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3 **Metabolic Perceptrons for Neural Computing in Biological Systems**  
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5 Amir Pandi<sup>1\*</sup>, Mathilde Koch<sup>1\*</sup>, Peter L Voyvodic<sup>2</sup>, Paul Soudier<sup>1,3</sup>, Jerome Bonnet<sup>2</sup>, Manish  
6 Kushwaha<sup>1†</sup>, and Jean-Loup Faulon<sup>1,3,4†</sup>  
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10 <sup>1</sup> Micalis Institute, INRA, AgroParisTech, Université Paris-Saclay, Jouy-en-Josas, France

11 <sup>2</sup> Centre de Biochimie Structurale, INSERM U1054, CNRS UMR 5048, University of Montpellier,  
12 Montpellier, France

13 <sup>3</sup> iSSB Laboratory, Génomique Métabolique, Genoscope, Institut François Jacob, CEA, CNRS, Univ  
14 Evry, Université Paris-Saclay, 91057 Evry, France

15 <sup>4</sup> SYNBIOCHEM Center, School of Chemistry, University of Manchester, Manchester, UK  
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18 \*Equal contributions

19 †To whom correspondence should be addressed: [jean-loup.faulon@inra.fr](mailto:jean-loup.faulon@inra.fr) or [manish.kushwaha@inra.fr](mailto:manish.kushwaha@inra.fr)  
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22 **Abstract**

23 Synthetic biological circuits are promising tools for developing sophisticated systems for medical,  
24 industrial, and environmental applications. So far, circuit implementations commonly rely on gene  
25 expression regulation for information processing using digital logic. Here, we present a new  
26 approach for biological computation through metabolic circuits designed by computer-aided tools,  
27 implemented in both whole-cell and cell-free systems. We first combine metabolic transducers to  
28 build an analog adder, a device that sums up the concentrations of multiple input metabolites.  
29 Next, we build a weighted adder where the contributions of the different metabolites to the sum  
30 can be adjusted. Using a computational model trained on experimental data, we finally implement  
31 two four-input “perceptrons” for desired binary classification of metabolite combinations by  
32 applying model-predicted weights to the metabolic perceptron. The perceptron-mediated neural  
33 computing introduced here lays the groundwork for more advanced metabolic circuits for rapid  
34 and scalable multiplex sensing.

1

## 2 **Introduction**

3 Living organisms are information-processing systems that integrate multiple input signals,  
4 perform computations on them, and trigger relevant outputs. The multidisciplinary field of  
5 synthetic biology has combined their information-processing capabilities with modular  
6 and standardized engineering approaches to design sophisticated sense-and-respond  
7 behaviors<sup>1-3</sup>. Due to similarities in information flow in living systems and electronic  
8 devices<sup>4</sup>, circuit design for these behaviors has often been inspired by electronic circuitry,  
9 with substantial efforts invested in implementing logic circuits in living cells<sup>4-6</sup>.  
10 Furthermore, synthetic biological circuits have been used for a range of applications  
11 including biosensors for detection of pollutants<sup>7,8</sup> and medically-relevant biomarkers<sup>9,10</sup>,  
12 smart therapeutics<sup>11,12</sup>, and dynamic regulation and screening in metabolic  
13 engineering<sup>13,14</sup>.

14

15 Synthetic circuits can be implemented at different layers of biological information  
16 processing, such as: (i) the genetic layer comprising transcription<sup>15</sup> and translation<sup>16</sup>, (ii)  
17 the metabolic layer comprising enzymes<sup>17,18</sup>, and (iii) the signal transduction layer  
18 comprising small molecules and their receptors<sup>19,20</sup>. Most designs implemented thus far  
19 have focused on the genetic layer, developing circuits that perform computations using  
20 elements such as feedback control<sup>21</sup>, memory systems<sup>22,23</sup>, amplifiers<sup>24,25</sup>, toehold  
21 switches<sup>26</sup>, or CRISPR machinery<sup>27,28</sup>. However, gene expression regulation is not the  
22 only way through which cells naturally perform computation. In nature, cells carry out  
23 parts of their computation through metabolism, receiving multiple signals and distributing  
24 information fluxes to metabolic, signaling, and regulatory pathways<sup>17,29,30</sup>. Integrating  
25 metabolism into synthetic circuit design can expand the range of input signals and  
26 communication wires used in biological circuits, while bypassing some limitations of  
27 temporal coordination of gene expression cascades<sup>31,32</sup>.

28

29 The number of inputs processed by synthetic biological circuits has steadily increased  
30 over the years, including physical inputs like heat, light, and small molecules such as  
31 oxygen, IPTG, aTc, arabinose and others<sup>21,33-36</sup>. However, most of these circuits process  
32 input signals using digital logic, which despite its ease of implementation lacks the power  
33 that analog logic can offer<sup>1,37,38</sup>. The power of combining digital and analog processing is  
34 exemplified by the “perceptron”, the basic block of artificial neural networks inspired by  
35 human neurons<sup>39</sup> that can, for instance, be trained on labelled input datasets to perform  
36 binary classification. After the training, the perceptron computes the weighted sum of  
37 input signals (analog computation) and makes the classification decision (digital  
computation) after processing it through an activation function.

38

39 Here we describe the development of complex metabolic circuitry implemented using  
analog logic in whole-cell and cell-free systems by means of enzymatic reactions. For

1 circuit design, we first employ computational design tools, Retropath<sup>40</sup> and Sensipath<sup>41</sup>,  
2 that use biochemical retrosynthesis to predict metabolic pathways and biosensors. We  
3 then build and model three whole-cell metabolic transducers and an analog adder to  
4 combine their outputs. Next, we transfer our metabolic circuits to a cell-free system<sup>42,43</sup> in  
5 order to take advantage of the higher tunability and the rapid characterization it offers<sup>44-</sup>  
6 <sup>46</sup>, expanding our system to include multiple weighted transducers and adders. Finally,  
7 using our integrated model trained on the cell-free metabolic circuits we build a more  
8 sophisticated device called the “metabolic perceptron”, which allows desired binary  
9 classification of multi-input metabolite combinations by applying model-predicted weights  
10 on the input metabolites before analog addition, and demonstrate its utility through two  
11 examples of four-input binary classifiers. Altogether, in this work we demonstrate the  
12 potential of synthetic metabolic circuits, along with model-assisted design, to perform  
13 complex computations in biological systems.

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## 16 **Results**

17

### 18 Whole-cell processing of hippurate, cocaine and benzaldehyde inputs

19 To identify the metabolic circuits to build, we use our metabolic pathway design tools,  
20 Retropath<sup>40</sup> and Sensipath<sup>41</sup>. These tools function using a set of sink compounds at the  
21 end of a metabolic pathway, here metabolites from a dataset of detectable compounds<sup>47</sup>,  
22 and a set of source compounds that can be used as desired inputs for the circuit. The  
23 tools then propose pathways and the enzymes that can catalyze the necessary reactions,  
24 allowing for promiscuity. Our metabolic circuit layers are organized according to the main  
25 processing functions: transduction and actuation (**Figure 1a**). Transducers are the  
26 simplest metabolic circuits that function as sensing enabling metabolic pathways  
27 (SEMP)<sup>48</sup>, consisting of one or more enzymes that transform an input metabolite into a  
28 transduced metabolite. The transduced molecule, in turn, is detected through an actuation  
29 function that is implemented using a transcriptional regulator.

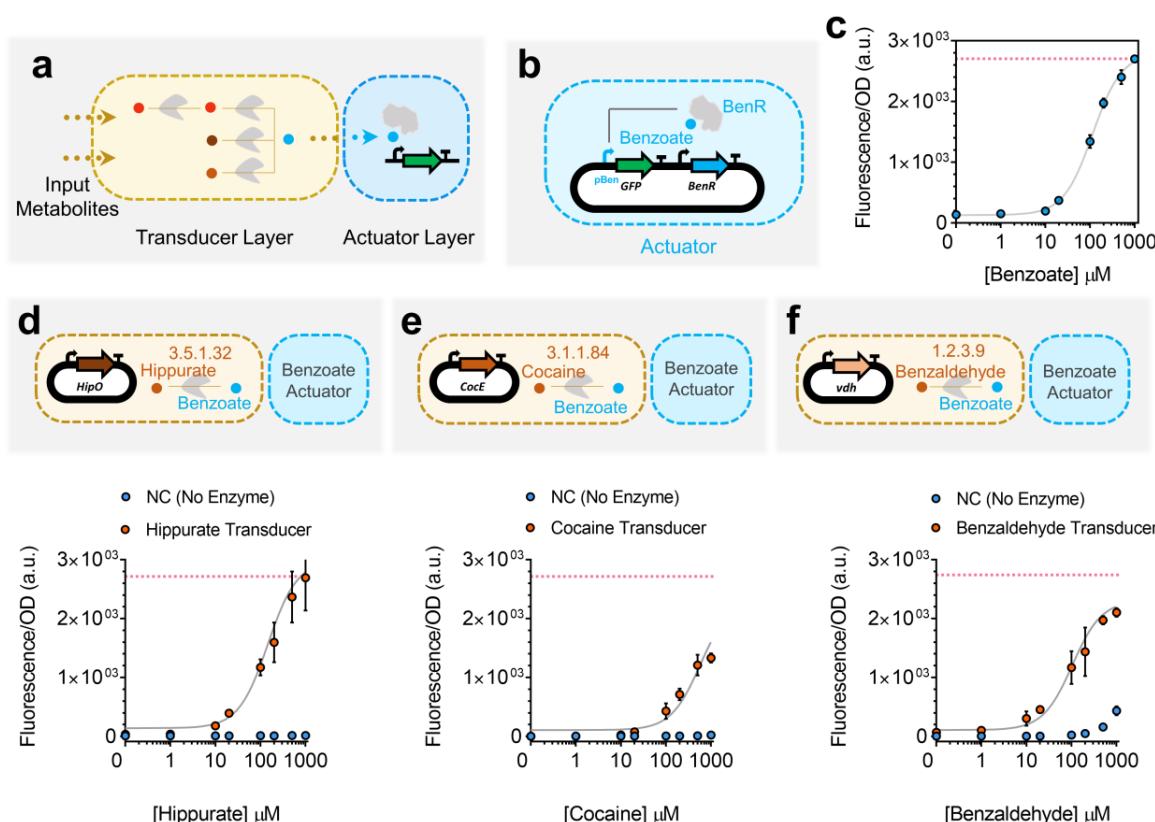
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31 We used benzoate as our transduced metabolite, its associated transcriptional activator  
32 BenR, and the responsive promoter pBen to construct the actuator layer of our whole-cell  
33 metabolic circuits<sup>49</sup>. To compare the shape of the response curve, we constructed the  
34 actuator layer in two formats: (i) an open-loop circuit (**Figure 1b**) and (ii) a feedback-loop  
35 circuit (**Figure S1**). When compared to the open-loop format, the feedback-loop circuit  
36 has previously been shown to exhibit linear dose-response to input<sup>21,50</sup>. We found that  
37 while the feedback-loop format does linearize the actuator response curve, apparent  
38 toxicity at high benzoate concentrations reduces the usable activator dynamic range  
39 (**Figure S1**). Therefore, we selected the open-loop format due to its higher dynamic range

1 of activation (**Figure 1c**), setting the maximum concentration of benzoate used in this  
2 work to the saturation point of this open-loop circuit.

3  
4 Building on our previous work<sup>48</sup>, we next implemented three upstream transducers that  
5 convert different input metabolites into benzoate for detection by the actuator layer  
6 already tested. The transducer layers were composed of enzymes HipO for hippurate  
7 (**Figure 1d**), CocE for cocaine (**Figure 1e**), and vdh for benzaldehyde (**Figure 1f**).  
8 Compared to the benzoate output signal, we found that the transduction capacities of the  
9 three transducers were 99.6%, 49.2%, and 77.8%, respectively (**Supplementary Figure**  
10 **S2**), indicating a partial dissipation in signal.

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14 **Figure 1. Whole-cell actuator and metabolic transducers.** (a) Designed synthetic metabolic circuits  
15 using Retropath<sup>40</sup> or Sensipath<sup>41</sup> consist of a transducer layer and an actuator layer. (b) Open-loop circuit  
16 construction of the benzoate actuator, which is used downstream of transducer metabolic circuits in this  
17 work. For the open-loop circuit, the transcription factor (TF) is expressed constitutively under control of the  
18 promoter J23101 and RBS B0032. (c) Dose-response plot of the open-loop circuit for the benzoate actuator.

1 The gray curve is a model-fitted curve (see Methods section) for the open-loop circuit. (d,e,f) Whole-cell  
2 metabolic transducers for hippurate (d), cocaine (e) and benzaldehyde (f) represented in dose-response  
3 plots (orange circles) and their associated dose-response when there is no enzyme present (blue circles).  
4 The red dotted lines refer to the maximum signal from the actuator (c). The transducer output benzoate is  
5 reported through the open-loop circuit actuator. The enzymes are expressed under constitutive promoter  
6 J23101 and RBS B0032. All data points and the error bars are the mean and standard deviation of  
7 normalized values from three measurements.

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10 **A Whole-cell metabolic concentration adder**

11 A metabolic concentration adder is a device composed of more than one transducer that  
12 converts their respective input metabolites into a common transduced output metabolite.  
13 For our whole-cell concentration adder, we combined two transducers to build a  
14 hippurate-benzaldehyde adder actuated by the benzoate circuit (**Figure 2a**). Unlike digital  
15 bit-adders that exhibit an ON-OFF digital behavior, our metabolic adders exhibit a  
16 continuous analog behavior that is natural for metabolic signal conversion<sup>51</sup> (**Figure 2b**  
17 and **Supplementary Figure S3**). Increasing the concentration of one of the inputs at any  
18 fixed concentration of the other shows an increase in the output benzoate, and thus in the  
19 resulting fluorescence (**Figure 2b** and **Supplementary Figure S3**).

