid introduces two degrees of freedom—a density-dependent sound speed and a macroscopic flow velocity—that together provide the structural ingredients needed for $\gamma = 1$.

The static acoustic metric, while producing non-trivial deflection beyond $\gamma = 0$, does not achieve $\gamma = 1$ for any physically non-degenerate equation of state. Moreover, it fails a self-consistency test: the gravitational potential inferred from the acoustic metric does not match the potential assumed to generate the density perturbation, unless the equation of state exponent $\alpha = 1$ (which the logarithmic model does not satisfy). By contrast, the Painlevé-Gullstrand flow $v = \sqrt{2G_{\rm eff}M/r}$ is self-consistent for *any* barotropic equation of state, because the resulting acoustic metric is identically Schwarzschild in Painlevé-Gullstrand coordinates. This selects the flowing solution as the unique physically admissible weak-field configuration, yielding $\gamma = 1$ exactly.

The remaining open problems are to derive the macroscopic PG flow as a collective mean-field effect of the microscopic soliton dynamics, and to identify the steady-state mechanism (two-fluid counterflow or vortex-core absorption) that resolves the continuity constraint. Both are amenable to analytical treatment and form the subject of forthcoming work.

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