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4 The more the merrier?

5 Multivariate approaches to genome-wide association analysis

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28 **Abstract**

29 The vast majority of genome-wide association (GWA) studies analyze a single trait while
30 large-scale multivariate data sets are available. As complex traits are highly polygenic, and
31 pleiotropy seems ubiquitous, it is essential to determine when multivariate association tests
32 (MATs) outperform univariate approaches in terms of power. We discuss the statistical
33 background of 19 MATs and give an overview of their statistical properties. We address the
34 Type I error rates of these MATs and demonstrate which factors can cause bias. Finally, we
35 examine, compare, and discuss the power of these MATs, varying the number of traits, the
36 correlational pattern between the traits, the number of affected traits, and the sign of the
37 genetic effects. Our results demonstrate under which circumstances specific MATs perform
38 most optimal. Through sharing of flexible simulation scripts, we facilitate a standard
39 framework for comparing Type I error rate and power of new MATs to that of existing ones.

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46 **Introduction**

47 Genome-wide association (GWA) studies aim to identify single nucleotide polymorphisms
48 (SNPs) that are associated with (i.e., explain variation in) continuous traits (e.g., height, blood
49 pressure, BMI), or in the liability underlying dichotomous (disease) traits (e.g., schizophrenia,
50 cancer, Type II diabetes). Most GWA studies are univariate in the sense that they focus on a
51 single trait. However, often data on multiple correlated traits are available and sometimes
52 traits treated as univariate are actually multivariate in nature. For instance, GWA studies on
53 metabolic syndrome (e.g., Zhu et al., 2017, Kristiansson et al., 2012) base the case-control
54 status on the joint evaluation of multiple measures (e.g., waist circumference, body mass
55 index, blood pressure, and various blood measures). Similarly, GWA studies on psychiatric
56 disorders like major depressive disorder (e.g., Howard et al., 2018, Wray et al., 2018)
57 generally use case-control status variables that originate in the joint evaluation of multiple
58 clinical criteria, and GWA studies on cognitive ability use cognitive scores that summarize
59 the performance on batteries of cognitive tests covering e.g., vocabulary, general knowledge,
60 and memory (e.g., Savage et al., 2018, Benyamin et al., 2014; Davis et al., 2010).

61 With increasing availability of multivariate information (e.g., UK Biobank), and
62 knowing that pleiotropy is wide-spread both within and between trait domains (Watanabe et
63 al., in revision), it is important to determine the circumstances in which a multivariate
64 approach has greater statistical power than the standard univariate test to detect an associated
65 SNP, which we henceforth will generally refer to as the genetic variants (GV, plural GVs). As
66 GWA studies use a stringent correction for multiple testing (usually α is set to 5×10^{-8} , Pe'er
67 et al., 2008, Sham & Purcell, 2014), and effect sizes of individual GVs are expected to be
68 small (e.g. Visscher et al., 2012, 2017; Psychiatric GWAS Consortium, 2009), statistical
69 power remains a pivotal concern in GWA studies, despite increasing sample sizes. Besides
70 increasing study sample sizes, exploiting the multivariate nature of GWA data sets may under
71 some circumstances, as we will demonstrate here, increase the *statistical power* to detect
72 GVs.

73 Numerous multivariate association tests (MATs) are available. We define a MAT as
74 any test that formalizes the statistical association between a GV and a set of m traits that are
75 measured in the same individual. MATs differ in several respects, such as their ability to
76 accommodate missing values or traits of different measurement levels (e.g., a mix of
77 continuous and dichotomous traits). The power of MATs has been subject of investigation,
78 but the scope of the settings in which power was studied was generally limited: simulation
79 scenarios often featured just a few (e.g., 2 or 3; He et al., 2013, Wu & Pankow, 2015),

80 uniformly correlated traits, only GVs that affect all traits in the analysis (Galesloot et al.,
81 2014; Van der Sluis et al., 2013; Aschard et al., 2014, Suo et al., 2013; Yang et al., 2016), or
82 only same-sign GV effects (e.g., Porter & O'Reilly, 2017). Reality is, however, often more
83 complex, and the *true genotype-phenotype* model (i.e., the model describing the relations
84 between the m traits and the GV as they are in reality) is usually unknown. To determine the
85 circumstances in which MATs perform best in terms of power the following should be
86 considered: the number of traits in the simulations, the correlational patterns between the
87 traits (e.g., both uniform and block-wise), the generality of GV effects (i.e., the number of
88 traits affected by the GVs), and the sign of the GV effects (i.e., allowing the reality of
89 opposite effects).

90 The aim of this Review is to provide a classification of available MATs, to give an
91 overview of their defining characteristics, to inspect their *Type I error rate*, and to compare
92 their statistical power to detect GVs under a multitude of realistic circumstances. We classify
93 MATs based on the underlying statistical model, and explicate their associated hypotheses.
94 We inspect Type I error rates in various circumstances, given various values of *criterion level*
95 α , and we identify the circumstances in which conducting multivariate analyses is
96 (dis)advantageous in terms of statistical power. We do so through extensive simulation in
97 which we investigate the effects of the factors mentioned above: the number of traits in the
98 analysis, the correlational pattern between the traits, and generality and sign of the GV effects.
99 We show that the power of MATs can vary considerably as a function of the true genotype-
100 phenotype model (e.g., in consequence of the presence of unaffected traits or opposite GV-
101 effects). Overall, these results facilitate the choice of the most appropriate and optimal MATs
102 in future multivariate GWA studies. Through sharing of flexible simulation scripts
103 (<https://ctg.cncr.nl/software/>), we facilitate prospective application of a standard verification
104 framework within which the statistical power and Type I error rate of new MATs can be
105 compared to that of existing ones.

106

107 **1. Classification of MATs**

108 A wide range of MATs are available (see Table 1 for an overview of the MATs included in
109 this paper). Following Yang and Wang's conceptual classification (Yang & Wang, 2012), we
110 distinguish transformation-based MATs, regression-based MATs, and combination tests. We
111 discuss each class of MATs and provide a short statistical description of the MATs included
112 in this review in Boxes 1-3. These descriptions provide a basic understanding of the statistical
113 properties of individual MATs, which furthers insight into their specific strengths and

114 weaknesses. For a non-statistical overview of all included MATs, we refer to Table 1. Note
115 that in each MAT, the predictor of main interest is a single genetic variant, i.e., a potential
116 GV. In practice, however, additional predictors (i.e., covariates) are standardly included in the
117 model such as the age and sex of participants, and genetic principle components (obtained
118 using e.g. Eigenstrat (Price et al., 2006) or FlashPC2 (Abraham & Inouye, 2014)) to correct
119 for population stratification.

120

121 *Transformation-based MATs*

122 The simplest way to deal with a multivariate problem is by reducing it to a univariate problem
123 through transformation of the multivariate information. Given N subjects and m traits $y_1 \dots y_m$,
124 a single new *variate* \tilde{y} for subject i can be created that is a linear combination of these m
125 traits:

126

$$127 \tilde{y}_i = b_0 + b_1 y_{1i} + b_2 y_{2i} + \dots + b_m y_{mi} \quad [1]$$

128

129 where the weights $b_1 \dots b_m$ determine how much each original trait contributes to the new
130 variate. All transformation-based MATs are aimed at variable reduction. The following
131 transformation-based MATs are included in this review and their characteristics (e.g., how the
132 weights $b_1 \dots b_m$ in Eq 1 are determined) are described in Box 1: sum-score analysis, Principal
133 Component Analysis (PCA), the Combined Principal Components test (CPC, Asschard et al.,
134 2014), and common factor analysis. Important to note is that all transformation-based MATs
135 determine the weights in Eq. 1 independently of the association of the m traits with the GV
136 (e.g., in factor analysis, the weights depend on the correlations among the m phenotypes
137 only). That is, all transformation-based MATs first transform the data solely based on the
138 phenotypic information, and only then consider the possible association of this new variate
139 with the GV (generally using a univariate regression model).

140

141 *Regression-based MATs*

142 In a multivariate GWA settings, one focusses on the association between a set of k predictors
143 (the GV and the covariates), and a set of m traits. Given N subject, m traits and k predictors,
144 this multivariate (referring to the number of dependent variables) multiple (referring to the
145 number of predictors) regression model can be represented as:

146

$$147 \mathbf{Y} = \mathbf{X}\mathbf{B} + \mathbf{E} \quad [2]$$

148

149 Here, \mathbf{Y} is the $N \times m$ matrix of trait scores. \mathbf{X} is the design matrix, i.e., a $N \times (k+1)$ matrix of
150 predictor scores in which the first column usually is a unit vector that serves to estimate the m
151 trait-specific intercepts. \mathbf{B} is a $(k+1) \times m$ matrix of regression weights with the first row
152 containing m trait-specific intercepts, and the subsequent k rows containing m trait-specific
153 regression weights for the k predictors. The m regression weights on the row corresponding to
154 the GV are usually all freely estimated, giving rise to an m degrees of freedom (df) omnibus
155 test (i.e., the GV is allowed to affect the m traits differently). The m weights may be
156 constrained to be equal, thus giving rise to a 1-df test (i.e., the GV is assumed to affect all m
157 traits similarly: in this case, the m traits should be measured on, or be transformed to the same
158 scale). Finally, \mathbf{E} is a $N \times m$ matrix of individual- and trait-specific zero-mean residuals, also
159 referred to as error or disturbance terms. Generally, homoscedasticity of the residuals is
160 assumed, and the $m \times m$ symmetrical background covariance matrix is denoted as $E[\mathbf{E}^t\mathbf{E}] =$
161 $\Sigma_{\mathbf{E}}$. That is, $\Sigma_{\mathbf{E}}$ is the residual variance-covariance matrix between the m traits conditional on
162 the k predictors, i.e., \mathbf{E} captures all sources of residual (co)variability. Note that matrix $\Sigma_{\mathbf{E}}$ is
163 usually not diagonal because, conditional on the k predictors, the m traits are generally still
164 correlated. Regression-based MATs mainly differ in their treatment of $\Sigma_{\mathbf{E}}$ (see Box 2). As
165 given in Eq. 2, the multivariate multiple regression model is thus a system of univariate
166 regression equations. By combining them all within one model, specific hypotheses can be
167 tested, and the model can be simplified by introducing constraints in matrices \mathbf{B} and \mathbf{E} .

