# Asymmetric Inertial Forcing with Phase-Locked Counter-Rotating Rotors: Distinguishing Ground-Reaction Motion from Free-Space Payload Push

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***Abstract*—This work analyzes an inertial actuation concept using phase-locked, counter-rotating rotor sets that produce time-asymmetric internal force waveforms. A distinction is made between two operational modes: (i) ground-reaction motion, where net external impulse is obtained via stick–slip rectification at contact patches; and (ii) free-space payload push, where net external impulse is delivered to a separate mass via a phase-selective couple/decouple interface. The device does not self-accelerate in free space without a reaction path to the mass it is pushing. A Hann-shaped force model yields the per-cycle impulse $I_{\text{cycle}} = (1-\alpha)F_{\text{peak}}T/4$. Conservation statements bounding the isolated system to zero net impulse are given, and timed coupling is shown to transfer constructive-window impulse to an external payload while isolating the destructive window. Energy accounting, expected output acceleration, timing requirements, and prototype plans are provided.**

***Index Terms*—inertial actuation, phase gating, couple/decouple, stick–slip, asymmetric forcing, conservation of momentum**

## I. Introduction

Net change of linear momentum requires forces transmitted to an external medium or mass. This work distinguishes two operational modes for an asymmetric inertial forcing concept:

- Ground-reaction motion: a wheeled platform gains net momentum via tribological rectification (stick during constructive lobes, slip during destructive lobes).

- Free-space payload push: the platform transfers net impulse to a separate payload via a phase-selective couple/decouple interface engaged during constructive windows and disengaged prior to destructive windows.

No claim is made for self-acceleration of an isolated, continuously coupled platform; conservation of momentum constrains such a system to zero net external impulse over each full cycle.

## II. System parameters and centripetal forces

Baseline parameters are used throughout:

- Frame mass: $m_{\text{frame}} = 50\,\text{kg}$.
- Two counter-rotating rotor sets mounted on the frame to suppress net reaction torque.
- Per rotor set A (point-mass): $m_1 = 0.908\,\text{kg},\ r_1 = 0.10\,\text{m},\ f_1 = 10\,\text{Hz}$.
- Per rotor set B (point-mass): $m_2 = 1.816\,\text{kg},\ r_2 = 0.20\,\text{m},\ f_2 = 5\,\text{Hz}$.

Angular speeds are $\omega_i = 2\pi f_i$. Tangential speeds are matched: $v = 2\pi r f \approx 6.283\,\text{m}\,\text{s}^{-1}$.

For a concentrated mass at radius $r$, centripetal force is

$$F = m\,\omega^2 r.$$

Evaluated:

$$F_{\text{fast}} = m_1(2\pi f_1)^2 r_1 \approx 358\,\text{N}, \tag{1}$$

$$F_{\text{slow}} = m_2(2\pi f_2)^2 r_2 \approx 358\,\text{N}. \tag{2}$$

The aligned constructive peak per rotor set is defined as

$$F_{\text{peak}} \equiv F_{\text{fast}} + F_{\text{slow}} \approx 716\,\text{N}.$$

## III. Force waveform and impulse per cycle

Let $T = 1/f_2 = 0.2\,\text{s}$ denote the slow-rotor period. Constructive and destructive half-cycles are modeled using Hann-shaped lobes.

Constructive half-cycle, $0 \le t < T/2$:

$$F_c(t) = F_{\text{peak}} \sin^2\!\Big(\frac{\pi t}{T}\Big).$$

Destructive half-cycle, $T/2 \le t < T$:

$$F_d(t) = -\,\alpha\,F_{\text{peak}} \sin^2\!\Big(\frac{\pi(t-T/2)}{T}\Big),$$

where $0 < \alpha < 1$ denotes destructive-phase attenuation (due to slip or isolation). Using $\sin^2 x = \frac{1-\cos(2x)}{2}$,

$$\int_0^{T/2} \sin^2\!\Big(\frac{\pi t}{T}\Big)\,dt = \frac{T}{4}.$$

Therefore

$$I_{\text{constructive}} = F_{\text{peak}}\frac{T}{4}, \qquad I_{\text{destructive}} = \alpha\,F_{\text{peak}}\frac{T}{4},$$

and the net per-cycle impulse is

$$I_{\text{cycle}} = (1-\alpha)\,F_{\text{peak}}\frac{T}{4}.$$

### A. Vector-sum justification for 59%/41%

Instantaneous force vectors are summed over each half-period and integrated to produce scalar impulses for constructive and destructive windows. If the destructive integral equals $0.41$ of the constructive integral ($\alpha = 0.41$), then the destructive lobe cancels 41% of the constructive impulse and 59% remains as the net per-cycle impulse:

$$I_{\text{cycle}} = (1 - 0.41)\, F_{\text{peak}} \frac{T}{4} = 0.59\, F_{\text{peak}} \frac{T}{4}.$$

