

# Evolution of microbial growth traits under serial dilution

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Selection of mutants in a microbial population depends on multiple cellular traits. In serial-dilution evolution experiments, three key traits are the lag time when transitioning from starvation to growth, the exponential growth rate, and the yield (number of cells per unit resource). Here we investigate how these traits evolve in laboratory evolution experiments using a minimal model of population dynamics, where the only interaction between cells is competition for a single limiting resource. We find that the fixation probability of a beneficial mutation depends on a linear combination of its growth rate and lag time relative to its immediate ancestor, even under clonal interference. The relative selective pressure on growth rate and lag time is set by the dilution factor; a larger dilution factor favors the adaptation of growth rate over the adaptation of lag time. The model shows that yield, however, is under no direct selection. We also show how the adaptation speeds of growth and lag depend on experimental parameters and the underlying supply of mutations. Finally, we investigate the evolution of covariation between these traits across populations, which reveals that the population growth rate and lag time can evolve a nonzero correlation even if mutations have uncorrelated effects on the two traits. Altogether these results provide useful guidance to future experiments on microbial evolution.

Laboratory evolution experiments in microbes have provided insight into many aspects of evolution [1–3], such as the speed of adaptation [4], nature of epistasis [5], the distribution of selection coefficients from spontaneous mutations [6], mutation rates [7], the spectrum of adaptive genomic variants [8], and the preponderance of clonal interference [9]. Despite this progress, links between the selection of mutations and their effects on specific cellular traits have remained poorly characterized. Growth traits — such as the lag time when transitioning from starvation to growth, the exponential growth rate, and the yield (resource efficiency) — are ideal candidates for investigating this question. Their association with growth means they have relatively direct connections to selection and population dynamics. Furthermore, high-throughput techniques can measure these traits for hundreds of genotypes and environments [10–13]. Numerous experiments have shown that single mutations can be pleiotropic, affecting multiple growth traits simultaneously [14, 15]. More recent experiments have even measured these traits at the single-cell level, revealing substantial non-genetic heterogeneity [10, 13, 16]. Several evolution experiments have found widespread evidence of adaptation in these traits [17–20]. This data altogether indicates that covariation in these traits is pervasive in microbial populations.

There have been a few previous attempts to develop quantitative models to describe evolution of these traits. For example, Vasi et al. [17] considered data after 2000 generations of evolution in *Escherichia coli* to estimate how much adaptation was attributable to different growth traits. Smith [21] developed a mathematical model to study how different traits would allow strains

to either fix, go extinct, or coexist; Wahl and Zhu [22] focused on how the fate of different trait-affecting mutations was determined by their time of occurrence during the growth cycle. However, simple quantitative results that can be used to interpret experimental data have remained lacking. More recent work [23, 24] derived a quantitative relation between growth traits and selection, showing that selection consists of additive components on the lag and growth phases. However, this did not address the consequences of this selection for evolution, especially the adaptation of trait covariation.

In this work we investigate a model of evolutionary dynamics with covariation across multiple growth traits. We consider a minimal model in which different strains of cells interact only by competition for a single limiting resource. We find that the fixation probability of a mutation, even in the presence of substantial clonal interference, is accurately determined by a linear combination of its change in growth rate and change in lag time relative to its immediate ancestor; the relative weight of these two components is determined by the dilution factor. Yield, on the other hand, is under no direct selection. We provide quantitative predictions for the speed of adaptation of growth rate and lag time as well as their evolved covariation. Specifically, we find that even in the absence of an intrinsic correlation between growth and lag due to mutations, these traits can evolve a nonzero correlation due to selection and variation in number of fixed mutations.

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## METHODS

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### Model of population dynamics

73 We consider a model of asexual microbial cells in a  
 74 well-mixed batch culture, where the only interaction be-  
 75 tween different strains is competition for a single limiting  
 76 resource [23, 24]. Each strain  $i$  is characterized by a lag  
 77 time  $L_i$ , growth rate  $r_i$ , and yield  $Y_i$  (see Fig. 1a for  
 78 a two-strain example). Here the yield is the number of  
 79 cells per unit resource [17]. Note that some of our nota-  
 80 tion differs from related models in previous work, some  
 81 of which used  $g$  for growth rate and  $\lambda$  for lag time [23],  
 82 while others used  $\lambda$  for growth rate [25]. Although it is  
 83 possible to extend the model to account for additional  
 84 growth traits such as a death rate or lag and growth on  
 85 secondary resources, here we focus on the minimal set  
 86 of traits most often measured in microbial phenotyping  
 87 experiments [10–12, 14–16, 18, 26].

88 When the population has consumed all of the initial re-  
 89 source, the population reaches stationary phase with con-  
 90 stant size. The time  $t_c$  at which this occurs is determined  
 91 by equating the total amount of resources consumed by  
 92 the population at that time with the total initial amount  
 93 of resources  $R$ :

$$\sum_{\text{strain } k} \frac{N_0 x_k e^{r_k(t_c - L_k)}}{Y_k} = R, \quad (1)$$

94 where  $N_0$  is the total population size and  $x_k$  is the fre-  
 95 quency of each strain  $k$  at the beginning of the growth  
 96 cycle. In Eq. 1 we assume the time  $t_c$  is longer than  
 97 each strain's lag time  $L_k$ . We define the selection coeffi-  
 98 cient between each pair of strains as the change in their  
 99 log-ratio over the complete growth cycle [27, 28]:

$$s_{ij} = \ln \left( \frac{N_i^{\text{final}}}{N_j^{\text{final}}} \right) - \ln \left( \frac{N_i^{\text{initial}}}{N_j^{\text{initial}}} \right) \quad (2) \\ = r_i(t_c - L_i) - r_j(t_c - L_j),$$

100 where  $N_i^{\text{initial}}$  is the population size of strain  $i$  at the  
 101 beginning of the growth cycle and  $N_i^{\text{final}}$  is the population  
 102 size of strain  $i$  at the end. After the population reaches  
 103 stationary phase, it is diluted by a factor of  $D$  into a fresh  
 104 medium with amount  $R$  of the resource, and the cycle  
 105 repeats (Fig. 1a). We assume the population remains  
 106 in the stationary phase for a sufficiently short time such  
 107 that we can ignore death and other dynamics during this  
 108 phase [29, 30].

