

1 Firing Rate-dependent Phase Responses of Purkinje 2 Cells Support Transient Oscillations

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8 Abstract

9 Both spike rate and timing transmit information in the brain, yet how rate-modulated cellular
10 properties affect spike timing is largely unexplored. Phase response curves (PRCs), quantifying
11 how a neuron transforms input to output by spike timing, exhibit strong rate-adaptation, but its
12 mechanism and relevance for network output are poorly understood. Using our Purkinje cell
13 (PC) model and pyramidal neuron model, we demonstrate that the rate-adaptation is caused by
14 rate-dependent subthreshold membrane potentials efficiently regulating the activation of Na^+
15 channels. Then we use a realistic PC network model to examine how rate-dependent responses
16 synchronize spikes in the scenario of reciprocal inhibition-caused high-frequency oscillations.
17 Large and broad PRCs at high rates increase oscillation power and spike correlations. The
18 irregularity of spiking and the network connectivity also regulate oscillations. The combination
19 of these factors enables transient oscillations between fast-spiking neurons. Our work
20 demonstrates that rate-adaptation of PRCs can spatio-temporally organize neuronal output.

21 Introduction

22 The propensity of neurons to fire synchronously depends on the interaction between cellular
23 and network properties (Ermentrout et al., 2001). A phase response curve (PRC) quantifies
24 how a weak stimulus exerted at different phases during the interspike interval can shift
25 subsequent spike timing in repetitively firing neurons (Ermentrout et al., 2001; Gutkin et al.,
26 2005). Essentially, the PRC measures how a neuron transforms an input to output by spike
27 timing. Therefore, it determines the potential of network synchronization (Ermentrout et al.,
28 2001; Ermentrout et al., 2008; Gutkin et al., 2005; Smeal et al., 2010). The PRC is not static
29 and shows significant adaptation with spike rate. It was theoretically predicted that PRCs
30 decrease at high firing rates in pyramidal neurons (Gutkin et al., 2005), which unfortunately
31 did not match later experimental observations showing an increased PRC peak at higher rates
32 (Tsubo et al., 2007). Similarly, for PRCs in Purkinje Cells (PCs), the responses to weak stimuli
33 at low spiking rates are small and surprisingly flat. With increased rates, responses in later
34 phases become phase-dependent, with onset-phases left-shifted and gradually increasing peak
35 amplitudes, which has never been theoretically replicated or explained (Couto et al., 2015;
36 Phoka et al., 2010), nor has its effect in synchronizing spike outputs been explored.

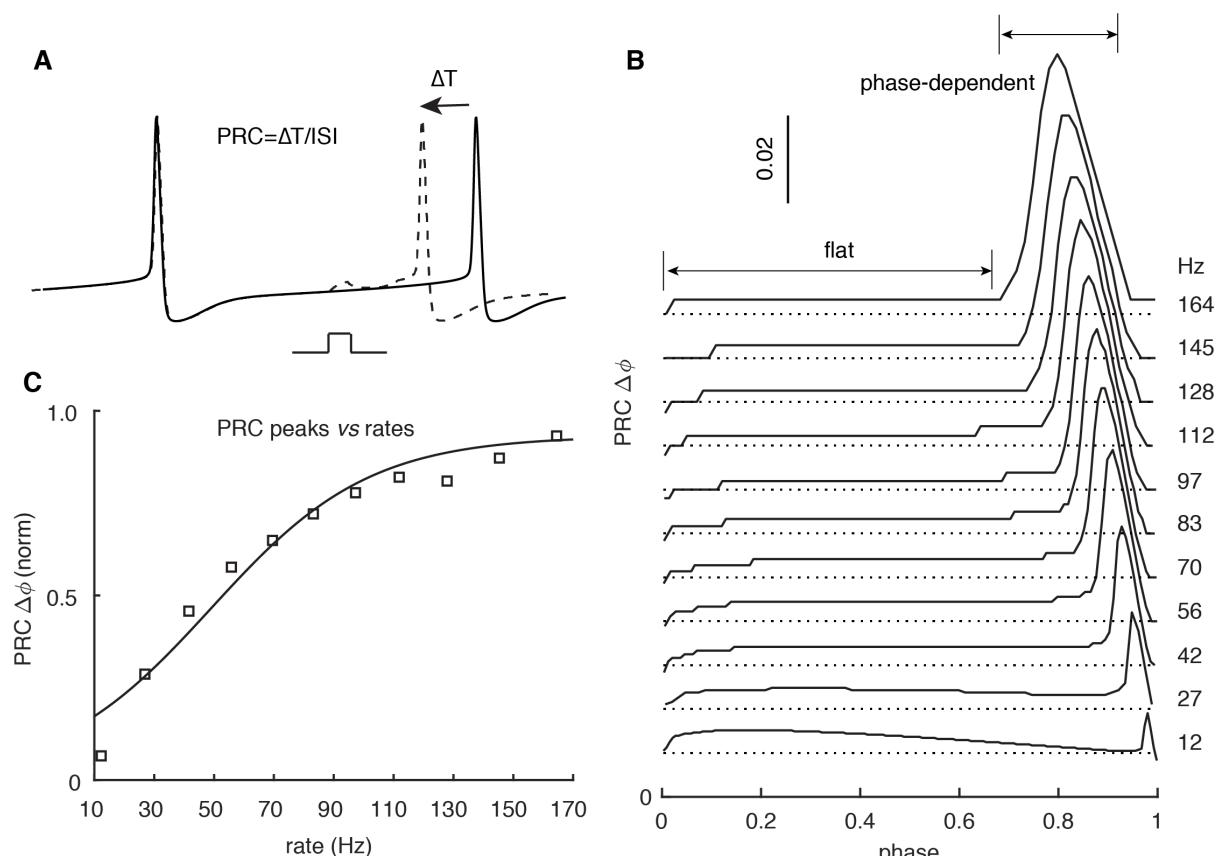
37 On the circuit level, reciprocal inhibition causing high frequency oscillations has been
38 observed in many regions of the brain such as cerebellum and hippocampus (Bartos et al., 2002;
39 Cheron et al., 2004; de Solages et al., 2008). However, how cellular properties such as PRCs
40 regulate the oscillations is still poorly understood. Furthermore, the functional importance of
41 oscillations in information transmission will be largely determined by their spatio-temporal
42 scale, which is difficult to predict given the hard-wired inhibitory connections. Since the PRC
43 is spike rate-dependent, it is interesting to explore whether this cellular property can
44 dynamically regulate the spatial range of oscillations based on spike rate changes.

45 To examine the mechanism of rate-dependent PRCs, we use our physiologically detailed PC
46 model (Zang et al., 2018) and a simple pyramidal neuron model to explore the rate adaptation
47 of PRCs. By analyzing simulation data and *in vitro* experimental data (Rancz and Häusser,
48 2010), we identify that rate-dependent subthreshold membrane potentials can modulate the
49 activation of Na^+ channels to shape neuronal PRC profiles. We also build a PC network model
50 connected by inhibitory axon collaterals to simulate high-frequency oscillations (de Solages et
51 al., 2008; Witter et al., 2016). Rate adaptation of PRCs can increase the power of oscillations
52 to link the rate with spike timing. Moreover, firing irregularity and network connectivity also
53 regulate the oscillation level. The combination of these factors enables PC spikes uncorrelated
54 at low basal rates to become transiently correlated in a subgroup of cells at high rates.

55 Results

56 PRC Exhibits Rate Adaptation in PCs

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59 Figure 1. PRC Exhibits Strong Rate Adaptation in PC model

60 (A) Schematic representation of the definition and computation of PRCs. The current pulse has
61 a duration of 0.5 ms and an amplitude of 50 pA. Different spike rates were achieved by somatic
62 current injection (Couto et al., 2015; Phoka et al., 2010). (B) The rate adaptation of flat and
63 phase-dependent parts of PRCs. (C) PRC peaks at different rates fitted by the Boltzmann
64 function.

65 PRCs were obtained by repeatedly exerting a weak stimulus at different phases of the
66 interspike interval (ISI). The resulting change in ISI relative to original ISI corresponds to the
67 PRC value at that phase (Fig. 1A). All previous abstract and detailed PC models failed to
68 replicate the experimentally observed rate-adaptation of PRCs (Akemann and Knopfel, 2006;
69 Couto et al., 2015; De Schutter and Bower, 1994; Khalil et al., 2003; Phoka et al., 2010). Our
70 recent PC model was well constrained against a wide range of experimental data (Zang et al.,

71 2018). Here, we explored whether this model can capture the rate-adaptation of PRCs under
72 similar conditions. When the PC model fires at 12 Hz, responses (phase advances) to weak
73 stimuli are small and nearly flat for the whole ISI (Fig. 1B). Only at a very narrow late phase
74 do the responses become phase-dependent, increasing slightly. With increased rates, the
75 responses remain small and flat during early phases. However, later, phase-dependent
76 responses gradually become larger, with onset-phases shifted left. In agreement with
77 experiments under the same stimulus conditions (Phoka et al., 2010), the peak of PRCs finally
78 became saturated at ~ 0.06 at high rates. The relationship between normalized PRC peaks and
79 rates can be fitted by the Boltzman function and matches experimental data (Fig. 1C, fitted
80 with $1/(1 + e^{-(rate-a)/b})$, $a = 49.1$, $b = 26.4$ in the model vs $a = 44.1$ and $b = 20.5$ in experiments
81 (Couto et al., 2015)). PRCs in our model show similar rate adaption with inhibitory stimuli
82 (phase delay, Fig. S1A). Rate-adaptive PRCs require the presence of a dendrite in the PC model
83 (not shown), but the dendrite can be passive (Fig. S1B). We also tested the effect of increasing
84 stimulus amplitude on PRC adaptation. Increasing stimulus amplitude consistently shifted
85 onset-phases of phase-dependent parts to the left and increased their amplitudes (Fig. S1C).