168 The following regression-based techniques are described in Box 2: Multivariate
169 Analysis of Variance (MANOVA), Generalized Estimating Equations (GEE), and MultiPhen
170 (O'Reilly et al., 2012). Assuming an additive codominant genetic model, MANOVA, GEE
171 models, and Linear Mixed Models (LMM, not included in this review, see Box 2) are specific
172 instances of the model presented in Eq. 2. In contrast, the regression-based MAT MultiPhen is
173 based on reversed ordinal regression with the m traits as the predictors and the GV as the
174 dependent variable.

175

176 *Combination tests*

177 We define a combination test as any test that combines the p-values or test statistics obtained
178 in m univariate analyses to test a multivariate hypothesis. The challenge characterizing
179 combination tests is to optimally handle the correlations between the m p-values or m test
180 statistics, resulting from the phenotypic correlations between the m traits. How the
181 information obtained in univariate tests is combined is described in Box 3 for the following

182 tests: Nyholt-Šidák and Bonferroni corrected p-values (min- P_{NS} , min- P_{Bonf} ; Nyholt, 2004), the
183 Simes test (Simes, 1986), its adjusted version TATES (Trait-based Association Test that uses
184 Extended Simes; Van der Sluis et al., 2013), two version of JAMP (Joint genetic Association
185 of Multivariate Phenotypes: JAMP_{mult} and JAMP_{min} (ctg.cncr.nl/software/), the meta-analysis
186 inspired techniques S_{Hom} and S_{Het} (Zhu et al., 2015), and the adjusted Fisher-combination test
187 FC-Pearson (Yan et al., 2016).

188 We emphasize the following important aspects of these combination tests. First, only 4
189 of the combination tests truly create, based on the univariate test statistics, a new multivariate
190 test statistics, and, as such, evaluate the *joint* association signal of the m traits to the GV
191 (JAMP_{mult}, S_{Hom} , S_{Het} , FC-Pearson). The others essentially constitute various types of
192 corrections for multiple testing. Second, Simes, TATES, min- P_{Bonf} , min- P_{NS} and JAMP_{min} do
193 not create a new test statistic, but simply select the smallest of m weighted univariate p-
194 values. Due to the weighting (i.e., effectively a correction for multiple testing), the p-values of
195 these combination tests are always larger than the original univariate p-values on which they
196 are based.

197
198 These three classes of MATs are conceptually distinguished. Alternatively, all transformation-
199 based and regression-based tests, and some combination tests, can be described from a
200 maximum-likelihood perspective, and within this framework, one could distinguish 1-df and
201 m -df tests. Specifically, 1-df tests either reduce all m traits to a single new variate (i.e., sum-
202 score analysis, PCA using PC1 only, and factor scores obtained in a single common factor
203 model), or constrain all m associations between the GV and the m -traits to be equal (S_{Hom} , and
204 the 1-df versions of GEE and MANOVA). In all these tests, the association between the GV
205 and the m traits is modelled via 1 parameter, which can be tested using a (1-df) likelihood
206 ratio test. In contrast, in m -df tests, the associations between the GV and the m traits are
207 allowed to vary, and the m parameters are subjected to a m -df likelihood ratio test, or a closely
208 related (F-) test (standard MANOVA, CPC, and the m -df versions of GEE). An alternative
209 classification, based on the underlying mathematical model and the structure of the resulting
210 test statistic, that matches distinction of MATs based on degrees of freedom, is outlined in the
211 Supplemental Information.

212 Irrespective of their statistical foundation, all MATs need to deal with the fact that the m
213 simultaneously modelled traits are often correlated conditional on the tested GV. The way
214 they do so differs: combination tests use either permutation or a correction factor, regression-
215 based tests either treat the m traits as predictors, avoiding the issue altogether (MultiPhen), or

216 accommodate the residual trait correlations in a background covariance matrix Σ_E
217 (MANOVA, GEE, LMM), and transformation-based tests explicitly use the covariance
218 between the m traits to create the new variate.

219

220 **2. Characteristics of MATS**

221 The classification in transformation-based tests, regression-based tests, and combination tests
222 is based on the statistical properties of the MATs. They differ, however, in various respects
223 that have a bearing on the their performance and applicability. We discuss these differences
224 briefly, and refer to Table 1 for an extensive summary.

225

226 ***Specific hypothesis tested***

227 While all MATs evaluate the statistical relationship between m traits and a GV, they differ
228 with respect to the exact hypothesis that they test. First, MATs can evaluate the omnibus
229 hypothesis that the *joint* association signal of the m traits to the GV deviates significantly
230 from 0. This omnibus test can be an m -df test, allowing for heterogeneity in the m GV-effects
231 regarding sign and size. By assuming the GV-effects to be homogeneous across the m traits,
232 the omnibus test reduces to a 1-df test, which can be more powerful if the homogeneity
233 assumption holds approximately. The 1-df tests are obtained through constraining of model
234 parameters (e.g., the regression weights are constrained to be equal), or through the use of
235 transformation-based techniques, in which the m traits are reduced to a single new variate
236 under the assumption that this new variate is representative of what the m traits have in
237 common. Second, MATs can test the hypothesis that at least one of the m traits is significantly
238 associated with the GV. These MATs generally concern combination tests that evaluate the
239 smallest of m weighted p-values as obtained in univariate GWA analyses.

240

241 ***Measurement level of the m traits***

242 The choice of MAT is often largely dictated by the *measurement levels* of the m traits.
243 Specifically, if all m traits are continuous (to reasonable approximation), PCA, CPC, and
244 MANOVA can be used directly. All MATs suited for continuous data assume the data to be
245 multivariate normally distributed. GEE-based generalized linear modeling can handle
246 continuous or categorical traits, but current standard implementation (e.g., GEE in SPSS or
247 the R library *gee*) cannot handle a mix of different measurement levels. Which measurement
248 levels factor analysis can handle, depends on the software package (e.g., when conducted in
249 MPlus (Muthén & Muthén, 2017) or OpenMx (Neale et al., 2016), factor analysis can in

250 principle handle all measurement levels as well as a mix). The sum score method is applicable
251 to continuous variables, or ordinal variables (including dichotomous) variables (i.e., “burden
252 score”), as long as all m aggregated traits are measured on the same scale. If the m traits have
253 different measurement levels, combination tests and MultiPhen can be used (but see Guo et
254 al., 2015 on power losses in MultiPhen when traits are non-normally distributed). The
255 strength of combination tests lies in their flexibility to combine results regardless of the traits’
256 measurement level. For instance, TATES has been shown to work well on a mix of non-
257 uniformly correlating dichotomous, ordinal, and continuous traits (Van der Sluis et al., 2013).
258 The current implementation of the permutation-based combination tests of JAMP is suited for
259 continuous data only, but is in principle amendable to traits with a mix of measurement levels.
260

261 ***Missingness***

262 In univariate analyses, missing values simply result in a smaller effective sample size N . In a
263 multivariate context, however, partial missingness can occur, i.e., participants having missing
264 values on a subset of the m traits. Not all software can handle partial missingness; methods
265 often resort to listwise deletion, basing analyses only on cases with complete data. As in
266 practice the probability of at least 1 of the m scores being missing increases with m , listwise
267 deletion can result in a substantial reduction of sample size and consequently a considerable
268 reduction in statistical power. Alternatively, however, one can use packages like OpenMx
269 (Neale et al., 2016) that use Full Information Maximum Likelihood (FIML, i.e., all available
270 data are used) to specify a wide variety of multivariate models (including MANOVA, PCA,
271 and factor analysis) while accommodating the missingness. This can, however, come with a
272 prohibitive computational burden in the GWA settings.

273 If one weights the m trait scores appropriately, sum scores can still be used if the data
274 show partial missingness: e.g., each individual sum score may be divided by the number of
275 observed trait scores. As this may result in heteroskedastic variance, weighted sum scores are
276 generally used in combination with a cut off criterion (e.g., no more than 20% of the m scores
277 can be missing), which also ensures approximate conceptual comparability between scores
278 over subjects with different numbers of observed scores.

279 The essentially univariate nature of the input of combination tests guarantees their
280 ability to handle missingness. However, if sample sizes differ greatly between the m traits, a
281 (sample size) weighted procedure (like S_{Hom} and S_{Het} offer) is desirable.

282 Generally, partial missingness lowers the power to detect GV, especially if the traits
283 with a relative large percentage of missingness are the traits with the strongest genetic

284 association. Additionally, in using methods that can accommodate the missingness, one
285 should realize that the multivariate association signal may be primarily driven by the traits
286 with the lowest percentage of missingness.

287 Imputation of the missing scores can be a convenient way to handle missing data, as
288 replacement of the missing values with imputed ones facilitates the use of all MATs.
289 Multivariate imputation, i.e., dealing with imputation of missing values in multiple variables
290 at once, can be done in many ways, but comes with its own challenges and can yield biased
291 results (see e.g. Nakai & Ke, 2011; van Buuren & Groothuis-Oudshoorn, 2011).

292

293 **Relateds**

294 GWA data sets may include data collected in families (e.g., trios of parents and one affected
295 off-spring, data of twins and their family members). In univariate analyses, inclusion of
296 family members can be useful to differentiate “between” from “within” family associations,
297 the latter being free of any effects of population stratification (Fulker et al., 1999). Also,
298 including all available data, even data of genetically similar monozygotic twin pairs, can be
299 beneficial in terms of power to detect GV-effects (e.g., Minica et al., 2014). However, if data
300 include family members, the data clustering induced by the relatedness must be
301 accommodated statistically to avoid inflated Type I error rates. In the univariate setting,
302 multiple linear mixed model approaches exist (see Eu-ahsunthornwattana et al., 2014 for
303 comparisons). When data only include a few relateds, one can chose to “correct for” the
304 familial relatedness rather than explicitly model it. For instance, PLINK (Purcell et al., 2007)
305 offers the option to correct for relatedness in the data by running GEE, which involves a
306 correction of standard errors¹. In principle, these univariate procedures can be used in the
307 context of transformation-based techniques (i.e., correcting the univariate analyses of the new
308 variate), and in the context of combination tests, in which case the corrected model
309 parameters of the m univariate GWA analyses are used as input for the combination tests (to
310 our knowledge, only the performance of the combination test TATES has been studied in the
311 context relatedness; Vroom et al., 2015). Combination tests using permutation, like JAMP,

¹ Specifically, the working correlation matrix is by default set to “independent” in PLINK (i.e., the family scores are assumed independent conditional on the GV under study) to minimize computational intensity. GEE’s standard sandwich correction then corrects the standard errors of all estimated parameters for model misspecification induced by ignoring relatedness. This procedure works well in terms of Type I error rates, but Minica et al (2015, see also Vroom et al., 2016) showed that considerable statistical power can be gained if the working correlation matrix is set to unstructured, although this is computationally more demanding.