109 Over many cycles of growth, as would occur in a lab-  
 110 oratory evolution experiment [1, 28, 31], the population  
 111 dynamics of this system are characterized by the set of  
 112 frequencies  $\{x_k\}$  for all strains as well as the matrix of se-  
 113 lection coefficients  $s_{ij}$  and the total population size  $N_0$  at  
 114 the beginning of each cycle. In Supplementary Methods

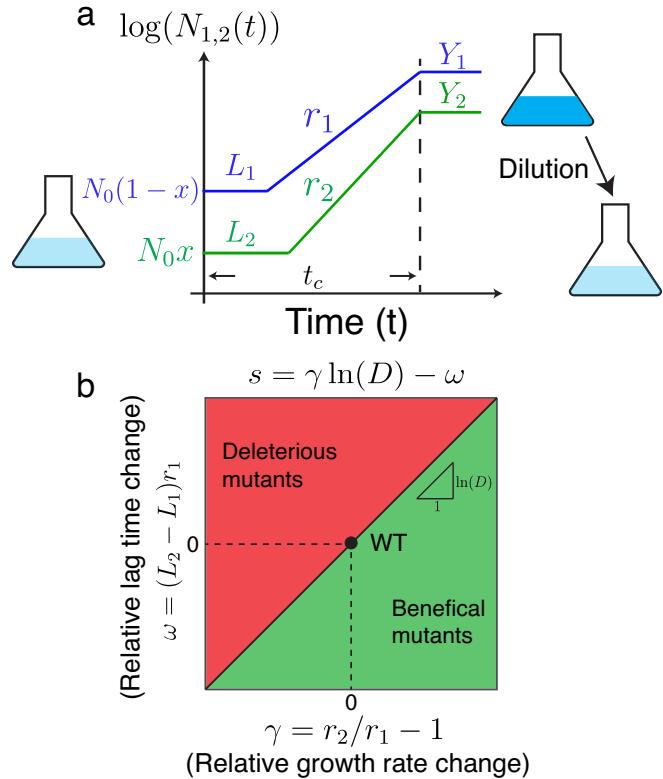


FIG. 1. **Model of selection on multiple microbial growth traits.** (a) Simplified model of microbial population growth characterized by three traits: lag time  $L$ , growth rate  $r$ , and yield  $Y$ . The total initial population size is  $N_0$  and the initial frequency of the mutant (strain 2) is  $x$ . After the whole population reaches stationary phase (time  $t_c$ ), the population is diluted by a factor  $D$  into fresh media, and the cycle starts again. (b) Phase diagram of selection on mutants in the space of their growth rate  $\gamma = r_2/r_1 - 1$  and lag time  $\omega = (L_2 - L_1)r_1$  relative to a wild-type. The slope of the diagonal line is  $\ln D$ .

115 we derive explicit equations for the deterministic dynam-  
 116 ics of these quantities over multiple cycles of growth for  
 117 an arbitrary number of strains. In the case of two strains,  
 118 such as a mutant and a wild-type, the selection coefficient  
 119 is approximately

$$s \approx \gamma \ln D - \omega, \quad (3)$$

120 where  $\gamma = (r_2 - r_1)/r_1$  is the growth rate of the mu-  
 121 tant relative to the wild-type and  $\omega = (L_2 - L_1)r_1$  is the  
 122 relative lag time. The approximation is valid as long as  
 123 the growth rate difference between the mutant and the  
 124 wild-type is small, which is true for most single muta-  
 125 tions [6, 32]. This equation shows that the growth phase  
 126 and lag phase make distinct additive contributions to the  
 127 total selection coefficient, with the dilution factor  $D$  con-  
 128 trolling their relative magnitudes (Fig. 1b). This is be-  
 129 cause a larger dilution factor will increase the amount of  
 130 time the population grows exponentially, hence increas-

ing selection on growth rate. Neutral coexistence between multiple strains is therefore possible if these two selection components balance ( $s = 0$ ), although it requires an exact tuning of the growth traits with the dilution factor (Supplementary Methods) [23, 24]. With a fixed dilution factor  $D$ , the population size  $N_0$  at the beginning of each growth cycle changes according to (Supplementary Methods):

$$N_0 = \frac{R\bar{Y}}{D}, \quad (4)$$

where  $\bar{Y} = (\sum_k x_k/Y_k)^{-1}$  is the effective yield of the whole population in the current growth cycle. In this manner the ratio  $R/D$  sets the bottleneck size of the population, which for serial dilution is approximately the effective population size [31], and therefore determines the strength of genetic drift.

#### Model of evolutionary dynamics

We now consider the evolution of a population as new mutations arise that alter growth traits. We start with a wild-type population having lag time  $L_0 = 100$  and growth rate  $r_0 = (\ln 2)/60 \approx 0.012$ , which are roughly consistent with *E. coli* parameters where time is measured in minutes [17, 31]; we set the wild-type yield to be  $Y_0 = 1$  without loss of generality. As in experiments, we vary the dilution factor  $D$  and the amount of resources  $R$ , which control the relative selection on growth versus lag (set by  $D$ , Eq. 3) and the effective population size (set by  $R/D$ , Eq. 4). We also set the initial population size to its steady state value of  $N_0 = RY_0/D$  (Supplementary Methods).