86 To unveil the biophysical principles governing rate-adaptive PRC profiles, we need to
87 answer three questions: Why are responses flat in early phases? Why do responses become
88 phase-dependent during later phases? What changes cause the rate adaptation of PRCs?

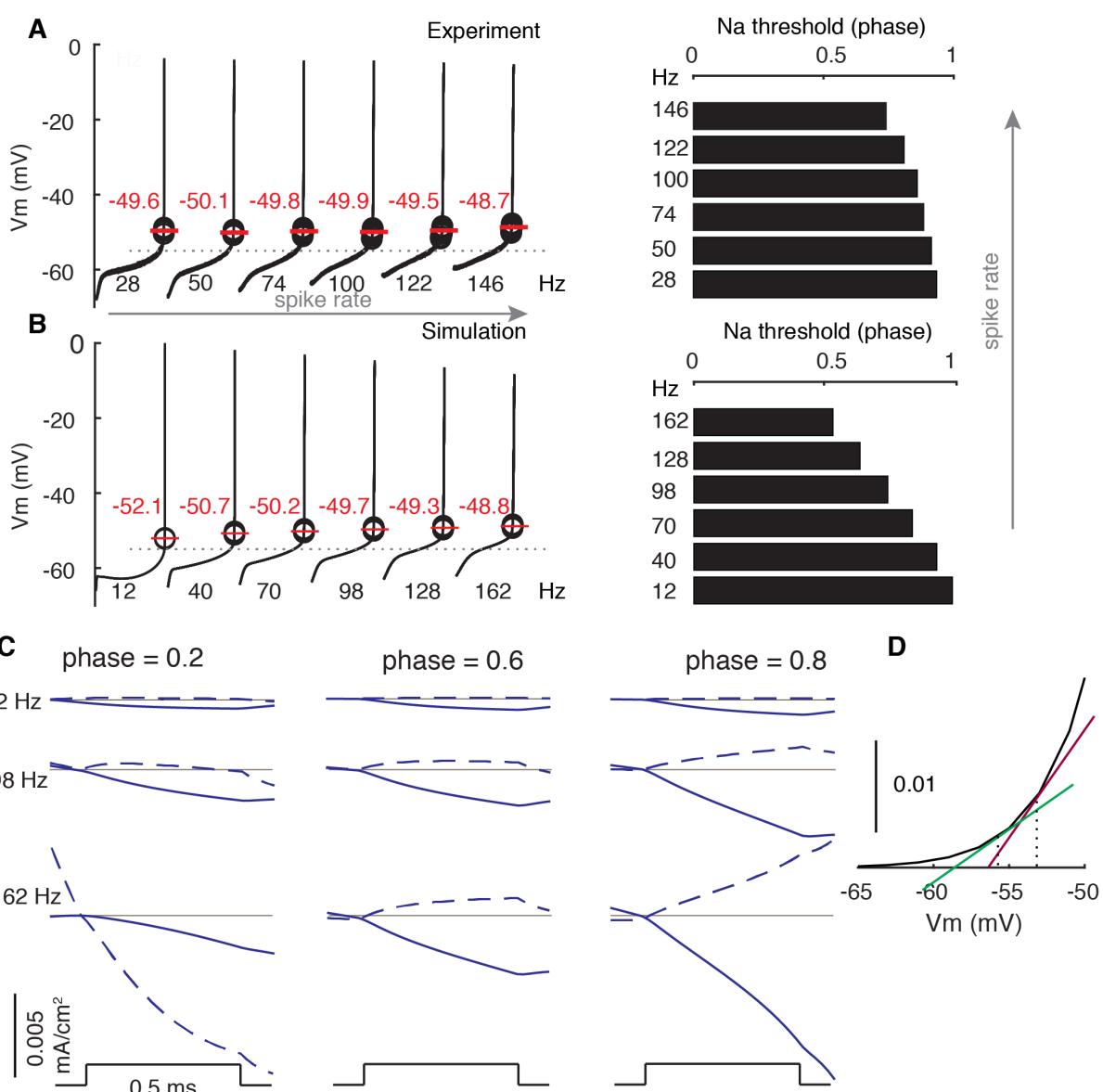
89 The Biophysical Mechanism of Rate Adaptation of PRCs in PCs

90 To answer the aforementioned questions, we examine how spike properties vary with rate
91 and find that the facilitation of Na^+ currents relative to K^+ currents, due to elevated subthreshold
92 membrane potentials at high rates, underlies the rate adaptation of PRCs. After each spike,
93 there is a pronounced after-hyperpolarization (AHP) caused by the large conductance Ca^{2+} -
94 activated K^+ current, and subsequently the membrane potential gradually depolarizes due to
95 intrinsic Na^+ currents and dendritic axial current (Zang et al., 2018). As confirmed by re-
96 analyzing *in vitro* somatic membrane potential recordings (shared by Ede Rancz and Michael
97 Häusser (Rancz and Häusser, 2010)), subthreshold membrane potential levels are significantly
98 elevated at high rates, but spike thresholds rise only slightly with rate (Fig. 2A). This means
99 the ISI phase where Na^+ activation threshold (~ -55 mV for 0.5% activation in PCs (Khaliq et
100 al., 2003; Zang et al., 2018)) is crossed shifts left and larger phase ranges of membrane
101 potentials are above the threshold at high rates (Fig. 2B).

102 During early phases of all rates, membrane potentials are distant from the Na^+ activation
103 threshold (Fig. 2A,B). The depolarizations to weak stimuli fail to activate sufficient Na^+
104 channels to speed up voltage trajectories, and phase advances are caused by just the passive
105 depolarizations (Fig. 2C). Consequently, phase advances in early phases are small and flat (or
106 phase independent). At later phases, membrane potentials gradually approach and surpass the
107 Na^+ activation threshold. Stimulus-evoked depolarizations activate more Na^+ channels to speed
108 up trajectories in return. Therefore, phase advances become large and phase- (actually voltage-)
109 dependent. At low rates, membrane potentials are below the Na^+ activation threshold during
110 nearly the entire interspike period (Fig. 2B). Responses are thus generally phase-independent.
111 At high rates, onset-phases of phase-dependent responses parallel the left shifts of Na^+
112 activation threshold-corresponding phases, due to elevated subthreshold membrane potentials.
113 Because high rate-corresponding elevated membrane potentials have larger slopes at the foot
114 of the Na^+ activation curve, a same ΔV (passive depolarization) activates more Na^+ channels
115 and consequently causes larger PRC peaks at high rates (Fig. 2C,D). Under all conditions
116 (except phase = 0.2, 162 Hz), stimulus-evoked depolarizations also increase outward currents,
117 but this increase is smaller than that of inward currents (mainly Na^+) due to the high activation
118 threshold of K^+ currents (mainly $\text{Kv}3$) in PCs (Martina et al., 2003; Zang et al., 2018). As the

119 stimulus becomes stronger, its triggered passive depolarization increases and the required pre-
 120 stimulus membrane potential (phase) to reach Na^+ activation threshold is lowered (left shifted).
 121 Thus, increasing the stimulus amplitude not only increases the PRC peaks, but also shifts the
 122 onset-phases of phase-dependent responses to the left (Fig. S1C).

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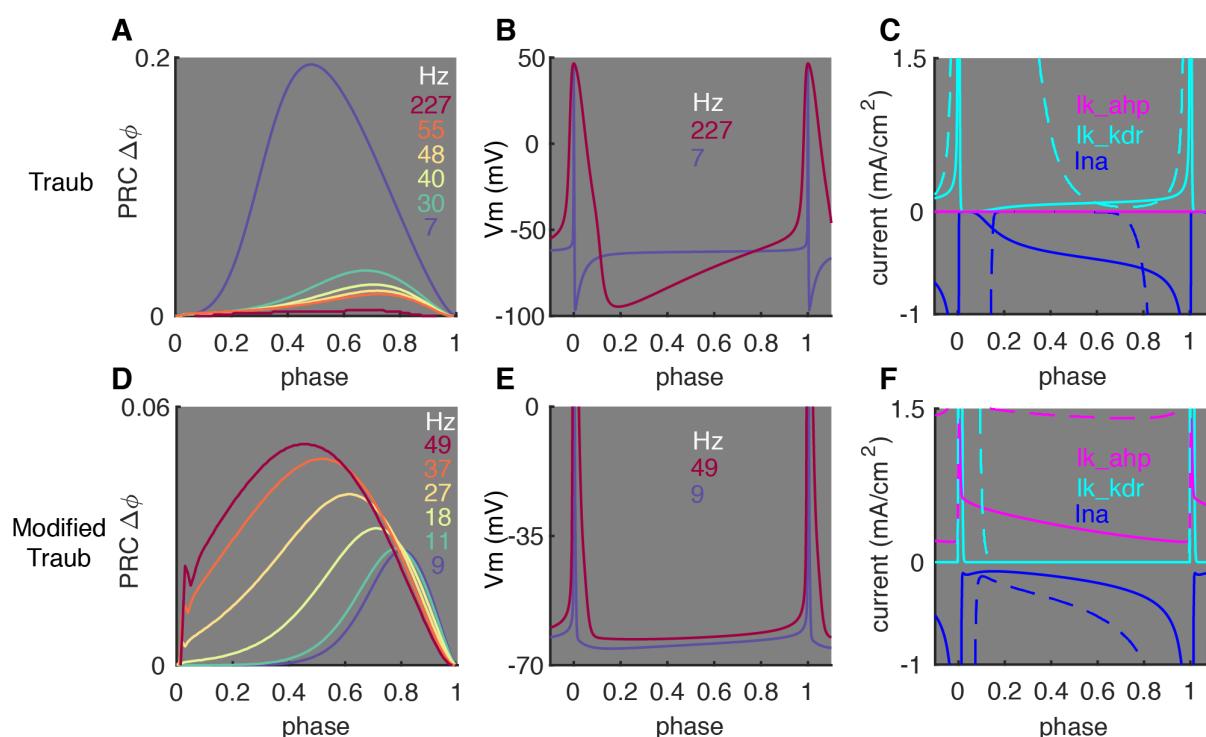
125 **Figure 2. Modulated Subthreshold Membrane Potentials Account for the Rate-
 126 adaptation of PRCs.**