312 need to permute the data not on an individual level but on the family-level to retain the
313 familial relatedness in the data. This is complicated if the families in the data set do not all
314 have the same size and composition.

315 In their standard form, MANOVA and MultiPhen cannot be used on data including
316 relatives. Theoretically, in case of familial clustering, multivariate multilevel modelling can be
317 used instead of MANOVA (Pituch and Stevens, 2016), and Structural Equation Modelling
318 can be used instead of MultiPhen, treating the m traits as exogenous variables. These
319 approaches are, however, computationally intensive.

320 As standard GEE software can handle only one source of clustering at the time, it can
321 handle either familial relatedness in a univariate setting, or multivariate data in a sample of
322 genetically unrelated individuals, but not both. In principle, LMM (Box 2) can handle
323 multiple sources of clustering or correlation.

324

325 ***Computational feasibility***

326 Given imputation of genetic variants, current GWA studies may include tens of millions of
327 SNPs. Cluster computers offer large computation capacity, but computation burden is an
328 important consideration in the choice of MAT. In theory, any of the MATs discussed here can
329 be applied using standard software. However, in practice, the use of dedicated software like
330 PLINK (Chang et al, 2015, Purcell et al, 2007) considerably facilitates running such vast
331 amounts of statistical tests on files containing multiple terabytes of data. From a
332 computational feasibility perspective, MATs that rely on univariate analyses (i.e.,
333 transformation-based tests and combination tests) or MATs that are built-in in dedicated
334 software (Canonical Correlation Analysis, i.e., MANOVA (see Box 1) as part of PLINK) may
335 be preferred over tests like GEE, MultiPhen, S_{Hom} and S_{Het} , or permutation-based tests like
336 JAMP_{mult} and JAMP_{min}. Due to their increased computational intensity, these latter options
337 are particularly attractive if they indeed come with clear advantages, like substantial gains in
338 power.

339

340 **3. Type I error rates of MATs**

341 A correct Type I error rate is a primary requirement of any statistical test. We studied the
342 Type I error rates of 17 MATs, excluding the JAMP-methods as the correctness of their Type
343 I error rates is guaranteed by their reliance on permutation. The 17 MATs were studied in 20
344 scenarios that are outlined in Table 2 (see Supplemental Information for simulation details).
345 The 20 scenarios varied with respect to the number of included variables ($m=4$ or $m=16$), the

346 strength of the correlations between the traits, and the correlational structure, i.e., uniformly
347 correlated traits (i.e., 1-factor model with compound symmetry), or two clusters of more or
348 less strongly correlated traits (i.e., 2-factor model). All simulated traits were standard
349 normally distributed. For each scenario, we ran $N_{\text{sim}}=1,000,000$ replications, allowing us to
350 reliably evaluate Type I error rates at α -levels of .05, .01, and .001. All Type I results are
351 available in Tables S7-S9.

352 We note that the large number of replications provides high statistical power to detect
353 small deviations from the expected Type 1 error rate (α), especially for the larger α values.
354 For instance, with 1 million replications, the 99% confidence interval (CI_{99}) for $\alpha=.05$ is very
355 narrow: .04944-.05056 (see Table S6 for the CI_{99} for all α -levels). As a result, merely
356 considering which MATs show Type I errors outside the CI_{99} paints a gloomy picture (Figure
357 S2a). Type I error rates of MANOVA, S_{Hom} , and all transformation-based (i.e., essentially
358 univariate) MATs are virtually always correct. However, when considered across all 20
359 scenarios and all three levels of α (.05, .01, .001, i.e., 60 scenarios in total), all other MATs
360 showed Type I error rates outside the CI_{99} , with overall percentages ranging from 22% (CPC)
361 to 92% (FC Pearson) and 100% ($GEE_{\text{uns_}m}$).

362 Figure 1 shows the Type I error rates of the 17 MATs given $\alpha=.05$ for 4 or 16
363 variables, split for scenarios with mostly low or mostly high trait correlations (see Table S2).
364 As many of these deviations outside the CI_{99} were (very) small (Tables S7-S9), we also
365 looked beyond the CI_{99} by summing the deviations from the expected α across all scenarios,
366 allowing us to determine which factors caused the largest deviations (Figure S2b). Overall,
367 the largest deviations are observed for TATES, min- P_{NS} , Simes, FC-Pearson, $GEE_{\text{uns_}m}$, and
368 min- P_{Bonf} . Interestingly, combination tests show mainly deviations from the expected when
369 the m traits are highly correlated, while the number of traits m mainly drives the deviations in
370 most other method. Taking the direction of the deviations into account, we see that CPC,
371 Simes and min- P_{Bonf} are always conservative, while S_{Het} , Tates and min- P_{NS} are conservative
372 when applied to many (highly correlated) traits, and liberal otherwise. All other methods that
373 do show deviations from the expected, always show inflation, with Type I error rates of
374 $GEE_{\text{uns_}m}$ and FC-Pearson especially being inflated when m is large, irrespective of the
375 correlations between the phenotypes.

376 Summarizing, due to the strong power to detect deviations from the expected, many
377 methods showed Type I error rates outside the CI_{99} . When considering the magnitude of the
378 deviations, especially application of Simes, min- P_{Bonf} , FC-Pearson, and m -df versions of GEE

379 warrant careful consideration, although even here the actual deviations are often quite small
380 (Tables S7-S9).

381

382 **4. Power of MATs**

383 The statistical power of a test is the probability that the null-hypothesis of no association is
384 correctly rejected when the GV is indeed statistically associated with the trait(s). In the context
385 of GWA studies, GV-effects are expected to be small, so in selecting a MAT for one's
386 analyses, power is an important consideration.

387 We studied the power of 19 MATs in 15 scenarios covering 270 settings of the true
388 genotype-phenotype model, which are summarized in Tables 3 and 4 (see Supplemental
389 Information for simulation details). The scenarios varied with respect to the number of traits
390 ($m=4, 8, \text{ or } 16$, all standard normally distributed), the correlational structure (i.e., uniformly
391 correlated or clustered, corresponding to 1- or 2-factor models), the strength and sign of the
392 correlations between the m traits, the number of traits affected by the GV (1, half, or all m),
393 and the presence or absence of opposite effects (i.e., GV affecting multiple traits but in
394 opposite direction). For each setting, we ran 1,000 simulations with a GV explaining .1, .2 or
395 .5% of the variance in each affected trait, and a sample size of $N=2000$.

396 The full results of the power simulations are available in Table S10-S12. Below, we
397 discuss the power results for a GV explaining .1% of the variance (Table S10), and emphasize
398 that these main findings hold for GV of different effect sizes (Tables S11-S12). We excluded
399 the 2 MATs with highly inflated Type I error rates ($\text{GEE}_{\text{uns}_m}$, and FC-Pearson) from
400 discussion as their power estimates can be biased upwards due to the inflated Type I error
401 rates (but see Tables S10-S12 for all power results of these test). We did include the two
402 conservative MATs (Simes, min- P_{Bonf}) in our discussion, as their deflated Type I error rates
403 will result in under- rather than overestimation of power which we can interpret as a lower
404 bound estimate.

405 Figure 2 depicts the power of these 17 MATs in all 15 scenarios for 4 and 16
406 variables. We note that the power of MATs can be compared within, but not always directly
407 between, scenarios as the total contribution of the GV to the m traits can differ across
408 scenarios as a function of the correlations between the m traits.

409

410 ***Univariate versus multivariate***

411 When testing the association of a GV to m traits, one could simply do m univariate analyses
412 and correct the m resulting p-values for multiple testing. We consider the power results of the

413 combination test min- P_{Bonf} an approximation of this approach (although min- P_{Bonf}
414 subsequently selects the smallest Bonferroni corrected p-value). The power results in Figure 2
415 reveals that when all or half of the m traits are affected by the GV (scenarios 1-4, 6-13),
416 MATs are very often (but not always!) more powerful than a for multiple testing corrected
417 univariate analysis. MATs even often outperform univariate analyses when only 1 of the m
418 trait is affected by the GV, especially when the trait correlations are generally high. Taken
419 over all scenarios, it is safe to conclude that multivariate approaches towards identification of
420 GV are generally worth pursuing.

421

422 ***Equivalence of MATs***

423 So far, we classified MATs based on their underlying statistical approach, the descriptions in
424 Boxes 1-3 outlining their differences. The power simulations, however, demonstrate that there
425 are 3 groups of MATs that function very similarly, i.e., have very similar power across all or
426 most of the scenarios (see Supplemental Information for detailed comparisons). First, the
427 combination tests min- P_{NS} , Simes, TATES, and JAMP_{min} demonstrate very similar power
428 throughout all 15 scenarios, with min- P_{Bonf} showing a very similar yet consistently lower
429 power profile. Second, the m -df tests MANOVA, CPC, and MultiPhen perform very similarly
430 (and very similar to the m -df variants of GEE), with S_{Het} generally does equally well or
431 slightly worse. Third, in the context of uniformly correlated traits (scenarios 1-5), tests that
432 can generally be referred to as 1-df tests group together, i.e., the transformation-based
433 techniques sum-score, PCA, and factor scores, and the 1-df variants of the regression-based
434 tests GEE (exchangeable and unstructured) and MANOVA. However, in the context of
435 clustered traits (scenarios 6-15), PCA and the factor scores perform much worse than the
436 other 1-df tests when the clusters correlate negatively. Interestingly, the combination test
437 JAMP_{mult} follows its own trend (which is very similar to that of the FC-Pearson test).

438

439 ***Relative insensitivity to the true genotype-phenotype model***

440 The true genotype-phenotype model provides the multivariate context in which one tests the
441 associations between the m traits and the GV. Our power simulations show that some MATs
442 are relatively insensitive to this context, i.e., their power varies much less across the different
443 scenarios compared to other MATs. These relatively insensitive MATs all concern
444 combination tests that are based on selection of the minimum weighted p-value: min- P_{Bonf} ,
445 min- P_{NS} , Simes, TATES, and JAMP_{pmin}. Mainly in the context of many uniformly correlated
446 traits and a *pleiotropic* variant affecting all m traits (scenarios 1-2), do these methods

447 demonstrate noticeable variation in power, i.e., their power to detect the GV decreases with
448 increasing correlations between the m traits, irrespective of the presence of opposite effects. In
449 all other scenarios, the power curves for these methods are rather flat, illustrating their relative
450 insensitivity.

