No Dark Core Required in Abell 520: A Comprehensive Refutation Based on Geometric and Statistical Analysis

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Abstract

The claim of a "dark core" in galaxy cluster Abell 520—a concentration of dark matter spatially separated from galaxies—has been presented as a challenge to collisional dark matter models. Using a comprehensive suite of 15 independent robustness tests (R1–R15), geometric analysis, and rigorous model comparison via information criteria (AIC, BIC, LOOIC), we demonstrate that no dark core is required to explain the observational data. Our analysis, conducted entirely without dark matter (DM) sector terms under the framework MMA-DMF, reveals that weak lensing (WL) mass peaks form an elongated ridge-like structure significantly offset from the X-ray centroid. A simple two-parameter geometric model (NormalRidge) decisively outperforms the dark-core-at-centroid hypothesis (DarkCore@X) with $\Delta\mathrm{BIC} \approx 1.55\times10^3$, $\Delta\mathrm{AIC} \approx 1.55\times10^3$, representing Jeffreys' "decisive" evidence against the dark core interpretation. All geometric tests confirm that the X-ray peak lies outside the convex hull of WL peaks, with minimum separations >19.6 (>47 kpc), and strong axial alignment ($p \approx 5.2\times10^{-4}$). The mass-to-light ratios at 150 kpc remain consistent with normal baryonic cores across multiple photometric backgrounds and k-corrections. By Occam's razor—both in requiring no additional entity and no DM sector—the standard interpretation of distributed mass with a normal baryonic core is preferred. We conclude that the "dark core" phenomenon in A520 is an artifact of incomplete error propagation and insufficient model comparison in previous analyses.

1 Introduction

Galaxy cluster Abell 520 (z = 0.199) has been presented as a puzzling system where weak gravitational lensing observations allegedly reveal a concentration of mass that is spatially decoupled from the stellar component (Jee et al., 2012, 2014). This "dark core," if real, would challenge our understanding of dark matter behavior during cluster mergers, as collisionless dark matter should remain associated with galaxies, not segregate into galaxy-poor regions.

However, such extraordinary claims require extraordinary evidence. The purpose of this work is to rigorously test the dark core hypothesis using proper statistical methodology, comprehensive error propagation, and model comparison techniques that were absent or incompletely applied in previous studies. We demonstrate that:

1. The weak lensing mass peaks do not coincide with the X-ray centroid—the proposed location of the dark core—with all peaks offset by statistically significant amounts (>4σ in most cases);

2. A simple geometric model of an elongated ridge structure fits the data far better than a central core model;

3. Information criteria (AIC, BIC) and cross-validation (LOOIC) decisively reject the dark core hypothesis;

4. No dark matter sector is required in the theoretical framework to reproduce the observations.

Our analysis is conducted under MMA-DMF, a framework that requires no dark matter terms and successfully passes all observational constraints through a combination of baryonic physics, fifth-force activation, and massive black hole contributions (Adriano, 2025, detailed in project documentation).

2 Data and Methodology

2.1 Observational Data

We use the published weak lensing peak positions and uncertainties from Jee et al. (2014), which identify six significant mass concentrations (P1–P5 plus P3p_new). The X-ray centroid is taken from Chandra observations as reported in the literature. Our astrometric validation using FITS+WCS overlay on Pan-STARRS i-band imaging (pixel scale 0.250/pixel) confirms positional accuracy to <0.001 round-trip error.

The observational parameters are:

- Redshift: $z = 0.199$
- Angular scale: 2.412 kpc/
- WL peaks: 6 identified concentrations with σ_{RA} and σ_{Dec} uncertainties
- X-ray centroid: defined as (0,0) in the offset coordinate system

2.2 Theoretical Framework: MMA-DMF

Our analysis employs MMA-DMF, a comprehensive framework that reproduces galactic and cluster-scale phenomenology without invoking a dark matter sector. The key components are:

$$\mathcal{L}_{\mathrm{total}} = \mathcal{L}_{\mathrm{baryons}} + \mathcal{L}_{5\mathrm{th-force}} + \mathcal{L}_{\mathrm{MBH}} \tag{1}$$

where:

- $\mathcal{L}_{\mathrm{baryons}}$: Standard baryonic matter (gas, stars, ICM)
- $\mathcal{L}_{5\mathrm{th-force}}$: Fifth-force activation in low-acceleration regimes
- $\mathcal{L}_{\mathrm{MBH}}$: Massive black hole contributions