127 (A and B) Experimental and simulated voltage trajectories in PCs during ISIs at different rates.
 128 The model used (Zang et al., 2018) was not fitted to this specific experimental data. Spike
 129 thresholds at different rates are labeled in plots. The Na^+ activation threshold is defined as -55
 130 mV (stippled line). Right plots show left-shifted Na^+ -activation threshold-corresponding
 131 phases at high rates. (C) Stimulus-triggered variations of inward ionic currents (solid) and
 132 outward ionic currents (dashed) at different phases and rates. Ionic currents are shifted to 0
 133 (grey line) at the time of stimulus to compare their relative changes. At phase = 0.2, the outward
 134 current is still decreasing due to the inactivation of the large conductance Ca^{2+} -activated K^+
 135 current at 162 Hz. (D) Larger slopes of the Na^+ activation curve at high membrane potentials.

136 General Effect of Subthreshold Membrane Potentials on Shaping PRCs

137 Here, we examine whether the critical role of subthreshold membrane potentials in shaping
138 PRC profiles also applies to other neuron types. A frequently used pyramidal neuron model,
139 the Traub model (Ermentrout et al., 2001) was tested. It shows an opposite rate-adaptation of
140 PRCs compared to PCs (Fig. 3A). In the Traub model, responses become smaller and relatively
141 phase-independent at high rates. In contrast to PCs, subthreshold membrane potentials are
142 significantly lower at high rates due to the accumulation of delayed rectifier K⁺ current (kdr,
143 Fig. 3B,C), which has a low activation threshold and large conductance. The lower
144 subthreshold membrane potentials are far below the Na⁺-activation threshold, making
145 responses to weak stimuli passive at high rates. Accordingly, PRCs in the model become
146 smaller and relatively phase-independent at high rates, this was not confirmed in more recent
147 experimental recordings (Tsubo et al., 2007). We minimally modified the Traub model by
148 reducing the conductance of the kdr current, raising its activation threshold and increasing the
149 AHP current (details in **Methods**) (Fig. 3D-F). With these modifications, subthreshold
150 membrane potentials are significantly elevated at high spiking rates. Accordingly, onset-phases
151 of phase-dependent responses shift left and peaks increase at high rates. These simulation
152 results confirm that, for any type of neuron, spike rate-dependent subthreshold membrane
153 potentials and their effect on nonlinear activation of Na⁺ currents are crucial in shaping
154 neuronal PRC profiles.

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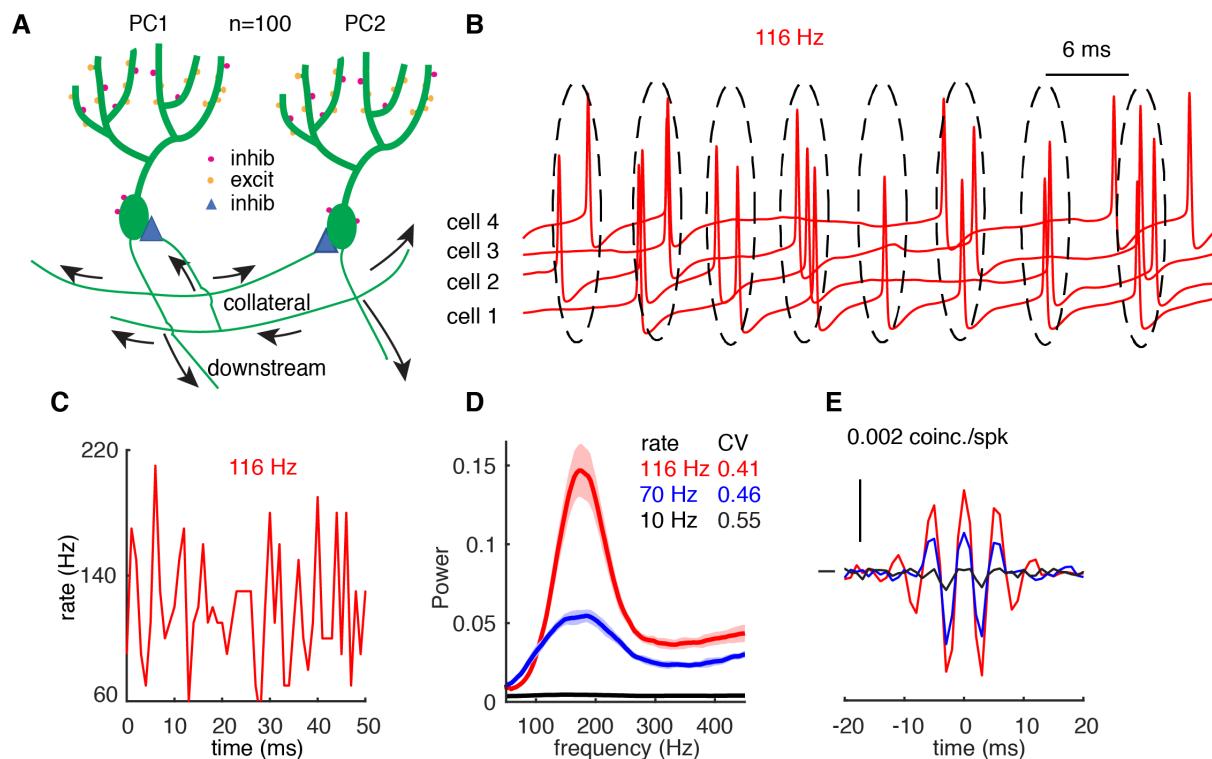
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158 **Figure 3. Subthreshold Membrane Potential Regulates PRC changes in Pyramidal
159 Neuron Models.**

160 (A) Rate-adaptation of PRCs in the original Traub model. (B) Lowered ISI membrane potential
161 at high rates. (C) Comparison of ionic currents at low (solid, 7 Hz) and high (dashed, 227 Hz)
162 rates. (D) Rate-adaptation of PRCs in the modified Traub model. (E) Elevated ISI membrane
163 potential at high rates. (F) Comparison of ionic currents at low (solid, 9 Hz) and high (dashed,
164 49 Hz) rates. In C and F, current peaks are truncated to show currents during ISIs. In E, spike
165 peaks are truncated to show the elevated ISI membrane potential at high rates.

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Rate-dependent High-frequency Oscillations



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Figure 4. High-frequency Oscillations Show Adaptation with Cellular Spike Rates.

(A) Schematic representation of the network configuration. (B) Example of sampled PC voltage trajectories in the network. (C) Example of population rates in the network. (D) The power spectrum of population rates of the network at different cellular rates and firing irregularity (CV of ISIs). (E) Averaged normalized cross-correlations at different cellular rates.