451 This relative insensitivity to the true genotype-phenotype model can be advantageous:
452 there are several settings in which these MATs generally outperform m -df tests and S_{Het} (e.g.,
453 scenarios 1, 6, 7, 10, 11), factor scores and PCA (e.g., scenarios 6-9, 12 and 13), JAMP_{mult} (3-
454 5,13-15), and sum scores, S_{Hom} and 1-df regression-based tests (e.g., 2-5,8,9,12 and 13).
455 However, some MATs actually benefit from specific characteristics of the true genotype-
456 phenotype model, such as the presence of unaffected or oppositely affected variables in the
457 analysis (see below). Under these circumstances, these relative insensitive MATs are,
458 sometimes substantially, outperformed. Because of their relative insensitivity, we exclude
459 these MATs from further discussion.

460

461 ***Clustered versus uniformly correlated traits***

462 When the m traits are uniformly correlated, all transformation-based techniques have very
463 similar power (scenarios 1-5). In this context, the power of transformation-based techniques
464 increases with decreasing correlation among the m traits. Specifically, the variance of the new
465 variates, summarizing the communality between the traits, is larger when the m traits correlate
466 more strongly and the contribution of the GV to that common variance is in that case
467 relatively small. That is, the *signal-to-noise ratio* is more optimal when the covariance
468 between the traits conditional on the GV is low (see Supplemental Information for an
469 elaborate discussion).

470 In the context of clustered correlated traits, however, PCA and factor scores perform
471 differently from the other transformation-based tests when the correlation between clusters of
472 positively correlated traits is negative (scenarios 6-11). In that case, the first PC from PCA
473 and the factor scores from a 1-factor model will only summarize 1 of the two clusters well,
474 while they do not capture information from the other cluster. Interestingly, in the calculation
475 of sum scores, the presence of negatively correlated variables can actually have a beneficial
476 effect on the detection of GV-effects (scenarios 6-11): the negative covariances between pairs
477 of traits reduce the total variance of the sum, which in turn improves the signal-to-noise ratio
478 (see Supplemental Information).

479 When the GV affects only half or 1 of the traits, the m -df tests MANOVA, MultiPhen
480 and CPC perform better when the m traits are uniformly correlated than when they are

481 clustered (scenarios 3 and 5 versus 10-11 and 14-15), but when the GV affects all m traits or
482 conveys opposite effects (scenarios 1,2,4 versus 6-7,8-9,12-13), the power of these tests does
483 not seem to suffer much from the clustering in the data.

484 In the context of uniformly correlated traits (scenarios 1-5), the power of JAMP_{mult} is
485 clearly a function of the trait correlations, with lower trait correlations resulting in higher
486 power. Similar results are observed for the clustered scenarios, if one compares the power in
487 the scenarios with within-cluster correlations of .3 (scenarios 6,8,10,12 and 14) to those with
488 within-cluster correlations of .7 (scenarios 7,9,11,13 and 15: always lower).

489

490 ***Pleiotropic versus local variants***

491 In evaluating GV-effects in a multivariate context, it is desirable to distinguish between the
492 detection of pleiotropic or *global genetic variants* (i.e., variants that affect all or multiple of
493 the m traits in the analysis) and *local genetic variants* (i.e., variants that effect only 1 or a few
494 of the m traits in the analysis). As we defined a MAT as any test that formalizes the statistical
495 association between a GV and a set of m traits that are measured in the same individual, one
496 may argue that MATs should be assessed based on their power to detect global variants.
497 Conducting multivariate analyses may then not only be lucrative with respect to power, but
498 can also aid theoretical development and biological understanding by revealing shared
499 underlying biology. However, a one-sided focus on global variants neglects the importance of
500 identifying local variants, which may be a source of genetic heterogeneity. Identification of
501 genetically homogeneous subsets of traits within the full set of m traits acknowledges the
502 contribution of more local variants and may be biologically informative (e.g., Nagel et al.,
503 2018).

504 In the context of uniformly correlated traits, the (transformation-based) 1-df tests work
505 best for the identification of global variants that affect all phenotypes in the same direction
506 (scenario 1), as these contribute most to the variance of the new variate. Here, the power to
507 detect global GVs decreases as the conditional correlations between the m traits increase (i.e.,
508 the *signal-to-noise ratio* decreases). Yet, GVs that affect only half or 1 of the m traits
509 (scenarios 3, 5) can hardly be detected through these 1-df tests: such GVs will generally
510 contribute little to the variance of the new variate and will therefore be (very) difficult to
511 identify using transformation-based approaches. When traits show clustering, we see a clear
512 difference between sum scores and other 1-df MATs, which do well in detection global
513 variants (scenarios 6,7), and PCA and factor scores, which do poorly. Clearly, the first PC and
514 factor scores based on a 1-factor model do not capture the clustered nature of the data well.

515 Interestingly, in a clustered context, 1-df tests do best in detecting GV affecting only half of
516 the m traits (scenarios 10,11), especially when the unaffected traits correlate negatively to the
517 affected traits: in that case, the negatively correlations lower the variance of the new variate
518 and as such improve the signal-to-noise ratio. Yet, truly local variants go undetected when
519 transformation-based or 1-df MATs are used.

520 Conceptually, MATs that evaluate the joint association signal of the m traits through
521 m -df omnibus tests truly test for global variants, i.e., Cross Phenotype (CP) associations, i.e.,
522 whether a genetic variant is associated with more than one trait (i.e., pleiotropic, see Solovieff
523 et al., 2013). Counter intuitively, however, our simulations demonstrate that in the context of
524 both uniformly correlated and clustered traits (scenarios 1,6,7), those m -df MATs do not have
525 the best power to detect global variants, and (like for all MATs) their power suffers especially
526 when the m traits correlate substantially (Minica et al., 2010; Medland & Neale, 2010). When
527 traits correlate uniformly, these m -df MATs do have the best power to detect local GVs
528 (scenario 5) and GVs that affect only half of the m traits (scenarios 3). In case of clustered
529 variables, the presence of negatively correlated variables can boost the power to detect global
530 GVs (scenarios 6,7), but their power to detect GVs that affect only half (scenarios 10,11) or 1
531 (scenarios 14,15) of the m traits is generally very low, although still superior to that of other
532 MATs.

533 JAMP_{mult} is quite good at picking up global GVs, especially when the trait correlations
534 are low (scenarios 1-4,6). In the context of uniformly correlated traits, JAMP_{mult} has
535 noticeably less power than the m -df tests to pick up GV that affect only 1 or half of the m
536 traits, especially with increasing correlations between the m traits. In clustered settings,
537 JAMP_{mult} can perform slightly better than m -df tests when GV affect only half of m traits
538 (e.g., scenarios 10,11).

539

540 ***Presence of unassociated traits***

541 In psychology and clinical research, it is common to observe mean group differences in some
542 but not all variables of a set of m moderately/highly correlated traits. For instance, Van der
543 Sluis et al (2008) observed significant gender differences in the means of 3 out of 12
544 substantially positively correlated cognitive subtests of the WISC-R (Carroll, 1993).
545 Similarly, gender differences in endorsement rates are often observed in some but not all of
546 positively correlated depression symptoms (see e.g. Lux & Kendler, 2010). In genetic
547 research, where GV-effects are generally small, it is likely that a GV affects correlated traits
548 differently. For instance, in a set of 12 phenotypically correlated neuroticism items (.17-.54),

549 Nagel et al (2018) identified many item-specific genome-wide significant genetic regions (see
550 their Supplementary Data 2). As the exact GV-trait relationship is generally unknown, it is
551 important to consider the effect of the presence of unassociated traits in the set of m traits on
552 the power of MATs.

553 To study the effect of the presence of unaffected traits on the power to detect as GV of
554 interest, we compare the power results of scenario 5 for 4, 8 and 16 variables, i.e., the power
555 to detect a local GV-effect in the presence of 3, 7, or 15 unaffected variables, respectively
556 (Table S10). In this context, the power to detect the GV is low for all methods, except the m -
557 df techniques MANOVA, MultiPhen and CPC, and S_{Het} , which do have some power if the
558 trait correlations are substantial (i.e., .5 or higher). For all MATs, the power to detect that
559 local GV deteriorates when more unaffected uniformly correlated traits are added to the
560 analysis.

561 Interestingly, the m -df tests MANOVA, MultiPhen and CPC, and S_{Het} have lower
562 power to detect a GV that affects all m traits (scenario 1) than to detect a GV affecting half of
563 the m traits (scenario 3), even though the total amount of signal is lower in the latter scenario.
564 Specifically, the presence of unaffected traits can boost the power to detect GV effects
565 considerably, but only if they are substantially correlated to the affected traits in the analysis.
566 In the Supplemental Information, we show graphically for $m=2$ (inspired on Cole et al., 1994)
567 how a GV that affects trait Y1 but not trait Y2 can aid discrimination between genotype
568 groups (and thus detection of the GV).

569

570 ***Opposite effects***

571 GV with opposite effects, in which an allele increases the value of/risk to one trait, while
572 decreasing the value of/risk to another, are not uncommon (Solovieff et al., 2013). For
573 instance, Sitora et al (2009) demonstrated such opposite effects in autoimmune diseases.
574 Given the existence of GVs with opposite effects, it is important to determine which MATs
575 can detect them.

576 Our simulations show that the power of all 1-df MATs (both reduction and regression-
577 based techniques, and S_{Hom}) suffers seriously from the presence of opposite effects. The
578 transformation-based tests all rely on the variance that is shared between the m traits, i.e., their
579 communality. While concordant effects contribute to this communality, opposite effects do
580 not and cancel out. Consequently, the opposite GV-effects are poorly represented in the new
581 variate (depending on the ratio concordant-to-opposite effects), thus resulting in decreased
582 power to detect them.