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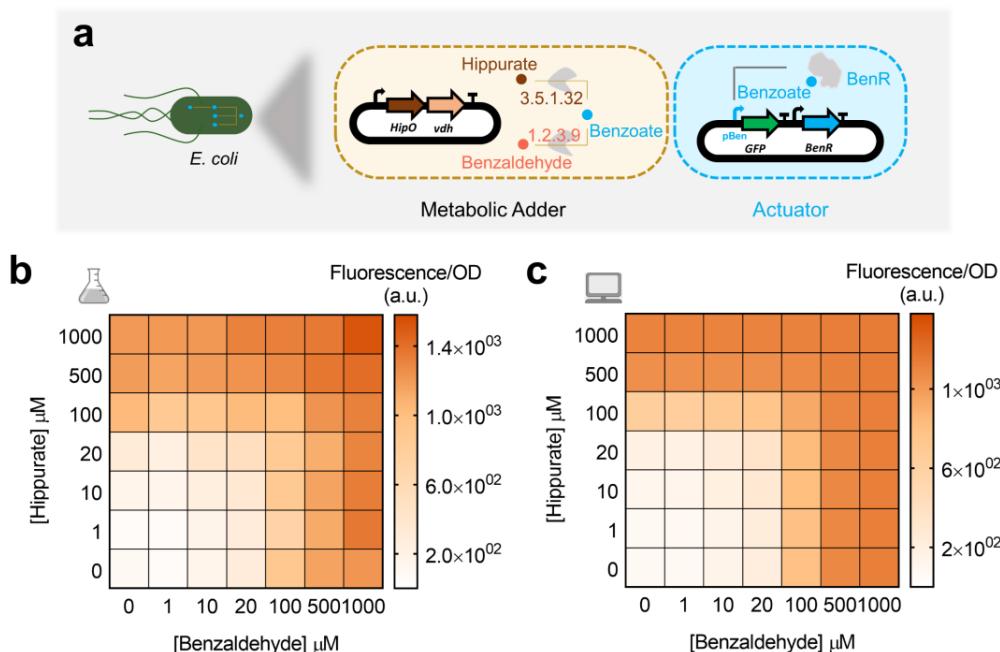
21 The maximum output signal for our adder, when hippurate and benzaldehyde were both  
22 at the maximum concentration of 1000  $\mu$ M, was lower than the maximum signal produced  
23 by hippurate and benzaldehyde transducers alone (**Supplementary Figure S2**).  
24 However, as seen above, the difference between the maximum signal of their transducers  
25 and the actuator was smaller. This dissipation in signal from the transducers to the adders  
26 and from the actuators to the transducers (**Supplementary Figure S2**) could either be  
27 because of resource competition (as a result of adding more genes) or because of  
28 enzyme efficiency (as a result of poorly balanced enzyme stoichiometries). To test these  
29 two hypotheses, we investigated the effect of the enzymes on cellular resource allocation.  
30 For this purpose, the cocaine transducer and the hippurate-benzaldehyde adder were  
31 characterized by adding benzoate to these circuits (**Supplementary Figures S4 and S5**).  
32 Comparing the results of these characterizations with the benzoate actuator reveals that  
33 dissipation in signal from the transducers to the adders is due to resource competition,  
34 whereas that from the actuators to the transducers is due to enzyme efficiency.

35

36 In order to gain quantitative understanding of the circuits' behavior, we empirically  
37 modeled their individual components to see if we were able successfully capture their  
38 behavior. We first modeled the actuator (gray curve in **Figure 1c**) using Hill formalism<sup>52</sup>  
39 as it is the component that is common to all of our outputs and therefore constrains the  
40 rest of our system. We then modeled our transducers, considering enzymes to be  
41 modules that convert their respective input metabolites into benzoate, which is then

1 converted to the fluorescence output already modeled above. This simple empirical  
2 modeling strategy reproduces our transducer data (results not shown). To incorporate  
3 observations made in **Supplementary Figure S4** and **S5**, we included resource  
4 competition in our models to explain circuits with one or more transducers. To this end,  
5 we extended the Hill model to account for resource competition following previous  
6 works<sup>53,54</sup>, with a fixed pool of available resources for enzyme and reporter protein  
7 production that is depleted by the transducers. This extension is further presented in the  
8 Methods section. We trained our model on all transducers, with and without resource  
9 competition (i.e. individual transducers, or transducers where another enzyme competes  
10 for the resources). This model (presented in gray lines in **Figure 1d,e,f** and **Figure 2c**),  
11 which was not trained on adder data but only on actuator, transducer, and transducers  
12 with resource competition data, recapitulates it well. This indicates that the model  
13 accounts for all important effects underlying the data. The full training process is  
14 presented in the Methods section, and a table summarising scores of estimated goodness  
15 of fit of our model is presented in **Supplementary Table S1**.

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17  
18 **Figure 2. Whole-cell metabolic adder of hippurate and benzaldehyde.** (a) Hippurate and benzaldehyde  
19 transducers are combined to build a metabolic adder producing a common output, benzoate, which is  
20 reported through the benzoate actuator. The enzymes are expressed in one operon under control of  
21 constitutive promoter J23101 and RBSs B0032 for HipO and B0034 for vdh. (b) Heatmap representing the  
22 output of the adder while increasing the concentration of both inputs, hippurate and benzaldehyde. All data  
23 points are the mean of normalized values from three measurements. (c) Model simulations for experimental  
24 conditions presented in (b). The model was fitted on transducer data and resource competition data.

25

26

1 **Cell-free processing of multiple metabolic inputs**

2 Cell-free systems have recently emerged as a promising platform<sup>42</sup> that provide rapid  
3 prototyping of large libraries by serving as an abiotic chassis with low susceptibility to  
4 toxicity. We took advantage of an *E. coli* cell-free system with the aim of increasing the  
5 computational potential of metabolic circuits in several ways (**Figure 3a**). Firstly, a higher  
6 number of genes can be simultaneously and combinatorially used to increase the  
7 complexity and the number of inputs for our circuits. Secondly, the lower noise provided  
8 by the absence of cell growth and maintenance of cellular pathways<sup>55</sup> improves the  
9 predictability and accuracy of the computation. Thirdly, having genes cloned in separate  
10 plasmids enables independent tunability of circuit behavior by varying the concentration  
11 of each part individually. Finally, cell-free systems are highly adjustable for different  
12 performance parameters and components. In all, these advantages of cell-free systems  
13 enable us to develop more complex computations than the whole-cell adder.

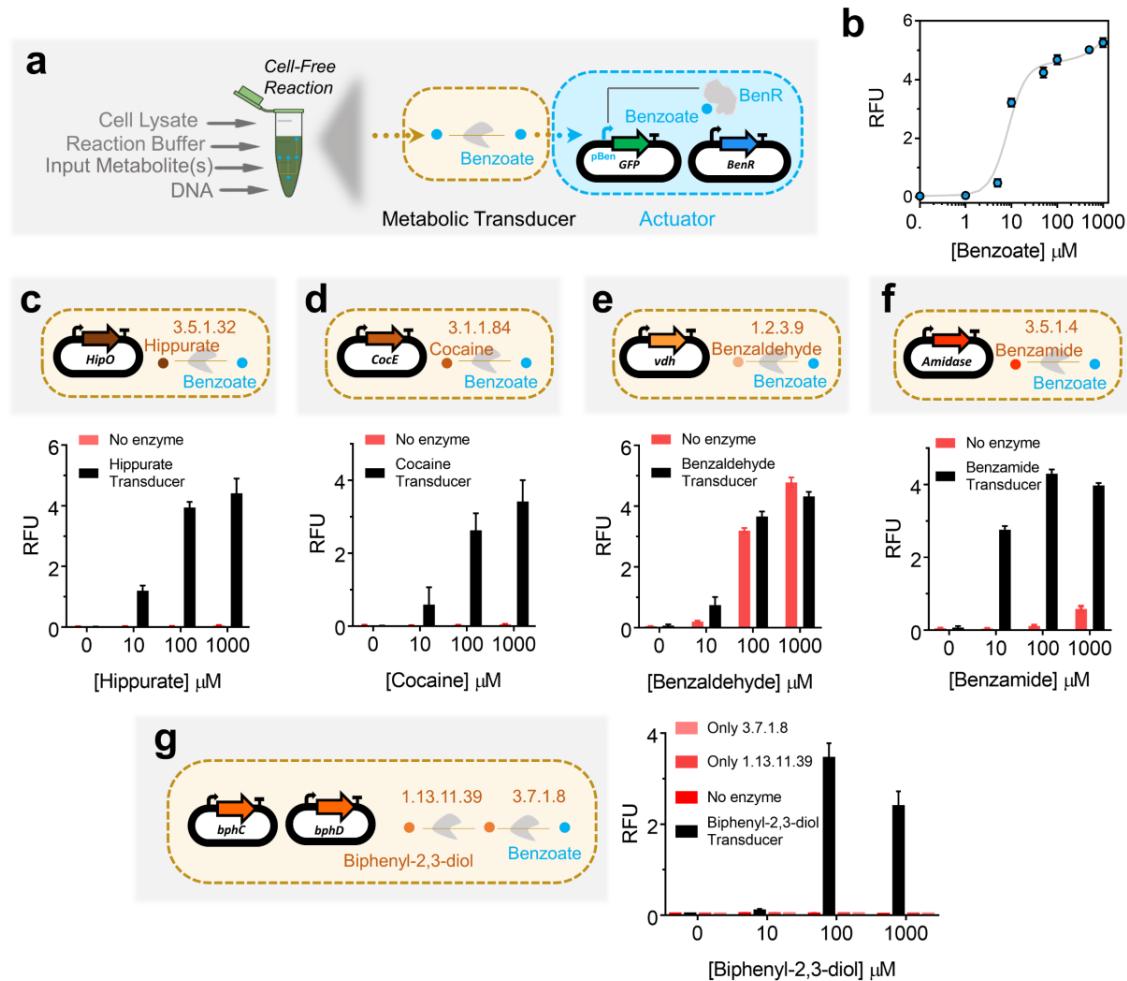
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15 Following from our recent work<sup>56</sup>, we first characterized a cell-free benzoate actuator to  
16 be used downstream of other metabolic transducers. **Figure 3a** shows a schematic of the  
17 cell-free benzoate actuator composed of a plasmid encoding the BenR transcriptional  
18 activator and a second plasmid expressing sfGFP reporter under the control of a pBen  
19 promoter. This actuator showed a higher operational range than the whole-cell  
20 counterpart (**Figure 1c**). The optimal concentration of the TF plasmid (30 nM) and the  
21 reporter plasmid (100 nM) were taken from our recent study<sup>56</sup>. Following successful  
22 implementation of the actuator, we proceeded to build five upstream cell-free transducers  
23 for hippurate, cocaine, benzaldehyde, benzamide, and biphenyl-2,3-diol (**Figure**  
24 **3c,d,e,f,g**) that convert these compounds to benzoate. Each of the five transducers used  
25 10 nM of enzyme DNA per reaction, except the biphenyl-2,3-diol transducer that used two  
26 metabolic enzymes with 10 nM DNA each.

27

28 Compared to its whole-cell counterpart (**Figure 1f**), in the cell-free transducer reaction  
29 (**Figure 3e**) benzaldehyde appears to spontaneously oxidise to benzoate without the  
30 need of the transducer enzyme vdh. This behavioral difference between the whole-cell  
31 and cell-free setups could be due to the difference in redox states inside an intact cell and  
32 the cell-free reaction mix<sup>57,58</sup>. Furthermore, benzamide and biphenyl-2,3-diol transducers  
33 exhibit inhibition in fluorescence outputs at very high (1000  $\mu$ M) input concentrations.

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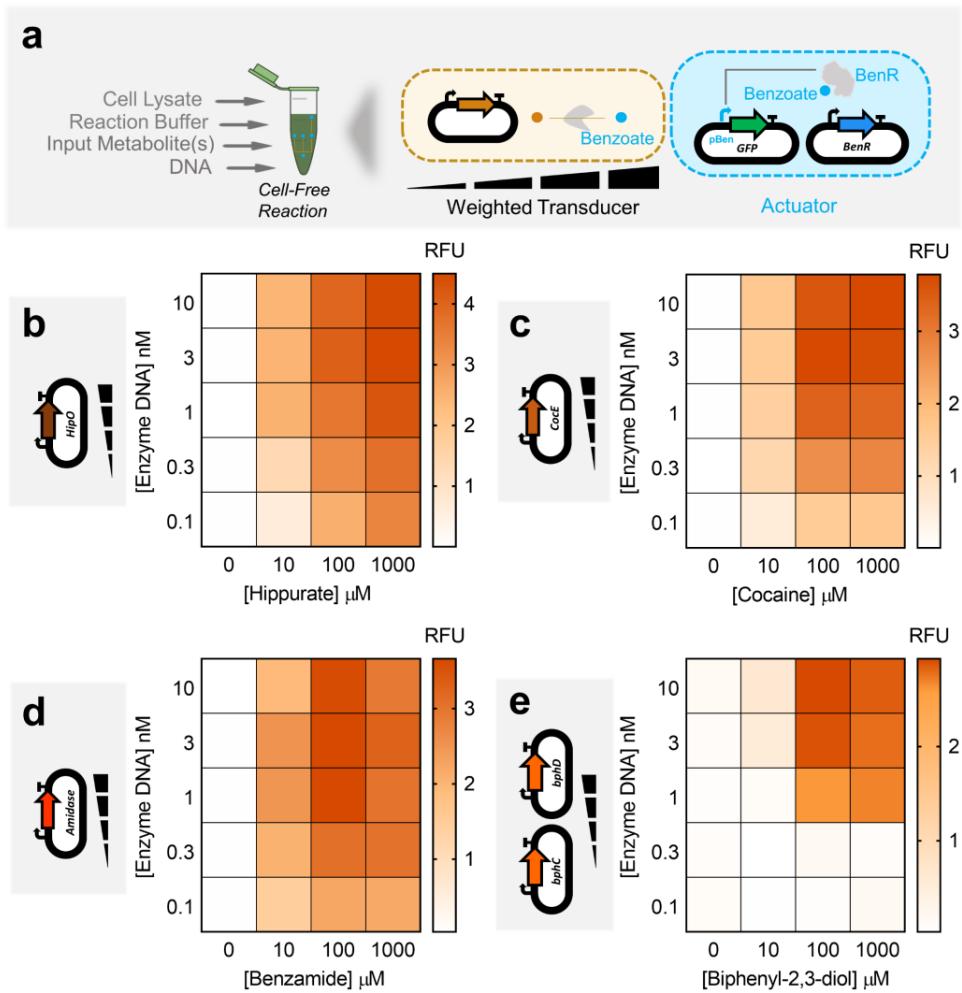
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2 **Figure 3. Cell-free actuator and metabolic transducers.** (a) Implementing benzoate actuator and  
3 transducers in *E. coli* transcription/translation (TXTL) cell-free system. Cell-free reactions are composed of  
4 cell lysate, reaction buffer (energy source, tRNAs, amino acids, etc.) and DNA plasmids. (b) Dose-response  
5 plot of the benzoate actuator in the cell-free system with 30 nM of TF-plasmid (constitutively expressed  
6 BenR) and 100 nM of reporter plasmid (pBen-sfGFP) per reaction. The data points represent the dose-  
7 response of the actuator to different concentrations of benzoate and the gray curve is a model-fitted curve  
8 on actuator data (c,d,e,f,g). Cell-free transducers coupled with the benzoate actuator for hippurate (c),  
9 cocaine (d), benzaldehyde (e), benzamide (f), and biphenyl-2,3-diol (g), which is composed of two  
10 enzymes. All enzymes are cloned in a separate plasmid under the control of a constitutive promoter J23101  
11 and RBS B0032. 10 nM of each plasmid was added per reaction. The bars are the response of the circuits  
12 to different concentrations of input with (transducers, black bars) and without enzyme (red bars). All data  
13 are the mean and the error bars are the standard deviation of normalized values from three measurements  
14 (RFU: Relative Fluorescence Unit).

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1 Cell-free weighted transducers and adders

2 After characterizing different transducers in the cell-free system that enable building a  
3 multiple-input metabolic circuit, we sought to rationally tune the transducers. Cell-free  
4 systems allow independent tuning of each plasmid by pipetting different amounts of DNA.  
5 We applied this advantage to weight the flux of enzymatic reactions in cell-free  
6 transducers (**Figure 4a**). The concentration range we used was taken from our recent  
7 study<sup>56</sup>, in order to have an optimal expression with minimum resource competition. We  
8 built four weighted transducers for hippurate (**Figure 4b**), cocaine (**Figure 4c**),  
9 benzamide (**Figure 4d**) and biphenyl-2,3-diol (**Figure 4e**). Increasing the concentration  
10 of the enzymes produces a higher amount of benzoate from the input metabolites, and  
11 hence higher GFP fluorescence. Compared to the others, the hippurate transducer  
12 reached higher GFP expression at a given concentration of the enzyme and the input,  
13 and biphenyl-2,3-diol reached the weakest signal. For the biphenyl-2,3-diol transducer  
14 built with two enzymes (**Figure 4e**), both enzymes are added at the same concentration  
15 (e.g., 1 nM of “enzyme DNA” indicates 1 nM each of plasmids encoding enzymes bphC  
16 and bphD).