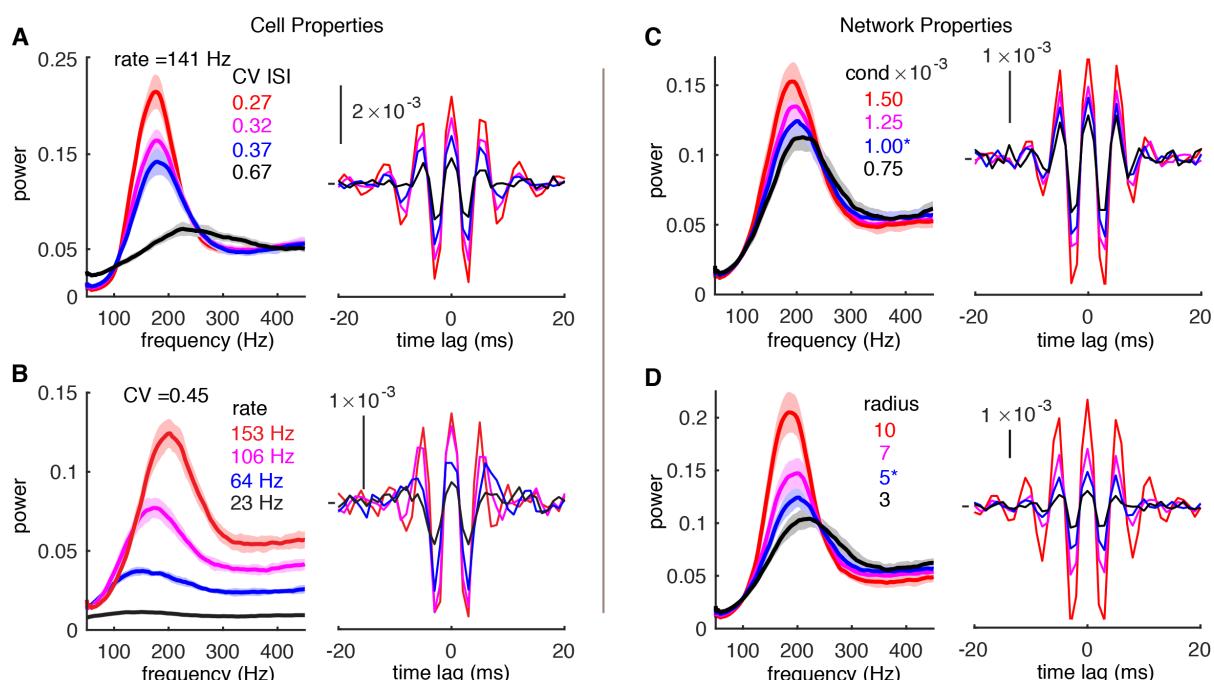
The potential effect of rate-caused variations of cellular response properties on population synchrony has been largely ignored in previous studies (Bartos et al., 2002; Brunel and Hakim, 1999; de Solages et al., 2008; Heck et al., 2007; Shin and De Schutter, 2006). Here, we examine whether rate correlates with synchrony in the presence of high-frequency oscillations that have been observed in the cerebellar cortex (Cheron et al., 2004; de Solages et al., 2008). We built a biophysically realistic network model composed of 100 PCs with passive dendrites distributed on the parasagittal plane (Witter et al., 2016). Each PC connects to the somas of its 5 nearest neighboring PCs through inhibitory axon collaterals on each side (Bishop and O'Donoghue, 1986; de Solages et al., 2008; Witter et al., 2016). Rates of each PC are independently driven by parallel fiber synapses, stellate cell synapses, and basket cell synapses (Fig. 4A). More details are in **Methods**.

When the average cellular rate is 116 Hz, PCs in the network tend to fire within interspaced clusters with time intervals of ~ 6 ms (Fig. 4B). However, individual PCs do not fire within every cluster. Therefore, spikes in the network show intermittent pairwise synchrony on the population level rather than spike-to-spike synchrony (Fig. 4B). Each peak in Fig. 4C corresponds to a 'cluster'. Based on the power spectrum, the network oscillates at a frequency of ~ 175 Hz (inverse of the cluster interval, ~ 6 ms), independent of cellular rates (116 Hz in red and 70 Hz in blue, Fig. 4D) because oscillation frequency is mainly determined by synaptic properties (Brunel and Hakim, 1999; de Solages et al., 2008). When cellular rates increase from 70 Hz to 116 Hz, the power of high-frequency oscillations significantly increases and the peak

195 becomes sharper. When individual PCs fire at low rates (10 Hz), the network fails to generate
 196 high-frequency oscillations and each PC fires independently, as evidenced by the flat power
 197 spectrum (Fig. 4D). High-frequency oscillations and their rate-dependent changes are also
 198 reflected in the average normalized cross-correlograms (CCGs) between PC pairs (Fig. 4E).
 199 When PCs fire at 70 Hz and 116 Hz, in addition to positive central peaks, two significant side
 200 peaks can be observed in the CCGs, suggesting correlated spikes with 0 ms time lag and ~ 6
 201 ms time lag. Amplitudes of the peaks increase with cellular rates and disappear when the rate
 202 is low (10 Hz). Notice that the computed CCG shows ‘excess’ correlation, which is computed
 203 by the raw correlation minus the shift predictor (Heck et al., 2007; Smith and Kohn, 2008).

204 Effect of Cell and Network Properties on Oscillations

205 In the previous section, the variation of cellular rates was driven by synaptic input to
 206 demonstrate the rate-adaptation of high-frequency oscillations. However, it is difficult to
 207 differentiate the relative contribution of PRC shapes and firing irregularity (measured by the
 208 CV of ISIs) since they covary with rate (Fig. 4D). Therefore, cellular rates were systematically
 209 varied by dynamic current injections, which were approximated by the Ornstein–Uhlenbeck
 210 (OU) process (Destexhe et al., 2001). This simulation protocol also causes the formation of
 211 high-frequency oscillations (Fig. S2). When PCs fire with low to moderate CV of ISIs, they
 212 show loose spike-to-spike synchrony at high rates, but not at low rates. With high CV of ISIs,
 213 spikes are jittered and spike-to-spike synchrony is disrupted. High-frequency oscillations were
 214 never observed under the condition of low cellular rates.



215
 216 **Figure 5. Effect of Cell and Network Properties on High-frequency Oscillations.**
 217 Both irregular spiking (high CV of ISIs) in A or low cellular rates in B decorrelate network
 218 output in the forms of reduced peaks of power spectrums (left) and CCGs (right). In A, the
 219 cellular rate is ~ 141 Hz. In B, the CV ISI is ~ 0.45 . Both small conductance (cond) of inhibitory
 220 synapses in C and a short connection radius (D) decorrelate network output in the forms of
 221 reduced peaks of power spectrums (left) and CCGs (right). In A&B, the cond is 1 nS and radius
 222 is 5. In C&D, the cellular rate is ~ 151 Hz and the CV ISI is ~ 0.45 .

223 Both spiking irregularity and rates of PCs covary with cerebellum-associated behaviors
 224 (Chen et al., 2016). Our results show that small spiking irregularity supports high-frequency
 225 oscillations. However, further increasing spiking irregularity reduces the power of high-

226 frequency oscillations and makes the power spectrum flatter when rates are the same (141 Hz)
227 (Fig. 5A). In average normalized CCGs, both central and side peaks decrease with increased
228 firing irregularity. Both results suggest reduced correlation when PCs fire very irregularly.
229 Next, we explore how rate-dependent PRCs regulate network output. The power of high-
230 frequency oscillations increases and the power peak becomes sharper with large, broad PRCs
231 at high rates (Fig. 5B). Peaks in the average CCGs also increase, suggesting more correlated
232 spike outputs at high rates. In Fig. S3, we demonstrate that the PRC peak amplitude at high
233 cellular rates controls the oscillation power.

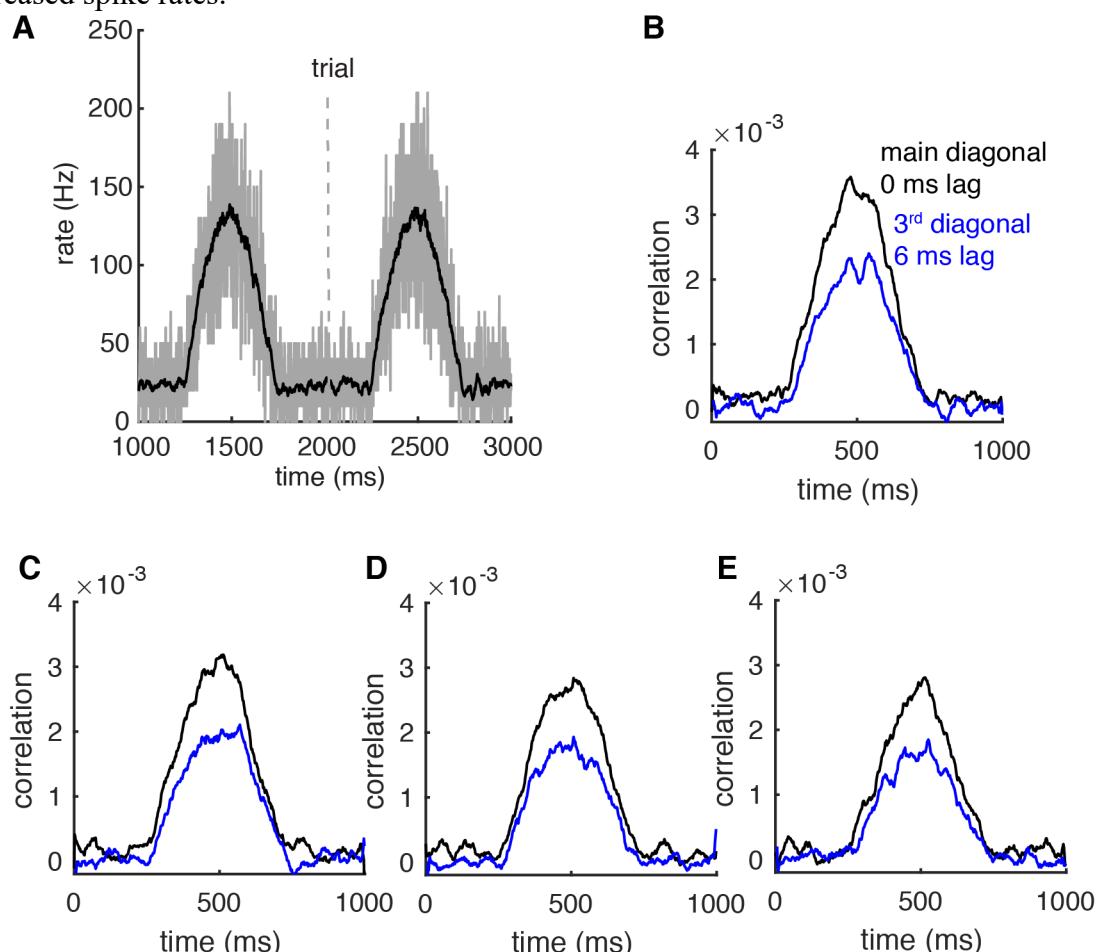
234 At the circuit level, the strength of inhibitory synapses and connection radius are difficult to
235 determine accurately, but their values are critical for the function of axon collaterals. Within
236 the ranges of experimentally reported synaptic conductance and connection radius (de Solages
237 et al., 2008; Fisyunov et al., 2006; Orduz and Llano, 2007; Watt et al., 2009; Witter et al.,
238 2016), the network generates robust high frequency oscillations (Fig. 5C, D). In addition, we
239 find that increasing the conductance of inhibitory synapses or their connection radius increases
240 the power of high-frequency oscillations and make the power spectrum sharper. The increased
241 oscillation power due to connectivity properties is also captured by the larger peaks in CCGs.
242 Both effects can be explained by larger phase responses due to larger inputs (synaptic
243 connections, Fig. S1C). Together, our simulation data suggest that the correlation between PC
244 spikes is strong under conditions of low to moderate spiking irregularity, high cellular rate,
245 high synaptic conductance, and large connection radius.