The population grows according to the dynamics in Fig. 1a. Each cell division can generate a new mutation with probability  $\mu$ , which we set to  $\mu = 10^{-6}$ ; note this rate is only for mutations altering growth traits, and therefore it is lower than the rate of mutations anywhere in the genome. We therefore generate a random waiting time  $\tau_k$  for each strain  $k$  until the next mutation with instantaneous rate  $\mu r_k N_k(t)$ . When a mutation occurs, the growth traits for the mutant are drawn from a distribution  $p_{\text{mut}}(r_2, L_2, Y_2 | r_1, L_1, Y_1)$ , where  $r_1, L_1, Y_1$  are the growth traits for the background strain on which the new mutation occurs and  $r_2, L_2, Y_2$  are the traits for the new mutant. We will assume mutational effects are not epistatic and scale with the trait values of the background strain, so that  $p_{\text{mut}}(r_2, L_2, Y_2 | r_1, L_1, Y_1) = p_{\text{mut}}(\gamma, \omega, \delta)$ , where  $\gamma = (r_2 - r_1)/r_1$ ,  $\omega = (L_2 - L_1)r_1$ , and  $\delta = (Y_2 - Y_1)/Y_1$  (Supplementary Methods). For simplicity, we focus on uniform distributions of mutational effects where  $-0.02 < \gamma < 0.02$ ,  $-0.05 < \omega < 0.05$ , and  $-0.02 < \delta < 0.02$ , but in Supplementary Methods we extend our main results to the case of Gaussian distributions as well. Note that since mutations only arise

during the exponential growth phase, beneficial or deleterious effects on lag time are not realized until the next growth cycle [20]. After the growth cycle ceases (once the resource is exhausted according to Eq. 1), we randomly choose cells, each with probability  $1/D$ , to form the population for the next growth cycle.

## RESULTS

### Fixation of mutations

We first consider the fixation statistics of new mutations in our model. In Fig. 2a we show the relative growth rates  $\gamma$  and relative lag times  $\omega$  of fixed mutations, along with contours of constant selection coefficient  $s$  from Eq. 3. As expected, fixed mutations all increase growth rate ( $\gamma > 0$ ), decrease lag time ( $\omega < 0$ ), or both. In contrast, the yield of fixed mutations is the same as the ancestor on average (Fig. 2d); indeed, the selection coefficient in Eq. 3 does not depend on the yields (Supplementary Methods). If a mutation arises with significantly higher or lower yield than the rest of the population, the bottleneck population size  $N_0$  immediately adjusts to keep the overall fold-change of the population during the growth cycle fixed to the dilution factor  $D$ . Therefore mutations that significantly change yield have no effect on the overall population dynamics.

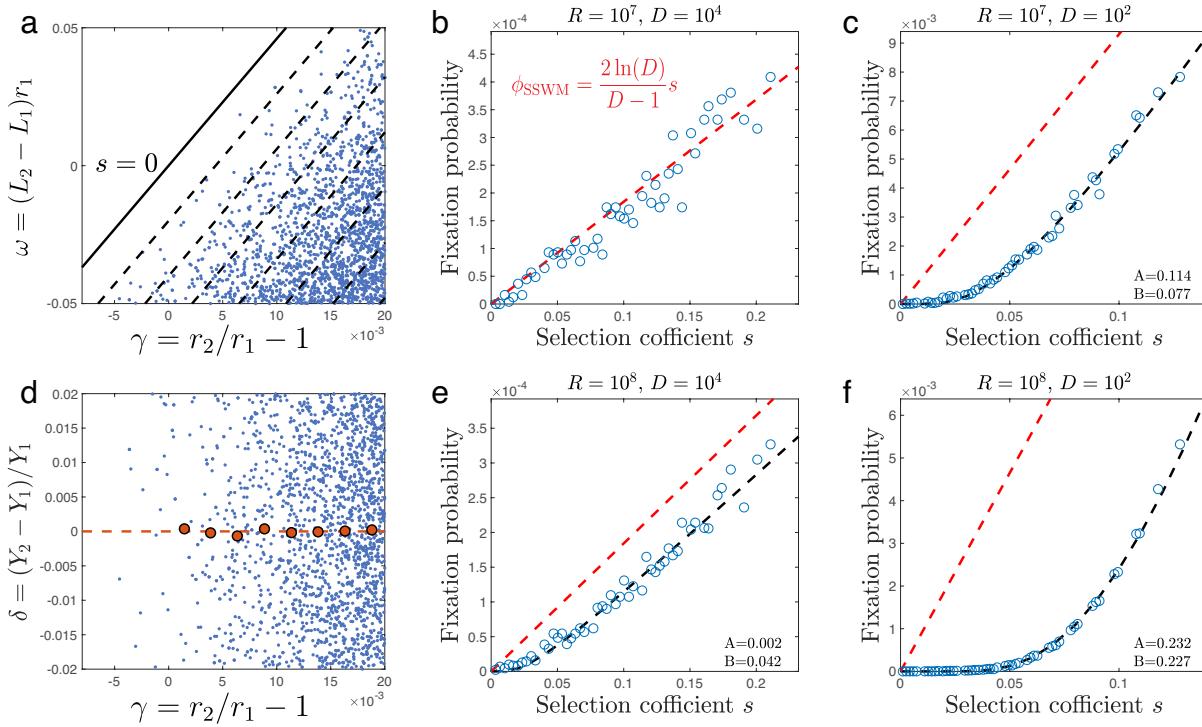
Figure 2a also suggests that the density of fixed mutations depends only on their selection coefficient  $s$ , rather than their individual combination of traits. We therefore plot the fixation probabilities of mutations as functions of their selection coefficients calculated by Eq. 3 (Fig. 2 b,c,e,f). We test this over a range of population dynamics regimes by varying the dilution factor  $D$  and the amount of resources  $R$ . For small populations, mutations generally arise and either fix or go extinct one at a time, a regime known as “strong-selection weak-mutation” (SSWM) [33]. In this case, we expect the fixation probability of a beneficial mutation with selection coefficient  $s > 0$  to be [22, 34, 35]

$$\phi_{\text{SSWM}}(s) = \frac{2 \ln D}{D - 1} s. \quad (5)$$

This is similar to the standard Wright-Fisher fixation probability of  $2s$  [36], but with a correction to account for the fact that the mutation can arise at different times in the exponential growth phase. Indeed, we see this predicted dependence matches the simulation results for the small population size of  $N_0 \sim R/D = 10^3$  (Fig. 2b).