17



1  
2 **Figure 4. Cell-free weighted transducers characterized by varying the concentration of the enzyme**  
3 **DNA.** (a) In the cell-free system, the circuits can be tuned by varying the amount of each enzyme pipetted  
4 per reaction. Weighted transducers are characterized by varying the concentration of the enzymes in  
5 transducers which then are reported through the benzoate actuator. The range of the concentrations was  
6 varied to get optimal expression and minimum resource competition. (b,c,d,e) Heatmaps representing  
7 weighted transducers at different concentrations of input molecules and enzymes DNA for hippurate (b),  
8 cocaine (c), benzamide (d) and biphenyl-2,3-diol (e). For the biphenyl-2,3-diol weighted transducer (e),  
9 concentrations represent those of each metabolic plasmid (e.g., 1 nM of “enzyme DNA” refers to 1 nM of  
10 bphC plus 1 nM of bphD). See **Supplementary Figure S6** for model results of each weighted transducer.  
11 All data are the mean of normalized values from three measurements. (RFU: Relative Fluorescence Unit).

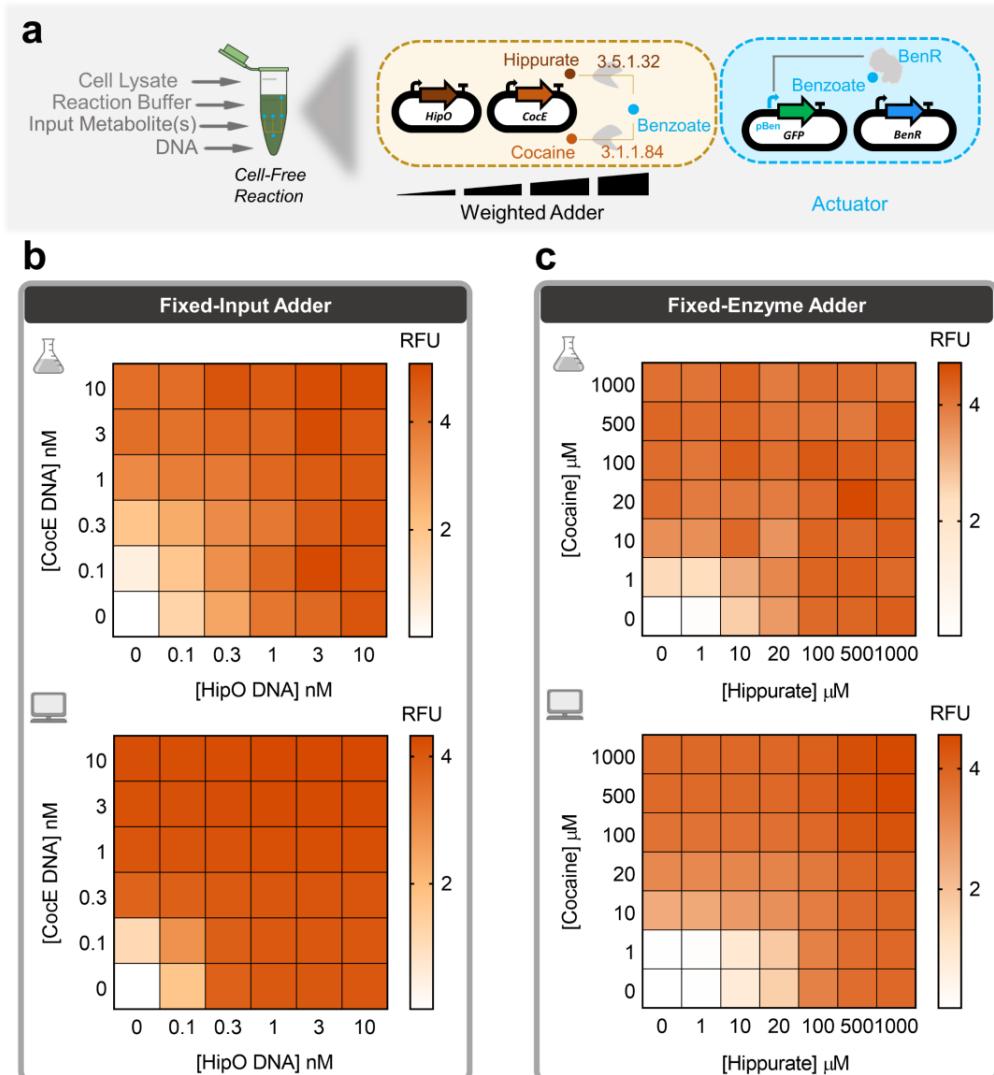
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1 Data in **Figure 4** show that similar output levels can be achieved for different input  
2 concentrations, provided the appropriate transducer concentrations are used. In the next  
3 step, we applied this finding to build hippurate-cocaine weighted adders by altering either  
4 the concentration of the enzymes or the concentration of the inputs (**Figure 5a**). The  
5 fixed-input adder is an adder in which the concentration of inputs, hippurate and cocaine,  
6 are fixed to 100  $\mu$ M and the concentration of the enzymes is altered (top panel in **Figure**  
7 **5b**). In this device, the weight of the reaction fluxes is continuously tunable. We then  
8 characterized a fixed-enzyme adder by fixing the concentration of the enzymes (1 nM for  
9 HipO, 3 nM for CocE; the cocaine signal is weaker, which is why a higher concentration  
10 of its enzyme is used) and varying the inputs, hippurate and cocaine (top panel in **Figure**  
11 **5c**).

12  
13 In order to have the ability to build any weighted adder with predictable results, we  
14 developed a model that accounts for the previous data. We first empirically modeled the  
15 actuator (gray curve in **Figure 3b**) since all other functions are constrained by how the  
16 actuator converts metabolite data (benzoate) into a detectable signal (GFP). We then  
17 trained our model with individual weighted transducers (**Supplementary Figure S6**) and  
18 predicted the behaviors of the weighted adders (bottom panel in **Figure 5b,c**). The results  
19 shown in **Figure 5b,c** indicate that our model describes the adders well, despite being  
20 trained only on transducer data. **Supplementary Table S2** summarizes the different  
21 scores to estimate goodness of fit of our model. Briefly, the model quantitatively captures  
22 the data but tends to overestimate values at intermediate enzyme concentration ranges  
23 and does not capture the inhibitory effect observed at the high concentration of  
24 benzamide or biphenyl-2,3-diol, as this was not accounted for in the model.

25  
26 Using the above strategy, we can build any weighted adder for which we have pre-  
27 calculated the weights using the model on weighted transducers. We use this ability in  
28 the following section to perform more sophisticated computation for a number of  
29 classification problems.

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3 **Figure 5. Multiple transducers are combined to shape an adder while weighing inputs or enzymes.**  
4 (a) Cell-free adder characterization by varying the concentration of either inputs or enzymes producing  
5 different levels of fluorescence through the actuator. (b) Heatmap showing fixed-input adder in which the  
6 inputs, hippurate and cocaine, are fixed to 100  $\mu$ M and concentrations of associated enzyme are altered  
7 by altering the concentration of plasmid DNA encoding them. Top: Cell-free experiment of hippurate-  
8 cocaine fixed-input (weighted) adder. Bottom: Model simulation (prediction) of hippurate-cocaine fixed-input  
9 (weighted) adder. (c) Fixed-enzyme adder with fixed concentrations of the enzyme DNAs, 1 nM for HipO  
10 and 3 nM for CocE, and various concentrations of the inputs, hippurate and cocaine. Top: Cell-free  
11 experiment of hippurate-cocaine fixed-enzyme adder. Bottom: Model simulations (prediction) of hippurate-  
12 cocaine fixed-enzyme adder. All data are the mean of normalized values from three measurements. (RFU:  
13 Relative Fluorescence Unit).

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1 **Cell-free perceptron for binary classifications**

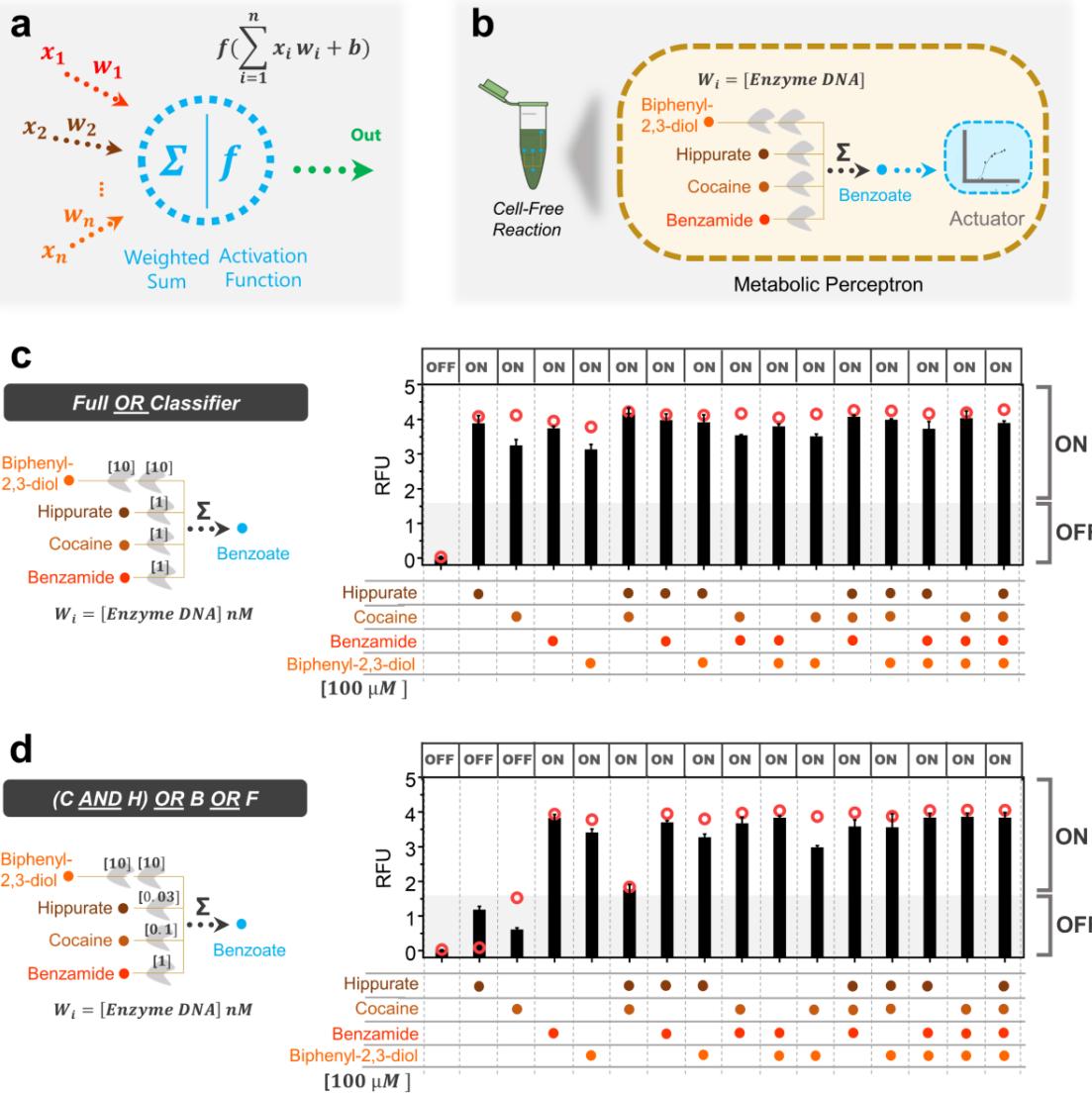
2 The perceptron algorithm was first developed to computationally mimic the neuron's  
3 ability to process information, learn, and make decisions<sup>59</sup>. Perceptrons are the basic  
4 blocks of artificial neural networks enabling the learning of deep patterns in datasets by  
5 training the model's input weights<sup>60</sup>. Like a neuron, the perceptron receives multiple input  
6 signals ( $x_i$ ) and triggers an output depending on the weighted ( $w_i$ ) sum of the inputs<sup>39</sup>. A  
7 perceptron can be used to classify a set of input combinations after it is trained on labeled  
8 data. In binary classification, the weighted sum is first calculated ( $\sum w_i \cdot x_i$ ) and an activation  
9 function ( $f$ ), coupled with a decision threshold  $d$ , finally makes the decision: ON if  $f(\sum w_i \cdot x_i) > d$ , OFF otherwise (**Figure 6a**). The activation function could be linear or non-linear  
10 (Sigmoid, tanh, ReLU, etc.) depending on the problem<sup>61</sup>, although a sigmoid is generally  
11 used for classification.

12  
13 Since our weighted transducer models have already been trained on the cell-free  
14 experimental data, we checked if we could use them to calculate the weights needed to  
15 classify different combinations of two inputs: hippurate and cocaine. We tested our model  
16 on five different binary classification problems, A to E (**Supplementary Figure 7**). For  
17 each problem, the two types of data were represented as a cluster of dots on the scatter  
18 plot. The trained model was then used to identify weights needed to be applied to the  
19 weighted transducers such that a decision threshold 'd' exists to classify the two clusters  
20 into red (ON,  $>d$ ) or blue (OFF,  $\leq d$ ). The lines shown in **Supplementary Figure 7** plots  
21 show three iso-fluorescence lines that represent the threshold that classifies the data into  
22 the binary categories: ON and OFF. These theoretical classification problems  
23 demonstrate the ability of our trained perceptron model to successfully carry out binary  
24 classification.

25  
26 Using the integrated model from our weighted transducers and adders, we next sought  
27 to design four-input classifiers using a metabolic perceptron, and test them  
28 experimentally. Our metabolic perceptron is a device enabling signal integration of  
29 multiple inputs with associated weights, represented by enzyme DNA concentrations  
30 (**Figure 6b**). The 4-input adder performs the weighted sum and the benzoate actuator  
31 acts as the activation function of the metabolic perceptron. The weights can be adjusted  
32 to implement different classification functions. To illustrate the potential of building  
33 perceptrons with metabolic weighted adders, we computed adder weights using our  
34 model for two different classifiers: a simple classifier equivalent to a "full OR" gate (**Figure**  
35 **6c**), and a more complex classifier equivalent to a "[cocaine AND hippurate] OR  
36 benzamide OR biphenyl-2,3-diol" gate (**Figure 6d**). Weight calculation methods are  
37 reported in the Methods section.

