Emergent Electromagnetism from the Gauged Logarithmic Superfluid Vacuum

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Abstract

The relativistic logarithmic Klein-Gordon equation for the vacuum condensate possesses a global $U(1)$ phase symmetry. We show that promoting this to a local gauge symmetry—the minimal coupling $\partial_\mu S \to \partial_\mu S - eA_\mu$—produces Maxwell's equations as the Euler-Lagrange equations for the gauge field A_μ, with the condensate current $J^\nu = (em_{\text{eff}}/\hbar_{\text{eff}}^2)\,\rho\,u^\nu$ as source. The Madelung decomposition of the gauged equation yields the acoustic metric equations (emergent gravity) and Maxwell's equations (emergent electromagnetism) simultaneously from a single Lagrangian. Gravity is sourced by the soliton energy density (mass); electromagnetism is sourced by the topological winding number of vortex-Gausson defects, which quantizes electric charge as $Q = ne$ with $n \in \mathbb{Z}$. Both fields propagate at the sound speed c on the same acoustic metric, automatically ensuring $c_{\text{grav}} = c_{\text{EM}} = c$ as observed. The London penetration depth of the condensate determines the electromagnetic screening length; in the electrically neutral vacuum background, it diverges, permitting free propagation of electromagnetic waves. Near vortex cores, the screening becomes significant, producing the Coulomb field as a condensate boundary effect. The framework unifies gravity and electromagnetism as two aspects of the same condensate dynamics—the acoustic metric and the gauge connection—while preserving their distinct physical origins: mass versus topology.

1 Introduction

In previous work, we showed that the Madelung decomposition of the relativistic logarithmic Klein-Gordon equation produces an acoustic metric equivalent to the Schwarzschild geometry in the weak-field limit, yielding the PPN parameter $\gamma = 1$ and an emergent gravitational constant $G_{\text{eff}} = \eta c^2/(4\pi\rho_0)$. The vacuum condensate's density ρ and flow velocity $u_\mu = \partial_\mu S/m_{\text{eff}}$ generate the gravitational field; topological defects (vortex-Gaussons) play the role of massive particles.

This framework has a conspicuous unused structure. The logarithmic wave equation

$$\Box\psi + \mu^2\psi - b\,\psi\,\ln\left(\frac{|\psi|^2}{\rho_0}\right) = 0 \tag{1}$$

is invariant under the global phase rotation $\psi \to \psi\, e^{i\alpha}$ for constant α. This $U(1)$ symmetry is associated, via Noether's theorem, with the conservation of the condensate particle number (the continuity equation $\partial_\mu(\rho\,\partial^\mu S) = 0$).

In the Standard Model, the fundamental interactions arise from *gauging* internal symmetries: promoting global invariances to local ones and introducing gauge fields to maintain covariance. Electromagnetism itself originates from gauging the $U(1)$ phase symmetry of the electron field [6].

In this paper, we apply the same principle to the vacuum condensate. Promoting the global $U(1)$ of Eq. (1) to a local symmetry requires the introduction of a gauge field A_μ and the minimal

coupling $\partial_\mu \to D_\mu = \partial_\mu - i(e/\hbar)A_\mu$. We show that the Euler-Lagrange equations for A_μ are precisely Maxwell's equations, with the condensate current as source. Together with the acoustic metric (gravity), this produces a unified description of two fundamental interactions from a single condensate Lagrangian.

2 The Gauged Lagrangian

The ungauged Lagrangian density for the logarithmic condensate is:

$$\mathcal{L}_0 = (\partial_\mu \psi^*)(\partial^\mu \psi) - \mu^2|\psi|^2 + b\,|\psi|^2 \ln\left(\frac{|\psi|^2}{\rho_0}\right) . \tag{2}$$

This is invariant under $\psi \to \psi\, e^{i\alpha}$ for constant α.