246 Transient Correlations Are Robust to Heterogeneous Spike rates

247 Although inhibitory connections loosely synchronize spike output and cause oscillations,
248 their functional role will depend on the following answers: When do they occur? How fast can
249 they be achieved? Are they robust to heterogeneous cellular spike rates? We have previously
250 simulated networks with a range of homogeneous stable cellular rates. Here, we first test
251 whether rate-dependent synchrony still holds when population rates change dynamically.
252 Population rates of the network approximate the half-positive cycle of a 1 Hz sine wave (peak
253 ~ 140 Hz) with the duration of each trial being 1 sec (Fig. 6A). We computed shuffle-corrected,
254 normalized joint peristimulus time histograms (JPSTHs) to reflect the dynamic synchrony
255 (Aertsen et al., 1989) (Fig. S4A). The main and the third diagonal of the JPSTH matrix,
256 corresponding to 0 ms time lag correlation and 6 ms time lag correlation respectively, are
257 plotted to show the dynamic synchrony at transiently increased rates (bin size is 2 ms, Fig. 6B).
258 At low basal rates, there are no correlations between spikes. Both correlations start to increase
259 ~ 250 ms after the onset of simulations and decrease again when the cellular rates drop, closely
260 following rate changes. It demonstrates that axon collateral-caused spike correlations can be
261 achieved transiently to transmit a timing signal conjunctive with temporal cellular rate
262 increases.

263 Although it remains unclear whether the population of PCs converging onto a same
264 cerebellar nuclei neuron are homogeneous or heterogeneous, simultaneous bidirectional PC
265 rate changes have been observed during cerebellum-related behaviors (Chen et al., 2016;
266 Herzfeld et al., 2015). It is very likely that neighboring PCs show heterogeneous spike rate
267 changes, which may reduce spike correlations (Markowitz et al., 2008). Therefore, we
268 distributed 10-30 extra cells with decreased spike rates in the network to test the effect of
269 heterogeneous neighboring rate changes on the transient correlations (The population firing
270 rates of increased-rate cells and decreased-rate cells are shown in Fig. S4B). They were
271 randomly scattered among the cells with increasing rates. Spike correlations still become larger
272 for the subgroup of PCs showing increased cellular rates, despite a slight decrease of the
273 correlation amplitude when more cells decrease their spike rates (Figs. 6C-E). The results
274 suggest that a population of PCs with increased spike rates can form subgroups to propagate

275 synchrony information even when they are surrounded by non-correlated neighboring PCs with
276 decreased spike rates.



277
278 **Figure 6. Correlations can be transient and robust to heterogeneous spike rates**
279 (A) Population spike rates of PCs. (B) The 0 ms- and 6 ms-time lag correlations increase with
280 population rates. (C-E) The rate-dependent correlation is robust to heterogeneous cellular rate
281 changes. From C to E, the number of decreased rate cells increases from 10 to 30.

282 Discussion

283 In this work, we dissected biophysical mechanisms shaping PRC profiles and explored their
284 role in synchronizing spikes in cerebellar PCs. We started by reproducing the rate adaptation
285 of PRCs in PCs and then identified rate-modulated interspike potentials as the underlying
286 mechanism. We further demonstrated rate-dependent phase responses can link spike rate with
287 spike timing to regulate neuronal output.

288 Biophysical Mechanisms Underlying Rate-dependent PRCs

289 The profiles of neuronal PRCs are rate-dependent (Couto et al., 2015; Ermentrout et al.,
290 2001; Gutkin et al., 2005; Phoka et al., 2010; Tsubo et al., 2007). In cortical neurons, slow
291 voltage-dependent K⁺ and Ca²⁺-activated K⁺ currents were proposed to mediate rate adaptation
292 of their PRCs and emergent synchrony between excitatory coupled neurons (Ermentrout et al.,
293 2001; Gutkin et al., 2005). These studies presumed that the decrease of PRC peaks with
294 increasing spike rates would be general for type-I PRCs (always advancing). However, this
295 prediction has not been confirmed in later experiments (Couto et al., 2015; Phoka et al., 2010;
296 Tsubo et al., 2007). In Layer 2/3 pyramidal neurons, PRCs tend to transit from type-I to type-
297 II with increasing rate (advanced or delayed depending on phase) (Tsubo et al., 2007). Even in

298 neurons preserving type-I PRCs (mainly Layer 5), peaks tend to increase with rate (see Figs.
299 3E&4 of Tsubo et al. (2007)). Cerebellar PCs exhibit a transition from small, phase-
300 independent responses to large, phase-dependent type-I responses with increasing rate (Couto
301 et al., 2015; Phoka et al., 2010), but the mechanism was unknown (Akemann and Knopfel,
302 2006; Couto et al., 2015; De Schutter and Bower, 1994; Khaliq et al., 2003; Phoka et al., 2010).
303 This work reproduces and explains the experimentally observed rate adaptation of PRCs. Note
304 that the slight increase of PRCs in the very narrow late phase in our model (low rate, Fig. 1B)
305 may be annihilated by noise in spontaneously firing neurons (Couto et al., 2015; Phoka et al.,
306 2010). Compared with previous work emphasizing the slow deactivation of K^+ currents in
307 cortical neurons (Ermentrout et al., 2001; Gutkin et al., 2005), here we unveil the general role
308 of rate-dependent subthreshold membrane potentials and their corresponding activation of Na^+
309 currents. In both pyramidal neurons and PCs, spike rates cause significant variation of the
310 subthreshold membrane potential during the ISI (Rancz and Hausser, 2010; Tsubo et al., 2007).
311 In response to a stimulus, both Na^+ and K^+ currents are activated. In PCs, the main K^+ current
312 is high-threshold activated (Martina et al., 2003; Zang et al., 2018); therefore, depolarization-
313 facilitated Na^+ currents dominate, causing larger PRCs at high rates (Fig. 2). However, previous
314 PC models (Akemann and Knopfel, 2006; Couto et al., 2015; De Schutter and Bower, 1994;
315 Khaliq et al., 2003; Phoka et al., 2010) possess low-threshold-activated K^+ currents, which
316 counteract facilitated Na^+ currents, explaining an absence of increased PRCs at high rates. In
317 the Traub model, slow deactivation of K^+ currents and consequent hyperpolarization
318 synergistically reduce the PRC peaks at high rates (Ermentrout et al., 2001; Gutkin et al., 2005).
319 By minimally modifying the Traub model, elevated subthreshold interspike potentials generate
320 larger PRCs peaks at high rates (Fig. 3) as in experiments (Tsubo et al., 2007). To the best of
321 our knowledge, decreased neuronal PRCs at high firing rates haven't been experimentally
322 observed yet.

323 Spike Synchronization Mechanisms

324 Whether spike timing carries critical information for cerebellar function is still controversial.
325 For smooth pursuit eye movement in monkeys, the movement was reported to be coded just by
326 PC spike rates (Payne et al., 2019). However, during saccades, the spike timing of some PCs
327 provides a temporally reliable signal for the onset of eye movement (Hong et al., 2016). In
328 mice, well-timed spiking in PCs and cerebellar nuclei neurons is critical in cerebellum-related
329 locomotion and whisking (Brown and Raman, 2018; Sarnak and Raman, 2018). Therefore,
330 spike timing is important for at least some, if not all, cerebellum-related behaviors. Both
331 synchronized complex spikes (De Gruijl et al., 2014; Tang et al., 2019) and simple spikes (de
332 Solages et al., 2008; Heck et al., 2007; Shin and De Schutter, 2006) have been observed *in vivo*.
333 Among all observations, the high-frequency oscillations (Cheron et al., 2004; de Solages et al.,
334 2008; Groth and Sahin, 2015) do not rely on synchronized afferent input and were proposed to
335 be caused by recurrent inhibitory axon collaterals (de Solages et al., 2008). Recent work has
336 confirmed the presence of recurrent axon collaterals in adult mice by direct imaging and
337 reconstruction (Witter et al., 2016), while they were previously thought to exist in young mice
338 only (Watt et al., 2009). For high-frequency oscillations, rate-dependent phase responses
339 increase the oscillation level at high rates, with no need to increase afferent input correlation.
340 Note that rate-related synchrony can also be achieved via common synaptic inputs (Heck et al.,
341 2007), gap junctions (Middleton et al., 2008), and ephaptic coupling (Han et al., 2018), when
342 the inputs or connections are weak. To achieve synchrony, neuronal PRCs are required to be
343 non-zero, regardless of the driving mechanism. For larger PRC values, advanced spikes, or
344 those delayed by inhibition, will be more clustered relative to the stimuli (Ermentrout et al.,
345 2008). Since synchronization mechanisms are not mutually exclusive, they may work
346 synergistically to achieve the required synchrony level. However, rate-dependent PRCs will

347 not help synchronize complex spikes triggered by common or correlated climbing fiber inputs
348 (De Gruijl et al., 2014; Tang et al., 2019), because the inputs are powerful enough to evoke
349 spikes immediately.