38  
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1 For the classifiers, the input metabolites are fixed to 100  $\mu$ M, as it allows the best ON-  
2 OFF behavior for all inputs and weight-tuning according to model simulations (results not  
3 shown). The model accurately predicted weights to obtain the simple “full OR” classifier  
4 behavior (**Figure 6d**), as well as cocaine, benzamide, and biphenyl-2,3-diol weights for  
5 the second complex classifier. The initial weights computed by the model are presented  
6 in **Supplementary Figure S8**. The optimal weight of HipO (hippurate transducing  
7 enzyme) was calculated to be 0.1 nM, which leads to higher signals than predicted,  
8 particularly for the “ON” behavior with only hippurate. To further characterize the HipO  
9 weights at still lower concentrations of the enzyme, we performed an additional  
10 complementary characterization (**Supplementary Figure S9**). Our aim here was to find  
11 a weight for HipO through which a classifier outputs a low signal (“OFF”) with only  
12 hippurate and high signal (“ON”) when coupled with other inputs. We arrived at 0.03 nM  
13 HipO which exhibited this shifting behavior between “OFF” and “ON” (**Figure 6d** and  
14 **Supplementary Figure S9**). Using our model-guided design and rapid cell-free  
15 prototyping on the HipO weight, we were able to design two 4-input binary classifiers. In  
16 **Figure 6c,d** red circles are the weights predicted with 0.03 nM for HipO and the bars are  
17 experimental results. All actual values of the model and the experiments are provided in  
18 **Supplementary Table S7**.



1  
2 **Figure 6. Cell-free perceptron enabling development of classifiers.** (a) A perceptron scheme showing  
3 the inputs and their associated weights, the computation core, and the output. The perceptron computes  
4 the weights and actuates the weighted sum through an activation function. (b) Metabolic perceptron  
5 integrating multiple inputs and actuating an output. The benzoate actuator acts as the activation function of  
6 the perceptron reporting the sum of benzoate produced by the metabolic perceptron. Hippurate, cocaine,  
7 benzamide, and biphenyl-2,3-diol are the inputs of the metabolic perceptron fixed to 100  $\mu\text{M}$ . The weights  
8 of the perceptron are the concentration of the enzymes calculated using the model made on weighted  
9 metabolic circuits (red circles). These weights are calculated to develop two classifiers using the metabolic  
10 perceptron and benzoate actuator. “Full OR” classifier (c), “[cocaine (C) AND hippurate (H)] OR benzamide  
11 (B) OR biphenyl-2,3-diol (F)” classifier (d) are the two classifiers built using this metabolic perceptron. The  
12 “Full OR” classifier (c) classifies to “OFF” when none of the inputs is present and it passes an arbitrary

1 threshold to “ON” when any of the inputs or their combinations are present. The second classifier (**d**)  
2 performs a more complex computation. The shading represents the arbitrary threshold that allows for  
3 perceptron decision making and the panel of “OFF” and “ON” at the top of the bars are the expected output  
4 of the classifiers. All data are the mean and the error bars are the standard deviation of normalized values  
5 from three measurements and red circles are the model predictions. (RFU: Relative Fluorescence Unit).

6

7

## 8 **Discussion**

9

10 Computing in synthetic biological circuits has largely relied on digital logic-gate circuitry  
11 for almost two decades<sup>5,62</sup>, treating inputs as either absent (0) or present (1). While such  
12 digital abstraction of input signals provides conceptual modularity for circuit design, it is  
13 less compatible with the physical-world input signals that vary between low and high  
14 values on a continuum<sup>37</sup>. As a result, digital biological circuits must carefully match input-  
15 output dynamic ranges at each layer of signal transmission to ensure successful signal  
16 processing<sup>2,30</sup>. More recently, the higher efficiency of analog computation on continuous  
17 input has been recognized<sup>63</sup>, and some analog biological circuits have started  
18 emerging<sup>21</sup>. In this regard, using metabolic pathways for cellular computing seems like a  
19 natural progression for analog computation in biological systems<sup>21,30</sup>.

20

21 In this study, we investigated the potential of metabolism to perform analog computations  
22 using synthetic metabolic circuits. To that end, we first established a benzoate actuator  
23 to report the output from our metabolic circuits in both whole-cell and cell-free systems  
**(Figures 1c and 3b)**. Upstream of the actuator, we constructed hippurate, cocaine, and  
25 benzaldehyde transducers in the whole-cell system (**Figures 1d,e,f**) and a metabolic  
26 adder by combining the benzaldehyde and hippurate transducers (**Figure 2**). Similarly,  
27 we constructed hippurate, cocaine, benzaldehyde, benzamide, and biphenyl-2,3-diol  
28 transducers in the cell-free system (**Figures 3c,d,e,f,g**) and weighted adders by  
29 combining them (**Figure 5**). Compared to the numerous digital biological devices, which  
30 compute through multi-layered genetic logic circuits, the metabolic adder is a simple one-  
31 layered device with fast execution times.

32

33 Our computational models trained only on the actuator and transducer data predicted  
34 adder behaviors with high accuracy (**Supplementary Tables S1 and S2**). This further  
35 enabled us to calculate the required weights for more complex “metabolic perceptrons”  
36 that compute weighted sums from multiple inputs and use them to classify the multi-input  
37 combinations in a binary manner (**Figures 6 and S7**). Although we used fixed  
38 concentrations of inputs to demonstrate the ability of our perceptrons to classify, models  
39 trained on characterization data from weighted transducers should enable one to build  
40 classifiers for other concentrations in the operational range of the transducers  
**(Supplementary Figure S10)**. Indeed, as shown in **Figures 4** and **5**, for different input

1 concentrations in the operational range the weight of the input can be tuned through the  
2 concentration of the enzyme DNA. To the best of our knowledge, the metabolic adders  
3 and perceptrons presented in this work are the first engineered biological circuits that use  
4 metabolism for analog computation.

5  
6 Unlike genetic circuits that experience expression delays<sup>2</sup>, metabolic circuits have the  
7 advantage of faster response times since the enzymes have already been expressed in  
8 the system. Yet, metabolic circuits can be connected with the other layers of cellular  
9 information processing (like genetic or signal transduction layers) when needed, to build  
10 more complex sense-and-respond behaviors. The actuator layer of our perceptrons is a  
11 good example of this, where the calculated weighted sum is converted to fluorescence  
12 output via the genetic layer. In addition, we took advantage of the properties of cell-free  
13 systems, such as higher tunability and lack of toxicity<sup>56,64</sup>, to rapidly build and characterize  
14 multiple combinations of transducer-actuator circuits. Cell-free systems can be lyophilized  
15 on paper and stored at ambient temperature for <1 year for diagnostic applications<sup>16</sup>. This  
16 expands the potential scope of cell-free metabolic perceptrons for use in multiplex  
17 detection of metabolic profiles in medical or environmental samples<sup>16,56</sup>.

18  
19 Here, we have built a single-layer perceptron, with positive weights, that can classify  
20 different profiles of input metabolites by applying different weights to each transducer. In  
21 the future, by adding competing or attenuating reactions that reduce the concentration of  
22 the transduced metabolite in response to an input, it may be possible to expand the  
23 training space by applying negative weights to certain inputs<sup>65</sup>. Furthermore, a single-  
24 layer perceptron can only classify data that is linearly separable<sup>66</sup>, which means that it  
25 should be possible to draw a line between the two classes of data points in order for the  
26 perceptron to classify them (**Supplementary Figure S7**). In contrast, multi-layer  
27 perceptrons, can approximate any function<sup>67</sup> and can be used for more complex pattern  
28 recognition tasks<sup>68</sup>. With the use of bioretrosynthesis-based computational tools for  
29 metabolic pathway design, like Retropath<sup>40</sup> and Sensipath<sup>41</sup>, it will be possible to build  
30 multiple layers of metabolic perceptrons that can classify complex patterns of metabolic  
31 states *in vivo*, or identify different metabolite concentrations in analytical samples. Finally,  
32 it may also be possible to apply *in situ* learning (within the whole-cell or cell-free  
33 environment) by applying winner selection strategies on successful classifiers<sup>69</sup>.

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10  
11

#### 12 **Author contributions**

13 AP, MKo, MKu and JLF designed the project. AP designed and cloned the constructs,  
14 and performed the whole-cell experiments. AP, PLV, and JB designed cell-free  
15 experiment platform. AP, PLV, and PS performed cell-free experiments. MKo performed  
16 computational model simulations. All authors contributed to the manuscript write-up and  
17 approved the final manuscript.

18  
19

#### 20 **Competing financial interests**

21 The authors declare no competing financial interest.

22  
23  
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#### 26 **Methods**

27

##### 28 Designing synthetic metabolic circuits

29 Retropath<sup>40</sup> and Sensipath<sup>41</sup> were used to design the metabolic circuits between potential  
30 input metabolites and detectable metabolites as outputs<sup>47</sup>. These tools function using a  
31 set of sink compounds, a set of source compounds, and a set of chemical rules<sup>47,70</sup>  
32 implementing enzyme-mediated chemical transformations. They then use retrosynthesis  
33 to propose pathways and the enzymes that can catalyze the necessary reactions,  
34 allowing promiscuity, between compounds from the sink and compounds from the source.  
35 To design the adder, the Retropath software was used with a set of detectable  
36 compounds as the sink and the molecules we wish to use as circuit inputs as the source.  
37 The results were potential pathways and the associated enzymes, which were then  
38 analyzed for feasibility. The sequences of the enzymes were codon-optimized,  
39 synthesized and implemented in *E. coli* or taken from a previous study.

1

2 Molecular biology

3 All plasmids were made using Golden Gate assembly in *E. coli* Mach1 chemically  
4 competent cells. Whole-cell constructs were cloned in BioBrick standard vectors pSB1K3  
5 (high-copy plasmid) and pSB4C5 (low-copy plasmid) and the TF and all the enzymes  
6 were constitutively expressed under constitutive promoter J23101 and RBS B0032. All  
7 cell-free plasmids were cloned in pBEAST<sup>56</sup> (a derived vector from pBEST<sup>71</sup>). BenR cell-  
8 free plasmid and its cognate responsive prompter, pBen, expressing super-folder GFP  
9 were taken from our recent work<sup>56</sup>. All other cell-free enzymes were cloned under  
10 constitutive promoter J23101 and RBS B0032. Sequence and source of all the genes and  
11 parts are available in **Supplementary Table S5**. Synthetic sequences were provided by  
12 Twist Bioscience. Enzymes for cloning including Q5 DNA polymerase, Bsal, and T4 DNA  
13 ligase were purchased from New England Biolabs. DNA plasmids for cell-free reactions  
14 were prepared using the Macherey-Nagel maxiprep kit.

15

16

17 Characterization of whole-cell circuits

18 For each circuit separate colonies of *E. coli* top10 strains harboring the circuit plasmids  
19 were cultured overnight at 37°C in LB with appropriate antibiotic. The next day each culture  
20 was diluted 100x in LB with antibiotics. 95 µL of fresh cultures were distributed in 96-well  
21 plate (Corning 3603) and the plate was incubated to reach the OD ~ 0.1 in a plate reader  
22 (Biotek Synergy HTX). Then 5 µL of the input metabolites (100x ethanol solutions 5x  
23 diluted in LB) were added and the plate was incubated for 18 hours at 37°C. During the  
24 incubation, the OD<sub>600</sub> and GFP fluorescence (gain: 35, ex: 458 nm, em: 528 nm) were  
25 measured. Benzoate, hippurate, cocaine hydrochloride, benzaldehyde, benzamide and  
26 biphenyl-2,3-diol (2,3-dihydroxy-biphenyl) were purchased from Sigma-Aldrich.  
27 Permission to purchase cocaine hydrochloride was given by the French drug regulatory  
28 agency (Agence Nationale de Sécurité du Médicament et des Produits de Santé). For all  
29 chemicals, serial dilutions of 100x concentrations were prepared in ethanol. The formula  
30 presenting the results of the circuits' characterization is shown in data normalization  
31 section. The mean and standard deviation of all normalized data are provided in  
32 **Supplementary Table S6**.

33

34

35 Cell-free extract and buffer preparation

36 Cell-free *E. coli* extract was produced as previously described<sup>56,72,73</sup>. Briefly, an overnight  
37 culture of BL21 Star (DE3)::RF1-CBD<sub>3</sub> *E. coli* was used to inoculate 4L of 2xYT-P media  
38 in six 2 L flasks at a dilution of 1:100. The cultures were grown at 37°C with 220 rpm  
39 shaking for approximately 3.5-4 hours until the OD 600 = 2-3. Cultures were centrifuged  
40 at 5000 x g at 4°C for 12 minutes. Cell pellets were washed twice with 200 mL S30A

1 buffer (14 mM Mg-glutamate, 60 mM K-glutamate, 50 mM Tris, pH 7.7), centrifuging after  
2 each wash at 5000 x g at 4°C for 12 minutes. Cell pellets were then resuspended in 40  
3 mL S30A buffer and transferred to pre-weighed 50 mL Falcon conical tubes where they  
4 were centrifuged twice at 2000 x g at 4°C for 8 and 2 minutes, respectively, removing the  
5 supernatant after each. Finally, the tubes were reweighed and flash frozen in liquid  
6 nitrogen before storing at -80°C.

7  
8 Cell pellets were thawed on ice and resuspended in 1 mL S30A buffer per gram of cell  
9 pellet. Cell suspensions were lysed via a single pass through a French press  
10 homogenizer (Avestin; Emulsiflex-C3) at 15000-20000 psi and then centrifuged at 12000  
11 x g at 4°C for 30 minutes to separate out cellular cytoplasm. After centrifugation, the  
12 supernatant was collected and incubated at 37°C with 220 rpm shaking for 60 minutes.  
13 The extract was re-centrifuged at 12000 x g at 4°C for 30 minutes, and the supernatant  
14 was transferred to 12-14 kDa MWCO dialysis tubing (Spectrum Labs; Spectra/Por4) and  
15 dialyzed against 2 L of S30B buffer (14 mM Mg-glutamate, 60 mM K-glutamate, ~5 mM  
16 Tris, pH 8.2) overnight at 4°C. The following day, the extract was re-centrifuged one final  
17 time at 12000 x g at 4°C for 30 minutes, aliquoted, and flash frozen in liquid nitrogen  
18 before storage at -80°C.

19  
20 The buffer for cell-free reactions is composed such that final reaction concentrations were  
21 as follows: 1.5 mM each amino acid except leucine, 1.25 mM leucine, 50 mM HEPES,  
22 1.5 mM ATP and GTP, 0.9 mM CTP and UTP, 0.2 mg/mL tRNA, 0.26 mM CoA, 0.33 mM  
23 NAD, 0.75 mM cAMP, 0.068 mM folinic acid, 1 mM spermidine, 30 mM 3-PGA, and 2%  
24 PEG-8000. Additionally, the Mg-glutamate (0-6 mM), K-glutamate (20-140 mM), and DTT  
25 (0-3 mM) levels were serially calibrated for each batch of cell-extract for maximum signal.  
26 One batch of buffer was made for each batch of extract, aliquoted, and flash frozen in  
27 liquid nitrogen before storage at -80°C.

28  
29 Characterization of cell-free circuits  
30 Cell-free reactions were performed in 15.75 µL of the mixture of 33.3% cell extract, 41.7%  
31 buffer, and 25% plasmid DNA, input metabolites, and water. The reactions were prepared  
32 in PCR tubes on ice and 15 µL of each was pipetted into 384-well plates (Thermo  
33 Scientific 242764). GFP fluorescence out of each circuit was recorded in the plate reader  
34 at 30°C (gain: 50, ex: 458 nm, em: 528 nm). The background (cell-free reaction without  
35 any plasmid) corrected fluorescence data were normalized by 20 ng/µL of a plasmid  
36 expressing strong constitutive sfGFP (under OR2-OR1-Pr promoter<sup>56</sup>) and were plotted  
37 after 8 hours incubation. The mean and standard deviation of all normalized data are  
38 provided in **Supplementary Table S7**.