Promoting to local gauge invariance $\psi \to \psi\, e^{i\alpha(x)}$ requires replacing $\partial_\mu \to D_\mu = \partial_\mu - i(e/\hbar)A_\mu$ and adding the gauge field kinetic term. The gauged Lagrangian is:

$$\boxed{\mathcal{L} = (D_\mu \psi)^*(D^\mu \psi) - \mu^2|\psi|^2 + b\,|\psi|^2 \ln\left(\frac{|\psi|^2}{\rho_0}\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}} \tag{3}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor and e is the gauge coupling constant.

The term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ is the unique Lorentz-invariant, gauge-invariant, parity-preserving kinetic term for A_μ with at most two derivatives [6]. Its inclusion is not optional: without it, A_μ would have no dynamics and could be gauged away.

3 Madelung Decomposition of the Gauged Equation

3.1 The gauge-invariant velocity

Applying the Madelung transformation $\psi = \sqrt{\rho}\, e^{iS/\hbar_{\text{eff}}}$, the covariant derivative becomes:

$$D_\mu \psi = \left(\frac{\partial_\mu \sqrt{\rho}}{\sqrt{\rho}} + \frac{i}{\hbar_{\text{eff}}}(\partial_\mu S - eA_\mu)\right)\psi\,. \tag{4}$$

The gauge-invariant superfluid four-velocity is:

$$\boxed{u_\mu = \frac{1}{m_{\text{eff}}}(\partial_\mu S - eA_\mu)} \tag{5}$$

This is the central modification. In the ungauged theory, $u_\mu = \partial_\mu S/m_{\text{eff}}$; the gauge field shifts the velocity by $-eA_\mu/m_{\text{eff}}$. This is the relativistic generalization of the London relation in superconductors: $m\mathbf{v}_s = \hbar\nabla\phi - e\mathbf{A}$ [7].

3.2 The modified fluid equations

Separating the Euler-Lagrange equation for ψ into real and imaginary parts yields the gauged Madelung equations:

Continuity:

$$\partial_\mu(\rho\, u^\mu) = 0\,. \tag{6}$$

Hamilton-Jacobi:

$$\frac{m_{\text{eff}}^2}{\hbar_{\text{eff}}^2}\, u_\mu u^\mu = \mu^2 - b\, \ln\!\left(\frac{\rho}{\rho_0}\right) + Q[\rho]\,, \tag{7}$$

where $Q = \Box\sqrt{\rho}/\sqrt{\rho}$ is the quantum potential (Bohm potential), unchanged from the ungauged case.

These equations are *identical in form* to the ungauged Madelung equations, with the single replacement $\partial_\mu S \to \partial_\mu S - eA_\mu$. The acoustic metric depends on ρ and u_μ, both of which are gauge-invariant. The entire gravitational sector—the PPN parameter $\gamma = 1$, the emergent G_{eff}, the Painlevé-Gullstrand flow—is preserved without modification.

4 Maxwell's Equations from the Gauge Field Variation

The Euler-Lagrange equation for A_μ is obtained by varying the Lagrangian (3) with respect to A^ν.

The condensate contribution:

$$\frac{\delta}{\delta A^\nu}\left[(D_\mu\psi)^*(D^\mu\psi)\right] = -\frac{e\, m_{\text{eff}}}{\hbar_{\text{eff}}^2}\,\rho\, u^\nu\,. \tag{8}$$

The gauge field contribution:

$$\frac{\delta}{\delta A^\nu}\left[-\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}\right] = \partial_\mu F^{\mu\nu}\,. \tag{9}$$

Setting the total variation to zero:

$$\boxed{\partial_\mu F^{\mu\nu} = J^\nu} \tag{10}$$

where the four-current is:

$$J^\nu = \frac{e\, m_{\text{eff}}}{\hbar_{\text{eff}}^2}\,\rho\, u^\nu\,. \tag{11}$$

Equation (10) is Maxwell's inhomogeneous equation. The homogeneous equations $\partial_{[\mu}F_{\nu\lambda]} = 0$ follow identically from the definition $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ (the Bianchi identity).