350 The Evidence Supporting Rate-dependent Correlations

351 There is no direct experimental data supporting rate-dependent synchrony in the cerebellum.
352 However, careful analysis of previous experimental data in the cerebellum provides some
353 evidence to support our findings. In the work of de Solages et al. (2008), units with lower
354 average rates (<10 Hz) did not exhibit significant correlations between neighboring PCs, for
355 unknown reasons. This can be explained by the small flat PRCs at low rates. Under extreme
356 conditions, when the PRC is constantly 0 (equivalent to disconnection), no correlations can be
357 achieved (Figs. 4-6). Additionally, the oscillation power increased by the application of WIN
358 55,212-2, which was intended to suppress background excitatory and inhibitory synapses (de
359 Solages et al., 2008). The increased power could be due to more regular spiking after inhibiting
360 the activity of background synapses (Fig. 5A). However, it could also be caused by increased
361 spike rates (Fig. 5B), because this agent also blocks P/Q type Ca^{2+} channels and consequently
362 P/Q type Ca^{2+} -activated K^+ currents, to increase spike rates (Fisyunov et al., 2006). Similarly,
363 enhanced oscillations have also been observed in calcium-binding protein gene KO mice,
364 which accompany significantly higher simple spike rates (Cheron et al., 2004).

365 Spatio-temporal Correlated Spiking

366 We built a link between oscillation level and firing irregularity (Fig. 5A). Given that
367 increased firing irregularity is usually linked to cerebellar dysfunction (Peter et al., 2016;
368 Walter et al., 2006), our results imply that irregularity-disrupted temporal population firing
369 patterns can be the ultimate reason. Although temporal synchrony may carry important timing
370 information for the onset of some movements (Brown and Raman, 2018; Hong et al., 2016;
371 Sarnak and Raman, 2018), optimal cerebellar function seems to occur between excessive
372 asynchrony and synchrony (Shakkottai, 2014). It has been shown that synchronized simple
373 spikes are time-locked to reaching-grasping movements in rats (Heck et al., 2007). Before and
374 after such movements, synchronized simple spikes were not observed. High-frequency
375 oscillations time-locked to lever-pressing in rats have been reported (Groth and Sahin, 2015).
376 Additionally, in humans high-gamma oscillations in the cerebellar cortex significantly increase
377 during a reaching task (Carver et al., 2019). Nonetheless, global and rhythmic increased
378 synchrony (Cheron et al., 2004) may abrogate normal separation of timing signals to different
379 muscle groups (for example agonist and antagonist muscles), causing impaired motor
380 coordination, such as dystonia (Shakkottai, 2014). In our model, PRCs are quantitatively close
381 to experimental data (Fig. 1). When cell and network parameters fall within physiological
382 ranges (Figs. 5,6), the network shows very weak oscillations at low basal cellular rates, but the
383 PC ensemble can dynamically increase the correlation level within a subgroup of PCs with
384 increased rates (Person and Raman, 2011) (Fig. 6). Local gamma oscillations have been shown
385 to selectively route input information in a cortical circuit model (Palmigiano et al., 2017).
386 Similarly, in the cerebellum, temporal information can be transiently separated and directed to
387 different subgroups of PCs to efficiently coordinate muscle movements. Thus, the spatial range
388 of axon collaterals ($\sim 210 \mu\text{m}$, each connecting 7 to 10 neighboring PCs (Bishop and
389 O'Donoghue, 1986; Watt et al., 2009; Witter et al., 2016)), the strength of their synapses (on
390 the order of 1 ns (de Solages et al., 2008; Orduz and Llano, 2007; Witter et al., 2016)), and
391 rate-dependent PRCs (Couto et al., 2015; Phoka et al., 2010) may well be configured to support
392 spatio-temporal synchrony at high rates. Furthermore, when PCs fire at high rates, the strong
393 facilitation of inhibitory axon collateral synapses may be a complementary mechanism to
394 strengthen this dynamic synchrony (Orduz and Llano, 2007).

395 Possible Rate-dependent Oscillations in Hippocampus

396 The high-frequency oscillations in the cerebellum are reminiscent of hippocampal ripple
397 oscillations involved in memory consolidation. Ripple oscillations are thought to originate
398 from parvalbumin-expressing GABAergic neuron networks (Bartos et al., 2002). PCs and
399 hippocampal GABAergic neurons both are fast-spiking inhibitory neurons. They have similar
400 f-I curve slopes and their principal repolarization currents both are Kv3 currents (Hu et al.,
401 2018). Thus, it will be interesting to explore whether PRCs of GABAergic neurons and the
402 power of ripple oscillations exhibit similar rate adaptations.

403 Conclusion

404 We have identified the subthreshold membrane potential as a general mechanism shaping
405 neuronal PRC profiles. It will help experimentalists and theorists to understand and reproduce
406 measured PRCs, and further explore their function in encoding temporal information in
407 different circuits. Rate-dependent phase responses couple spike rate with spike timing, which
408 may be a significant neuronal property to spatio-temporally regulate their outputs.

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412 by Steven Douglas Aird.

413

414 Methods

415 The detailed PC model and the interconnected network model were implemented in
416 NEURON 7.5 (Carnevale and Hines, 2006). The Traub model was implemented in MATLAB.
417 The code used in this work will be available from ModelDB.

418 PRC computations

419 Our recently developed compartment-based PC model was used (Zang et al., 2018). To
420 compute the PRCs in Fig. 1, brief current pulses with a duration of 0.5 ms and an amplitude of
421 50 pA were administered at the soma at different phases of interspike intervals. The resulting
422 perturbed periods were then used to calculate phase advances by the formulation
423 $PRC = (\langle ISI \rangle - ISI_{perturb}) / \langle ISI \rangle$. Different cellular rates were achieved by somatic
424 holding currents (Couto et al., 2015; Phoka et al., 2010). To compute PRCs in response to
425 negative stimuli, the amplitudes of the pulses were changed to - 50 pA. To compute PRCs of
426 our PC model with passive dendrites, only H current and leak current were distributed on the
427 dendrites with the same parameters as in the active model (Zang et al., 2018). The Traub model
428 (Traub et al., 1999) was implemented according to the work of Ermentrout et al (Ermentrout
429 et al., 2001; Gutkin et al., 2005). In the modified version of this model, the conductance of the
430 kdr current was reduced from 80 to 40. Activation and deactivation rates of this current were
431 shifted to the right by 30 mV, $\alpha_n(v) = 0.032*(v+22)/(1-exp(-(v+22)/5))$; $\beta_n(v) = 0.5*exp(-(v+27)/40)$;
432 the conductance of AHP current was increased from 0 to 0.1.

433 Network simulations

434 We implemented our recurrent inhibitory PC layer network using the Watts-Strogatz model
435 (Watts and Strogatz, 1998) to avoid boundary effects. To reduce simulation time, we used the
436 PC model with passive dendrites, which exhibits similar rate-dependent PRCs as the PC model
437 with active dendrites (Fig. S1B). In the baseline version of the network, 100 PCs were