39

1 Data normalization:

2 For whole-cell data, we use the following normalization:

3

4 
$$\text{Fluorescence}(\text{input}) = \frac{\text{GFP}(\text{input}) - \text{GFP}(\text{LB})}{\text{OD}(\text{input}) - \text{OD}(\text{LB})} - \frac{\text{GFP}(\text{empty\_plasmid}) - \text{GFP}(\text{LB})}{\text{OD}(\text{empty\_plasmid}) - \text{OD}(\text{LB})}$$

5

6 Reference: cells harboring empty plasmids

7

8 For cell-free data, we consider Relative Fluorescence Unit (RFU):

9

10 
$$\text{RFU}(\text{input}) = \frac{\text{GFP}(\text{input}) - \text{GFP}(\text{extract})}{\text{GFP}(\text{reference}) - \text{GFP}(\text{extract})}$$

11 Reference: 20 ng/µL of a plasmid expressing the constitutive sfGFP under OR2-OR1-Pr  
12 promoter<sup>56</sup>.

13

14 Simulation tools and parameter fitting:

15 All data analysis and simulations were run on R (version 3.2.3)<sup>74</sup>. Dose-response curves  
16 were fitted using ordinary least squares errors and the R optim function (from Package  
17 stats version 3.2.3, using the L-BFGS-B method implementing the Limited-memory  
18 Broyden Fletcher Goldfarb Shanno algorithm, which is a quasi-Newton method). For the  
19 random parameter sampling around the mean fit, values were sampled from within +-1.96  
20 standard error of the mean of the parameter estimation. The seed was set so as to ensure  
21 reproducibility. All simulations were run in the Rstudio development environment<sup>75</sup>.

22 All parameters are presented in **Supplementary Tables S3 and S4**.

23

24

25 Whole-cell model

26 The whole-cell model is composed of three parts: the actuator, the transducers (which all  
27 obey the same law) and the resource competition.

28

29 
$$\text{Actuator}(\text{total}) = \left( \frac{(\text{total})^{\text{hill\_transfer}}}{(\text{K}_M)^{\text{hill\_transfer}} + (\text{total})^{\text{hill\_transfer}}} * \text{fold}_{\text{change}} + 1 \right) * \text{baseline}$$

30

31 where *total* is the concentration of the considered input (in µM), *K<sub>M</sub>* is the concentration  
32 that allows for half-maximum induction (in µM), also termed IC<sub>50</sub>, *hill\_transfer* is the Hill  
33 coefficient that characterizes the cooperativity of the induction system, *fold\_change* is the  
34 dynamic range (in AU) and *baseline* is the basal GFP fluorescence without input  
35 (benzoate).

36

1

$$Transducer(inducer) = inducer * range\_enzyme$$

2

Where *input* is the input concentration in  $\mu\text{M}$  and *range\_enzyme* is a dimensionless number characterizing the capacity of the enzyme to transduce the signal. When combining transducers with the actuator, transducer results are added before being fed into the actuator equation, just as benzoate concentrations are added before being converted to a fluorescent signal in the cell.

3

To account for resource competition, given our experimental results where there is little competition with one enzyme and significant competition with two, we used an equation including cooperativity of resource competition. This reduces the fold change of the actuator as there are less resources available for producing transcription factors and GFP.

4

5

$$Result(value) =$$

6

$$range_{resources} * value * \left( \frac{(totalenzyme)^{cooperativity\_resources}}{(totalenzyme)^{cooperativity\_resources} + (coce + benz + ratiohipbenz * hipo)^{cooperativity\_resources}} \right)$$

7

where *value* is the result of the actuator transfer function before accounting for resource competition, *range\_resources*, *total\_enzyme*, *cooperativity\_resources* characterize the Hill function that accounts for competition, *coce*, *benz* and *hipo* are the enzyme plasmid concentrations. *ratio\_hip\_benz* accounts for the differences in burden from different enzymes, its value around 0.8 is close to the ratio between enzyme lengths (1500 for benzaldehyde transducing enzyme and 1200 for HipO).

8

9

### Cell-free model

10

The model is composed of two parts: the actuator and the transducers.

11

12

$$Actuator(total) = \left( \frac{(total)^{hill\_transfer}}{(K_M)^{hill\_transfer} + (total)^{hill\_transfer}} * fold\_change + 1 \right) * baseline + slower\_slope * 0.0001 * total$$

13

where *total* is the concentration of the considered input metabolite (in  $\mu\text{M}$ ), *Km* is the concentration that allows for half-maximum induction (in  $\mu\text{M}$ ), also termed  $IC_{50}$ , *hill\_transfer* is the Hill coefficient that characterizes the cooperativity of the induction system, *fold\_change* is the dynamic range (in AU) and *baseline* is the basal GFP fluorescence without input (benzoate). *Slower\_slope* accounts for the linearity observed in the actuator behavior at concentrations saturating the Hill transfer function.

14

15

$$Transducer(input) = range_{enzyme} * \left( \frac{(E)^{hill_E}}{(K_E)^{hill_E} + (E)^{hill_E}} \right) * \left( \frac{(input)^{hill_{input}}}{(K_I)^{hill_{input}} + (input)^{hill_{input}}} \right)$$

1 Where *range\_enzyme* is a dimensionless number characterizing the capacity of the  
2 enzyme to transduce the signal. The activity of the enzyme is characterized by a Hill  
3 function as increasing concentrations do not lead to a linear increase but enzymes  
4 saturate ( $E$  is the enzyme quantity in nM,  $K_E$  and  $hill_E$  are its Hill constants), and similarly,  
5 *input* is the input metabolite concentration in  $\mu$ M with  $K_i$  and  $hill\_input$  as its Hill constants.  
6

7 When combining transducers, transducer results are added before being fed into the  
8 actuator equation, just as benzoate concentrations are added before being converted to  
9 the fluorescent signal in the cell.

10

#### 11 Full model training process

12 Our training process is detailed in the Readme files supporting our modeling scripts  
13 provided in GitHub and is summarized here.

14

15 As the first step, the actuator transfer function model (benzoate transformed into  
16 fluorescence) is fitted 100 times on the actuator data, with all actuator parameters allowed  
17 to vary. The mean, standard deviation, standard error of the mean and confidence interval  
18 were saved at 95% of the estimation of those parameters. For transducer fitting (all  
19 transducers in cell-free and all except cocaine in whole-cell), we constrained the actuator  
20 characteristics in the following way: upper and lower allowed values are within the 95%  
21 confidence interval (or plus or minus one standard deviation from the mean for fold  
22 change and baseline in cell-free as it allowed a wider range, accounting for the decrease  
23 in actuator signal in transducer experiments without affecting the shape of the sigmoid).  
24 The initial values for the fitting process were sampled from a Gaussian distribution  
25 centered on the mean parameter estimation and spread with a standard deviation equal  
26 to the standard error of this parameter estimation. We then allowed fitting of all transducer  
27 parameters freely and of the actuator parameters within their 95% confidence interval.  
28

29

30 Once this is done, all common parameters (actuator transfer function and resource  
31 competition) were sampled using the same procedure and fitting on the cocaine  
32 transducer was performed. To show that parameters are well constrained (proving they  
33 minimally explain the data), **Supplementary Figures S11 and S12** show results of  
34 sampling parameters from the final parameters distribution (without fitting at that stage)  
35 and how they compare to the data.

36

#### Objective functions and model scoring:

37 In order to evaluate and compare our models, we used the following functions.

38

$$RMSD = \sqrt{\frac{\sum_1^n (y_i^{true} - y_i^{pred})^2}{n}}$$

39

1 It measures how close the model is to the experiments. It allows for comparison of  
2 different models on the same data, the one with the smaller RMSD being better, but does  
3 not allow comparison between experiments.

4

$$R^2 = 1 - \frac{\sum_1^n (y_i^{true} - y_i^{pred})^2}{\sum_1^n (y_i^{true} - y_{mean}^{true})^2}$$

5  $R^2$  allows measuring the goodness of fit. When the prediction is only around the sample  
6 mean,  $R^2 = 0$ . When the predictions are close to the real experimental value,  $R^2$  gets  
7 closer to 1, whereas it can have important negative values when the model is really far  
8 off.

9

$$Weighted\ R^2 = 1 - \frac{\sum_1^n \frac{(y_i^{true} - y_i^{pred})^2}{std_i^2}}{\sum_1^n \frac{(y_i^{true} - y_{mean}^{true})^2}{std_i^2}}$$

10  
11 It is a variant of  $R^2$  that weights samples according to their experimental error, giving more  
12 weight to more certain samples. It otherwise has the same properties as  $R^2$ .

13  
14

$$Error\ percentage = abs(\frac{y_i^{true} - y_i^{pred}}{y_i^{true}}) * 100$$

15 This measures the percentage of error for each point. We present the average on all  
16 experiments in Supplementary Tables S1 and S2.

17  
18  
19  
20 Perceptron weights calculation  
21 In order to calculate the weights for the classifiers presented in **Figure 6**, we followed the  
22 following procedure. First, we defined the expected results (expressed in “OFF”s and  
23 “ON”s). We also defined a list of weights to test for each enzyme (here, between 0.1 nM  
24 and 10 nM, as tested in our weighted transducers). Then, for each combination of enzyme  
25 weights, we simulated the outcome of the classifiers for all possible input combinations.  
26 We then tested various possible thresholds and kept the enzyme combinations for which  
27 a threshold exists that allows for the expected behavior. As the last step, we manually  
28 analyzed the classifier to keep the ones both a high difference between ON and OFF,  
29 and a minimal enzyme weight to prevent resource competitions issues that could arise as  
30 we are adding more genes than previous experiments. In order to perform clusterings  
31 presented in **Supplementary Figure S8**, we sampled values uniformly within the stated  
32 ranges ([0, 2μM] for low values and [80, 100μM] for high values). We then simulated the  
33 results to assess the robustness of our designs.

34  
35  
36



1 Code and data availability:

2 All scripts and data for generating results presented in this paper are available at  
3 <https://github.com/brsynth>.

4

5

6 Biological and chemical identifiers

7 In order to allow easier parsing of our article by bioinformatics tools, we provide here the  
8 identifiers of our biological sequences and chemical compounds.

9

10 *Benzoate (Benzoic acid):* InChI=1S/C7H6O2/c8-7(9)6-4-2-1-3-5-6/h1-5H,(H,8,9)

11 *Hippurate (Hippuric acid):* InChI=1S/C9H9NO3/c11-8(12)6-10-9(13)7-4-2-1-3-5-7/h1-  
12 5H,6H2,(H,10,13)(H,11,12)

13 *Cocaine:* InChI=1S/C17H21NO4/c1-18-12-8-9-13(18)15(17(20)21-2)14(10-12)22-  
14 16(19)11-6-4-3-5-7-11/h3-7,12-15H,8-10H2,1-2H3/t12-,13+,14-,15+/m0/s1

15 *Benzaldehyde:* InChI=1S/C7H6O/c8-6-7-4-2-1-3-5-7/h1-6H

16 *Biphenyl-2,3-diol:* InChI=1S/C12H10O2/c13-11-8-4-7-10(12(11)14)9-5-2-1-3-6-9/h1-  
17 8,13-14H

18 *Benzamide:* InChI=1S/C7H7NO/c8-7(9)6-4-2-1-3-5-6/h1-5H,(H2,8,9)

19

20 *BenR identifier:* UniProtKB - Q9L7Y6

21 *HipO identifier:* UniProtKB - P45493

22 *CocE identifier:* UniProtKB - Q9L9D7

23 *vdh identifier:* UniProtKB - D0RZT4

24 *bphC identifier:* UniProtKB - P17297

25 *bphD identifier:* UniProtKB - Q52036

26 *Benzamide transforming enzyme identifier:* UniProtKB - B4XEY3

27

28 Sequence and source of all the genes and parts are available in **Supplementary Table**  
29 **S5**

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9

Supplementary information for:

## **Metabolic Perceptrons for Neural Computing in Biological Systems**

Amir Pandi<sup>1\*</sup>, Mathilde Koch<sup>1\*</sup>, Peter L Voyvodic<sup>2</sup>, Paul Soudier<sup>1,3</sup>, Jerome Bonnet<sup>2</sup>,  
Manish Kushwaha<sup>1†</sup>, and Jean-Loup Faulon<sup>1,3,4†</sup>

<sup>1</sup> Micalis Institute, INRA, AgroParisTech, Université Paris-Saclay, Jouy-en-Josas, France

<sup>2</sup> Centre de Biochimie Structurale, INSERM U1054, CNRS UMR 5048, University of Montpellier, Montpellier, France

<sup>3</sup> iSSB Laboratory, Génomique Métabolique, Genoscope, Institut François Jacob, CEA, CNRS, Univ Evry, Université Paris-Saclay, 91057 Evry, France

<sup>4</sup> SYNBIOCHEM Center, School of Chemistry, University of Manchester, Manchester, UK

\*Equal contributions

†To whom correspondence should be addressed: [jean-loup.faulon@inra.fr](mailto:jean-loup.faulon@inra.fr) or [manish.kushwaha@inra.fr](mailto:manish.kushwaha@inra.fr)

The supplementary information contains:

**Supplementary Figure S1.** Feedback-loop circuit design of the benzoate actuator.

**Supplementary Figure S2.** 2D plots for the data presented in the heatmap in Figure 2b.

**Supplementary Figure S3.** Comparison of maximum signal of whole-cell circuits.

**Supplementary Figure S4.** Examining the effect of resource competition and enzyme efficiency on the whole-cell cocaine transducer.

**Supplementary Figure S5.** Examining the effect of resource competition and enzyme efficiency on the whole-cell metabolic adder.

**Supplementary Figure S6.** Weighted transducers model results for experimental results presented in Figure 4.

**Supplementary Figure S7.** Five different binary classification problems using a metabolic perceptron for hippurate and cocaine

**Supplementary Figure S8.** Model simulations for classifiers in Figure 6.

**Supplementary Figure S9.** Further characterization of HipO enzyme (hippurate transforming enzyme) at lower concentrations of the enzyme.

**Supplementary Figure S10.** Exploring Hippurate-Cocaine on-off behavior with different weights and input concentrations for hippurate.

**Supplementary Figure S11.** Simulations from the random sampling of estimated parameters in whole-cell system.

**Supplementary Figure S12.** Simulations from the random sampling of estimated parameters in the cell free system.

**Supplementary Table S1.** Goodness of fit scores for the whole-cell models.

**Supplementary Table S2.** Goodness of fit scores for the cell-free models.

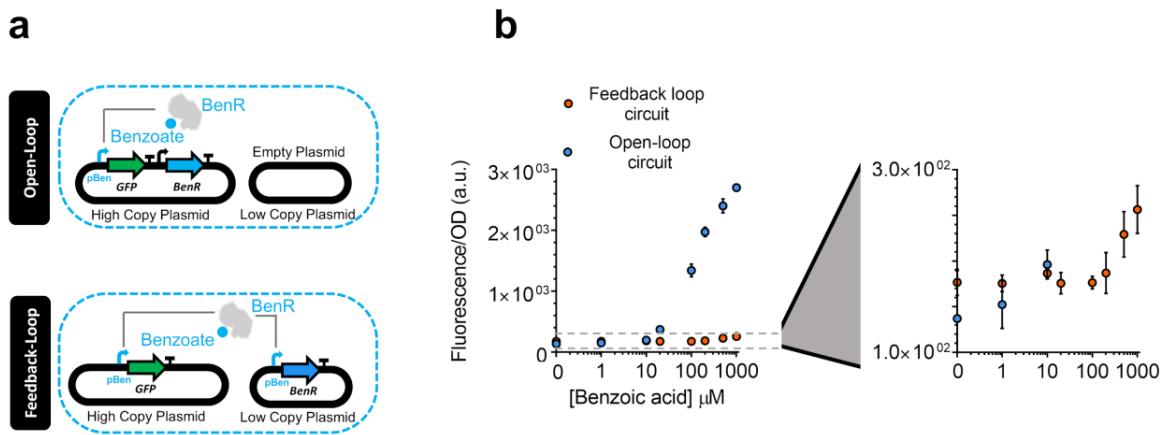
**Supplementary Table S3.** Parameter estimations for whole-cell model.