For larger populations, multiple beneficial mutations will be present simultaneously in the population and interfere with each other, an effect known as clonal interference [37, 38]. We find that the fixation probability of a mutant in this clonal interference regime is well fit by

$$\phi_{\text{CI}}(s) = A s e^{-B/s}, \quad (6)$$



**FIG. 2. Selection coefficient determines fixation probability.** (a) The relative growth rate  $\gamma$  and lag time  $\omega$  of fixed mutations. Dashed lines mark contours of constant selection coefficient, while the solid line marks  $s = 0$ . (d) Same as (a) but for relative growth rate  $\gamma$  and relative yield  $\delta$ . The red dots mark the relative yield of fixed mutations averaged over binned values of the relative growth rate  $\gamma$ . In (a) and (d),  $D = 10^2$  and  $R = 10^7$ . (b,c,e,f) Fixation probability of mutations against their selection coefficient for different amounts of resource  $R$  and dilution factors  $D$  as indicated in the titles. The red dashed line shows the fixation probability predicted in the SSWM regime (Eq. 5), while the black line shows a numerical fit of the data points to the fixation probability under clonal interference (Eq. 6), with the resulting fitting parameters  $A$  and  $B$  shown in the lower right corner of each panel. In all panels mutations randomly arise from a uniform distribution  $p_{\text{mut}}$  (Supplementary Methods).

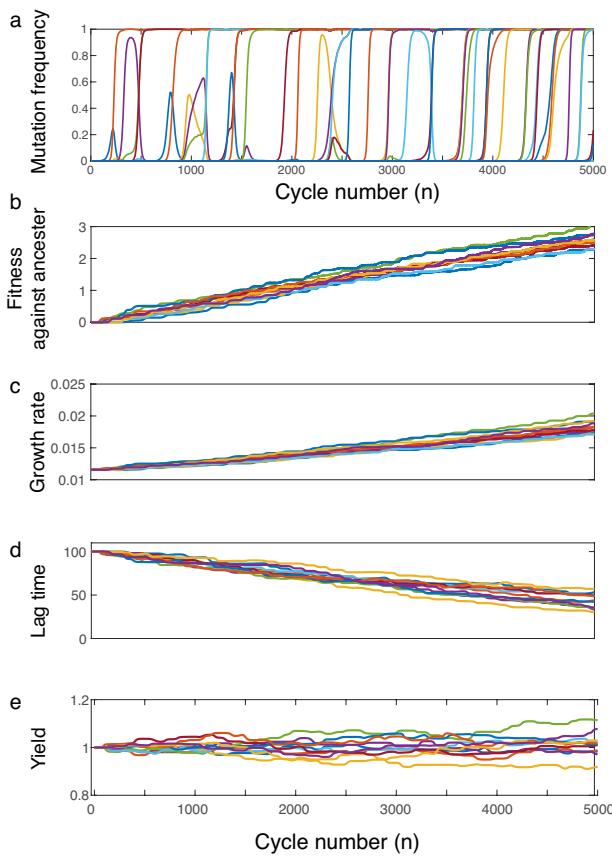
229 where  $A$  and  $B$  are two constants that depend on other  
 230 parameters of the population; we treat these as empirical  
 231 parameters to fit to the simulation results, although Ger-  
 232 rish and Lenski [37] predicted  $A = 2 \ln D / (D - 1)$ , i.e.,  
 233 the same constant as in the SSWM case (Eq. 5). Concep-  
 234 tually, this means that interference from other beneficial  
 235 mutations suppresses the SSWM fixation probability by  
 236 an exponential factor, where the  $1/s$  term comes from  
 237 the time between the appearance of mutation and its fix-  
 238 ation [37]. Equation 6 matches our simulation results well  
 239 for larger population sizes  $N_0 \sim R/D > 10^4$  (Fig. 2c,e,f).  
 240 Furthermore, the constant  $A$  we fit to the simulation data  
 241 is indeed close to the predicted value of  $2 \ln D / (D - 1)$ ,  
 242 except in the most extreme case of  $N_0 \sim R/D = 10^6$   
 243 (Fig. 2f).

244 Altogether Fig. 2 shows that mutations with different  
 245 effects on cell growth — for example, a mutant that in-  
 246 creases growth rate and a mutant that decreases lag time  
 247 — can nevertheless have the same fixation probability as  
 248 long as their overall effect on selection is the same ac-  
 249 cording to Eq. 3. In Supplementary Methods we show  
 250 that these results also hold for a Gaussian distribution of  
 251 mutational effects  $p_{\text{mut}}(\gamma, \omega, \delta)$ , including the presence of

252 correlated mutational effects (Fig. S1). While the depen-  
 253 dence of fixation probability on the selection coefficient is  
 254 a classic result of population genetics [39], the existence  
 255 of a simple relationship here is nontrivial since, strictly  
 256 speaking, selection in this model is not only frequency-  
 257 dependent [23] (i.e., selection between two strains de-  
 258 pends on their frequencies) but also includes higher-order  
 259 effects [24] (i.e., selection between strain 1 and strain 2  
 260 is affected by the presence of strain 3). Therefore in  
 261 principle, the fixation probability of a mutant may de-  
 262 pend on the specific state of the population in which it  
 263 is present, while the selection coefficient in Eq. 3 only  
 264 describes selection on the mutant in competition with its  
 265 immediate ancestor. However, we see that, at least for  
 266 the parameters considered in our simulations, these ef-  
 267 fects are negligible in determining the eventual fate of a  
 268 mutation.

