

Ultra-High Fidelity Floquet Time Crystals in a $Cl(0,10)^{90}$ Clifford Subspace

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I report a Floquet discrete time crystal (DTC) in a 30×30 spin lattice within a $Cl_{0,10}^{90}$ Clifford subspace, achieving fidelities of 0.999671 at 30 periods and 0.999018 at 50 periods with $\theta_{\max} = \pi/512$ in an optimized run ($J = 1.0$). From a Néel state under $h(t) = h_0 + h_1 \cos(\omega t/2 + \pi/4) + h_{1,\text{fb}}$ ($\omega = 2$), with feedback-tuned h_1 (base 0.568 or 2.5, $sgain = 1900$) and $J = 1.0$ or 4.4, period doubling persists to 50T. A spiral twist ($iJ \sin(\theta_{ij}) \sigma_i^z \sigma_j^z$) stabilizes the phase, yielding real $\langle H_{zz} \rangle$ (e.g., -0.850054 at 30T, optimized). Fidelities surpass prior clean DTCs (0.5 at 30T [3]), enhancing quantum simulation potential. Stabilizers, however, are unexpectedly low (0.0011074 at 30T, 0.000751738 at 1090T, optimized) compared to prior reports (0.9997 to 0.672708), suggesting integrable dynamics without many-body localization (MBL) spectral pairing, contrasting with MBL DTCs (0.9 [1, 11]).

INTRODUCTION

Discrete time crystals (DTCs) break time-translation symmetry in Floquet systems ($U(T)^2 \approx I$) [1, 2]. In clean systems, fidelity typically decays to ~ 0.5 by 30 periods due to heating [3], often requiring disorder [4]. I present a clean DTC in a 30×30 lattice, evolved in a $Cl_{0,10}^{90}$ Clifford subspace, achieving > 0.999 fidelity at 30T and 50T with $\theta_{\max} = \pi/512$ in an optimized configuration [5]. Feedback and a spiral twist exceed prior clean DTCs (e.g., 0.5 at 30T in 2D theory [3]), leveraging 2D scalability.

MODEL AND METHODS

A 30×30 lattice (900 spins, periodic boundaries) uses:

$$H(t) = iJ \sum_{\langle ij \rangle} \sin(\theta_{ij}) \sigma_i^z \sigma_j^z - h(t) \sum_i \sigma_i^x, \quad (1)$$

with $J = 4.4$ (unoptimized) or $J = 1.0$ (optimized), $h(t) = h_0 + h_1 \cos(\omega t/2 + \pi/4) + h_{1,\text{fb}}$, base $h_1 = 2.5$ (or $2.5/4.4 \approx 0.568$ for $J = 1.0$), $h_0 = 0$, and $\omega = 2.0$ ($T = \pi$). The Hamiltonian features an imaginary spiral twist $iJ \sum_{\langle ij \rangle} \sin(\theta_{ij}) \sigma_i^z \sigma_j^z$, where J scales the twist amplitude applied solely to the imaginary component, omitting the conventional real $-J \sum \sigma_i^z \sigma_j^z$ term. Dynamics evolve in a $Cl_{0,10}^{90}$ Clifford subspace, reducing the 2^{900} -dimensional space to $\sim 10^{10}$ states via projections onto a 10D Clifford algebra subset preserving $\sigma^z \sigma^z$ correlations [9]. The $Cl_{0,10}^{90}$ subspace exploits Néel state symmetry, aligning with the initial antiferromagnetic order to enable efficient simulation [9]. The subspace adapts to system size (e.g., a 40×40 lattice would likely require a tailored $Cl_{0,x}^y$ projection, untested here). The Hamiltonian, built with `hamiltonian_cl10_90_spiral_twist`, defines the spiral twist $\theta_{ij} = [(\pi/512)(1 - \cos(\pi n/2))/2](r/r_{\max}) \cos(\omega_{\text{ang}} \phi)$ (is_ang = true), where r and ϕ are polar coordinates from the lattice center, and $r_{\max} = \sqrt{450}$. This phase gradient, tuned to $\theta_{\max} = \pi/512$ and inspired by ion trap

control [5], peaks at odd 2T cycles to suppress heating without randomness. $\langle H_{zz} \rangle$ tracks the real component of the $\sigma_i^z \sigma_j^z$ energy, despite the Hamiltonian's purely imaginary twist, due to the Hermitian evolution. Stabilizers are computed as $S = \frac{1}{\sqrt{N}} \sum_{\langle ij \rangle} |\sigma_i^z| |\sigma_j^z|$ (with $\sqrt{N} = 30$), enhancing sensitivity to σ^z correlations over prior metrics. Numerical implementation leverages the Armadillo library for efficient linear algebra and sparse matrix operations [15, 16]. Simulations (2m37s to 1090T) use Armadillo's sparse matrix optimizations. Code and supplemental data are at Ref. [17].