**Supplementary Table S4.** Parameter estimations for cell-free model.

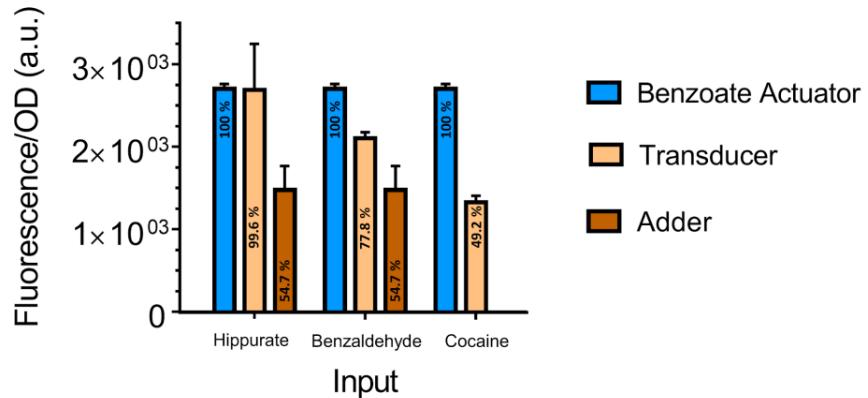
**Supplementary Table S5.** List of sequences and their source used in this study.

**Supplementary Table S6.** Excel file containing the mean and standard deviation of the normalized data of whole-cell experiments, and model simulated/predicted results associated with each experiment.

**Supplementary Table S7.** Excel file containing the mean and standard deviation of the normalized data of cell-free experiments and model simulated/predicted results associated with each experiment.

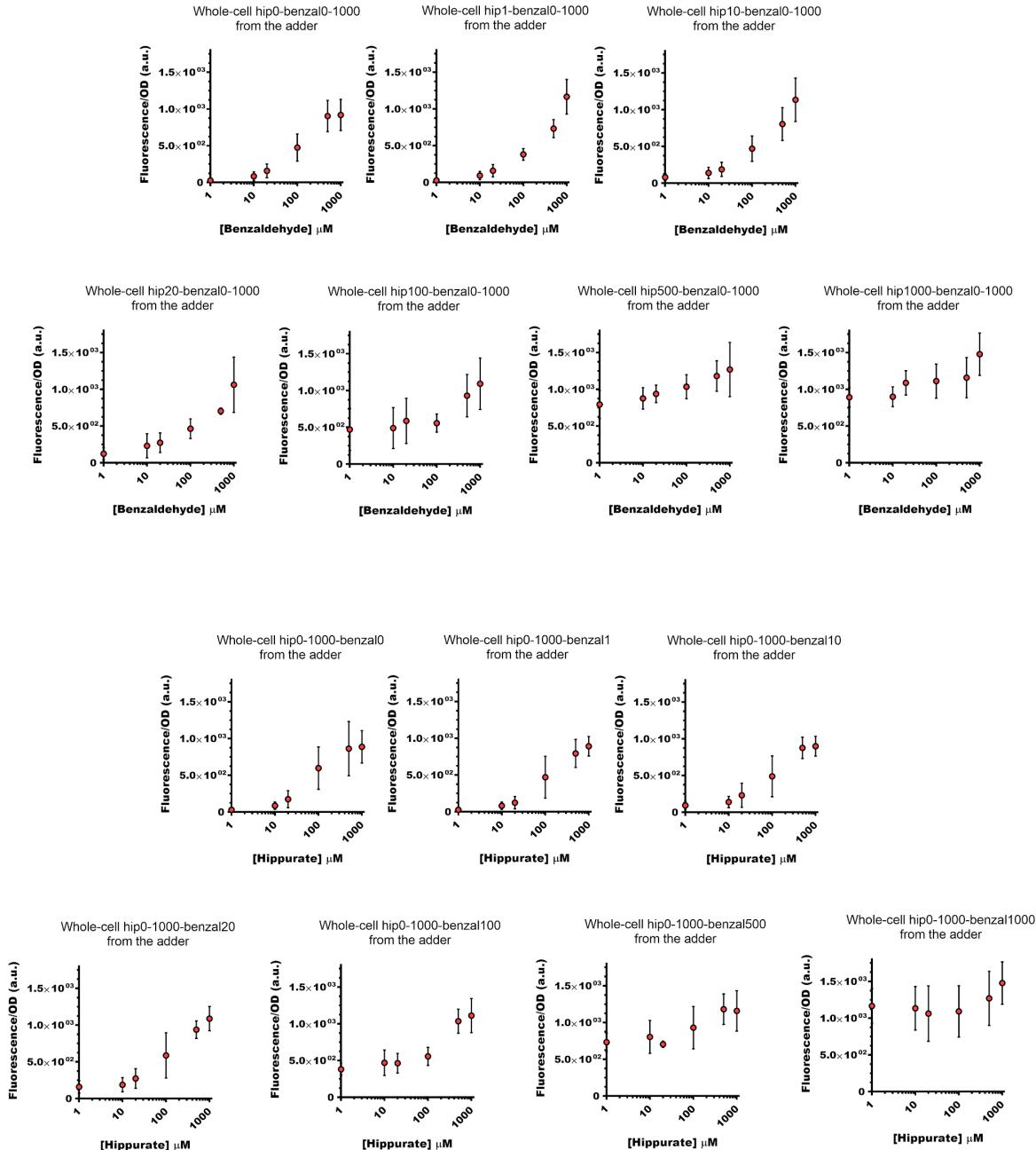


**Supplementary Figure S1. Feedback-loop circuit design of the benzoate actuator.** (a) The open-loop circuit (Figure 1b) versus a feedback-loop circuit for the benzoate actuator. In the feedback-loop actuator the TF is expressed under its responsive promoter, pBen, in a low copy plasmid and sfGFP reporting the signal in a high copy plasmid<sup>1</sup>. (b) The dose-response of the feedback-loop versus the open-loop circuit (Figure 1c) to different concentrations of benzoate. All data points and the error bars are the mean and standard deviation of normalized values from three measurements.

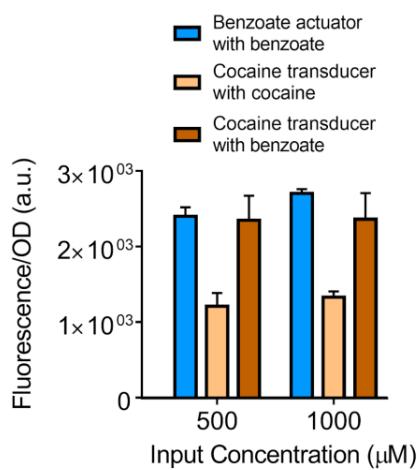


**Supplementary Figure S2. Comparison of the maximum signal of whole-cell circuits.**

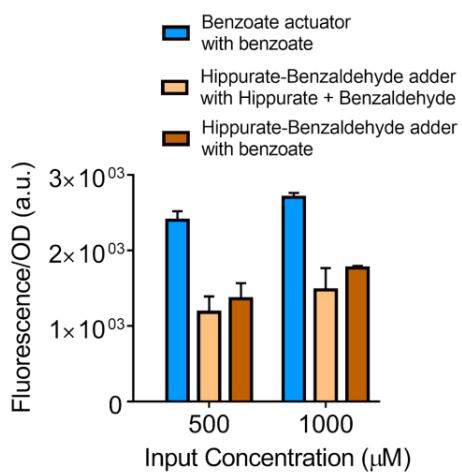
Comparison of the maximal signal of hippurate, benzaldehyde, and cocaine transducers (beige) as well as hippurate-benzaldehyde adder (orange) with benzoate actuator (blue). The maximum signal of all the circuits are at maximum concentration of their inputs (1000  $\mu$ M). The percentage in each bar represents its value with regard to maximum signal of benzoate in benzoate actuator. All data points and the error bars are from the results presented in **Figures 1 and 2**.



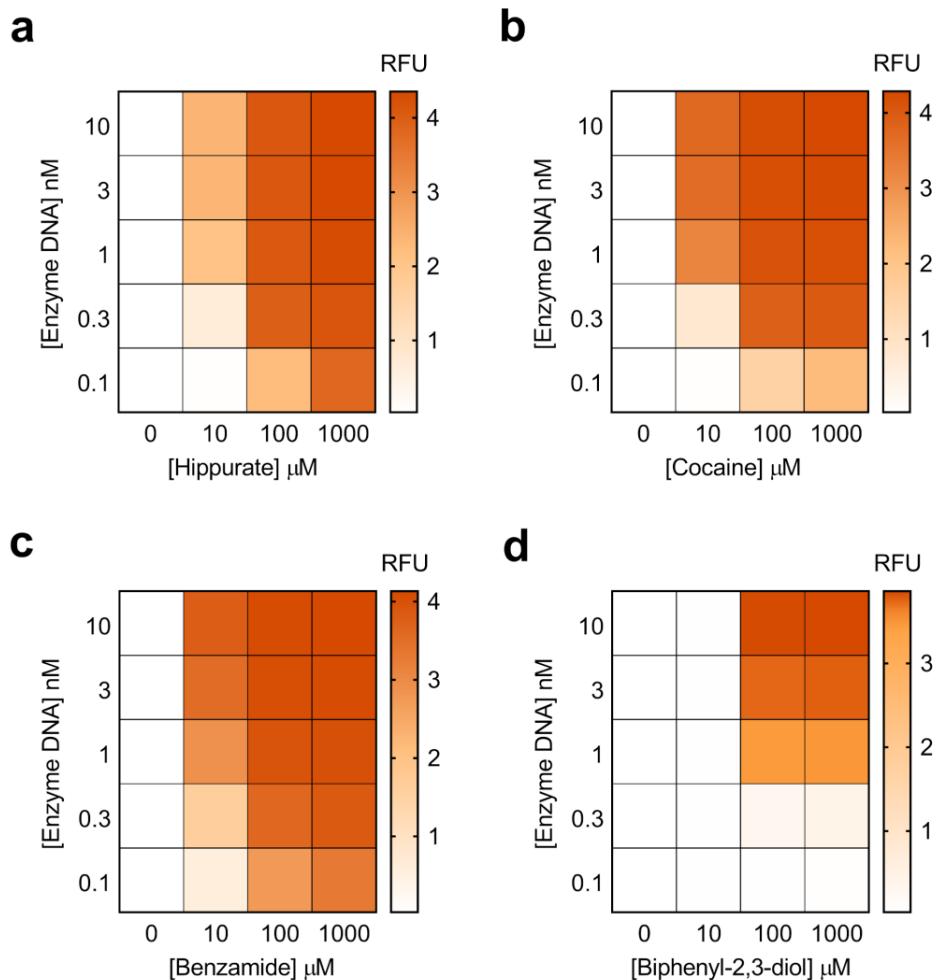
**Supplementary Figure S3. 2D plots for the data presented in heatmap in Figure 2b.** These 14 plots help visualize the linearity of metabolic addition. At the top of each plot the columns/rows of the heatmap in **Figure 2b** have been addressed.



**Supplementary Figure S4. Examining the effect of resource competition on the whole-cell cocaine transducer.** To study these effects on the single-enzyme metabolic circuit, the following experiment was performed: cocaine transducer (with the highest signal dissipation among the three tested in **Figure 1**) was supplied with benzoate input, to test the effect of enzymes on only cellular resource allocation but not conversion of inputs to benzoate. The cocaine transducer with benzoate input shows a behavior similar or close to the benzoate actuator. All data points and the error bars are the mean and standard deviation of normalized values from three measurements.

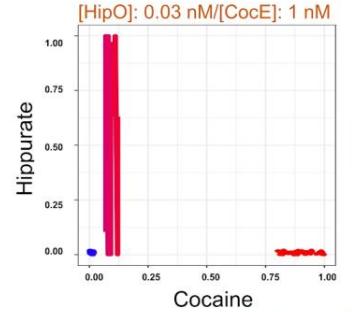
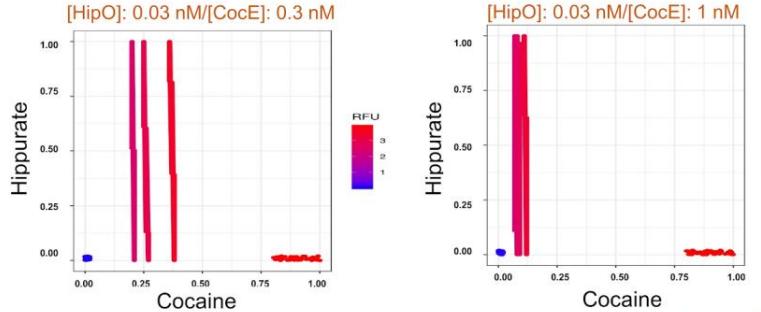


**Supplementary Figure S5. Examining the effect of enzyme efficiency on the whole-cell metabolic adder.** To study these effects on the two-enzyme metabolic circuit (adder) the following experiment was performed: hippurate-benzaldehyde adder was supplied with benzoate input, to test the effect of enzymes on only cellular resource allocation but not conversion of inputs to benzoate. The adder with benzoate input shows a behavior similar to the adder inputted with hippurate and benzaldehyde. All data points and the error bars are the mean and standard deviation of normalized values from three measurements.

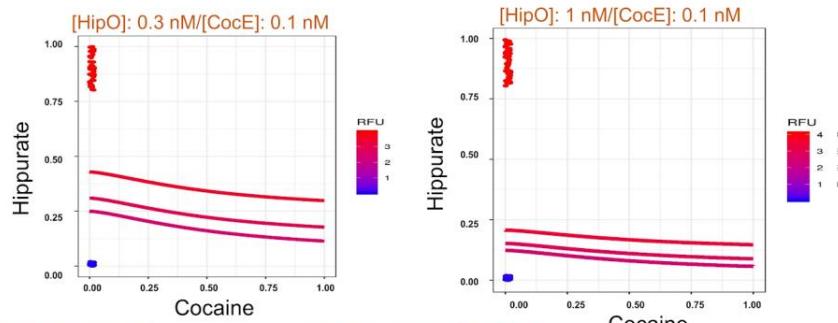


**Supplementary Figure S6. Weighted transducers model results.** The model simulations for experimental conditions presented in **Figure 4**. **(a,b,c,d)** Heatmaps representing model simulations for weighted transducers at different concentrations of input molecules and enzymes DNA for hippurate **(a)**, cocaine **(b)**, benzamide **(c)** and biphenyl-2,3-diol **(d)**.