583 Under the assumptions that the GV-effects are concordant across all m traits, 1-df
584 MATs constrain them to be equal and then test whether this single parameters deviates
585 significantly from 0. When the assumption holds, this reduced model has increased power to
586 detect the GV compared to univariate procedures (e.g., scenarios 1,6,7). However, if the GV-
587 effects are opposite in reality, constraining them to be identical will cancel individual effects
588 out, thus drastically reducing the power of 1-df MATs (e.g., scenarios 2,4,12,13).
589 Interestingly, when clusters of traits correlate negatively (e.g., scenarios 8,9), the GV-effects
590 can contribute to the communality if the difference in sign of the GV-effect is in concordance
591 with the difference in sign of the correlations, in which case GV with opposite effects can be
592 picked up by these methods.

593 In contrast, JAMP_{mult} handles opposite effects much better than transformation-based
594 and 1-df tests, while the m -df MATs MANOVA, MultiPhen, and CPC ,and S_{Het} actually seem
595 to benefit from the presence of opposite effects (scenarios 2,4,8,9,12,13). That is, the power to
596 identify opposite-effect GVs that affect all or half of the m traits is actually higher than the
597 power to detect a GV that has concordant effects on half or all of the m traits (pairwise
598 compare scenarios 1 to 2, 3 to 4, 6 to 8, 7 to 9, 10 to 12, 11 to 13). As m -df tests evaluate the
599 m association parameters individually, the effects do not cancel each other out. Cole et al
600 (1994) already showed that for MANOVA, the critical consideration is not simply the sign of
601 the GV-effects, but the sign of the correlation between the traits as well. In the Supplemental
602 Information, we show graphically for $m=2$ (inspired on Cole et al., 1994) how a GV that
603 increases the mean of trait Y1 while decreasing the mean of trait Y2 can aid discrimination
604 between genotype groups (and thus detection of the GV) if these traits are positively
605 correlated.

606

607 ***Number of traits***

608 In planning multivariate analyses, one important question is whether the power to detect the
609 GV depends on the number of traits. Our simulations show that when the GV affects only 1 of
610 the m traits, the power is generally slightly better if m is smaller (scenario 5,14,15; Figure 2).
611 For all other scenarios (i.e., scenarios concerning GVs that affect half or all of the m traits),
612 including more traits is generally beneficial for the power of all MATs, except for the m -df
613 tests. For the m -df tests, including more traits is only beneficial when the GV transmits
614 opposite effects (scenarios 2,4,8,9,12,13). Yet, when a GV affects all of the m traits similarly
615 (scenarios 1,6,7), or only 1 of the m traits (scenarios 5,14,15), then these m -df tests have better
616 power when m is small because in that case the number of degrees of freedom is smaller.

617

618

619 **5. Discussion**

620 Researchers often employ MATs with the aim to discover pleiotropic GVs, i.e., GVs that are
621 statistically associated to multiple traits, which possibly points towards a shared biological
622 substrate (Solovieff et al., 2013). The general finding of our simulations that the power to
623 detect such global variants decreases for all MATs as the phenotypic correlations between the
624 traits increase (e.g. Minica et al., 2010, Medland & Neale, 2010; as would be expected with
625 increasing genetic relatedness), demonstrates that currently available MATs are actually not
626 optimised to identify true pleiotropic GVs (see also Porter et al., 2017).

627 The considerable variation in power displayed by MATs across multiple scenarios
628 demonstrates that the choice of MAT is no trivial matter. The optimal choice is determined by
629 multiple factors that define the true genotype-phenotype model, such as the strength and sign
630 of the correlations between the traits, sign and generality of the GV-effect, and the presence of
631 unaffected traits. Many of these factors are unknown prior to analysis, which hampers the
632 formulation of globally applicable recommendations. As Zhou & Stephens (2014) noted "...in
633 a GWAS setting no single test will be the most powerful to detect the many different types of
634 genetic effects that could occur. Indeed, it is possible to manufacture simulations so that any
635 given test is most powerful. Thus different multivariate and univariate tests should be viewed
636 as complementary to one another, rather than competing." Consequently, identifying the
637 circumstances in which specific MATs perform strongly or poorly, and indicating which
638 (classes of) MATs are most versatile, is the best we can do for now. Overall, the *m*-df MATs
639 outperform both transformation-based tests and combination tests in 10 out of the 15
640 scenarios (2-5, 8,9,12-15) represented in our study. That is, the *m*-df MATs are better at
641 identifying GVs that convey opposite effects or GV that affect only a subset of the modelled
642 traits, but are often outperformed when GV are truly pleiotropic (scenarios 1,6-7).

643 As previously pointed out concerning MANOVA (Cole et al., 1994), the power of *m*-
644 df MATs can, somewhat counter-intuitively, improve from the inclusion of traits that are
645 unassociated to the GV, if these are correlated with the affected traits. In the context of
646 experimental studies, this knowledge can be put to use given prior or theoretical knowledge of
647 which traits are expected to be affected or unaffected by a given manipulation. In the context
648 of GWAS, however, such theory to guide in- or exclusion of traits is usually lacking (see
649 Supplemental Information for a short discussion on the (dis)advantages of increasing the
650 number of traits *m*).

651 In our simulations, we considered only additive codominant GVs and normally
652 distributed continuous traits. These choices fit the (distributional) assumptions underlying
653 most MATs. We note that Type I error rates of various techniques (e.g., MANOVA,
654 univariate regression) may not be correct when standard assumptions are violated (e.g.,
655 severely non-normal or non-continuous data, see e.g. O'Reilly et al, 2012, Yang et al., 2016,
656 Gasperik, 2010), and that some MATs may have better power to identify non-additive GVs
657 than others. Yet for a selection of MATs, Porter & O'Reilly (2017) showed that for those
658 methods amendable to dichotomous case-control data, the pattern of results was remarkably
659 similar to that obtained using continuous data.

660 In the current review, we focused only on frequentist-based MATs that do not rely
661 greatly on permutation or bootstrapping. MATs based on Bayesian modeling do, however
662 exist (e.g. multivariate version of SNPtest (Marchini et al., 2007) and BIMBAM (Stephens,
663 2013)) or bootstrapping (e.g., PCHAT, Klei et al, 2008), and we refer to Galesloot et al (2012)
664 and Porter et al (2017) for power simulations including these MATs. Similarly, we focused on
665 MATs that formalize the statistical association between a GV and a set of m traits that are all
666 measured on the same individual. Recently, multiple methods were developed that allow
667 estimation of the genetic covariance between traits using genome-wide association signal
668 (e.g., GCTA (Yang et al., 2011), BOLT-REML (Loh et al., 2015), LD Score Regression
669 (Bulik-Sullivan et al., 2015)), alongside multivariate methods like Multi-Trait Analysis of
670 GWAS (MTAG: Turley et al., 2018) and genomic SEM (Grotzinger et al., in press), which
671 use this genetic covariance among traits to boost the statistical power to detect GVs for (sets
672 of) target traits. As these techniques are not primarily SNP-level multivariate tests of traits
673 measured on the same individual (although genomic SEM can be used as such), they were not
674 included in this review.

675 Summarizing, we presented a classification on MATs based on both their underlying
676 statistical approach and the associated degrees of freedom, alongside a summary of their main
677 characteristics. We showed that MATS vary considerably in their power to detect associated
678 GVs, that under many circumstances, MATs are often more powerful than multiple testing
679 corrected univariate analyses even when only 1 of the m traits is affected by the GV, and that
680 in many scenarios m -df MATs are the most powerful. We also demonstrated for all current
681 MATs, the power to identify truly pleiotropic GVs decreases with increasing trait-
682 correlations, i.e., particularly when pleiotropy is expected. With increasing availability of
683 multivariate information from large publicly accessible biobanks (e.g., UK Biobank,
684 23andMe, deCODE), and knowing that pleiotropy is wide-spread both within and between

685 trait domains (Watanabe et al., in revision), we believe that development of new MATs that
686 focus specifically on detection of pleiotropic GVs is crucial. Through sharing of flexible
687 simulation scripts, we facilitate a standard framework for comparing Type I error rate and
688 power of new MATs to that of existing ones.

689

690

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692

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Type I error rate given $\alpha=.05$

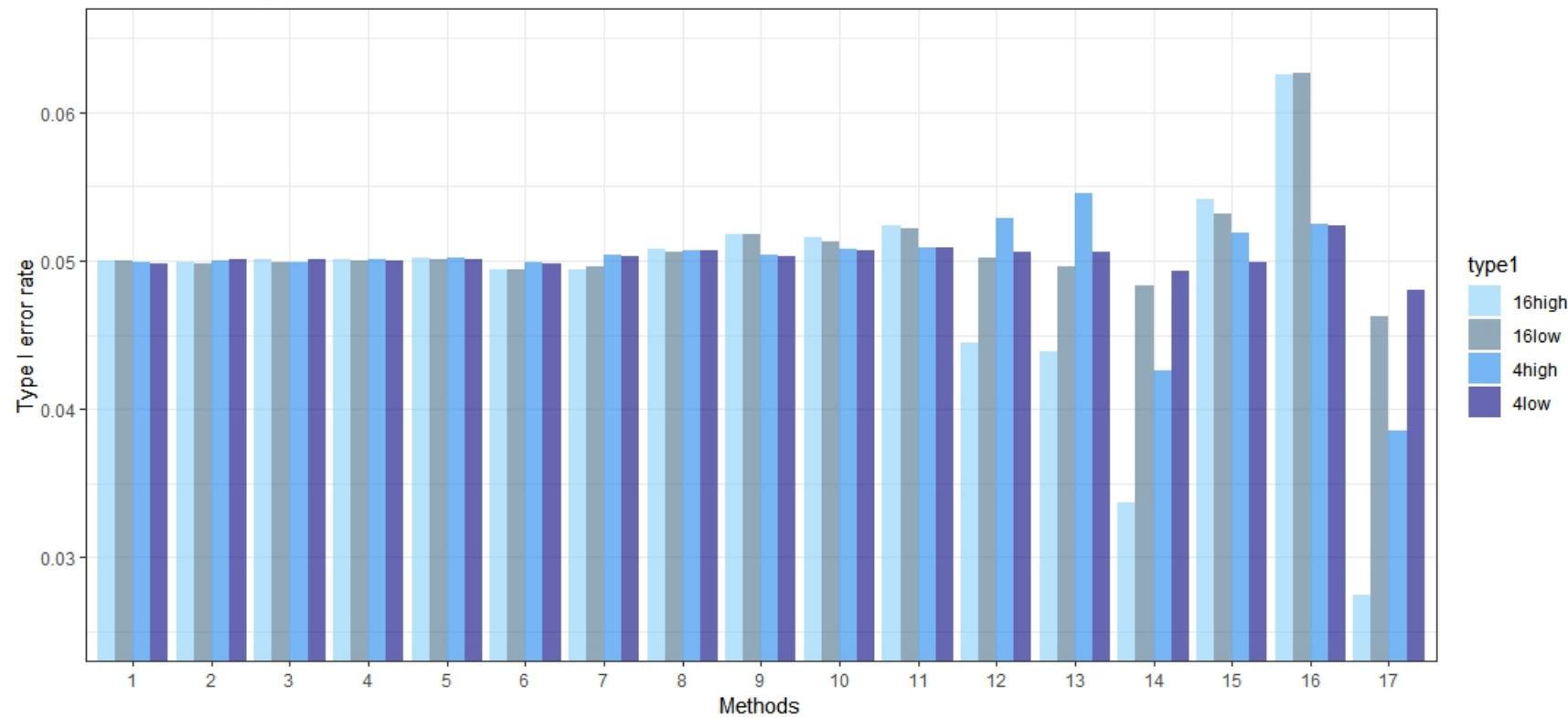


Figure 1: Type I error.