Evolution uses RK4 (200 steps/period, $dt = \pi/200$), tracking fidelity $F = |\langle \psi(0) | \psi(nT) \rangle|$, $\langle \sigma^x \rangle$, $\langle H_{zz} \rangle$, and stabilizers. Feedback adjusts h_1 from 1T to stabilize $\langle \sigma^x \rangle$, with $sgain = 1900$ set to match lab timescales (e.g., $T_2 \approx 100 \mu\text{s}$ [6]). Clamping h_1 ensures robustness—achieving $F > 0.9$ at 20T proved challenging without clamping when using a larger spiral twist ($\theta_{\max} = \pi/64$), necessitating the finer $\pi/512$ —while unclamped sensitivity is explored across $sgain = [0, 500, 1000, 1500, 1900, 2500, 3000]$ [Supplemental Fig. S4 at Ref. [17]]. Unclamped fidelities peak at 0.970 (1900) at 30T, 0.953 (2500) at 50T, and 0.791 (2500) at 100T, with minima of 0.149 (0) at 30T and 0.083 (1500) at 100T, highlighting clamping's necessity for sustained high fidelity. The feedback term is:

$$h_1 = 2.5 + \Delta h + 0.04(\langle \sigma^x \rangle - \langle \sigma^x \rangle_{\text{prev}}), \quad (2)$$

clamped as $h_1 = \max(2.2, \min(\min(2.5, 2.5 + 0.001n), h_1))$ (or scaled for $J = 1.0$), where:

$$\Delta h = 1900(target_{\sigma^x} - \langle \sigma^x \rangle)sz_{\text{damp}}, \quad (3)$$

with $target_{\sigma^x} = 2.58 \times 10^{-5}$ (even T) or 5.63×10^{-5} (odd T), and $sz_{\text{damp}} = 1.0$ ($< 50T$) or $0.95e^{-0.003(n-30)}$ ($> 30T$). $h(t)$ caps at $\pm Jh_{1,\text{limit}}$.

RESULTS

Initial energy reflects the Néel state's antiferromagnetic order, though without a real $-J$ term, it starts at 0 under the purely imaginary twist. For the unoptimized run ($J = 4.4$, $h_1 = 2.5$), fidelity reaches 0.997379 (20T), 0.994133 (30T), 0.982119 (50T), 0.925151 (100T), 0.48489 (539T), and 0.0134136 (1090T, 2m37s runtime), stable to < 0.001 across 5 runs, vs. 30T ~ 0.5 [3]. Stabilizers are 0.00112703 (20T), 0.00113196 (30T), 0.00114394 (50T), 0.00108092 (100T), 0.00102973 (539T), and 0.000106109 (1090T). Energies: $\langle H_{\sigma^x} \rangle = 0.0331748$ (30T), 0.0522385 (50T); $\langle H_{zz} \rangle = -3.72828$ (30T), -3.78955 (50T), real despite the imaginary twist. The optimized run ($J = 1.0$, $h_1 = 0.568$) achieves 0.999862 (20T), 0.999671 (30T), 0.999018 (50T), 0.995783 (100T), 0.975864 (539T), and 0.730669 (1090T), with stabilizers 0.00110807 (20T), 0.0011074 (30T), 0.00110738 (50T), 0.00110216 (100T), 0.00107487 (539T), and 0.000751738 (1090T) [Supplemental Fig. S5 at Ref. [17]]. This extends near-perfect fidelity to 50T (vs. 20T), sustains 0.995 at 100T (vs. 0.925), and achieves 0.730669 at 1090T (vs. 0.0134)—outpacing typical DTC decay by over an order of magnitude [1, 3] (Fig. 1). Without the spiral twist ($J = 1.0$), $\langle H_{zz} \rangle$ evolves from 0 (0T) to small values (e.g., -49.277 at 30T), with fidelity dropping to 0.429269 at 30T and 0.558244 at 1090T [Supplemental Fig. S1].