#### Adaptation of growth traits

269 As Fig. 3a shows, many mutations arise and fix over  
 270 the timescale of our simulations, which lead to pre-



**FIG. 3. Dynamics of evolving populations.** (a) Frequencies of new mutations as a function of the number  $n$  of growth cycles. Example trajectories of (b) the fitness of the evolved population relative to the ancestral population, (c) the evolved population growth rate, (d) the evolved population lag time, and (e) the evolved population yield. In all panels the dilution factor is  $D = 10^2$ , the amount of resource at the beginning of each cycle is  $R = 10^7$ , and mutations randomly arise from a uniform distribution  $p_{\text{mut}}$  (Supplementary Methods).

dictable trends in the quantitative traits of the population. We first determine the relative fitness of the population against the ancestral strain by simulating competition between an equal number of evolved and ancestral cells for one cycle, analogous to common experimental measurements [1, 31]. The resulting fitness trajectories are shown in Fig. 3b. To see how different traits contribute to the fitness increase, we also calculate the average population traits at the beginning of each cycle, e.g.,  $r_{\text{pop}}(n) = \sum_i r_i/N_0(n)$  (where the sum is over all cells), as a function of the number  $n$  of growth cycles. As expected from Eq. 3, the average growth rate increases (Fig. 3c) and the average lag time decreases (Fig. 3d) for all simulations. In contrast, the average yield evolves without apparent trend (Fig. 3e), since Eq. 3 indicates no direct selection on yield.

Figure 3 suggests relatively constant speeds of adapta-

tion for relative fitness, growth rate, and lag time. For example, we can calculate the adaptation speed of growth rate as the average change in population growth rate per cycle:

$$W_{\text{growth}} = \langle r_{\text{pop}}(n+1) - r_{\text{pop}}(n) \rangle, \quad (7)$$

where the bracket denotes an average over replicate populations and cycle number. In Supplementary Methods we calculate the adaptation speeds of these traits in the SSWM regime to be

$$\begin{aligned} W_{\text{growth}} &= \sigma_{\gamma}^2 r_0 (\ln D) \left( \frac{\mu R Y_0 \ln D}{D - 1} \right), \\ W_{\text{lag}} &= -\frac{\sigma_{\omega}^2}{r_0} \left( \frac{\mu R Y_0 \ln D}{D - 1} \right), \\ W_{\text{fitness}} &= \frac{W_{\text{growth}}}{r_0} \ln D - W_{\text{lag}} r_0, \end{aligned} \quad (8)$$

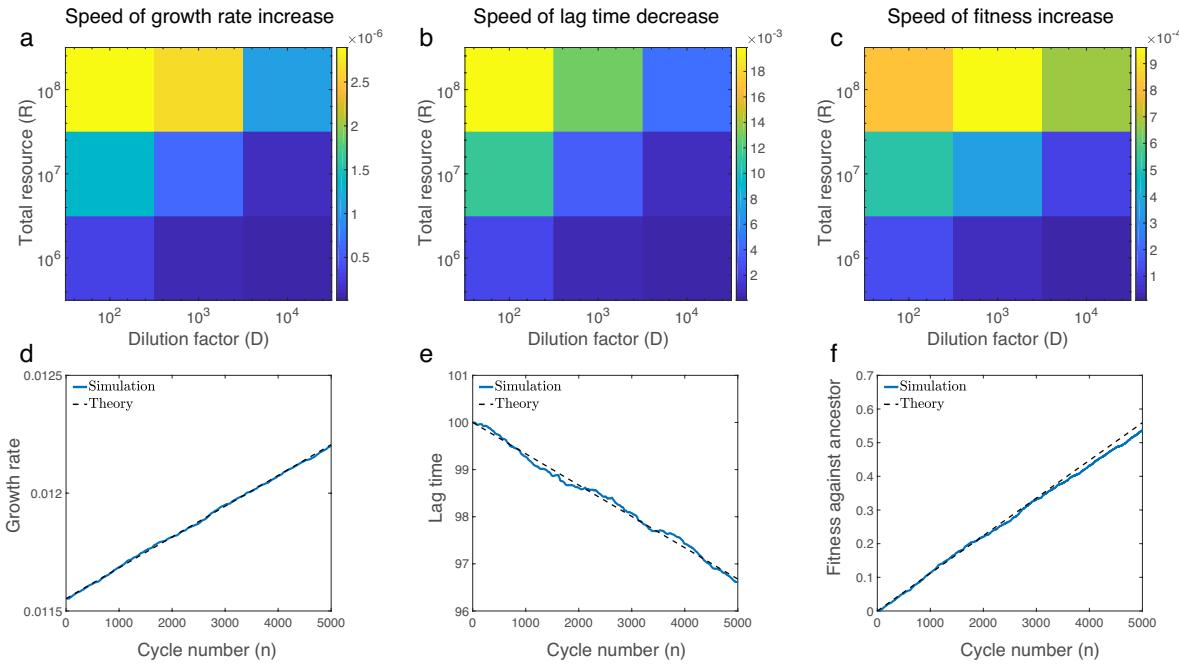
where  $\sigma_{\gamma}$  and  $\sigma_{\omega}$  are the standard deviations of the underlying distributions of  $\gamma$  and  $\omega$  for single mutations ( $p_{\text{mut}}(\gamma, \omega, \delta)$ ), and  $r_0$  is the ancestral growth rate and  $Y_0$  the ancestral yield (which we assume does not change on average according to Fig. 3e). Note the adaptation speeds are proportional to the variances of the traits, which is formally similar to the multivariate breeder's equation [40], Fisher's fundamental theorem of natural selection [41], and the Price equation [42]; however, in our case these are variances across single mutants in the SSWM regime, rather than variances of traits within a population. Furthermore, the ratio of growth and lag adaptation rates is independent of the amount of resource and mutation rate in the SSWM regime:

$$\frac{W_{\text{growth}}}{W_{\text{lag}}} = -r_0^2 \frac{\sigma_{\gamma}^2}{\sigma_{\omega}^2} \ln D. \quad (9)$$