Type I error rates for 17 MATs given $N_{var}=4$ or $N_{var}=16$, plotted separately for scenarios with mostly low and scenarios with mostly high correlations (see Supplemental Table S2). Methods are numbered: 1=MANOVA, 2=factor score, 3=PCA, 4=sum score, 5=SHom, 6=CPC, 7=SHet, 8=GEEex-1, 9=MultiPhen, 10=MANOVA 1df, 11=GEEun-1, 12=TATES, 13=min-PNS, 14=Simes, 15=FCPearson, 16=GEEun-m, 17=min-PBonf. See Supplemental Tables for Type I error rates given $\alpha=.01$ and $\alpha=.001$. Note: the two JAMP-methods were excluded from the Type I error rate study as the correctness of their Type I error rates is guaranteed by their reliance on permutation.

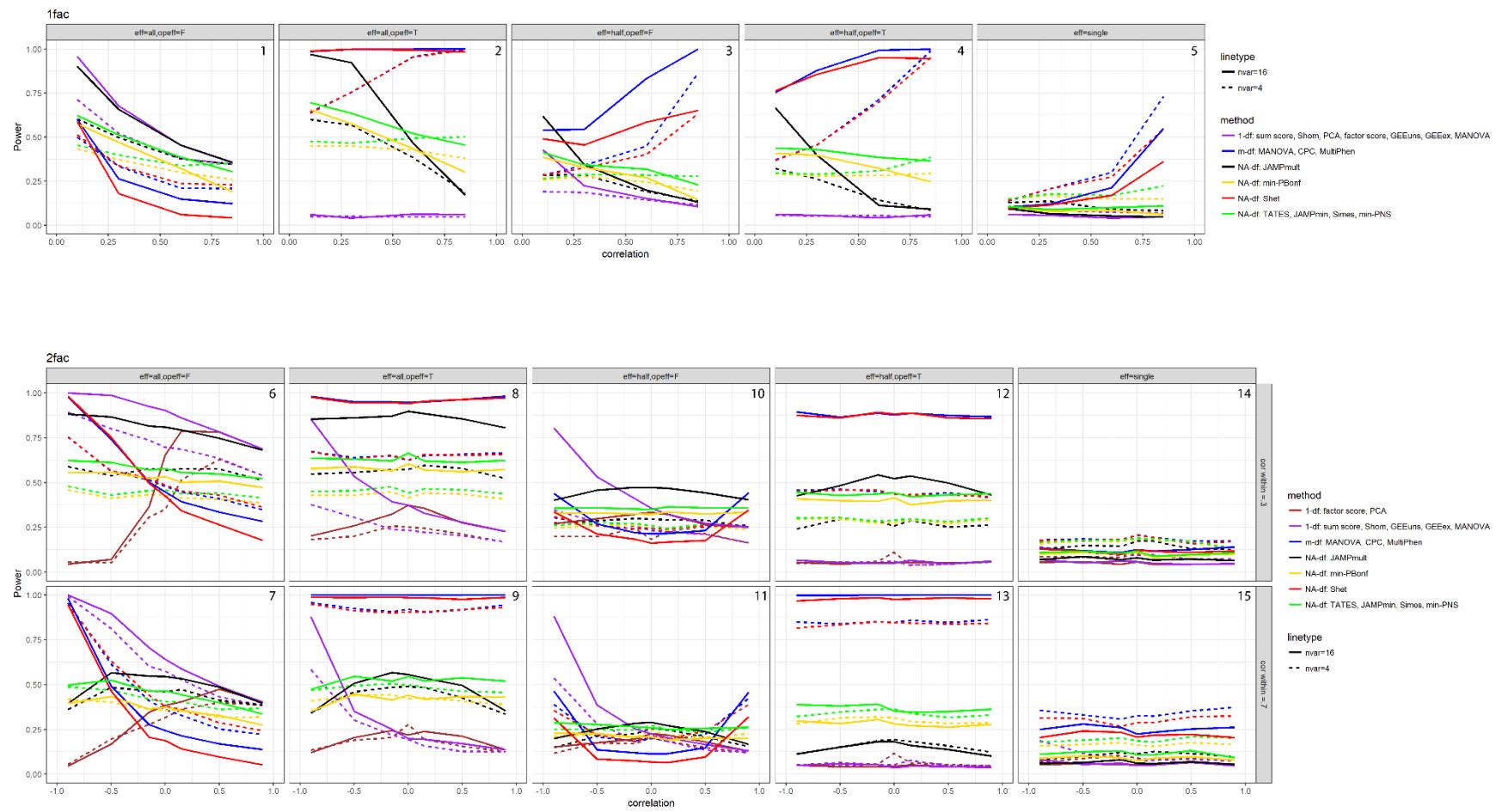


Figure 2: Power

Panels **a** and **b** show the power to detect a GV that explains .1% of the variance (see Supplemental Information) as a function of the number of traits (4 or 16; see Table S10 for results for 8 traits) and the correlations among the traits. Power curves are shown for 17 MATs in the 15 scenarios outlined in Table 4.

Table 1 - Classification of multivariate methods

Method	Input	Permutation	H _A	Data type	Short summary	df	Type I
Reduction-based							
Sum scores	Raw data	No	Joint effect	Any except nominal, but should be the same for all phenotypes	All observed scores on the m traits are summed and this sum is regressed on the DSL.	1	OK
PCA	Raw data	No	Joint effect	Continuous	The first principal component (PC1) obtained in regular Principal Component Analysis (PCA) is the weighted linear combination of the m traits that maximizes the amount of variation accounted for in the original data. PC1 is regressed on the DSL.	1	OK
CPC	Raw data	No	Joint effect	Continuous	Regular PCA on m traits yields maximally m orthogonal PCs. The association signals of all m individual PCs with the DSL are combined to form a non-central m df χ^2 test.	m	OK
Factor analysis	Raw data	No	Joint effect	Continuous	A factor score is obtained in factor analysis by fitting a single common factor model to the m traits, where the factor maximally explains the variance common to the m phenotypes. Individual scores on this factor are regressed on the DSL.	1	OK
Regression-based							
MANOVA	Raw data	No	Joint effect	Continuous	A multivariate regression model that tests the association between m traits and a DSL (i.e., treated as a 0/1/2 coded continuous predictor, assuming an additive codominant model), while accounting for the residual variance of and covariances between the m traits. In MANOVA, all elements in this residual covariance matrix are estimated freely, and an omnibus test determines whether the DSL affects the m traits.	m	OK
alternative MANOVA	Raw data	No	Joint effect	Continuous	A 1-df MANOVA, in which the m regression weights of the DSL are constrained to be equal, and the statistical significance of this one regression coefficient is evaluated. This model can be fitted in dedicated software like OpenMx (Neale et al., 2016).	1	OK
Generalized Estimating Equation	Raw data	No	Joint effect	Continuous or categorical (but not a mix)	A generalized linear model that estimates the association between m traits and a DSL while accounting for the residual variances of and covariances between the traits. This residual matrix can be set to <i>exchangeable</i> , assuming all residual covariances to be equal, or to <i>unstructured</i> , estimating all residual covariances freely. Unlike MANOVA, standard GEE software assumes the variances of all m traits to be equal, and uses sandwich correction of the standard errors of estimated parameters to correct for misspecification in the residual variance-covariance matrix. In GEE, the associations between the DSL and the m traits can be estimated freely (m -df test) or constrained to be equal (1-df test).	1 or m^{**}	The m -df variants can be liberal
MultiPhen	Raw data	No	Joint effect	Any	The 0/1/2 coded DSL functions as an ordinal dependent variable and the m traits as predictors (i.e., proportional odds logistic	m^*	OK

					regression model). An omnibus test is performed to test whether the full set of predictors is significantly associated to the DSL.		
Combination tests							
Min-P _{Bonf}	Univariate p-values	No	At least 1 of m traits	Any	The m p-values obtained in univariate regressions of the m traits on the DSL, are Bonferroni corrected to account for multiple testing, and then the smallest corrected p-value is selected.	NA	Can be conservative
Min-P _{NS}	Univariate p-values	No	At least 1 of m traits	Any	The m p-values obtained in univariate regressions of the m traits on the DSL, are Nyholt-Šidák corrected to account for multiple testing, and then the smallest corrected p-value is selected.	NA	Can be liberal or conservative, depending on trait correlations and number of traits
Simes	Univariate p-values	No	At least 1 of m traits	Any	Each j^{th} p-value of the m p-values obtained in univariate regressions of the m phenotypes on the DSL, is weighted by m/j . The p-value of the Simes test then corresponds to the smallest weighted p-value.		Can be conservative
TATES	Univariate p-values and the phenotypic correlation matrix	No	At least 1 of m traits	Any	In an iterative procedure, the top j of the m p-values obtained in univariate regressions of the m traits on the DSL, are sorted and weighted as a function of the eigenvalues of the correlation matrix between the top j traits. The p-value of the TATES test then corresponds to the smallest weighted p-value.	NA	Can be liberal or conservative, depending on trait correlations and number of traits
S_{hom}	Univariate t-statistics	No	Joint effect	Continuous and dichotomous	In a meta-analytic fashion, the Wald test statistics obtained in m univariate regressions (and possibly across k cohorts) are used to create a new test statistic that follows a χ^2 distribution with 1 df, while accounting for heterogeneity in sample size and for correlations between the test statistics. S_{Hom} constraints all DSL effects to be equal, and then evaluates the statistical significance of this one parameter.	1	OK
S_{het}	Univariate t-statistics	No	Joint effect	Continuous and binary	Unlike S_{Hom} , S_{Het} handles heterogeneity in DSL-effects across the m traits by calculating the new test statistic only for the subset of traits showing a Wald statistic above a certain threshold. This new test statistic is calculated for a range of thresholds, and the maximally obtained value corresponds to S_{Het} , which is evaluated against a gamma distribution.	NA	OK
JAMP _{min}	Raw data	Yes	At least 1 of m traits	Any	In the original data, the smallest of the m p-values, obtained in univariate regressions of the m traits on the DSL, is determined. Then, the multivariate trait scores are permuted K times across genotypes, retaining the correlations between the m traits. For each permutation, the smallest of the m p-value is determined. The smallest p-value from the original data is then evaluated against the p-values obtained in the K permutations.	NA	OK
JAMP _{mult}	Raw data	Yes	Joint effect	Any	In the original data, the sum of the -log10 transformed p-values, obtained in univariate regressions of the m traits on the DSL, is calculated across all m traits. Then, the multivariate trait scores are permuted K times across genotypes, retaining the correlation between the m traits. For each permutation, the $\Sigma(-\text{log10}(p))$ is	NA	OK