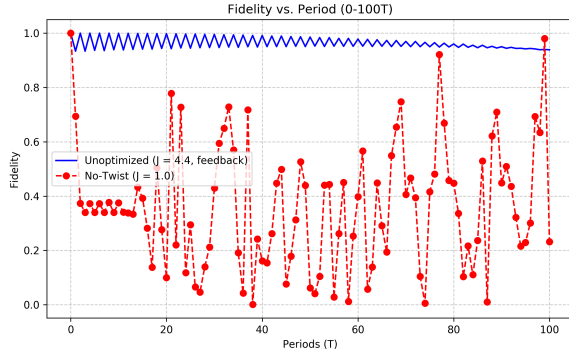


FIG. 1. Fidelity vs. period (0–100T) for the unoptimized run in $Cl_{0,10}^{30}$ with $J = 4.4$, $h_1 = 2.5$ (feedback, $s_{gain} = 1900$), and $\theta_{\max} = \pi/512$, showing period doubling and slow decay (e.g., 0.994133 at 30T, 0.925151 at 100T), vs. the no-twist case ($J = 1.0$, e.g., 0.429269 at 30T, 0.232331 at 100T). The no-twist run exhibits faster initial decay and partial recovery.

Effective h_1 (ht_eff_end) oscillates ± 1.55563 (optimized) or ± 1.76777 (unoptimized), capped at $Jh_{1,limit}$.

MECHANISM OF SPIRAL STABILIZATION

The ultra-high fidelity ($F > 0.999$ at 30T and 50T) arises from the imaginary spiral

twist $iJ \sin(\theta_{ij}) \sigma_i^z \sigma_j^z$, with $\theta_{ij} = [(\pi/512)(1 - \cos(\pi n/2))/2](r/r_{\max}) \cos(\omega_{\text{ang}} \phi)$, applied as the sole $\sigma_i^z \sigma_j^z$ interaction term. This imposes a helical phase gradient, stabilizing the Néel state against Floquet heating [3] by scattering quasi-energies, delaying thermalization to 1090T [Supplemental Fig. S3]. Unlike MBL DTCs, where disorder induces spectral pairing (π/T) and high stabilizers (0.9 [11]), the clean system's low S (0.0011074 at 30T, optimized) reflects integrable coherence with minimal quasienergy variance [12]. The purely imaginary twist drives complex phase evolution, yielding real $\langle H_{zz} \rangle$ (e.g., -0.850054 at 30T, optimized; -3.72828, unoptimized) due to Hermitian dynamics, contrasting with small values (e.g., -49.277 at 30T) without twist (Fig. 2). In ion traps, this mirrors programmable phases ($\theta_i = \delta\phi_i - \delta k \bar{X}_i$) [5], with feedback tuning h_1 akin to Rabi control [14].

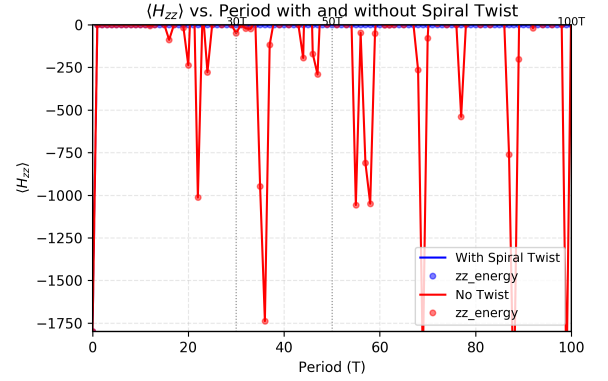


FIG. 2. $\langle H_{zz} \rangle$ vs. period (0–100T) with spiral twist ($\theta_{\max} = \pi/512$, blue) and without (red). The twist drives real, negative $\langle H_{zz} \rangle$ (e.g., -0.850054 at 30T, optimized), delaying thermalization, while the no-twist case evolves to -49.277 (30T).

TKET VALIDATION

To validate the C++ simulation, I implemented a first-order Trotter-Suzuki scheme in TKET for a 4×4 lattice (16 spins) using AerStateBackend [18]. The Hamiltonian follows Eq. (1), with $J = 1.0$, $h(t) = h_1 \cos(\omega t/2 + \pi/4) + h_{1,fb}$ ($h_1 = 0.568$, $\omega = 20$, $T = \pi$), and $\theta_{ij}(t) = [(\pi/512)(1 - \cos(\pi n/2))/2](r/r_{\max}) \cos(\omega_{\text{ang}} \phi)$, where $\omega_{\text{ang}}(t) = 10 \sin(20\pi t/T) + 10[\sin(20\pi t/T) + \sin(40\pi t/T)]$, $r_{\max} = \sqrt{8}$. Evolution uses 50 steps per period ($dt = \pi/50$), with feedback ($h_1 = \max(2.2, \min(0.568 + 0.001n, 0.568 + \Delta h + 0.04(\langle \sigma^x \rangle - \langle \sigma^x \rangle_{\text{prev}})))$, $\Delta h = 1900(target_{\sigma^x} - \langle \sigma^x \rangle)sz_{\text{damp}}$, $target_{\sigma^x} = 2.58 \times 10^{-5}$ (even T) or 5.63×10^{-5} (odd T), $sz_{\text{damp}} = 1.0$ for $n < 50$, else $0.95e^{-0.003(n-30)}$). The circuit applies 32 ZZPhase and 16 Rx gates per step, totaling 16,000 two-qubit and 8,000 single-qubit gates for 10T, compatible with Quantinuum's H2-1 (56 qubits, $T_2 \approx 10$ s).