## IV. Expected output acceleration and numerical example

Per-cycle velocity increment for the platform is

$$\Delta v = \frac{I_{\text{cycle}}}{m_{\text{frame}}}.$$

Average acceleration at cycle frequency $f_{\text{cycle}} = 1/T$ is

$$\bar{a} = \frac{I_{\text{cycle}}\, f_{\text{cycle}}}{m_{\text{frame}}}.$$

With nominal values $F_{\text{peak}} \approx 716\,\text{N}$, $T = 0.2\,\text{s}$, $\alpha = 0.41$, and $m_{\text{frame}} = 50\,\text{kg}$,

$$I_{\text{cycle}} \approx 20.9\,\text{N}\,\text{s}, \quad \Delta v \approx \frac{20.9}{50} \approx 0.418\,\text{m}\,\text{s}^{-1},$$

and at $f_{\text{cycle}} = 5$ Hz,

$$\bar{a} \approx \frac{20.9 \times 5}{50} \approx 2.09\,\text{m}\,\text{s}^{-2}.$$

Sensitivity: $\Delta v$ and $\bar{a}$ scale approximately with $(1 - \alpha)$ and with net transmission efficiency; ranges should be reported for ±20% variation in $\alpha$ and expected loss factors.

## V. Conservation statements

For an isolated platform with only internal forces $\{F_i(t)\}$,

$$\sum_i F_i(t) = 0 \quad \Rightarrow \quad \int_0^T \sum_i F_i(t)\, dt = 0,$$

and therefore $I_{\text{net,platform-only}} = 0$. Net external impulse requires exchange across an interface (ground or payload).

## VI. Use mode A: ground-reaction motion (stick–slip)

Let $N$ denote normal force and $\mu_s, \mu_k$ the static and kinetic friction coefficients with $\mu_s > \mu_k$. A rectification model at the contact patch is

$$F_{\text{trans}}(t) = \begin{cases} F(t), & |F(t)| \le \mu_s N \quad \text{(stick)} \\ \text{sgn}(F(t))\, \mu_k N, & |F(t)| > \mu_s N \quad \text{(slip)} \end{cases}$$

Design objectives for ground demonstrations are to keep constructive lobes within the static-friction limit and to allow destructive lobes to enter kinetic-friction-limited transmission so that

$$I_{\text{cycle,ground}} = \int_0^T F_{\text{trans}}(t)\, dt \approx (1 - \alpha) F_{\text{peak}} \frac{T}{4}.$$

## VII. Use mode B: free-space payload push (couple/decouple)

In free space, a reaction path to another mass is required for net impulse transfer. Let $m_p$ denote the platform mass and $m_L$ the payload mass. The couple/decouple interface is engaged on $[t_e - \Delta, t_e + \Delta]$ centered on the constructive peak.

Payload impulse during engagement is bounded by

$$I_L = \int_{t_e - \Delta}^{t_e + \Delta} F_{\text{eng}}(t)\, dt \le I_{\text{constructive}} = F_{\text{peak}} \frac{T}{4}.$$

Action and reaction imply

$$\Delta p_L = +I_L, \qquad \Delta p_p = -I_L, \qquad \Delta p_L + \Delta p_p = 0.$$

Resulting velocity increments after one engaged window are

$$\Delta v_L = \frac{I_L}{m_L}, \qquad \Delta v_p = -\frac{I_L}{m_p}.$$

Timing selection is specified by $t_e = t_{\text{peak}} - \tau$ to compensate actuator latency $\tau$. Recommended control targets are timing repeatability $\pm 1\,\text{ms}$ and a control loop rate $\ge 1\,\text{kHz}$.

## VIII. Energy accounting

Rotational energy for a concentrated mass at radius $r$ is

$$E_{\text{rot}} = \tfrac{1}{2} m r^2 \omega^2.$$

Using the exemplar parameters, one rotor set stores approximately $44\,\text{J}$ and two sets $\approx 89\,\text{J}$. To sustain $I_{\text{cycle}} f_2 \approx 105\,\text{N}\,\text{s}\,\text{s}^{-1}$ requires on the order of $212\,\text{W}$ ignoring losses; bearing friction, windage, tire hysteresis, and interface actuation increase required input power.

## IX. Prototype plan and validation

Ground test: a four-wheel cart with adjustable tire pressure and compliance; instrumentation: IMU $\ge 1\,\text{kHz}$, wheel encoders, under-wheel force sensors. Protocols: phase sweeps, locked-rotor baseline, symmetric-duty baseline, randomized-phase control, and dummy-mass runs. Free-space test: low-friction sled or air-bearing table with a fast couple/decouple interface (magnetic keeper, kinematic latch, or voice-coil latch); measure $I_L$, platform counter-impulse, timing repeatability, and perform full energy audits.

## X. Discussion and limitations

All net external effects require an explicit reaction path: ground contact or a separate payload. The isolated, continuously coupled platform cannot self-accelerate in free space. Numerical values are idealized; experimental validation is required to quantify losses, sensitivity to $\alpha$, timing jitter, actuator limits, and friction variability.

## XI. Conclusion

A clear operational distinction has been presented between ground-reaction motion (stick–slip rectification) and free-space payload push (timed couple/decouple). Net external impulse is realized only via an external reaction path; timed coupling enables impulse transfer to a payload while remaining consistent with conservation laws.

## References


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