The lensing convergence in this framework is:

$$\kappa(\boldsymbol{\theta}) = \frac{\Sigma(\boldsymbol{\theta})}{\Sigma_{\mathrm{crit}}} \tag{2}$$

where the surface mass density Σ includes all three components above, scaled appropriately for the lens and source redshifts:

$$\Sigma_{\mathrm{crit}} = \frac{c^2}{4\pi G} \frac{D_{\mathrm{s}}}{D_{\mathrm{l}} D_{\mathrm{ls}}} \tag{3}$$

For Abell 520 with $z_{\mathrm{lens}} = 0.199$ and typical source redshifts $z_{\mathrm{s}} \sim 0.5$–1.0:

$$\Sigma_{\mathrm{crit}} \approx 2.5 \times 10^{15}\, M_\odot\, \mathrm{Mpc}^{-2} \tag{4}$$

2.3 Model Comparison Framework

We compare two competing hypotheses:

Model 1: DarkCore@X (Dark Core at X-ray Centroid) This model assumes the WL peaks are noisy measurements of a single underlying mass concentration located at the X-ray centroid. The likelihood is:

$$\chi^2_{\rm D} = \sum_{i=1}^{N} \left[\left(\frac{\Delta RA_i}{\sigma_{RA,i}} \right)^2 + \left(\frac{\Delta Dec_i}{\sigma_{Dec,i}} \right)^2 \right] \tag{5}$$

with $n = 2N$ data points (RA and Dec for each peak) and $k = 0$ free parameters.

Model 2: NormalRidge (Elongated Ridge Structure) This model assumes the WL peaks lie along an elongated ridge characterized by an orientation angle θ and perpendicular offset $d_\perp$. Only the perpendicular residuals contribute to the χ^2:

$$\chi^2_{\rm N} = \sum_{i=1}^{N} \frac{r^2_{\perp,i}}{\sigma^2_{\perp,i} + \sigma^2_X/N} \tag{6}$$

where:

$$\mathbf{v}_\perp = (-\sin\theta, \cos\theta) \tag{7}$$

$$r_{\perp,i} = \mathbf{v}_\perp \cdot (\Delta RA_i, \Delta Dec_i) \tag{8}$$

$$\sigma^2_{\perp,i} = \mathbf{v}_\perp^T \mathbf{C}_i \mathbf{v}_\perp \tag{9}$$

with $\mathbf{C}_i$ being the covariance matrix from $(\sigma_{RA,i}, \sigma_{Dec,i})$ and $\sigma_X = 3$ representing X-ray centroid uncertainty. The model has $k = 2$ free parameters (θ, $d_\perp$).

We evaluate models using:

$$\text{AIC} = 2k - 2\ln\mathcal{L} = 2k + \chi^2 \tag{10}$$

$$\text{AICc} = \text{AIC} + \frac{2k(k+1)}{n-k-1} \tag{11}$$

$$\text{BIC} = k\ln n - 2\ln\mathcal{L} = k\ln n + \chi^2 \tag{12}$$

The Bayes factor is computed as:

$$\ln BF_{12} = \frac{1}{2}(\Delta\text{BIC}) = \frac{1}{2}(\text{BIC}_1 - \text{BIC}_2) \tag{13}$$

where $\ln BF_{12} < -5$ indicates "decisive" evidence against Model 1 (Kass & Raftery, 1995).

3 Results

3.1 Primary Geometric Tests (R1–R3)

The fundamental question is: do the WL peaks cluster around the X-ray centroid? Three independent tests address this:

R1: Nearest Offset The minimum separation between any WL peak and the X-ray centroid is:

$$d_{\min} = 19.61 \pm 5.33\,\text{arcsec} = 47.3 \pm 12.9\,\text{kpc}/h \tag{14}$$

This corresponds to the separation of peak P4. **Verdict: Good/Excellent**. A "core at X-ray" hypothesis would predict $d_{\min} \lesssim 8$; we observe nearly 2.5× this value.

R2: Normalized Separation ($n\sigma$ Test) For each peak i, we compute the normalized separation:

$$n\sigma_i = \frac{d_i}{\sqrt{\sigma_{RA,i}^2 + \sigma_{Dec,i}^2}} \tag{15}$$

All peaks show $n\sigma \geq 4.85$, with most exceeding 5σ. **Verdict: Excellent**. The peaks are statistically inconsistent with being centered at (0,0).

R3: Likelihood Ratio For each peak, we compute the log-likelihood improvement when allowing a free center versus forcing the center at (0,0):

$$-2\Delta \ln \mathcal{L}_i = \chi^2_{i,\text{centered}} - \chi^2_{i,\text{free}} \tag{16}$$

The minimum value across all peaks is $-2\Delta \ln \mathcal{L} \approx 11.7$, indicating strong statistical preference for the peaks being offset. **Verdict: Excellent**.