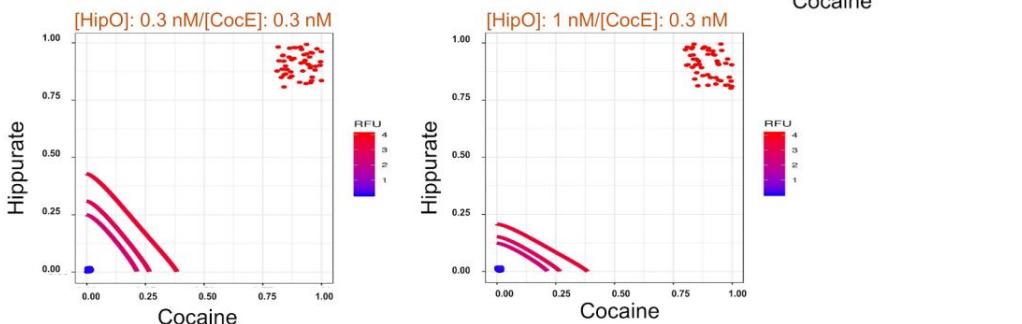
**A**



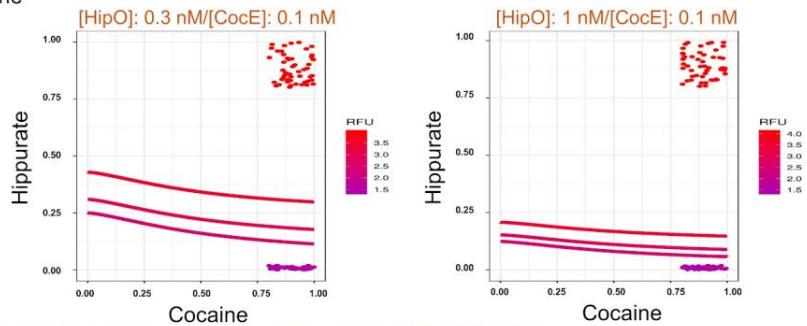
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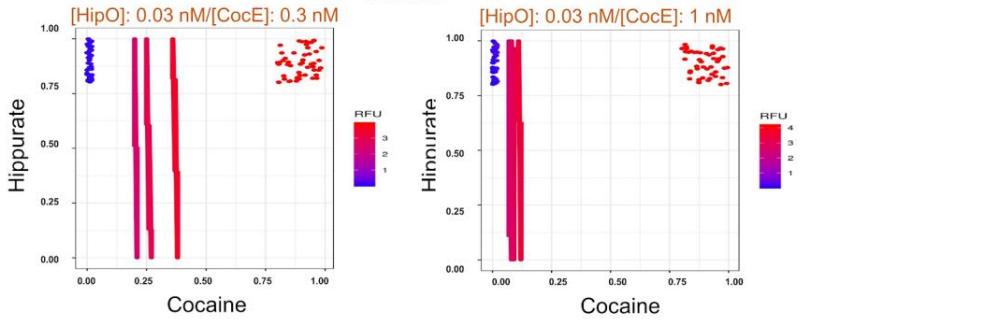
**C**



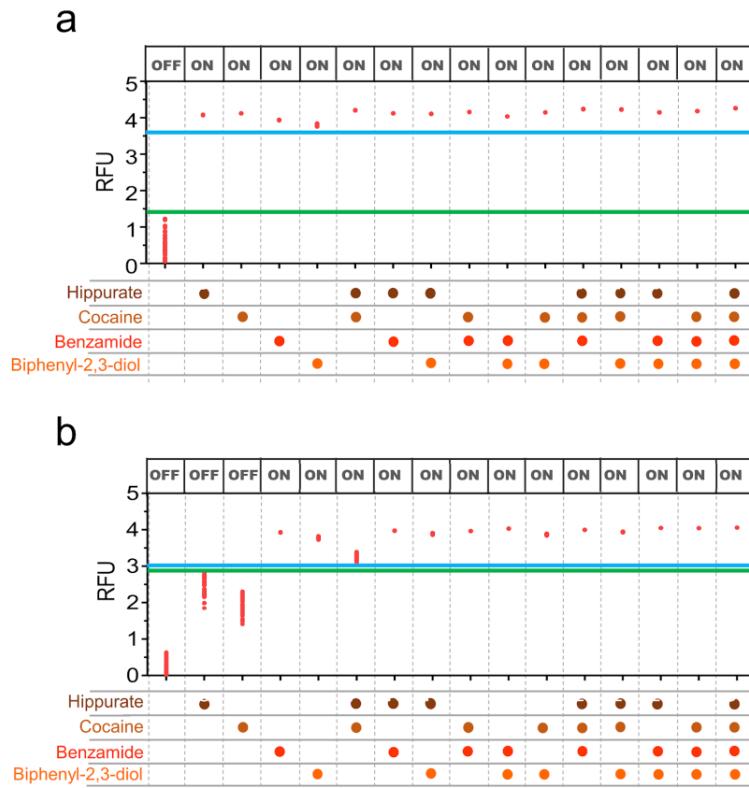
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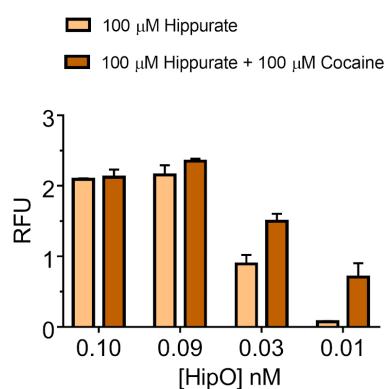
**E**



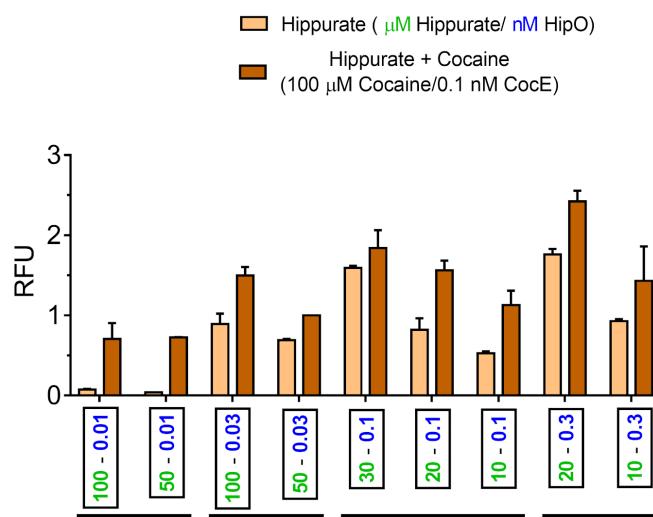
**Supplementary Figure S7. Five different binary classification problems using a metabolic perceptron for hippurate and cocaine. (A to E).** For each problem, the scatter plot shows multiple data points that represent a combination of input values of cocaine and hippurate. The concentrations for those points are sampled between 0 and 2 $\mu$ M for low values and 80 and 100  $\mu$ M for high values. The data points in each problem belong to two different sets that can be separated by a threshold line into two separate clusters. The trained model is then used to identify weights needed to be applied to the weighted transducers such that a decision threshold 'd' classifies the two clusters into red (ON,  $>d$ ) or blue (OFF,  $\leq d$ ). The threshold lines shown in the plots represent three iso-fluorescence lines that successfully classify the data into the binary categories: ON and OFF.



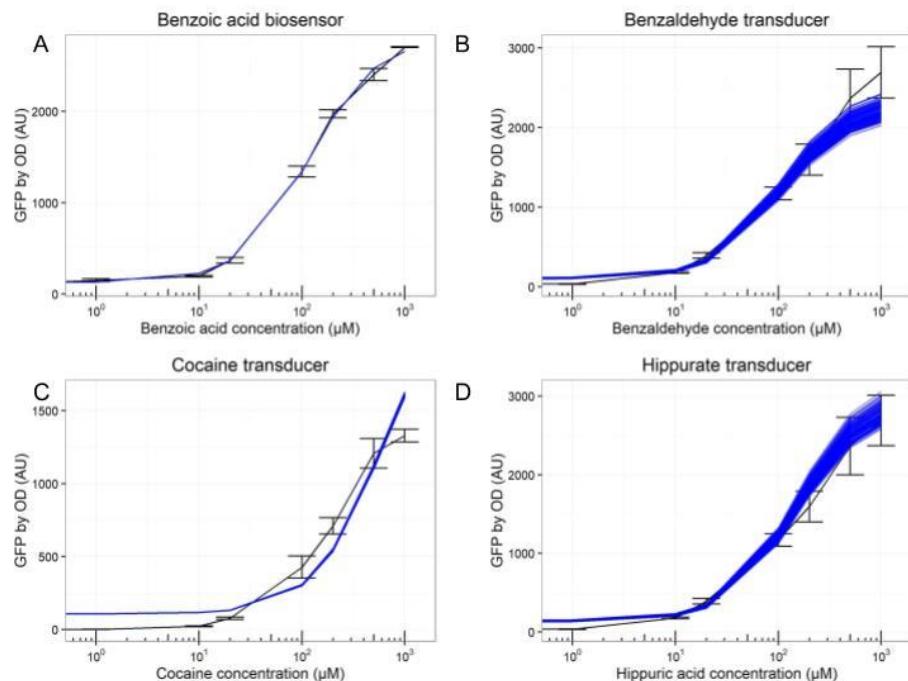
**Supplementary Figure S8. Model simulations for classifiers in Figure 6.** Predictions associated with (a) the full OR classifier (Figure 6c) and (b) the first calculation for “[cocaine (C) AND hippurate (H)] OR benzamide (B) OR biphenyl-2,3-diol (F)” classifier with 0.1 nM HipO weight with (instead of 0.03 as experimentally tested and presented in Figure 6d). In order to perform the clusterings, we sampled values uniformly within the stated ranges ([0, 2 $\mu$ M] for low values and [80, 100 $\mu$ M] for high values). We then simulated the results to assess the robustness of our designs. The blue and green lines refer to the thresholds separating “OFF” and “ON” states. The panel of “OFF” and “ON” at the top of the plots are the expected outputs. (RFU: Relative Fluorescence Unit).



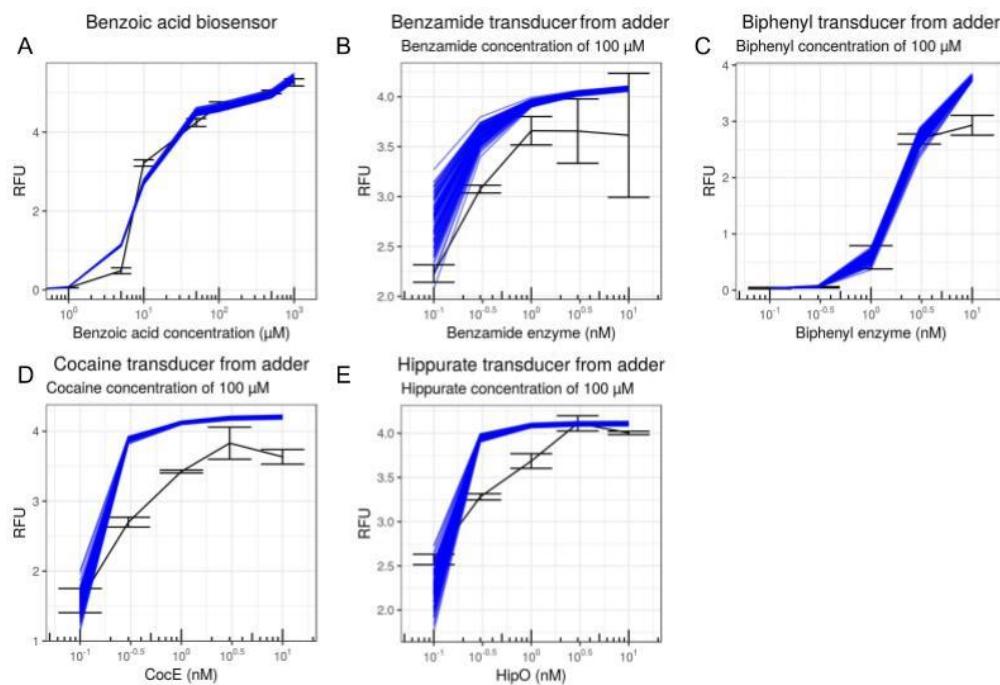
**Supplementary Figure S9. Further characterization of HipO enzyme (hippurate transforming enzyme) at lower concentrations of the enzyme and 100 μM hippurate.** HipO enzyme which for its weight led to higher signals than predicted, needed to be further characterized at concentrations lower than the minimum concentration used for the weighted metabolic circuits (0.1 nM). For this characterization, this figure shows the effect of 100 μM hippurate input alone and its additive effect when coupled with 100 μM cocaine at the weight (CocE enzyme concentration) of 0.1 nM. All data are the mean and the error bars are the standard deviation of normalized values from three measurements. (RFU: Relative Fluorescence Unit).



**Supplementary Figure S10. Exploring Hippurate-Cocaine ON-OFF behavior with different weights and input concentrations for hippurate.** All these experiments were done while Cocaine is at concentration of 100  $\mu$ M and weight of 0.1 nM CocE. All data are the mean and the error bars are the standard deviation of normalized values from three measurements. (RFU: Relative Fluorescence Unit).



**Supplementary Figure S11. Simulations from the random sampling of estimated parameters in whole-cell system.** Representation of the experimental data with SEM ( $n = 3$ ) in black, and in blue, the results from 100 simulations of the model with parameters drawn from the final parameters estimation without refitting. The combination of various parameters within our estimations correctly recapitulates the data. (A) benzoate actuator, (B) benzaldehyde transducer, (C) cocaine transducer, and (D) hippurate transducer. Scripts provided in GitHub also allow for visualization of those results for each axis of the adder in Figure 2.



**Supplementary Figure S12. Simulations from the random sampling of estimated parameters in the cell-free system.** Representation of the experimental data with SEM ( $n = 3$ ) in black, and in blue, the results from 100 simulations of the model with parameters drawn from the final parameters estimation without refitting. The combination of various parameters within our estimations correctly recapitulates the data. (A) benzoate actuator, (B) benzamide transducer, (C) biphenyl-2,3-diol transducer, (D) cocaine transducer, and (E) hippurate transducer. The simulation of the transducers were performed with 100  $\mu\text{M}$  of the input metabolites as will be used in the classifier experiments. Scripts provided in GitHub also allow for visualisation of those results for other axis of the various heatmaps in Figure 4. (RFU: Relative Fluorescent/expression Unit of GFP).

**Supplementary Table S1. Goodness of fit scores for the whole-cell models.** The correlation (from the R cor function), Weighted R squared and R squared between the experimental data and the model. Exact definition of the weighted R squared and the R squared are provided in the Methods section, as well as the RMSD that is used to compare models.