Equation 8 predicts that the adaptation speeds of growth rate, lag time, and relative fitness should all increase with the amount of resources  $R$  and decrease with the dilution factor  $D$  (if  $D$  is large); even though this prediction assumes the SSWM regime (relatively small  $N_0 \sim R/D$ ), it nevertheless holds across a wide range of  $R$  and  $D$  values (Fig. 4a,b,c), except for  $R = 10^8$  where the speed of fitness increase is non-monotonic with  $D$  (Fig. 4c). The predicted adaptation speeds in Eq. 8 also quantitatively match the simulated trajectories in the SSWM case (Fig. 4d,e,f); even outside of the SSWM regime, the relative rate in Eq. 9 remains a good prediction at early times (Fig. S2).

### Evolved covariation between growth traits

We now turn to investigating how the covariation between traits evolves. We have generally assumed that individual mutations have uncorrelated effects on different



**FIG. 4. Speed of adaptation.** The average per-cycle adaptation speed of (a) growth rate, (b) lag time, and (c) fitness relative to the ancestral population as functions of the dilution factor  $D$  and total amount of resources  $R$ . The adaptation speeds are averaged over growth cycles and independent populations. Average population values of (d) growth rate, (e) lag time, and (f) fitness relative to the ancestral population as functions of the number  $n$  of growth cycles. The dilution factor is  $D = 10^4$  and the total resource is  $R = 10^7$ , so the population is in the SSWM regime. The blue solid lines are simulation results, while the dashed lines show the mathematical predictions in Eq. 8. All panels show averages over 500 independent simulated populations, with mutations randomly arising from a uniform distribution  $p_{\text{mut}}$  in which  $-0.02 < \gamma < 0.02$ ,  $-0.05 < \omega < 0.05$ ,  $-0.02 < \delta < 0.02$  (Supplementary Methods).

328 traits (Supplementary Methods). Nevertheless, selection 329 may induce a correlation between these traits in evolved 330 populations. In Fig. 5a we schematically depict how the 331 raw variation of traits from mutations is distorted by se- 332 lection and fixation of multiple mutations. Specifically, 333 for a single fixed mutation, selection induces a positive 334 (i.e., antagonistic) correlation between growth rate and 335 lag time. Figure 2a shows this for single fixed mutations, 336 while Fig. 5b, c shows this positive correlation for popu- 337 lations that have accumulated the same number of fixed 338 mutations. We can calculate the Pearson correlation co- 339 efficient from the covariation of growth effects  $\gamma$  and lag 340 effects  $\omega$  for a single fixed mutation:

$$\rho_{\text{fixed}} = \frac{\langle \gamma \omega \rangle_{\text{fixed}} - \langle \gamma \rangle_{\text{fixed}} \langle \omega \rangle_{\text{fixed}}}{\sqrt{(\langle \gamma^2 \rangle_{\text{fixed}} - \langle \gamma \rangle_{\text{fixed}}^2)(\langle \omega^2 \rangle_{\text{fixed}} - \langle \omega \rangle_{\text{fixed}}^2)}}, \quad (10)$$

341 where  $\langle \cdot \rangle_{\text{fixed}}$  is an average over the distribution of sin- 342 gle fixed mutations (Supplementary Methods). We can 343 explicitly calculate this quantity in the SSWM regime, 344 which confirms that it is positive for uncorrelated muta- 345 tional effects (Supplementary Methods).

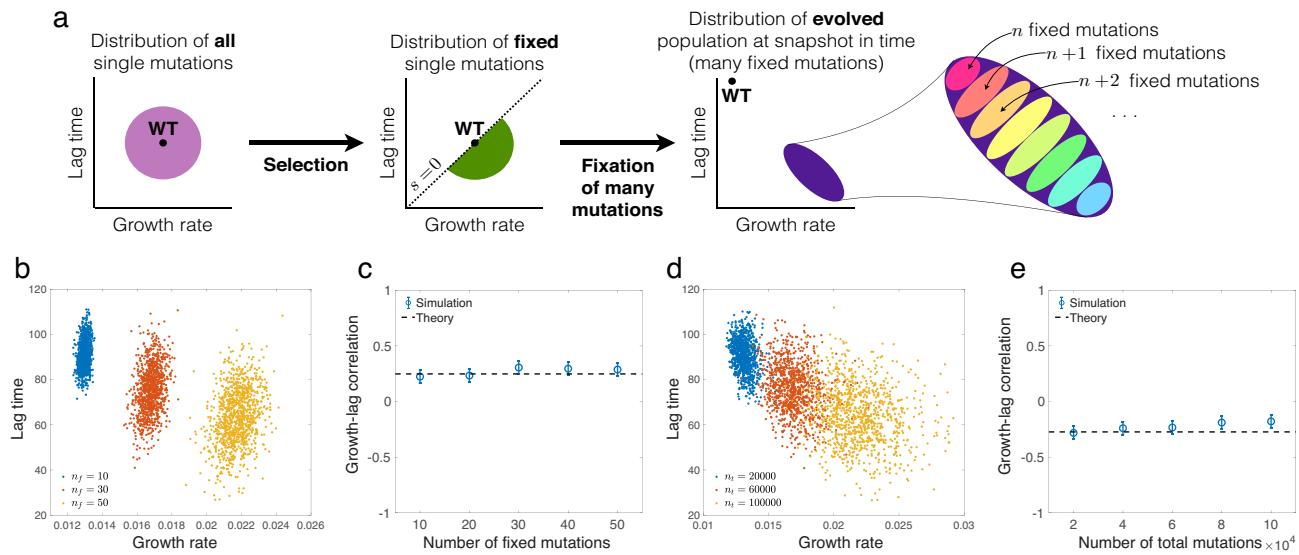
346 However, in evolution experiments we typically observe 347 populations at a particular snapshot in time, such that 348 the populations may have a variable number of fixed mu- 349 tations but the same number of total mutations that