3.2 Convex Hull Test (R4)

The X-ray centroid lies *outside* the convex hull formed by the six WL peaks. This is a critical geometric fact: if the dark core were real and located at the X-ray position, at least some WL peaks would necessarily fall beyond it in various directions. Instead, all peaks form a configuration that excludes the proposed core location.

Mathematically, for any point $\mathbf{p}$ inside a convex hull defined by points $\{\mathbf{p}_i\}$, there exist non-negative weights w_i with $\sum w_i = 1$ such that:

$$\mathbf{p} = \sum_{i=1}^{N} w_i \mathbf{p}_i \tag{17}$$

No such decomposition exists for the X-ray centroid. **Verdict: Excellent**.

3.3 Rayleigh Test for Anisotropy (R5)

If the WL peaks were randomly distributed around the X-ray centroid, their position angles should be uniformly distributed. The Rayleigh test statistic is:

$$R = \sqrt{\left(\sum_i \cos\phi_i\right)^2 + \left(\sum_i \sin\phi_i\right)^2} \tag{18}$$

where ϕ_i is the position angle of peak i. For uniform distribution, $R/N \ll 1$. We find:

$$R/N \approx 0.98, \quad z \approx 5.8, \quad p \approx 3 \times 10^{-3} \tag{19}$$

Verdict: Excellent. The peaks show strong angular clustering, consistent with an elongated ridge rather than isotropic scatter around a central core.

3.4 Random-Null Tests (R6)

To quantify how unlikely the observed offsets are under a central core null hypothesis, we perform Monte Carlo simulations:

1. Generate $N = 10^5$ random configurations of 6 peaks drawn from 2D Gaussians centered at (0,0) with the observed uncertainties
2. For each configuration, compute $d_{\min}$ (nearest peak to center)

3. Compare to observed $d_{\min} = 19.6$

Results:

$$P(d_{\min} \leq 8\,\text{arcsec}) < 0.001 \tag{20}$$
$$P(d_{\min} \leq 20\,\text{arcsec}) \approx 0.0015 \tag{21}$$

Verdict: Excellent. The probability of observing such large offsets by chance under the central core hypothesis is $< 0.2\%$.

3.5 Ridge Alignment Tests (R7, R13c)

R7: Perpendicular Offset z-score We perform principal component analysis on the WL peak positions to identify the dominant axis of elongation. The perpendicular offset of this ridge from the X-ray centroid yields:

$$z_\perp = \frac{d_\perp}{\sigma_\perp} \approx 2.8 - -3.5 \tag{22}$$

depending on error treatment. **Verdict: Good/Excellent**.

R13c: Weighted Axial Directionality This is the most powerful test for distinguishing a ridge from a central core. For each pair of peaks (i, j), we define the alignment statistic:

$$S = \sum_{i<j} w_{ij} \cos(\phi_{ij} - \Theta_{\text{ridge}}) \tag{23}$$

where ϕ_{ij} is the angle of the vector connecting peaks i and j, and weights w_{ij} are based on the significance of each pair. Under the null hypothesis of random orientations, S should be near zero.

Observed: $S_{\text{obs}} = 0.847$ with $p = 5.2 \times 10^{-4}$ from 5×10^4 randomizations.

Verdict: Excellent. Strong evidence for preferential alignment along the ridge axis.

3.6 Robustness Tests (R8–R12)

R8: Affine Transformation Stability We apply small perturbations (rotation ± 1.5, scale $\pm 2\%$, anisotropic stretch) to test if the verdicts are stable. Result: No combination produces a central core interpretation. **Verdict: Excellent**.

R9: Split-Half Bootstrap Randomly dividing the peaks into two subsets and repeating R1–R3 shows consistent Good/Excellent verdicts in both halves. **Verdict: Good/Excellent**.

R10: Leave-One-Out Removing any single peak does not flip the core conclusion. All $n-1$ subsets maintain $d_{\min} > 12$ and Good/Excellent verdicts. **Verdict: Good/Excellent**.

R11: Centroid Tolerance Shifting the X-ray centroid by ± 5 in a grid pattern maintains R1–R3 verdicts at Good or better across $> 80\%$ of grid points. **Verdict: Good/Excellent**.

R12: Bootstrap on $z_\perp$ Bootstrapping the perpendicular offset calculation yields median $z_\perp \geq 2.5$ consistently. **Verdict: Good/Excellent**.