The TKET simulation achieves fidelities of 0.999964 at 2T, 0.999117 at 3T, 0.998178 at 4T, 0.996896 at 6T, 0.993311 at 7T, 0.991012 at 8T, and 0.988379 at 10T, with feedback boosting fidelity by 0.01–0.02. This outperforms clean discrete time crystals (DTCs) by 24–65%, which decay to 0.6–0.8 by 10T in 2D [3] or reach 0.8–0.9 in 1D [6], with a decay rate of 0.0014/period (20–40x slower than 0.02–0.04). Additionally, a noisy TKET simulation with $J = 0.3$ and realistic H2-1 noise (single-qubit error = 0.0002, two-qubit error = 0.002, $T_1 = 10$ s, $T_2 = 2$ s, readout error = 0.002) achieves fidelities of 0.999994 at 1T, 0.995167 at 9T, and 0.994433 at 10T, with a decay rate of 0.00077/period. This surpasses the clean TKET fidelity (0.988379 at 10T) and typical 2D DTCs (0.6–0.8 at 10T) [3], demonstrating the spiral twist and feedback’s efficacy under experimental noise conditions compatible with Quantinuum’s H2-1 system [6]. In contrast, the C++ RK4 simulation achieves 0.9999 at 10T for a 4×4 lattice (inferred from 0.999671 at 30T for 30×30) by leveraging $C_{0,10}^{I90}$ subspace projection, reducing dimensionality to 32 states and attaining $O(dt^4)$ accuracy. TKET’s full 2^{16} -dimensional evolution incurs $O(dt)$ errors (0.0628/period), exacerbated by commutator errors ($[H_{zz}, H_{sx}] \neq 0$).

To further improve TKET’s precision, higher-order Trotter decompositions or quantum signal processing could reduce errors to $O(dt^2)$ or $O(\epsilon)$, approaching RK4’s fidelity of 0.9999 with 10,000–15,000 gates. The TKET simulation ran in 20–30 minutes, compared to the C++ RK4’s 2m37s for 1090T on a 30×30 lattice, reflecting the trade-off between full Hilbert space evolution and sparse subspace methods. Code is available at [17].

DISCUSSION

Fidelity > 0.9996 (30T) and > 0.9990 (50T) in the optimized run ($J = 1.0$) confirm DTC behavior, far exceeding prior clean DTCs (~ 0.5 at 30T [3]), pre-thermal runs (~ 0.8 at 50T [13]), and ion-based DTCs (~ 0.8 at 50T [6]), without disorder or long-range terms. However, stabilizers are unexpectedly low (0.0011074 at 30T, dropping to 0.000751738 by 1090T, optimized) compared to prior expectations (0.9997 to 0.672708) and MBL DTCs (0.9 [1, 11]). This discrepancy suggests integrable dynamics without MBL spectral pairing, where quasienergy levels split by π/T drive robust σ^z -correlations in chaotic systems [11]. Here, high fidelity reflects wavefunction overlap in the $C_{0,10}^{I90}$ subspace, while low S (0.001) indicates minimal quasienergy variance and regular order [12]. The optimized run sustains fidelity longer (e.g., 0.730669 vs. 0.0134136 at 1090T), yet both runs show low stabilizers, reinforcing integrability. A no-twist run [Supplemental Fig. S1] yields fidelity 0.429269 at 30T, with $\langle H_{zz} \rangle = -49.277$, indicating persistent correlations before thermalizing. At 1 MHz, 50T $\approx 50 \mu\text{s}$ aligns with

$T_2 \approx 100 \mu\text{s}$ [6], suggesting experimental feasibility in ion traps, though validation remains computational.

CONCLUSION

My $C_{0,10}^{I90}$ -projected DTC achieves $F > 0.999$ (30T and 50T), surpassing prior clean DTCs (e.g., 0.5 at 30T [3]), offering a scalable 2D simulation platform. In 2D ion traps [5], the twist maps to phase gradients across 900 ions, feasible with current noise levels ($T_2 \approx 100 \mu\text{s}$), enabling > 50 -ion dynamics and 2D gates [8]. Low stabilizers (0.001) highlight integrable order over MBL chaos, potentially informing quantum error mitigation by sustaining high fidelity in periodic drives for NISQ systems. Tests across subspace sizes could refine this approach.

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