350 arose (and either fixed or went extinct). Interestingly, 351 the variation in number of fixed mutations at a snapshot 352 in time causes the distribution of growth rates and lag 353 times across populations to stretch into a negative cor- 354 relation; this is an example of Simpson's paradox from 355 statistics [43]. Figure 5a shows this effect schematically, 356 while Fig. 5d,e show explicit results from simulations. 357 In Supplementary Methods, we calculate this evolved 358 Pearson correlation coefficient across populations in the 359 SSWM regime to be approximately

$$\rho_{\text{evo}} \approx \frac{\langle \gamma \omega \rangle_{\text{fixed}}}{\sqrt{\langle \gamma^2 \rangle_{\text{fixed}} \langle \omega^2 \rangle_{\text{fixed}}}}. \quad (11)$$

360 That is, the correlation of traits across populations with 361 multiple mutations is still a function of the distribution 362 of single fixed mutations, but it is not equal to the corre- 363 lation of single fixed mutations (Eq. 10). Indeed, in Sup- 364 plementary Methods we explicitly calculate this quantity 365 in the SSWM regime and show that it must be negative 366 for uncorrelated mutational effects.

367 The predicted correlations in Eqs. 10 and 11 quantita- 368 tively match the simulations well in the SSWM regime 369 (Fig. 5c,e). While they are less accurate outside of the 370 SSWM regime, they nevertheless still produce the correct 371 sign of the evolved correlation (Fig. S3a,b,c). However,



**FIG. 5. Evolved patterns of covariation among growth traits.** (a) Schematic of how selection and fixation of multiple mutations shape the observed distribution of traits. The sign of the Pearson correlation coefficient between the average growth rate and lag time depends on whether we consider an ensemble of populations with the same number of fixed mutations or the same number of total mutation events. (b) Distribution of average growth rate and lag time for 1000 independent populations with the same number of fixed mutations. Each color corresponds to a different number of fixed mutations ( $n_f$ ) indicated in the legend. (c) Pearson correlation coefficient of growth rate and lag time for distributions in panel (b) as a function of the number of fixed mutations. The dashed line is the prediction from Eq. 10 (Supplementary Methods). (d) Same as (b) except each color corresponds to a set of populations at a snapshot in time with the same number of total mutation events. Each color corresponds to a different number of total mutations events ( $n_t$ ) indicated in the legend. (e) Same as (c) but for the set of populations shown in (d). The dashed line is the prediction from Eq. 11 (Supplementary Methods). In (c) and (e) the error-bars represent 95% confidence intervals. In both (b) and (d), we consider the SSWM regime with  $D = 10^3$ .

372 the signs of the correlations can indeed change depend-  
 373 ing on the underlying distribution of mutational effects  
 374  $p_{\text{mut}}(\gamma, \omega, \delta)$ . For example, in Supplementary Methods  
 375 we explore the effects of varying the mean mutational  
 376 effects (Fig. S3d) — e.g., whether an average mutation  
 377 has positive, negative, or zero effect on growth rate —  
 378 as well as the intrinsic mutational correlation between  
 379 growth and lag (Fig. S3e).

## 380 DISCUSSION

381 We have investigated a model of microbial evolution  
 382 under serial dilution, which is both a common protocol  
 383 for laboratory evolution experiments [1, 6, 31, 44, 45]  
 384 as well as a rough model of evolution in natural envi-  
 385 ronments with feast-famine cycles. While there has been  
 386 extensive work to model population and evolutionary dy-  
 387 namics in these conditions [2, 34, 35, 37], these models  
 388 have largely neglected the physiological links connecting  
 389 mutations to selection. However, models that explicitly  
 390 incorporate these features are necessary to interpret ex-  
 391 perimental evidence that mutations readily generate vari-  
 392 ation in multiple cellular traits, and that this variation  
 393 is important to adaptation [17–20].

394 In this paper, we have studied a model where muta-  
 395 tions can affect three quantitative growth traits — the

396 lag time, exponential growth rate, and yield (Fig. 1a) —  
 397 since these three traits are widely measured for micro-  
 398 bial populations. In particular, we have derived a simple  
 399 expression (Eq. 3) for the selection coefficient of a mu-  
 400 tation in terms of its effects on growth and lag and a  
 401 single environmental parameter, the dilution factor  $D$ .  
 402 While previous work showed that this selection coeffi-  
 403 cient determines the fixation probability of a single mu-  
 404 tation in the SSWM regime [23], here we have shown  
 405 that this holds even in the presence of clonal interference  
 406 (Fig. 2b,c,e,f), which appears to be widespread in these  
 407 experiments [9, 28, 46]. Our result is therefore valuable  
 408 for interpreting the abundant experimental data on mu-  
 409 tant growth traits. We have also calculated the adapta-  
 410 tion rates of growth traits per cycle in the SSWM regime,  
 411 which turn out to increase with the amount of resource  
 412  $R$  and decrease with the dilution factor  $D$ . These results  
 413 are confirmed by numerical simulations and remain good  
 414 predictions even outside of the SSWM regime.