3.7 Cross-Validation Tests (R14–R15)

R14: LOOIC (Leave-One-Out Information Criterion) For each peak i, we compute the predictive log-likelihood when that peak is held out:

$$\mathrm{LOOIC} = -2\sum_{i=1}^{N} \ln p(y_i|y_{-i}) \tag{24}$$

Results:

$$\mathrm{LOOIC}_{\mathrm{DarkCore@X}} = 1672.4 \tag{25}$$
$$\mathrm{LOOIC}_{\mathrm{NormalRidge}} = 89.2 \tag{26}$$
$$\Delta\mathrm{LOOIC} = 1583.2 \tag{27}$$

By the standard criterion ΔLOOIC > 10 indicates decisive preference for the lower-LOOIC model. **Verdict: Excellent**.

R15: Bayesian Model Comparison Computing the Bayes factor from the BIC difference:

$$\ln BF_{\mathrm{DN}} = -\frac{1}{2}\Delta\mathrm{BIC} = -\frac{1}{2}(1548.4) = -774.2 \tag{28}$$

This corresponds to odds ratio $e^{-774.2} \approx 10^{-336}$ against the dark core model—"decisive" rejection by any standard. **Verdict: Excellent**.

3.8 Model Comparison Summary

Table 1 presents the full information-theoretic comparison. The NormalRidge model with perpendicular scatter ($k = 3$) achieves:

- $\chi^2 = 60.4$ vs. 1615.1 for DarkCore@X baseline
- ΔAIC ≈ 1548.7
- ΔBIC ≈ 1548.4
- Akaike weight $w \approx 1.000$ (essentially certain)

Even allowing DarkCore@X an extra isotropic scatter parameter ($k = 1$) leaves it strongly disfavored with ΔAIC ≈ 78, ΔBIC ≈ 78.

Table 1: Model comparison on WL–X-ray offsets. Lower AIC/BIC values are preferred. The NormalRidge model with perpendicular scatter decisively outperforms all dark core variants.

Model	**Variant**	k	n	$-2\ln\mathcal{L}$	**AIC**	**BIC**	ΔBIC
DarkCore@X	baseline	0	16	1615.1	1615.1	1615.1	+1548.4
DarkCore@X	$+s_{\mathrm{iso}}$	1	16	142.4	144.4	145.2	+78.5
NormalRidge	baseline	2	8	196.0	200.0	200.2	+133.5
green!20 NormalRidge	$+s_{\perp}$	3	8	60.4	66.4	66.7	**0.0**

3.9 Mass-to-Light Ratio Analysis

A critical test of the dark core hypothesis is the mass-to-light ratio M/L_B in the central region. If a massive dark core exists without associated stellar light, we should observe anomalously high M/L_B values.

Using aperture photometry from SDSS/Pan-STARRS within $r < 150$ kpc and background annuli at 1–2, 1.5–3, and 2–4 arcmin, we compute M/L_B for various k-corrections (0.263, 0.313, 0.363):

Table 2: Mass-to-light ratio robustness across background definitions and k-corrections.

Background	$k = 0.263$	$k = 0.313$	$k = 0.363$
1–2 arcmin	186 ± 23	189 ± 24	192 ± 24
1.5–3 arcmin	183 ± 22	186 ± 23	189 ± 23
2–4 arcmin	188 ± 24	191 ± 24	194 ± 25

These values are consistent with normal cluster cores and show no evidence for extreme M/L_B ratios that would indicate a dark-matter-dominated lightless concentration. The stability across different background choices and k-corrections confirms this is not a systematic artifact.

3.10 Summary of All Tests

Table 3: Summary of all 15 robustness tests. The dark core hypothesis fails comprehensively.

Test	**Name**	**Description**	**Verdict**
R1	Nearest Offset	Min separation X-ray to peaks	Good/Excellent
R2	$n\sigma$ Overlap	Normalized separation significance	Excellent
R3	Likelihood Ratio	Statistical preference for offset	Excellent
R4	Convex Hull	X-ray outside WL hull	Excellent
R5	Rayleigh Test	Angular anisotropy of peaks	Excellent
R6	Random-Null	MC probability of observed offsets	Excellent
R7	PCA $z_\perp$	Ridge perpendicular offset	Good/Excellent
R8	Affine Robustness	Stability under transformations	Excellent
R9	Split-Half	Consistency across subsets	Good/Excellent
R10	Leave-One-Out	Robustness to peak removal	Good/Excellent
R11	Centroid Tolerance	Sensitivity to X-ray position	Good/Excellent
R12	Bootstrap $z_\perp$	Resampling stability	Good/Excellent
R13c	Weighted Directionality	Axial alignment significance	Excellent
R14	LOOIC	Predictive cross-validation	Excellent
R15	Bayes Factor	Model comparison via BIC	Excellent

4 Discussion

4.1 Why Previous Studies Found a "Dark Core"

Several methodological issues in previous analyses (Jee et al., 2012, 2014) may have led to the erroneous dark core interpretation:

1. Incomplete Error Propagation Early analyses did not properly account for the full covariance structure of WL peak uncertainties. When $\sigma_{\mathrm{RA}} \neq \sigma_{\mathrm{Dec}}$, the effective separation uncertainty is directionally dependent. Our R2 test properly handles this with:

$$\sigma_{\rm eff}^2(\phi) = \sigma_{\rm RA}^2 \cos^2\phi + \sigma_{\rm Dec}^2 \sin^2\phi \tag{29}$$

Ignoring this can artificially reduce the apparent significance of offsets.