In three-vector notation:

$$\nabla\cdot\mathbf{E} = \rho_{\text{ch}}/\epsilon_0 \qquad \text{(Gauss)}\,, \tag{12}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \text{(no monopoles)}\,, \tag{13}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \text{(Faraday)}\,, \tag{14}$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t} \qquad \text{(Ampère-Maxwell)}\,, \tag{15}$$

where $\rho_{\text{ch}} = J^0$ is the charge density and $\mathbf{J} = (J^1, J^2, J^3)$ is the current density. These are the full Maxwell equations, derived from the gauge field variation of the condensate Lagrangian.

5 Charge Quantization from Vortex Topology

The vortex-Gausson solitons of the logarithmic condensate have quantized phase winding. For a vortex of winding number n, the phase increases by $2\pi n\hbar_{\text{eff}}$ around any closed loop encircling the core:

$$\oint \partial_\mu S \, dx^\mu = 2\pi n \hbar_{\text{eff}} \,, \qquad n \in \mathbb{Z} \,. \tag{16}$$

The gauge-invariant circulation is:

$$\oint u_\mu \, dx^\mu = \frac{1}{m_{\text{eff}}} \oint (\partial_\mu S - eA_\mu) \, dx^\mu = \frac{2\pi n \hbar_{\text{eff}}}{m_{\text{eff}}} - \frac{e}{m_{\text{eff}}} \oint A_\mu \, dx^\mu \,. \tag{17}$$

In the absence of an external gauge field ($A_\mu = 0$ at infinity), the total charge enclosed by the loop is determined by the winding number:

$$\boxed{Q = n\, e \,, \qquad n \in \mathbb{Z}} \tag{18}$$

Electric charge is quantized because vortex winding numbers are integers. This is the same mechanism by which magnetic flux is quantized in superconductors [8]: the single-valuedness of the order parameter forces the enclosed quantity to be an integer multiple of the fundamental quantum.

The physical identification is:

- Vortex-Gaussons with $n = +1$: positively charged particles (charge $+e$).
- Anti-vortices with $n = -1$: negatively charged particles (charge $-e$).
- Pure Gaussons with $n = 0$: electrically neutral particles (charge 0).

The vortex-Gausson thus carries both mass (from its soliton energy E_v, generating gravity) and charge (from its winding number n, generating electromagnetism).

6 Electromagnetic Waves in the Condensate

In the vacuum background ($\rho = \rho_0$, $u_\mu = 0$, no net charge), the linearized Maxwell equations (10) reduce to:

$$\Box A_\mu = 0 \qquad \text{(in Lorenz gauge)} \,, \tag{19}$$

where $\Box = \partial_t^2/c^2 - \nabla^2$ and $c = c_s(\rho_0)$ is the sound speed at the background density. Electromagnetic waves propagate at the speed of sound in the condensate.

This provides a unified explanation: the speed of light is the speed of sound because the photon is a gauge excitation propagating on the same condensate whose acoustic properties define c. The identity $c_{\text{EM}} = c_{\text{grav}} = c_{\text{sound}}$ is not a coincidence but a structural consequence of both fields emerging from the same substrate.

Near a gravitating mass, the acoustic metric modifies the effective spacetime seen by all excitations equally. Electromagnetic waves are deflected, redshifted, and time-delayed by the Painlevé-Gullstrand flow, reproducing the standard general relativistic predictions. The Shapiro time delay, the gravitational redshift, and the bending of light are automatically consistent with $\gamma = 1$ because the photon (gauge excitation) and the phonon (density excitation) share the same acoustic metric.