					calculated for the specific DSL. Finally, the number of hits (H) is calculated for each specific DSL by dividing the number of times the $\sum(-\log_{10}(p))$ obtained after permutation exceeds or equals the $\sum(-\log_{10}(p))$ from the original analysis. The empirical p-value is calculated as H/K .		
FC-Pearson	Univariate p-values and the phenotypic correlation matrix	No	Joint effect	Any	The original Fisher combination test defines test statistic T as the sum of the m -2log transformed p-values obtained in the univariate regressions of the m traits on the DSL. When the m p-values are not independent (e.g., because the m traits are correlated), T follows a gamma distribution with shape μ^2/σ^2 and scale σ^2/μ , where μ and σ^2 are the expected mean and variance of T .	NA	Can be liberal

Note. H_A : The alternative hypothesis of each MAT: the MATs either test whether at least 1 of the m traits is associated to the DSL, or they evaluate the joint effect of the DSL on all m traits. *Data type*: Does the method allow only continuous traits or also (a mix of) traits with different measurement levels (e.g., ordinal, dichotomous). *Df*: When described from a maximum-likelihood perspective, most tests can be classified as a 1-df test or an m -df test. *Type I*: denotes whether the Type I error rate, or false positive rate, of a MAT was found to be correct in our simulations.

* Ordinal predictors need to be properly dummy-coded. When the m traits contain g ordinal phenotypes with l levels each, the number of degrees of freedom equals $(m-g) + g(l-1)$.

** In these techniques, one can allow the DSL effect to vary across phenotypes (m -df test) or constrain it to be identical for each phenotype (1 df test).

Table 2 – Overview Type I simulation settings

	<i>Uniform (1-factor)</i>	
# variables	4	16
Phenotypic correlations	.1	.85
	<i>Clustered (2-factor)</i>	
# variables	4	
Phenotypic correlations within	.3	.7
Phenotypic correlations between	-.9	-.15 .15 .9

Note. All Type I simulations were run 1 million times with sample size of N=2000.

Table 3 - Overview Power simulation settings

	<i>Uniform (1-factor)</i>		
# variables	4	8	16
Phenotypic correlations	.1	.3	.6 .85
Affected	All	Half	1
Effect size	.01	.02	.05
Opposite	True	False	
	<i>Clustered (2-factor)</i>		
# variables	4	8	16
Phenotypic correlations within	.3	.7	
Phenotypic correlations between	-.9	-.5	-.15 0 .15 .5 .9
Affected	All	Half	1
Effect size	.01	.02	.05
Opposite	True	False	

Note. All power simulations were run 1000 times with sample size of N=2000.

The effect size is expressed as percentage of variance explained in each affected standardized trait (see Supplemental Information for details).

Table 4 – Overview 15 main power simulation scenarios

Scenario	Structure data	Location DSL effect	Opposite effects	Corr_within
1	Uniform	All	F	NA
2	Uniform	All	T	NA
3	Uniform	Half	F	NA
4	Uniform	Half	T	NA
5	Uniform	1	NA	NA
6	Clustered	All	F	.3
7	Clustered	All	F	.7
8	Clustered	All	T	.3
9	Clustered	All	T	.7
10	Clustered	Half	F	.3
11	Clustered	Half	F	.7
12	Clustered	Half	T	.3
13	Clustered	Half	T	.7
14	Clustered	1	NA	NA
15	Clustered	1	NA	NA

Note. *Structure data*: Uniform refer to data with a phenotypic 1-factor structure and uniformly correlated traits. Clustered refers to a phenotypic 2-factor structure with traits that correlate either .3 or .7 within clusters, while correlation between clusters vary. *Location DSL effect*: refers to whether the DSL affects 1, half or all the simulated traits. *Opposite effects*: refers to whether the DSL affects some traits positively and some negatively (Opposite is True: T) or whether the DSL-effect has the same sign for all affected traits (Opposite is FALSE: F). *Corr_Within*: describes the values of the correlations between traits belonging to the same cluster (i.e., loading on the same factor). NA refer to "Not Applicable".

Box 1 – Transformation-based techniques

Sum-scores. In psychology and psychiatry, sum scores are often used to summarize multivariate responses to items on tests (e.g., cognitive ability), questionnaires (e.g., personality), and clinical instruments and interviews (e.g., depression). In psychiatric studies, the sum-score is often dichotomized to obtain a binary case-control status variable, although this may lower the power to detect a possible GV (e.g. Van der Sluis et al., 2012; Lee & Wray, 2013). In the case of an unweighted sum score (i.e., b_1 to b_m in Eq 1 are set to one), the variance of a sum score equals the sum of all entries of the $m \times m$ variance-covariance matrix of the m traits. How well the GV can be detected through the sum score thus not only depends on the effect size of the GV, but also on the number of traits it affects. The contribution of *global* GVs, i.e., GVs that affect all or multiple of the m traits, to the variance of the sum is generally larger than to the variance of the underlying elements, so that the power to detect global GVs can benefit from using a sum-score. In contrast, GVs that affect only 1 or a few of the traits (i.e., *local* variants) contribute relatively little to the variance of the sum. Importantly, however: how well a sum-score reflects the GV-effect(s) also depends on the magnitude of the variances and covariances conditional on the GV: if these conditional (co)variances are relatively small, then the *signal-to-noise ratio* will be better than if the conditional (co)variances are large (see Supplemental Information for a more formal discussion of this topic).

Principal Component Analysis (PCA). PCA is used to transform a set of m correlated standardized traits into a set of maximally m orthogonal (i.e., uncorrelated) linear combinations of these traits, the new variates being denoted as Principal Components (PC). For the first PC (PC1), the weights $b_1 \dots b_m$ in Eq. 1 are chosen such that the variance of PC1 is maximized. If the correlations between the m traits are equal (i.e., homogeneous), then PC1 will correlate 1 with the sumscore (as, $b_1 = b_2 = \dots = b_m$). PC1 provides a summary of the full set of m traits. Additional PCs may be considered if the variance of PC1 is judged to be too small. In the psychometric context, where the m traits are generally items measuring a given latent trait (e.g., neuroticism), PC1 is viewed as a proxy of that latent trait. Assuming that PCA was used to reduce multivariate information, we focus on the analysis of PC1 (see Supplemental Information).

Combined PC test (CPC test). As PCA is conducted on the trait information and does not involve genetic information, of all PCs obtainable in PCA of a set of m traits, PC1 does not necessarily have the strongest association with the GV. In PCA's iterative procedure, the variance in $y_1 \dots y_m$ that is not accounted for by preceding PCs, can be accounted for by successive PCs. The weights of successive PCs are chosen such that again their variance is maximized and that they are uncorrelated with preceding PCs. Capitalizing on the fact that the m extracted PCs are uncorrelated (orthogonal), the combined PC test (CPC test) evaluates the association of the GV to all m PCs simultaneously by reference to a χ^2 -distribution with m degrees of freedom (Aschard et al., 2014).

Common factor analysis. As a data transformation method, factor analysis resembles PCA: just like one may use PC1, one can also fit a single common factor model to the m traits, calculate the scores on the common factor (i.e., factor scores), and use this factor score as dependent variable in GWA studies. In the single common factor model, the weights $b_1 \dots b_m$ in Eq. 1 are chosen such that the variance explained by the new variate \tilde{y} in the set of m traits is maximized, i.e., \tilde{y} maximally represent the variance common to the m traits. While PCA concerns the total variance of the traits, factor analysis thus focusses on the covariance shared by the m traits (also denoted as 'communality'). This common factor obtained in factor analysis may be viewed as a substantive variable: a common cause of (and as such a source of covariance among) the m traits (Lawley & Maxwell, 1971). For instance, the covariance between m neuroticism symptoms is assumed to originate in the fact that all m symptoms are caused by the

underlying latent trait “neuroticism”. PCA and factor analysis are thus conceptually different: PCA components are merely statistically optimal linear variates, while the factors in factor analysis are often assumed to actually represent a theoretical construct (e.g., neuroticism). In addition, the residuals of the m traits, i.e., the unique parts of $y_1 \dots y_m$ that are not explained by the variate \tilde{y} , are assumed to be uncorrelated in factor analysis, while no such assumption is made in PCA. In practice, however, PCA and factor analysis often yield very similar result, e.g. when the communality of the traits is high (i.e., the variance shared by the m traits is high compared to the unique variance of the traits). Assuming that factor analysis was used to reduce multivariate information, we focus on the analysis of factor scores obtained in a single common factor model (see Supplemental Information).

Canonical Correlation Analysis. Canonical Correlation Analysis (CCA) extracts for each GV under study the linear combination of m traits (i.e., variate) that explains the largest amount of covariance with that specific GV (Solovieff et al., 2013). The weights of the new variate thus differ between GV, and reveal which traits are the most strongly associated to a specific GV. CCA is thus the only transformation-based technique that uses the information from the GV to create the new variate. CCA is implemented in the widely used GWA package PLINK (Ferreira & Purcell, 2009). However, assuming an *additive codominant genetic model* in which the GV, coded 0/1/2 for the number of minor alleles, is treated as a continuous predictor (i.e., a “covariate”, rather than a “factor”), CCA is known to perform identically to MANOVA and therefore does not feature as a separate MAT in our study.