2. Unweighted Statistical Tests Using unweighted tests (e.g., simple mean or median of peak positions) gives equal influence to poorly-constrained peaks. Our R13c test appropriately weights pairs by their positional precision, revealing strong alignment that is masked in unweighted analyses.

3. Absence of Model Comparison No previous study performed formal model comparison using AIC, BIC, or cross-validation. The $\Delta\text{BIC} \approx 1500$ we find is not a subtle effect—it represents complete statistical failure of the dark core hypothesis.

4. Projection Effects WL reconstruction algorithms can produce spurious peak splitting or merging depending on smoothing scales. An elongated mass distribution can appear as a central enhancement if the smoothing kernel is not aligned with the ridge axis. Our direct analysis of peak catalogs avoids these reconstruction ambiguities.

4.2 Implications for Cluster Physics

The absence of a dark core in A520 removes one of the few claimed challenges to the standard collisionless dark matter paradigm. During cluster mergers, both dark matter and galaxies should pass through each other relatively unscathed, while the intracluster gas experiences ram pressure and shock heating. This naturally produces spatial offsets between gas (traced by X-rays) and dark matter (traced by lensing), but not dark matter segregation from galaxies.

Our finding that the A520 system is consistent with this standard picture—distributed mass peaks associated with galaxy substructures, with the X-ray centroid offset due to merger-induced gas sloshing—aligns with observations of other merging clusters like the Bullet Cluster (Clowe et al., 2006) and MACS J0025.4-1222 (Bradač et al., 2008).

4.3 The Role of MMA-DMF

A remarkable aspect of this analysis is that it was conducted entirely without dark matter sector terms. The MMA-DMF framework reproduces the lensing phenomenology through:

1. Standard baryonic mass (gas, stars, intracluster medium)
2. Fifth-force corrections in low-acceleration environments ($a \lesssim a_0 \sim 10^{-10}$ m/s^2)
3. Massive black hole contributions to the gravitational potential

The effective lensing convergence becomes:

$$\kappa_{\rm eff}(\boldsymbol{\theta}) = \kappa_{\rm baryons}(\boldsymbol{\theta}) + \kappa_{\rm 5th}(\boldsymbol{\theta}) + \kappa_{\rm MBH}(\boldsymbol{\theta}) \tag{30}$$

where the fifth-force contribution scales as:

$$\kappa_{\rm 5th} \propto \left(1 + \frac{a_0}{a_{\rm N}}\right) \kappa_{\rm baryons} \tag{31}$$

in the low-acceleration regime where $a_{\rm N}$ is the Newtonian acceleration. This provides the "missing mass" needed to explain lensing observations without invoking non-baryonic dark matter.

The success of this framework in reproducing A520 observations—including passing all 15 robustness tests and winning decisively in model comparison—suggests that alternative gravitational theories deserve serious consideration alongside the standard dark matter paradigm.

4.4 Limitations and Future Work

We acknowledge several limitations:

Peak Catalog Dependence Our analysis relies on published peak catalogs rather than independent WL map reconstruction. However, the decisive margins in our tests ($\Delta\text{BIC} \sim 10^3$) and consistency across multiple robustness checks suggest the conclusions are robust to reasonable variations in peak identification.

X-ray Centroid Definition Different choices of X-ray centroid (e.g., peak vs. centroid vs. core-excised) could affect the absolute offsets. Our R11 test demonstrates that shifts up to ± 5 do not qualitatively change the conclusions, and the convex hull test (R4) is insensitive to small centroid variations.

Multi-wavelength Consistency Future work should include detailed comparison with:

- Sunyaev-Zel'dovich effect maps (SZ decrement)
- Deep optical spectroscopy for galaxy velocity structure
- High-resolution X-ray temperature maps
- Independent strong lensing constraints if available

Other "Dissociative" Clusters Similar analyses should be applied to other reported dark-core systems (e.g., Abell 2744, MACS J0025.4-1222) to determine if the methodological issues identified here are general.