438 distributed on the parasagittal plane (Witter et al., 2016), corresponding to 2 mm of folium with
439 a distance of 20 μm between neighboring PC soma centers. 100 PCs are within the estimated
440 range of PCs converging to a same cerebellar nuclei neuron (Person and Raman, 2011). Each
441 PC was connected to its nearest 2*radius neighboring PC somas and connections had 0 rewiring
442 probability. The PCs were interconnected, according to anatomical data showing collaterals
443 present toward both the apex and the base of the lobule with only slight directional biases
444 (Witter et al., 2016). The baseline value of radius was 5 within the range of experimental
445 estimates (de Solages et al., 2008; Watt et al., 2009; Witter et al., 2016). The inhibitory
446 postsynaptic current (IPSC) was implemented using the NEURON built-in point process,
447 $\text{Exp2Syn. G} = \text{weight} * (\exp(-t/\tau_2) - \exp(-t/\tau_1))$, with $\tau_1 = 0.5$ ms (rise time) and $\tau_2 = 3$ ms (decay
448 time). The reversal potential of the IPSC was set at -85 mV (Watt et al., 2009). The conductance
449 was 1 nS (de Solages et al., 2008; Orduz and Llano, 2007; Witter et al., 2016). The delay
450 between onset of an IPSC and its presynaptic spike timing was 1.5 ms (de Solages et al., 2008;
451 Orduz and Llano, 2007; Witter et al., 2016). To test the effect of rate-dependent PRCs on high-
452 frequency oscillations, we varied the cellular rates in two paradigms. In the first (Fig. 4), each
453 PC is contacted by 4,000 excitatory parallel fiber synapses (PF, on spiny dendrites), 18
454 inhibitory stellate cells (STs, on spiny dendrites) and 4 inhibitory basket cells (BSs, on the
455 soma). Activation of excitatory and inhibitory synapses in each PC was approximated as an
456 independent Poisson process with different rates. We simulated 3 conditions: PC rate = 10 Hz
457 when PF rate = 0.27 Hz, ST rate = 14.4 Hz, BS rate = 14.4 Hz; PC rate = 70 Hz when PF rate
458 = 2.16 Hz, ST rate = 28.8 Hz, BS rate = 28.8 Hz; PC rate = 116 Hz when PF rate = 3.24 Hz,
459 ST rate = 28.8 Hz, BS rate = 28.8 Hz. To more systematically explore different factors
460 regulating network outputs, after Fig. 4, we used a second paradigm. Cellular rates of each PC
461 were manipulated by injecting stochastic currents on the soma. The stochastic current was
462 approximated by the commonly used Ornstein-Uhlenbeck random process (Destexhe et al.,
463 $\tau \frac{dI}{dt} = -I + \sigma\sqrt{\tau}\eta_i(t)$). σ represents the amplitude of the fluctuation; η_i represents
464 uncorrelated white noise with unit variance; $\tau = 5$ ms. In this paradigm, we systematically
465 varied the rates and firing irregularities of PCs (CV of ISIs) to explore their importance for
466 network output. Phase response is a result of input current and response gain of the cell. No
467 existing data support decreased PRC at high cellular rates. In our model, in a physiological
468 range, it is not available either. We therefore reduce the phase response by halving the input
469 current (synaptic conductance) to achieve a smaller response at high firing rates (Fig. S3). We
470 also explored the effect of connection radius with the values of 3, 5, 7 and 10 in Fig. 5D. The
471 conductance of inhibitory synapses was tested with the values of 0.75, 1.0, 1.25 and 1.5 nS in
472 Fig. 5C. To test the spatio-temporally increased correlation, we randomly distributed extra 10
473 - 30 PCs with decreased cellular rates into the original network (Fig. 6), including 100
474 increased-rate cells. Their mean population firing rates are shown in Fig. S4B.

475 Data analysis

476 The power spectrum of the spike trains of the network was estimated by Welch's method,
477 which calculates the average of the spectra of windowed segments (window size 128 points).
478 In each trial under each specific stimulus condition, the length of the signal was 2 sec, with a
479 time resolution of 1 msec. The final result was the average of 14 trials.

480 To compute the cross-correlogram (CCG) under each specific stimulus condition, we first
481 computed pairwise correlations between the spike trains of two neurons and then corrected
482 them by shift predictors, which removed the 'chance correlations' due to rate changes. Then
483 correlations were divided by the triangular function $\Theta(\tau) = T - |\tau|$ and $\sqrt{\lambda_i \lambda_j}$. T was the
484 duration of each trial and τ was the time lag. $\Theta(\tau)$ corrects for the degree of overlap between
485 two spike trains for each time lag τ . λ_i was the mean firing rate of neuron i (Kohn and Smith,

486 2005). Finally, the CCGs between all pairs in the network were averaged to reflect the
487 population level spike correlations. Thus, similar with previous work (Heck et al., 2007), the
488 computed CCGs reflect the 'excess' correlation caused by axon collaterals in our work. To
489 measure the dynamic correlation over the time course of the stimulus, we computed JPSTHs.
490 Here we used a 2-ms time bins due to the narrow central peak and side peaks. Larger time bins
491 annihilated the positive peaks due to the significant negative correlations in paired spikes.
492 Therefore, we simulated 1992 trials to compute the raw JPSTH between PC pairs. Similar with
493 CCGs, the raw JPSTH was corrected by subtracting the shift predictor (cross-product matrix
494 of individual peri-event time histograms) to remove the coincident spikes due to rate changes
495 and co-stimulus. The corrected JPSTH was then normalized by the squared root of product of
496 each neuron's PSTH standard deviations (Aertsen et al., 1989). The corrected matrix values
497 become correlation coefficients, with values between -1 and +1. Finally, all pair-wise JPSTHs
498 were averaged to reflect the population level dynamic correlations.

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629 **Supplementary figures**

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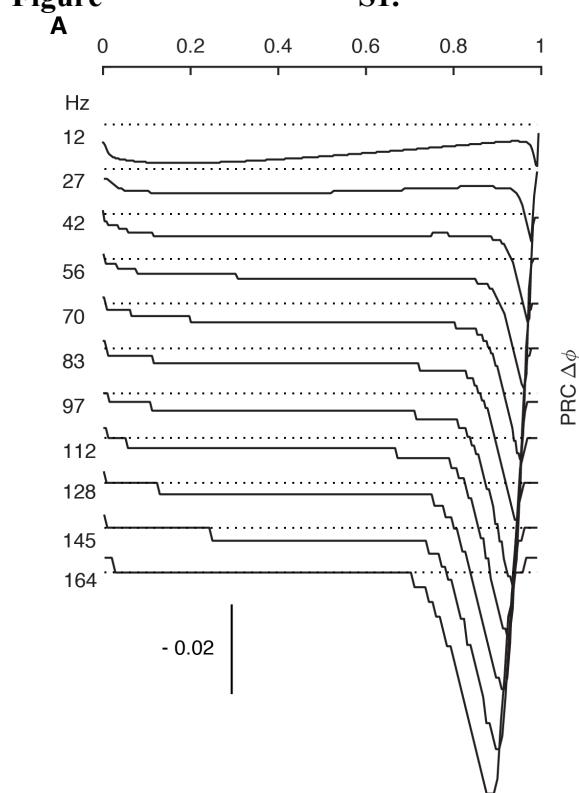
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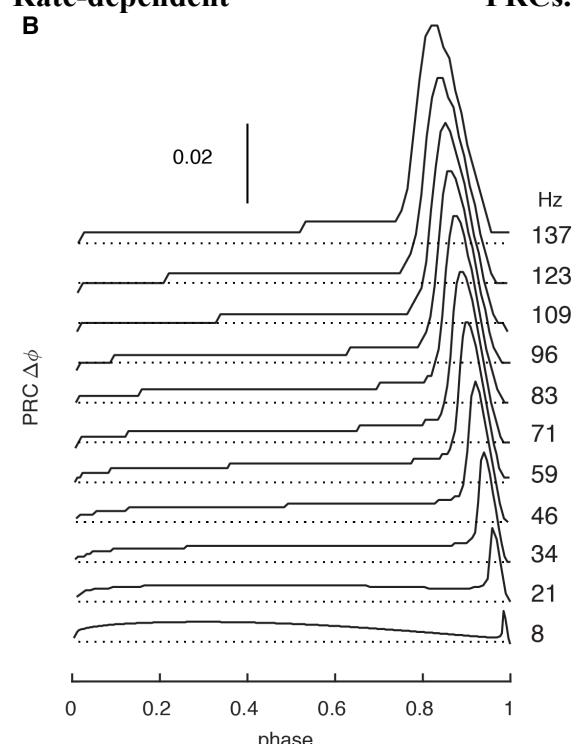
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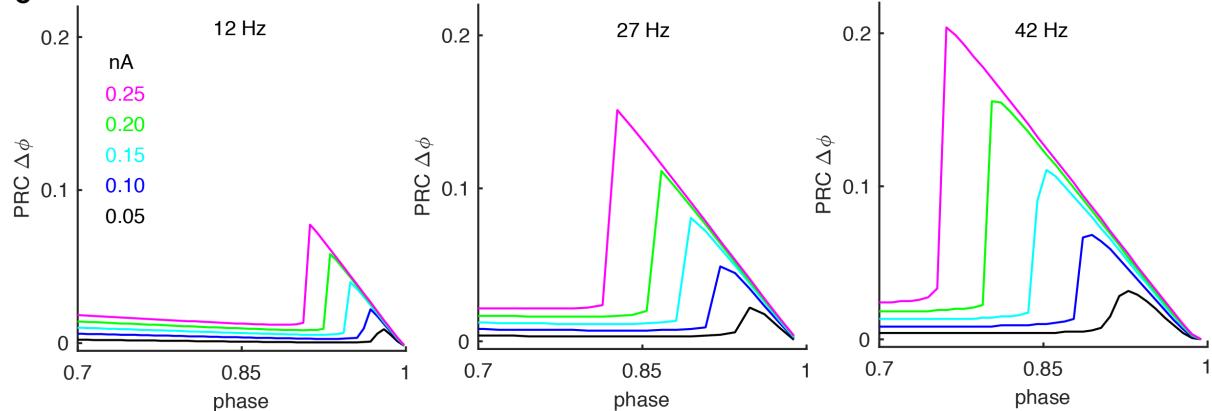
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Rate-dependent