415 An important difference with the previous work on this  
 416 model is that here we used a fixed dilution factor  $D$ ,  
 417 which requires that the bottleneck population size  $N_0$   
 418 fluctuates. In contrast, previous work used a fixed  $N_0$   
 419 and variable  $D$  [23, 24]. We observed two important dif-  
 420 ferences between these regimes. First, in the case of fixed  
 421  $N_0$  and variable  $D$ , the fold-change of the population dur-  
 422 ing a single growth cycle ( $R\bar{Y}/N_0$ ) determines the rela-

423 tive selection between growth and lag, since it determines  
424 how long the population undergoes exponential growth.  
425 Therefore one can experimentally tune this relative selec-  
426 tion by varying either the total amount of resources  $R$  or  
427 the fixed bottleneck size  $N_0$ . However, when the dilution  
428 factor  $D$  is fixed, the population fold-change is always  
429 constrained to exactly equal  $D$  (Eq. 4; Supplementary  
430 Methods), and therefore  $D$  alone determines the relative  
431 selection on growth and lag (Eq. 3). The second differ-  
432 ence is that, with fixed  $N_0$  and variable  $D$ , the selection  
433 coefficient depends explicitly on the effective yield  $\bar{Y}$  and  
434 is therefore frequency-dependent (Supplementary Meth-  
435 ods), which enables the possibility of stable coexistence  
436 between two strains [23, 24]. However, for the fixed  $D$   
437 case, the frequency dependence of  $\bar{Y}$  is exactly canceled  
438 by  $N_0$  (Eq. 4). Therefore there is only neutral coexis-  
439 tence in this case, requiring the growth and lag traits of  
440 the strains to follow an exact constraint set by  $D$  (Sup-  
441 plementary Methods).

442 A major result of our model is a prediction on the  
443 evolution of covariation between growth traits. In par-  
444 ticular, we have shown that correlations between traits  
445 can emerge from selection and accumulation of multiple  
446 mutations even without an intrinsic correlation between  
447 traits from individual mutations (Figs. 5 and S3). We  
448 have also shown that selection alone produces no corre-  
449 lation between growth and yield, in the absence of corre-  
450 lated mutational effects (Figs. 2d and 3e). This is impor-  
451 tant for interpreting evolved patterns of traits in terms  
452 of selective or physiological tradeoffs. Specifically, it em-  
453 phasizes that the evolved covariation between traits con-  
454 flates both the underlying supply of variation from muta-  
455 tions as well as the action of selection and other aspects  
456 of population dynamics (e.g., genetic drift, spatial struc-  
457 ture, recombination), and therefore it is difficult to make  
458 clear inferences about either aspect purely from the out-  
459 come of evolution alone. For example, simply observing a  
460 negative correlation between two traits from evolved pop-  
461 ulations is insufficient to infer whether that correlation  
462 is due to a physiological constraint on mutations (e.g.,  
463 mutations cannot improve both traits simultaneously) or  
464 due to a selective constraint (e.g., selection favors spe-  
465 cialization in one trait or another).

466 These questions, of course, have been the foundation of  
467 quantitative trait genetics [47]. Historically this field has  
468 emphasized polymorphic populations with abundant re-  
469 combination as are applicable to plant and animal breed-  
470 ing. However, this regime is quite different from micro-  
471 bial populations which, at least under laboratory con-  
472 ditions, are often asexual and dominated by linkage be-

473 tween competing mutations [9, 28, 46]. We therefore need  
474 a quantitative description of both between-population as  
475 well as within-population covariation of traits of micro-  
476 bial populations in this regime. Recent work has de-  
477 veloped some mathematical and simulation results along  
478 these lines [48–51], but so far it has not been applied to  
479 specific microbial traits.

480 Microbial growth traits should indeed be an ideal set-  
481 ting for this approach due to abundant data, but con-  
482 clusions on the nature of trait covariation have remained  
483 elusive. Physiological models have predicted a negative  
484 correlation between growth rate and lag time across geno-  
485 types [52, 53], while models of single-cell variation in lag  
486 times also suggests there should be a negative correla-  
487 tion at the whole-population level [54]. However, ex-  
488 perimental evidence has been mixed, with some stud-  
489 ies finding a negative correlation [13, 16], while others  
490 found no correlation [10, 11, 14]. Studies of growth-yield  
491 correlations have long been motivated by  $r/K$  selection  
492 theory, which suggests there should be tradeoffs between  
493 growth rate and yield [55]. For instance, metabolic mod-  
494 els make this prediction [56–58]. However, experimental  
495 evidence has again been mixed, with some data show-  
496 ing a tradeoff [26, 59, 60], while others show no correla-  
497 tion [15, 18, 19, 61] or even a positive correlation [11, 44].  
498 Some of this ambiguity may have to do with dependence  
499 on the environmental conditions [19] or the precise defin-  
500 ition of yield. We define yield as the proportionality con-  
501 stant of population size to resource (Eq. 1) and neglect  
502 any growth rate dependence on resource concentration.  
503 Under these conditions, we predict no direct selection  
504 on yield, which means that the only way to generate a  
505 correlation of yield with growth rate is if the two traits  
506 are constrained at the physiological level, so that muta-  
507 tional effects are correlated. In such cases higher yield  
508 could evolve but only as a spandrel [62, 63]. Ultimately,  
509 we believe more precise single-cell measurements of these  
510 traits, both across large unselected mutant libraries as  
511 well as evolved strains, are necessary to definitively test  
512 these issues.

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