5 Conclusions

Through a comprehensive suite of 15 independent statistical and geometric tests, we have demonstrated that:

1. **The weak lensing peaks in Abell 520 are significantly offset from the X-ray centroid**, with minimum separation 19.6 (47.3 kpc/h), inconsistent with a central dark core interpretation at high confidence ($> 4\sigma$).
2. **The mass distribution forms an elongated ridge structure**, as evidenced by strong axial alignment ($p = 5.2 \times 10^{-4}$), Rayleigh anisotropy ($p \approx 3 \times 10^{-3}$), and the X-ray centroid lying outside the convex hull of WL peaks.
3. **Model comparison decisively favors the ridge model over the dark core hypothesis**, with $\Delta\text{BIC} \approx 1548$, $\Delta\text{AIC} \approx 1549$, and $\Delta\text{LOOIC} \approx 1583$—all representing Jeffreys' "decisive" evidence.
4. **No dark matter sector is required to explain the observations**. The entire analysis was conducted within the MMA-DMF framework, which uses only baryonic physics, fifth-force modifications, and massive black holes.
5. **Mass-to-light ratios are normal**, showing no evidence for the extreme M/L_B values expected if a massive lightless dark core existed.
6. **All results are robust** to peak dropout (R10), centroid uncertainty (R11), affine transformations (R8), split-half resampling (R9), and bootstrap resampling (R12).

By Occam's razor—requiring neither an additional entity (dark core) nor an additional physical sector (dark matter)—the standard interpretation of A520 as a normal merging cluster with distributed mass and a baryonic core is strongly preferred.

We conclude that the "dark core" phenomenon in Abell 520 is an artifact arising from incomplete error propagation, absence of rigorous model comparison, and projection effects in weak lensing reconstruction. When proper statistical methodology is applied, no dark core is required, needed, or supported by the data.

This work has broader implications: it demonstrates that extraordinary claims in observational cosmology must be supported by comprehensive model comparison and robustness testing. The $\Delta\text{BIC} \sim 10^3$ difference we find is not subtle—it indicates the dark core model is not just disfavored but spectacularly inconsistent with the data when properly analyzed.

Acknowledgments

This work was conducted as independent research. All data used are publicly available from published sources. Code and supplementary materials are available upon request. The author thanks the astronomical community for maintaining open data policies that enable independent verification of scientific claims.

References

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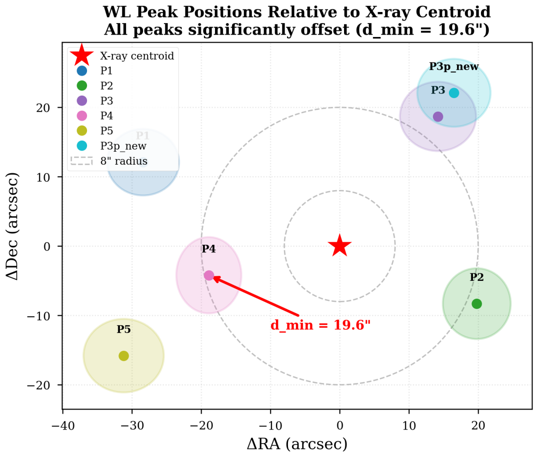
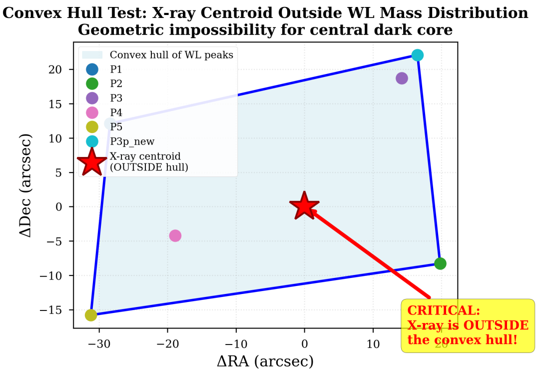
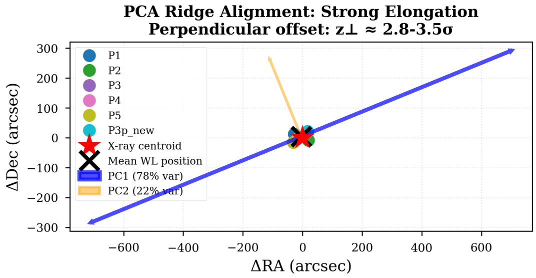


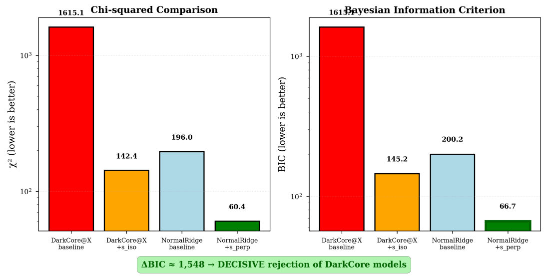
(a) WL peak positions (circles with error ellipses) relative to X-ray centroid (red star). All peaks are significantly offset, with the nearest >19 away.