Score	Correlation	Weighted R squared	R squared	Error percentage	Fit or prediction
Actuator	0.999	0.999	0.999	NA	Fit
Benzaldehyde transducer	0.995	0.992	0.980	NA	Fit
Hippurate Transducer	0.997	0.990	0.983	NA	Fit
Cocaine Transducer	0.965	0.950	0.924	NA	Fit
Adder complete	- 0.958	0.982	0.916	16.8 %	Fit (on inducer = 0) and prediction
Adder - both inputs present	0.947	0.931	0.889	15.3 %	Prediction

**Supplementary Table S2. Goodness of fit scores for the cell-free models.**

Score	Correlation	Weighted R squared	R squared	Error percentage	Fit or prediction
<b>Actuator</b>	0.990	0.999	0.980	NA	Fit
<b>Cocaine Transducer</b>	0.923	0.999	0.574	NA	Fit
<b>Hippurate Transducer</b>	0.984	0.999	0.962	NA	Fit
<b>Benzamide Transducer</b>	0.946	0.991	0.659	NA	Fit
<b>2,3 biphenyl Transducer</b>	0.965	0.998	0.762	NA	Fit
<b>Fixed enzyme Adder</b>	0.910	0.998	0.653	10.1%	Prediction
<b>Fixed inducer adder</b>	0.919	0.986	0.784	16.0%	Prediction
<b>Full OR classifier</b>	0.973	0.980	0.823	9%	Prediction
<b>(C AND H) OR B Or F- Fig7</b>	0.985	0.999	0.913	16.9 %	Prediction

**Supplementary Table S3. Parameter estimations for in vivo model.** Mean value plus and minus 95% Confidence Interval

Parameter	Mean Value +- 95 Confidence Interval
Hill_transfer	1.34 +- 1 e-6
Km	114 +- 1 e-4
Fold_change	20.6 +- 3 e-5
Baseline	130 +- 2 e-4
Range_BenZ	1.1 +- 1 e-6
Range_HipO	0.787 +- 1 e-6
Range_CocE	0.201 +- 2.97 e-3
total_enzyme	4.22 +- 0.193
Ratio_hip_benz	0.776 +- 3.7 e-3
Cooperativity_resource	1.956 +- 4.56 e-2
Range_resource	1.973 +- 0.107

**Supplementary Table S4. Parameter estimations for cell-free model.** Mean value plus and minus 95% Confidence Interval (Standard Deviation for fold change and baseline)

Parameter	Mean Value +- 95 CI
Hill_transfer	2.2 +- 0.1
Km	8.40 +- 9 e-3
Fold_change	137 +- 1.84 (sd : 9.41)
Baseline	3.29 e-2 +- 4 e-4 (sd : 2 e-3 )
Slower_slope	8.19 +- 9.3 e-2
Range_HipO	488 +- 35
HipO_constant	0.396 +- 0.022
Hippurate_constant	245 +- 29
Hill_HipO	1.82 +- 0.052
Hill_hippurate	1.205 +- 0.046
Range_CocE	337 +- 28
CocE_constant	0.799 +- 0.00017
Cocaine_constant	54 .4 +- 5.04
Hill_CocE	1.713 +- 0.055
Hill_cocaine	1.44 +- 0.047

range_benzamid_enz	234 +- 20
benzamid_enz_constant	3.73 +- 0.27
benzamid_constant	48.6 +- 5.5
hill_benzamid_enz	0.683 +- 0.072
hill_benzamid	0.906 +- 0.087
range_biphenyl_enz	63.7 +6- 4.79
biphenyl_enz_constant	8.63 +- 0.31
biphenyl_constant	56.3 +- 4.92
hill_biphenyl_enz	1.25 +- 0.067
hill_biphenyl	3.05 +- 0.192

**Supplementary Table S5. List of sequences and their source used in this study.**

Sequence	Description//Nucleotide sequence
<b>BenR</b> <i>UniProtKB - Q9L7Y6</i> Taken from Libis et al. <sup>2</sup>	Transcription factor for benzoate, an activator from <i>Pseudomonas putida</i> <sup>3</sup>  <b>ATG</b> GAATCTCGCTGCTGACGTTCTGTTCCACCAACGCTGACCGTACGCTGTTCTGACTACGTTAA CCAGCACGTTGGTCAGCACTGCATCGCTGCTCGTACCCACCCGAGGCTCTCTGCTCACCGTAA GCTGACTGGACCTGTCCGTATCTTACGGTGGTCTGTTCTGTTACCTCTCCGGCTCTGGAAACCATCTACC ACCTGCAGGTTCTGCTGACGGTAACGGTACTGCCTGTGGCGTGGTACAAACGTGAAACAGCACCTGGTTCCGGGTGAAC TGCTGCTGATCAACCCGGACGACCCGGTTGACCTGACCTACTCTGAAAGACTGCGAAAAATTCTACCTCTGAAAGTTCC GACCCGTCTGGACTCTATCTGCACGAACAGCGTTGGCAGCGTCCGGACGGTGGTCTGTTCTGCTGCGTAA CCACTACCGCTCTGGACGAACGGTGGTCTGTTACCTGCTGGCTATGGTTGACGAAAGCTGAAGTTCTGAC TCTCTGCCGCGTGTTCAGGGTCAACTCTCAGATCGTGTCTAAACTGCTGACCCCTGATGTCTACCAACATCCG TCGTGAATCTCTGCTGCTCCGCAGGCTGGTCAACGTGACTACATGCAACGTAACCTGAAACTGGAA CTGCTGCTGAAGTTCTGGCTGACACGGCTTGACATGCTCTGCTGTTCTGTTACGCTCTGTTGACCCGACCTGG GTATCACCCGAAACACTACGTTCTGCAACGGTGGACTACGGTTCTGCACCTGGTGTCTGACCCGACCTGG TGTTCTGTTACCGAACGGCTCTGGACTACGGTTCTGCACCTGGTGTCTGAAATCTACCGTCAGC AGTTGGTGAACCTGGCTCAGACCTCAAACGTCGTCT <b>TAA</b>
<b>pBen</b> Taken from Libis et al. <sup>2</sup>	Promoter responsive to benzoate-BenR  ACTGTTCGAACGCATTGCCATTCTGAAGTTACCGAAAAAGTACCGAACATCCGAAATCTGGATAACGTTCTGCAC AATCCGGATAGCCCCCGCCAGCCGCTCCCTAACCTGACCGAGGTCTAACAAACAAGGGAGAGTCTGGCC <b>AT</b> <b>G</b>
<b>Superfolder GFP (sfGFP)</b>	<b>ATG</b> CGTAAGGCCAAGAGCTGTTCACTGGTGTGTCCTATTCTGGTGGAACTGGATGGTATGTCACCGTCATA AGTTTCCGTCGCTGGCGAGGGTGAAGGTGACGCAACTAATGGTAAACTGACGCTGAAGTTCATCTGACTACTGG TAAACTGCCGGTACCTTGGCCACTCTGGTAACGACGCTGACTTATGGTGTCTGTTCTGCTTATCCGGAC CATATGAAGCAGCATGACTCTTCAAGTCCGCCATGCCGAAGGCTATGTCAGGAACGACGACGATTCCCTTAAGG ATGACGGCACGTACAAACCGCGTGCAGAAGTGAAGGCGATACCTGGTAACCGCATTGAGCTGAAG GCATTGACTTTAAAGAAGACCGCAATATCCTGGGCCATAAGCTGGAATACAATTAAACAGCCACAATGTTACATC ACCGCCGATAAACAAAAAAATGGCATTAAAGCGAATTAAATCCGCCAACAGTGGAGGATGGCAGCGTGCAGC TGGCTGACTACCAGCAACACTTCAATCGGTATGGTCTGTTCTGCTGCCAGACAATACTATCTGAGCAC GCAAAGCGTTCTGCTAAAGATCCGAACGAGAAACCGCGATCATGGTTCTGCTGGAGTTGTAACCGCAGCGGG CATACGCATGGTATGGACTGTACAA <b>TGATGA</b>
<b>HipO</b> <i>UniProtKB - P45493</i> Taken from Libis et al. <sup>2</sup>	Hippurate hydrolase (EC: 3.5.1.32), <i>Campylobacter jejuni</i> <b>Hippurate to benzoate</b>  <b>ATG</b> AACCTGATCCCGAAATCCTGGACCTGCAGGGTGAATTGCAAAAGATCCGTACCCAGATCCACGAAACCCGG AACTGGGTTTCGACGAACCTGTGCACCGCTAAACTGGTGTCTGAGAAACTGAAAGAATCGGTTACGAAAGTTACGA AGAAATCGGTAAACCGGTGTTGGTCTGAAAAAAGGTAACTCTGACAAAAAATCGGTCTCGTGTGACT TGGACGCTCTGCCGCTGAGAACGACCAACCTGCCGTACAAATCTAAAAAAGAAAACGTTATGCACGCTTCCGG TCACGCGGTACACCAACCTCTGCTGCTGGCTGCTAAATACCTGGCTCTCGACGAACTTCAACCGTGTCTGAAC CTGACTTCCAGCCGGCTGAAGAAGGTCTGGTGGTCTGCTAAAGCTATGACGAAAGACGGTCTGTTGCAAAATTG ACTCTGACTACGTTTCTGGTGGCACACATGCCGTTCTGACAAAAAATTCTACCTGAAAAAAGGTGCTATG ATGGCTTCTCTGACTCTACTATCGAACGTTATGGTGTGGTACGGTTCTGCTCCGGAAAAAGCTAAAGA CCGATCTACGCTGCTCTGCTGATCGTTGCTGCTGAGCTATCGTTCTGCTGTAACGTTGACCCGAGAACCTG CTGTTGTTCTACGGTGCTTCAACGCTGGTACCGCTTCAACATCATCCGGACATCGTACCATCAAATGTCT GTTCTGCTCTGGACAAACGAAACCGTAAACTGACCGAAGAAAAAATCTACAAATCTGCAAAGGTATCGCTCAGG CTAACGACATCGAAACAAATCAACAAAAACGTTGTTGCTCCGGTACCATGAAACAAACGACGAAAGCTGGACTTC GCTTCTGAAAGTTGCTAAAGAAACTGTTCTGGTAAAAAAACTCGCAATTCAACCCACCGTCCGCTATGGCTCTGAAAG ACTTCGGTTCTCTGCGAAATGAAAAATGCGCTACGCTTCTGGAAAACGAAAACGACATCTACCTGACACAC TCTTCTACGTTTCAACGACAAACTGCTGGCTGCTGCTTACTACGCTAAACTGGCTCTGAAATACCTGAA <b>ATAA</b>



	CCAAGCCCCATGATTGAGGTCGAATTGGTGGGGCCCGTACAGTGGATTCCCTTGGACCGTAGCGCGTCAC TCGCGCACCGCTATGGGGCATAAGTCTGTCGCGGACAACGCTAA
<b>bphD</b> <i>UniProtKB - Q52036</i>  Codon optimized and chemically synthesized	2-Hydroxy-6-oxo-6-phenylhexa-2,4-dienoate hydrolase ( <b>EC</b> : 3.7.1.8), <i>Pseudomonas putida</i> 2-hydroxy-6-oxo-6-phenylhexa-2,4-dienoate to <b>benzoate</b>  <b>ATG</b> ACAGCATTGACTGAAAGCTCTACTAGCAAATTCTAACATCAAAGAGAAAGGCTTGTCCGACTTAAGATTCA TATAATGAAGCGGGCAACCGGTGAAACTGTCTCATCATGCTGATGGCGGTGGACCGGGAGCCGGAGGATGGTC CTATTATCGTAATATCGGACCGTCTGGTAAAGCCGGTACCGTGTCTTGAAGGATTCAACCGGGCTTAACAA CCGATGCTGTCGTCATGGATGAACACGCTGGGATTCAATGGGAGGTGCAACCGCGCTTAACCTGCCATCGAG TACGCTTGGAAAATTATCCTTATGGGTCGGGAGGTTGGGACCCCTCATGTTGCCCAATGCCCTAGAGGGAA TTAAATTATTATTAAGTTATGCAGAGCCGTGATGAAAATCTGAAACAGATGATCCAAGTGTCCCTTATGATCA ATCTCTGATTACTGAGGAACCTTACAAGGACGCTGGGAAGGCATTACGCGTCAACCAGAACATCTTAAACCTCC TGATTCTGCGAGAAGGCGCCCTGAGTACGTGGGATGTTACCGCCCTTGGGAGAGATTAAGGCGAAGACCT TCATTACATGGGTCGTGACGCGCTTCTGTGCCGTTAGACCATGGCTGAAACTTTGTGGAATTGATGACGC ACGTTGACGTTTCAAGTGCAGCATTGGCACAATGGGAGCATGCTGACGAGTTAACCGCTTAGCCATT GACTTCTGCGCCAGGCTAA
<i>UniProtKB - B4XEY3</i>  Codon optimized and chemically synthesized	Amidase ( <b>EC</b> : 3.5.1.4), <i>Rhodococcus erythropolis</i> <b>Benzamide</b> to <b>benzoate</b>  <b>ATG</b> GCGACAATCCGTCCCGATGACAACGCAATTGACACGGCGGCCATTATGGCATTACCCCTGACCAAAGC GCGCGTCTTGAGTGGCCCGACTTATTGACGGAGCCTAGGGAGCTACGACGTTGACCAGCTGTACGCTGAT GAAGCCACGCCCAACAACGTCGCGTGAACATACTGTCCTACTGCTAGCAGAAAATCCCTTCCGCTGGTAC GTTACGACCTCTATCCCCCCCACAAGTGAACGGCTCGCGTACCGTTGAGGGATTACTCGTCACGCCACTGTAGTC ACTCGCCTGCTGGCTGCTGGTCAACAGTAGCTGGAAAGGCTGCTGTGAGGACTTATGCTTCTGGCTTAGTT TTACCCAGCCTCGGGACCTGTCGAATCCCTGGGATCCGCAAGCAGCAGGGTGGCTCATCGTATCCGGCT GCCTTTGCGCGTAGTCGGCACAAGCCTACATTGGACTTGTACCATATACGGGAGCCTCCAAATCGAACGCA CGATTGACCACCTGGACCGATTACACGCACTGTCATGACGCTGACTTATGCTGTCAGTTATGCCAGGCCGA TGGAAACGACCCCTGTAAGCGGATAAGTGTGGAAGCGGGGACTACCTTAGTACTTTAGATAGCGACGTCGACGG GTTACGTATCGGAATCGTACGTGAGGGTTGGCACCGCAGCTCAGCCAACCGGAGGTTAGACGACGCCGGTTCG AGCGGCTCACAGCTTAGCAGAAATCGATGACAGTGGAAAGAATGAAACATCCCATGGCACCTGCTGATCGTT ATCTGGAATGTTGATTGCCACCGATGGGGTCTTACCAATGTTAGACGGGAACGGTTATGGAATGAATGAG GTTTATCGACCCCTGAACCTATGGCTACTTCGATCTCGTCGCTTCAACATGCAAGATGCCCTGCTGAAACCGTT AAGCTTGTAGCTGACCGGCCACACGGGATTACGACATTAGGGGCGCTCGTACGGGAAAGCCCGCAACTTG GTTCCGTTAGCGCGTGCAGCTTACGACACCGCGCTCGTACGTTGACGTGCTTGTATGCCAACTTACCTTATG TCGCGCTCAGAATTACCGCCAATGATGTCGACCGTGAACCTTACTAAGGCCTGGGTATGTCGCTAACACA GCACCTTTCGATGTAACAGGGCACCCGAGCTTACGTTCCAGCTGGCCTTGTAAATGGGTTACCTGTCGGTATGA TGATTACTGGAAAGACTTTGATGATGCCACAGTCTCGTAGGGCGCTTGGAGAAAATTACGTGGGGCCTT TCCGACCCCTGCAAGATCACATTGCGATAGTGCCTCGACAATGCGCTTAA
<b>J23101-B0032</b>  From iGEM registry <sup>4</sup>	Constitutive promoter-RBS  AGGATACTAGAGGATGACCCATCTGTTACAGCTAGCTAGTCAGTCCAGGTATTATGCTAGCTAGTAGAGTCACACAG GAAAGTAGTAG <b>ATG</b>

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