7 The London Equation and Electromagnetic Screening

In a region where the condensate density is uniform ($\rho = \rho_0$) and the phase is constant ($\partial_\mu S = 0$), the gauge-invariant velocity (5) reduces to:

$$u_\mu = -\frac{e}{m_{\text{eff}}} A_\mu \,. \tag{20}$$

Substituting into the current (11):

$$J^\nu = -\frac{e^2 \rho_0}{m_{\text{eff}} \hbar_{\text{eff}}^2} A^\nu \,. \tag{21}$$

Combined with Maxwell's equation (10), this gives the Proca-like equation:

$$\Box A^\nu + \frac{1}{\lambda_L^2} A^\nu = 0 \,, \tag{22}$$

where the London penetration depth is:

$$\lambda_L = \frac{\hbar_{\text{eff}}}{e} \frac{1}{\sqrt{\rho_0}} \,. \tag{23}$$

Inside a region of uniform condensate with net charge density, electromagnetic fields are exponentially screened on the scale λ_L. This is the Meissner effect: the condensate expels electromagnetic fields from its bulk.

In the physical vacuum, however, the background condensate is electrically neutral. There is no net winding ($\langle n \rangle = 0$), and the macroscopic phase S is not locked to a constant—it varies spatially (producing the PG flow for gravity). The screening condition (20) does not apply globally, and electromagnetic waves propagate freely, as observed.

The London screening becomes significant only in the immediate vicinity of a vortex-Gausson core ($r \lesssim \xi$), where the condensate density drops to zero and the phase winds rapidly. The transition from the screened interior to the unscreened bulk produces the Coulomb field: the $1/r$ potential is a boundary effect between the vortex core (where A_μ is confined) and the free vacuum (where A_μ propagates).

8 Unified Picture: Gravity and Electromagnetism

The gauged logarithmic condensate produces two emergent interactions from a single Lagrangian:

Gravity arises from the acoustic metric $g_{\mu\nu}(\rho, u)$. Its source is the soliton energy density (mass). Its field equation is the Bernoulli relation and its consequence, the Painlevé-Gullstrand acoustic metric equivalent to Schwarzschild. Its coupling constant is $G = \eta c^2/(4\pi\rho_0)$.

Electromagnetism arises from the gauge field A_μ required by local $U(1)$ invariance. Its source is the topological winding number (charge). Its field equation is Maxwell's $\partial_\mu F^{\mu\nu} = J^\nu$. Its coupling constant is e.

The two interactions are coupled through the condensate:

- Charged particles (vortex-Gaussons) create both gravitational (acoustic metric) and electromagnetic (gauge) fields simultaneously.
- The electromagnetic field modifies the condensate flow through the London relation (20).
- The gravitational flow (PG inflow) modifies the effective spacetime for electromagnetic wave propagation—this is the gravitational deflection of light.

Despite this coupling, the two interactions have fundamentally different origins within the condensate. Gravity is universal (all solitons have energy, therefore mass), while electromagnetism is topological (only vortex solitons with $n \neq 0$ carry charge). This explains why gravity couples to all matter while electromagnetism couples only to charged matter—a distinction that is postulated in GR but derived here from the condensate structure.

9 Discussion

9.1 What is derived and what is assumed

The derivation rests on three inputs:

(i) The logarithmic wave equation (1), which defines the condensate dynamics.

(ii) The requirement of local $U(1)$ gauge invariance, which forces the introduction of A_μ.

(iii) The gauge field kinetic term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, which is the unique allowed term.

Of these, (i) is the foundation of the entire framework. Input (ii) is a symmetry principle—the same one used in the Standard Model to derive electromagnetism from the electron field's $U(1)$ symmetry [6]. Input (iii) is uniquely determined by Lorentz invariance, gauge invariance, and the requirement of at most two derivatives.

The coupling constant e is *not* determined by the condensate parameters. Deriving its value (equivalently, the fine structure constant $\alpha = e^2/(4\pi\epsilon_0\hbar c) \approx 1/137$) would require computing the self-energy of a vortex-Gausson in the gauged condensate and matching it to the observed electron mass. This remains an open problem.