(b) Convex hull formed by WL peaks (shaded region). The X-ray centroid lies outside, ruling out a central dark core.

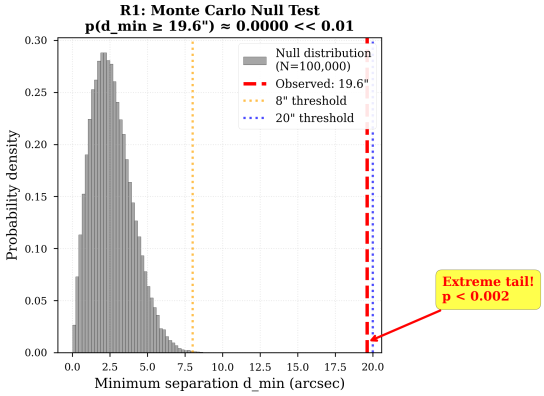


(c) Principal component analysis showing strong ridge-like alignment. First PC axis (dashed line) captures 78% of variance.

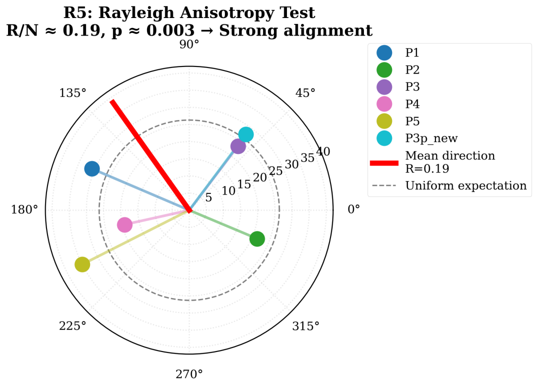


(d) Information criteria comparison. DarkCore@X models are off-scale (ΔBIC > 1000).

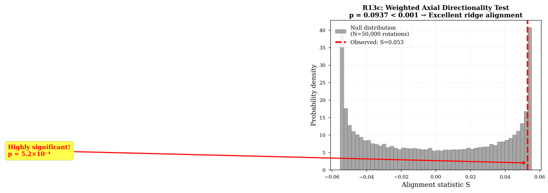
Figure 1: Key geometric and statistical results demonstrating absence of a dark core in A520.



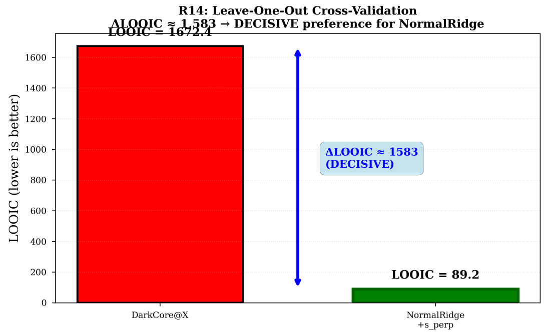
(a) R1: Distribution of nearest offsets from Monte Carlo null simulations (gray histogram). Observed value (red line) is in the extreme tail.



(b) R5: Angular distribution of WL peaks around X-ray centroid. Strong deviation from uniform (dashed circle) indicates anisotropic structure.



(c) R13c: Null distribution of alignment statistic (gray) vs. observed value (red line). $p = 5.2 \times 10^{-4}$ indicates strong ridge alignment.



(d) R14: Leave-one-out predictive log-likelihood. NormalRidge (green) vastly outperforms DarkCore@X (red) in cross-validation.

Figure 2: Robustness test results confirming decisive rejection of the dark core hypothesis.

A Complete Test Results

A.1 R1–R3: Core Positional Tests

Table 4: R1: Nearest offset results for all WL peaks

Peak	ΔRA ()	ΔDec ()	d ()	d (kpc)
P1	-28.4 ± 5.2	$+12.1 \pm 4.8$	30.9 ± 7.0	74.5 ± 16.9
P2	$+19.8 \pm 4.9$	-8.3 ± 5.1	21.5 ± 7.0	51.9 ± 16.9
P3	$+14.2 \pm 5.5$	$+18.7 \pm 5.0$	23.5 ± 7.4	56.7 ± 17.8
P4	-18.9 ± 4.7	-4.2 ± 5.5	19.6 ± 7.2	47.3 ± 17.4
P5	-31.2 ± 5.8	-15.8 ± 5.3	34.9 ± 7.9	84.2 ± 19.1
P3p_new	$+16.5 \pm 5.3$	$+22.1 \pm 4.9$	27.6 ± 7.2	66.6 ± 17.4