Box 2 – Regression-based techniques

All regression-based techniques described here assume that conditional on the effect of the GV, the data of the m traits follow a multivariate normal distribution.

MANOVA. In standard MANOVA, the $m \times m$ symmetrical background covariance matrix Σ_E is unconstrained, i.e., it has $((m+1)*m)/2$ freely estimated elements (covariances and variances). In terms of a likelihood ratio test (asymptotically equal to the F-test used to evaluate MANOVA), standard MANOVA is an m -df omnibus test of the null hypothesis that the m regression coefficients are all zero (no association). For comparison, we also ran simulations for a 1-df MANOVA (fitted in the R package OpenMx (Neale et al., 2016), in which the m regression weights of the GV are constrained to be equal, and the null-hypothesis is that this regression coefficient is zero (no association).

Generalized Estimating Equations (GEE). In GEE, one can specify various structures for Σ_E , which is modeled as $\Delta_E \mathbf{P}_E \Delta_E$, where \mathbf{P}_E is the residual correlation matrix between the m traits conditional on all predictors in the model, and Δ_E is a diagonal matrix with the m residual standard deviations of the m traits constrained to be equal. In GEE, the structure of correlation matrix \mathbf{P}_E , i.e., the working correlation matrix, is user-specified. In order of parsimony, plausible choices for \mathbf{P}_E are “independent” ($\mathbf{P}_E = \mathbf{I}$; the m traits show no correlation conditional on the GV), “exchangeable” (all conditional correlations between the m traits are equal), and “unstructured” (i.e., all conditional correlation are freely estimated). Standard GEE software uses sandwich correction of the standard errors of estimated parameters to correct for the possible misspecification of Σ_E (ref Dobson). As demonstrated elsewhere (e.g., Minica et al. 2015), the degree of misspecification does have a bearing on the power of the sandwich corrected test. In our simulations, we specified 1-df versions of ‘exchangeable’ and ‘unstructured’ GEE models (i.e., the m regression weights of the modelled GV were constrained to be identical). As m -df versions of ‘exchangeable’ and ‘unstructured’ GEE models yield identical results (see Supplemental Information), we

only included the results of GEE-unstructured m -df models in our main discussion, but results for the GEE ‘exchangeable’ m -df model are available in the Tables S7-S12.

Linear Mixed Models (LMM): Linear mixed effects models are an extension of the multivariate regression model, in which fixed effects are used to estimate the effects of the GV, and additional random effects account for the correlations among the m phenotypes (see e.g., Yang & Wang, 2012). In the genetics literature, LMM are frequently employed to model population substructure and relatedness in a univariate settings (e.g., EMMA, GenABEL, FaST-LMM, Mendel, GEMMA and MMM, see Euhusunthornwattan et al (2014) for comparisons, and Yang et al (2014) for a discussion of potential pitfalls), but LMM can also be used to model e.g. multivariate gene-environment interaction (Moore et al., 2018) or to accommodate multivariate data (e.g., Zhou & Stephens, 2014). In principle, LMM can handle multiple sources of clustering or correlation (e.g., multivariate data and familial relatedness or population substructure simultaneously). Because LMM often failed to converge in our simulations (especially with larger m), and Type I error rates were severely off for the m -df variant, we excluded LMM from our main discussion, but all results are available in the Tables S7-S12.

Multiphen: reversed ordinal multiple regression. The MultiPhen procedure (O'Reilly et al., 2012) reverses the regression model by treating the GV as an ordinal dependent variable, and the m traits as predictors. This has the practical advantage of rendering distributional assumption concerning the phenotypes (e.g., conditional multivariate normality, see Table 1) unnecessary; the m phenotypes can be a mix of continuous and categorical (appropriately dummy-coded) variables. The procedure is implemented in an R-package ('MultiPhen'). MultiPhen tests the m df null-hypothesis that the m regression coefficients are zero.

Box 3 – Combination tests

Minimal p-values: $\text{min-}P_{\text{NS}}$ and $\text{min-}P_{\text{Bonf}}$. Minimal p-value approaches use the m p-values obtained in univariate analyses, correct these p-values for multiple testing, and then select the smallest. Specifically, to obtain the Bonferroni-corrected minimal p-value, $\text{min-}P_{\text{Bonf}}$, first all original p-values are multiplied by m to obtain the Bonferroni-corrected p-values, and then the minimal Bonferroni-corrected p-value is selected (Simes, 1986). To obtain the Nyholt-Šidák corrected minimal p-value, $\text{min-}P_{\text{NS}}$ (O'Reilly et al., 2012), one first establishes the *effective number* of traits m_e , and this effective number of traits is then used to calculate the Sidak-corrected p-values as $(1 - (1 - p_{\text{org}}))^{m_e}$. Nyholt (2004) proposed to calculate m_e as a function of the variance of all eigen values, which can be derived from the correlation matrix between the m traits.

Simes. To obtain the p-value for the original Simes test (Simes, 1986), P_S , the m p-values obtained in m univariates association tests are first sorted ascendingly. Subsequently, each j th p-value (j running from 1 to m) is weighted with m/j , such that the lowest p-value is weighted with the largest weight (i.e., $m/1$) and the highest p-value is weighted with the smallest weight (i.e., $m/m=1$). The Simes p-value then corresponds to the smallest weighted p-value, i.e., $P_S = \min\left(\frac{mp_j}{j}\right)$.

TATES: adjusted Simes test. As the original Simes test is conservative (Simes, 1986), and becomes more so with increasing correlations and increasing m (van der Sluis et al., 2018), Van der Sluis et al (2012) developed an adjusted Simes procedure denoted TATES (Trait-based Association Test that uses Extended Simes: based on Li et al., 2011). TATES weights in a fashion similar to Simes, except that the observed

number of p-values m and j are replaced with the effective number of p-values m_e and m_{ej} . Specifically, the TATES p-value P_T is obtained as $P_T = \min\left(\frac{m_e p_j}{m_{ej}}\right)$, where m_e denotes the effective number of independent p-values, and m_{ej} the effective number of p-values among the top j p-values. The effective number of p-values m_e and m_{ej} is established from eigenvalue decomposition of the correlation matrix between the m p-values, which can be approximated from the correlation matrix between the m traits (see Van der Sluis et al., 2012, 2018).

JAMP: The permutation-based software tool JAMP (Joint genetic Association of Multivariate Phenotypes, <https://ctg.cnrc.nl/software/jamp>) incorporates two different multivariate tests: one that tests whether at least one of the m traits is associated to the GV (JAMP_{min}), and one that assesses the joint association signal of the m traits to the GV (JAMP_{mult})². Specifically, to calculate the empirical p-value for multivariate association, JAMP_{mult} uses permutation to control the Type I error rate and to adjust for correlations between the m traits. First, the univariate associations between the m traits and a GV are evaluated, and the GV-specific statistic G_o is calculated as $G_o = \sum_{i=1}^m -\log_{10}(p_i)$, aggregating the signal across the m traits. Second, the m traits scores are permuted J times across the GV, keeping the correlations between the m traits intact. For each permutation, $G_J = \sum_{i=1}^m -\log_{10}(p_i)$ is calculated for the specific GV. Finally, the number of hits (H) is calculated for each GV by dividing the number of times G_J obtained on permuted data exceeds or equals G_o obtained on the original data. The empirical p-value (P_{mult}) is then calculated as $P_{mult} = H/J$.

In contrast, JAMP_{min} produces an empirical p-value (P_{min}) associated with the hypothesis that at least one of the m traits is significantly associated with the GV. For each GV, the smallest of the m univariate p-values obtained in the original data is evaluated against the smallest of m univariate p-values obtained in each of the J permutations. In our simulations, the number of permutations J was set to 1000.

S_{Hom} . In a meta-analytic fashion, S_{Hom} (Zhu et al., 2015) uses the Wald test statistics obtained in m univariate GWASs (and possibly across k cohorts) to create a new test statistic that follows a χ^2 distribution with 1 df. S_{Hom} accounts for heterogeneity in sample size and for correlations between the test statistics. As a 1 df test, S_{Hom} constraints all GV effects to be the same, and then tests the omnibus hypothesis that this 1 GV-parameter is 0. S_{Hom} is thus most powerful when the GV effects are homogeneous in size and sign across the m traits.

S_{Het} . S_{Het} is equivalent to S_{Hom} but specifically handles heterogeneity in GV-effects across the m traits by calculating the new test statistic only for the subset of traits showing a Wald statistic above a certain threshold. This new test statistic is calculated for a range of thresholds, and the maximally obtained value corresponds to S_{Het} . The significance of S_{Het} is obtained through simulation of a Gamma distribution (see Supplemental Information for details). Like S_{Hom} , S_{Het} tests the omnibus hypothesis that all included effects are zero. Because of the selection, S_{Het} is expected to be more powerful than S_{Hom} when the GV-effects are heterogeneous in size and/or sign across the m traits.

FC-Pearson test: adjusted Fisher Combination test. Let $p_1 \dots p_m$ be the p-values obtained in the univariate regressions of the m traits on a GV. The original FC-test is calculated as $T = -2 \sum_{i=1}^m \ln(p_i)$ (Fisher,

² Note that the JAMP software also calculates an empirical p-value that controls for the family wise error due to testing multiple SNPs. This family-wise corrected p-value tends to be less conservative than the Bonferroni corrected p-value, as it properly takes into account the correlational structure of the genomic data. This family-wise corrected p-value was not used in the current study.

1932). If the m traits are uncorrelated, the original FC test statistic T is chi-squared distributed with $2m$ dfs. However, if the m traits are correlated, this original test has highly inflated Type I error rate (Fisher, 1932; van der Sluis et al., 2012). For m correlated traits, it can be shown (Brown and Yang, ref 27/28 in Yang et al, 2016) that, under the null hypothesis of no association between the GV and the m traits, T follows a scaled chi-squared distribution, or equivalently a specific gamma distribution with shape parameter that can be derived from the mean (μ) and variance (σ^2) of test statistic T . Yang et al. (2016) established an approximation of μ and σ in case of m continuous correlated traits. Just like the original FC-test, this adjusted test, referred to as the FC-Pearson test, tests the hypothesis that the aggregated GV-signal present in the set of m traits deviates significantly from 0.