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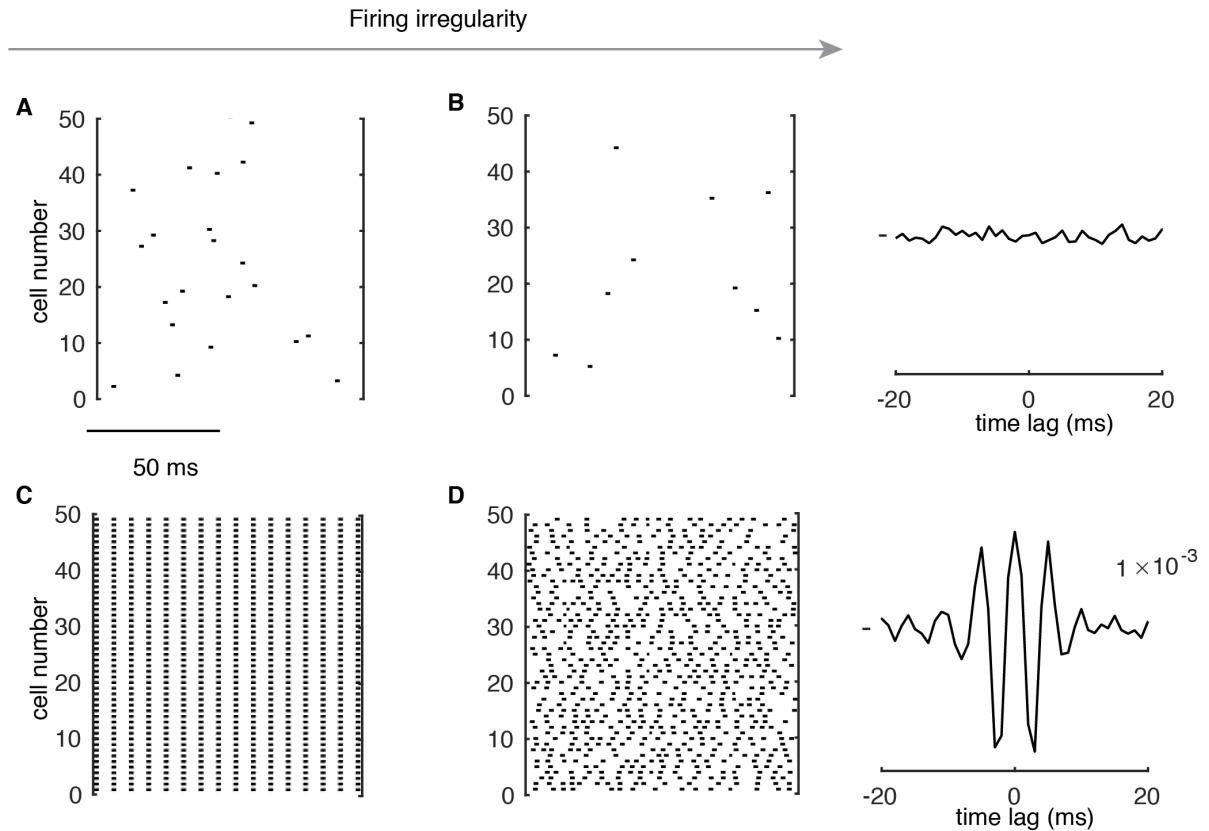
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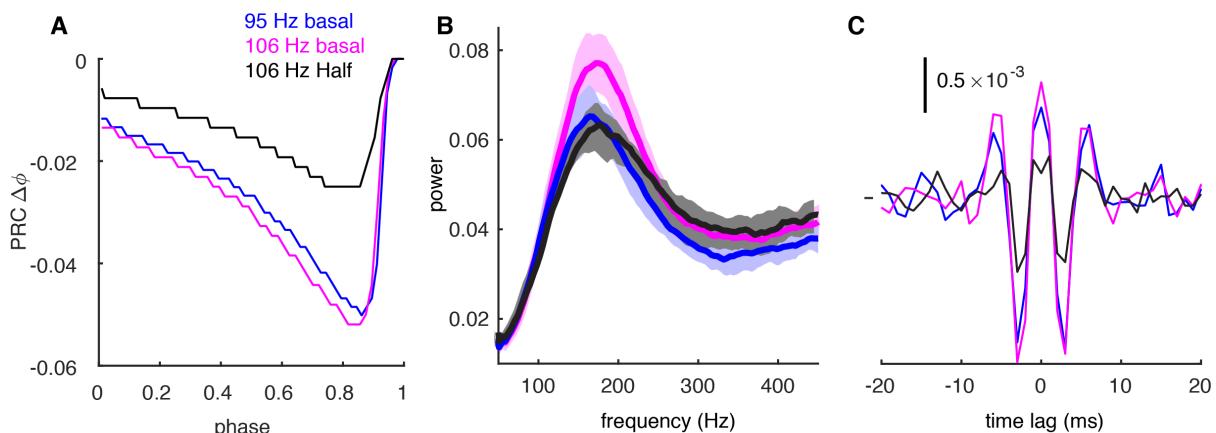
(A) Negative stimulus-triggered responses (phase delay) parallel positive stimuli (see Fig. 1). Onset-phases of phase-dependent responses shift left at high rates with gradually larger amplitudes. (B) The PC model with passive dendrites shows similar rate adaptation as in PC models with active dendrites (Fig. 1). (C) Larger stimulus amplitudes increase the peak of the phase-dependent PRCs and shift their onset phases to the left. Simulation results at rates of 12-, 27- and 42-Hz are shown with increased stimulus amplitudes from 0.05 nA to 0.25 nA.



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Figure S2. Formation of High-frequency Oscillations at High Rates.

(A&B) Raster plots of random PC spikes when they fire regularly (CV ISI ~ 0.07) and irregularly (CV ISI ~ 0.4) at low rates (here, ~ 2 Hz in the network, but 12 Hz in isolated cells). In the right plot of B, average CCG is shown. (C) PCs show spike-to-spike synchrony when they fire regularly (CV ISI ~ 0.02) at high rates (154 Hz). (D) PC spikes show high-frequency oscillations when they fire irregularly (CV ISI ~ 0.44) at high rates (153 Hz). The right plot of D shows the average CCG with a significant central peak and side peaks.



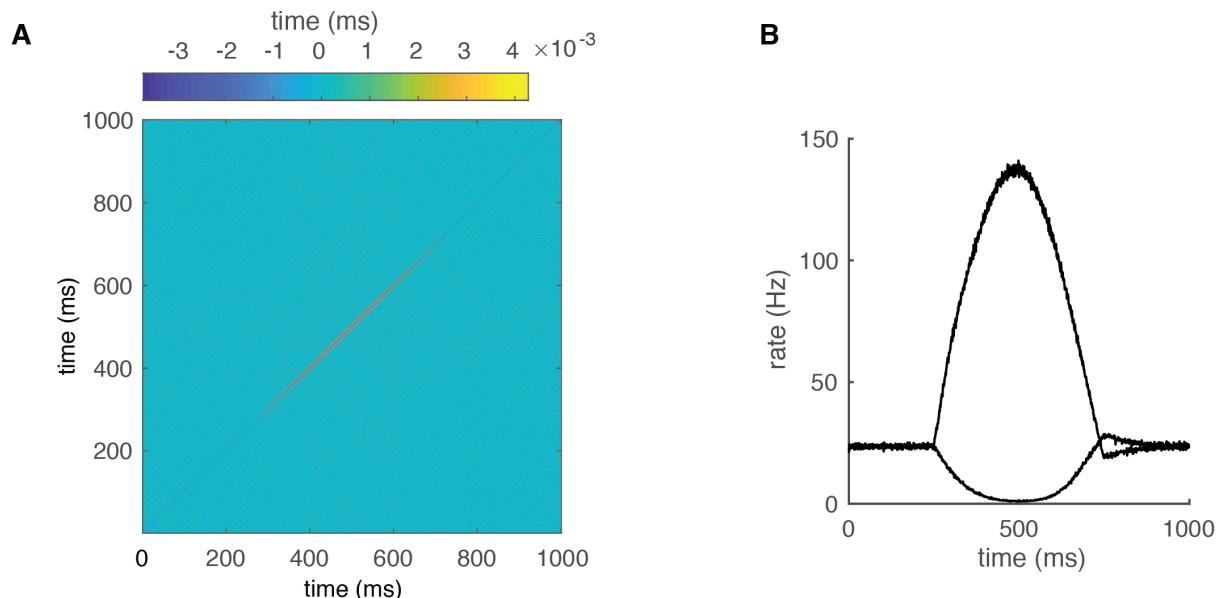
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Figure S3. Decreased PRC at High Firing Rates Can Weaken Oscillations.

A. PRCs for negative stimulus (Fig. 1A) when PCs fire at 95 Hz (blue) and 106 Hz (purple) with basal values of synaptic conductance. Reduced PRC at 106 Hz was achieved by 50% of basal synaptic conductance (black). B. The power spectrum of spike trains with the cellular rate of 95 Hz (blue, basal conductance), 106 Hz (purple, basal conductance) and 113 Hz (black,

659 50% of basal conductance, firing rate increased to 113 Hz due to the reduced inhibition, with
660 other conditions the same as 106 Hz with basal conductance). In all cases, the CV of ISI is 0.45.
661 The power spectrum at high firing rates gets flatter with lower amplitude when the PRC
662 amplitude is reduced (black trace). **C.** The CCGs of spike trains with the same condition as in
663 B. Central and side peaks reduce at high firing rates when phase response is smaller.

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669 **Figure S4. Dynamic Correlations of the PC Network Outputs (Correspond to Fig. 6).**
670 A. the JPSTH used to produce Fig. 6B. B, Population Firing rates of Increased-rate Cells and
671 Decreased-rate Cells for Fig. 6C-E.