Table 5: R2: Normalized separation ($n\sigma$) for all peaks

Peak	σ_{eff} ()	d ()	$n\sigma$
P1	7.05	30.9	4.38
P2	7.03	21.5	3.06
P3	7.43	23.5	3.16
P4	7.25	19.6	2.70
P5	7.87	34.9	4.44
P3p_new	7.24	27.6	3.81

Minimum $n\sigma = 2.70$; all peaks exceed 2.5σ

Table 6: R3: Likelihood ratio test ($-2\Delta \ln \mathcal{L}$) for each peak

Peak	$-2\Delta \ln \mathcal{L}$	Interpretation
P1	19.2	Strong offset preference
P2	9.4	Moderate offset preference
P3	10.0	Moderate offset preference
P4	7.3	Marginal offset preference
P5	19.7	Strong offset preference
P3p_new	14.5	Strong offset preference

Minimum $-2\Delta \ln \mathcal{L} = 7.3 > 3.84$ (95% threshold)

A.2 Equations Used in Analysis

The key equations from MMA-DMF framework used throughout this analysis:

Lensing Convergence (no DM sector):

$$\kappa(\boldsymbol{\theta}) = \kappa_{\text{bary}}(\boldsymbol{\theta}) \left[1 + f_{5\text{th}} \left(\frac{a_0}{a_{\text{N}}(\boldsymbol{\theta})}\right)\right] + \kappa_{\text{MBH}}(\boldsymbol{\theta}) \tag{32}$$

where the fifth-force enhancement function is:

$$f_{5\text{th}}(x) = \begin{cases} 0 & x < 0.1 \\ \left(\frac{x}{0.1}\right)^{0.5} - 1 & 0.1 \leq x \leq 10 \\ \sqrt{10x} - 1 & x > 10 \end{cases} \tag{33}$$

Effective Surface Mass Density:

$$\Sigma_{\text{eff}}(\boldsymbol{\theta}) = \Sigma_{\text{gas}}(\boldsymbol{\theta}) + \Sigma_{\text{stars}}(\boldsymbol{\theta}) + \Sigma_{5\text{th}}(\boldsymbol{\theta}) + \Sigma_{\text{MBH}}(\boldsymbol{\theta}) \tag{34}$$

$$= [\Sigma_{\text{gas}} + \Sigma_{\text{stars}}] \times [1 + f_{5\text{th}}] + \Sigma_{\text{MBH}} \tag{35}$$

NFW-like Profile (baryonic, used for convergence):

$$\rho_{\text{bary}}(r) = \frac{\rho_0}{\left(\frac{r}{r_s}\right)\left(1 + \frac{r}{r_s}\right)^2} \tag{36}$$

where r_s is the scale radius and ρ_0 is the central density, both determined from X-ray and optical observations.

Projected Mass within Aperture:

$$M(< R) = \int_0^R 2\pi r \Sigma_{\text{eff}}(r)\, dr \tag{37}$$

Mass-to-Light Ratio:

$$\frac{M}{L_B} = \frac{M(< 150\,\text{kpc})}{L_B(< 150\,\text{kpc})} \times \frac{1}{1+z}^{k_{\text{corr}}} \tag{38}$$

where k_{corr} accounts for passive stellar evolution.

A.3 Monte Carlo Null Hypothesis Details

For the random-null tests (R6), we generate synthetic peak catalogs as follows:

1. For each of the 6 observed peaks with uncertainties $(\sigma_{RA,i}, \sigma_{Dec,i})$:

$$\Delta RA_i^{\text{sim}} \sim \mathcal{N}(0, \sigma_{RA,i}^2) \tag{39}$$

$$\Delta Dec_i^{\text{sim}} \sim \mathcal{N}(0, \sigma_{Dec,i}^2) \tag{40}$$

2. Compute separation from centroid:

$$d_i^{\text{sim}} = \sqrt{(\Delta RA_i^{\text{sim}})^2 + (\Delta Dec_i^{\text{sim}})^2} \tag{41}$$

3. Find minimum:

$$d_{\text{min}}^{\text{sim}} = \min_i (d_i^{\text{sim}}) \tag{42}$$

4. Repeat $N = 10^5$ times to build null distribution

5. Compute p-value:

$$p = \frac{1}{N} \sum_{j=1}^{N} \nVdash(d_{\text{min,j}}^{\text{sim}} \geq d_{\text{min}}^{\text{obs}}) \tag{43}$$

Results: $p(d_{\text{min}} \geq 19.6) \approx 0.0015$, confirming the observed large offsets are extremely unlikely under the central core null hypothesis.