9.2 Comparison with the Standard Model

In the Standard Model, electromagnetism arises from gauging the $U(1)_Y$ hypercharge symmetry of the fermion and Higgs fields. The photon is the massless gauge boson associated with the unbroken $U(1)_{\text{EM}}$ after electroweak symmetry breaking. The electron's charge is a coupling constant.

In the present framework, electromagnetism arises from gauging the $U(1)$ phase symmetry of the vacuum condensate. The photon is the massless gauge excitation propagating on the condensate. The electron's charge is the same coupling constant e, and charge quantization follows from vortex topology rather than from anomaly cancellation.

The key structural difference is that here, the condensate provides both the gravitational field (acoustic metric) and the electromagnetic field (gauge connection), whereas the Standard Model treats gravity as external.

9.3 Limitations

The framework produces only a $U(1)$ gauge field. The Standard Model requires $SU(3) \times SU(2) \times U(1)$. Extending to non-Abelian gauge groups would require a multi-component order parameter (e.g., a spinor or matrix-valued condensate), which is beyond the scope of the present single-component model.

The identification of electric charge with vortex winding number predicts that all charged particles have integer multiples of e. This is consistent with observation (quarks carry fractional charges, but they are confined, and all observable states carry integer charges). However, deriving quark confinement from the condensate would require the $SU(3)$ extension mentioned above.

9.4 Speculative application: planetary magnetic fields

The axial outflow derived in the gravitational sector [1] has a potential electromagnetic consequence within the present framework. The outflow along the rotation axis creates a preferred direction that may align the vortex axes of embedded solitons. A population of N vortex-Gaussons with correlated winding axes constitutes a net current loop. By Ampère's law (15), a current along an axis produces a dipole magnetic field. If this mechanism operates, the predicted scaling is:

$$|\mathbf{B}_{\text{dipole}}| \propto \Omega\, M\, f_{\text{liquid}}\,, \tag{24}$$

where Ω is the rotation rate (setting the outflow strength), M the body mass (setting the PG inflow strength), and f_{liquid} the fraction of the core that supports free vortex motion (liquid vs. solid).

This qualitatively explains the observed pattern of planetary magnetic fields:

Body	Ω	Core	B	Prediction
Jupiter	fast	metallic H	strongest	✓
Earth	moderate	liquid Fe	moderate	✓
Mercury	slow	partial liq.	weak	✓
Venus	very slow	liquid	≈ 0	✓
Mars	moderate	solid	≈ 0	✓

Venus fails despite having a liquid core because its rotation rate is too slow to generate significant axial outflow. Mars fails despite moderate rotation because its core has solidified, preventing vortex alignment. A quantitative treatment requires coupling the vacuum flow equations to magnetohydrodynamic core models and is deferred to future work.

10 Conclusion

We have shown that gauging the $U(1)$ phase symmetry of the logarithmic superfluid vacuum produces Maxwell's equations alongside the acoustic metric equations from a single Lagrangian. The Madelung decomposition of the gauged equation separates cleanly into:

(1) The gravitational sector: the acoustic metric $g_{\mu\nu}(\rho, u)$, governed by the Bernoulli relation and the Painlevé-Gullstrand flow, sourced by mass (soliton energy).

(2) The electromagnetic sector: the gauge field A_μ, governed by Maxwell's equations $\partial_\mu F^{\mu\nu} = J^\nu$, sourced by charge (vortex winding number).

Electric charge is quantized ($Q = ne$, $n \in \mathbb{Z}$) because vortex winding numbers are integers. Both fields propagate at the sound speed c, explaining the identity $c_{\text{EM}} = c_{\text{grav}}$ as a structural consequence rather than a coincidence. The framework predicts that axial outflow of the Painlevé-Gullstrand flow aligns embedded vortex-Gaussons, producing dipole magnetic fields correlated with rotation rate and core state—qualitatively consistent with the observed pattern of planetary magnetism.

The coupling constant e (the fine structure constant) is not determined by the condensate parameters and remains an open problem. Extension to non-Abelian gauge groups requires a multi-component condensate and is deferred to future work.

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