

1 **Signal integration and information transfer**

2 **in an allosterically regulated network**

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10 **ABSTRACT**

A biological reaction network may serve multiple purposes, processing more than one input and impacting downstream processes via more than one output. These networks operate in a dynamic cellular environment in which the levels of network components may change within cells and across cells. Recent evidence suggests that protein concentration variability could explain cell fate decisions. However, systems with multiple inputs, multiple outputs, and changing input concentrations have not been studied in detail due to their complexity. Here, we take a systems biochemistry approach, combining physiochemical modeling and information theory, to investigate how cyclooxygenase-2 (COX-2) processes simultaneous input signals within a complex interaction network. We find that changes in input levels affect the amount of information transmitted by the network, as does the correlation between those inputs. This, and the allosteric regulation of COX-2 by its substrates, allows it to act as a signal integrator that is most sensitive to changes in relative input levels.

Keywords: network dynamics, allosteric regulation, systems biochemistry, Information theory, Cyclooxygenase-2

11 Many biological signaling networks process multiple inputs and yield multiple outputs. Examples
12 of multiple-input multiple-output (MIMO) biochemical systems include the mitogen-activated protein
13 kinase (MAPK) network, which can respond to numerous ligands and yield a range of outputs including
14 proliferation and differentiation(Santos et al., 2007); the NF- κ B pathway, which triggers pro- and anti-
15 inflammatory responses to a variety of ligands (Lawrence, 2009); and myriad metabolic networks, which
16 respond to multiple substrates and allosteric regulators by producing energy and the building blocks of
17 cellular components (Lorenz et al., 2015). Recent work (Adlung et al., 2017; Spencer et al., 2009; Shi
18 et al., 2016; Huang, 2009; Waite et al., 2016; Mitchell et al., 2018; Chen et al., 2012) has highlighted
19 the fact that modulation of input concentrations in intracellular networks can yield markedly different
20 outcomes. Despite this clear indication that MIMO systems are crucial to biological processes, few reports
21 exist to date to explain how multiple inputs modulate reaction flux and information flow in a network to
22 allow signal processing with a range of adaptive outputs.

23 To explore the properties of MIMO systems in biology, we chose to study the dynamics of cyclooxygenase-
24 2 (COX-2), a key enzyme that controls the balance between pro- and anti-inflammatory signals in mam-
25 malian organisms. COX-2 lies at the interface of the eicosanoid and endocannabinoid signaling pathways
26 (Alhouayek and Muccioli, 2014; Rouzer and Marnett, 2011) and is itself the target of the widely used
27 nonsteroidal anti-inflammatory drugs (NSAIDs). Although COX-2 is a structural homodimer, it behaves
28 as a heterodimer. One subunit in the dimer harbors the catalytically active site, while the other subunit
29 contains an allosteric site that modulates the overall activity of the enzyme (Dong et al., 2013, 2011;
30 Kulmacz and Lands, 1984). An array of substrates, inhibitors, and allosteric modulators can bind to,
31 and thus compete for, either site, giving rise to highly complex reaction kinetics (Kudalkar et al., 2015;
32 Kulmacz and Lands, 1985; Mitchener et al., 2015; Rimon et al., 2010; Yuan et al., 2009; Dong et al.,
33 2016a). The various products from COX-2 activity drive multiple downstream pro- and anti-inflammatory

34 processes that lead to diverse cellular fates including stress responses and apoptosis (Funk, 2001; Rouzer
35 and Marnett, 2003; Smith et al., 2000).

36 It is clear that COX-2 orchestrates a complex interplay between a variety of substrates (the enzyme
37 *inputs*), various allosteric regulators, and the concentration of downstream products (the enzyme *outputs*)
38 that control processes such as inflammation (Funk, 2001; Rouzer and Marnett, 2003; Smith et al., 2000).
39 Previously, most studies of COX-2 function have used simplified models based on Michaelis-Menten
40 kinetics (Briggs and Haldane, 1925). Not surprisingly, these approaches have proved insufficient to
41 capture the rich complexity of the COX-2 network of reactants, intermediates and products (Mitchener
42 et al., 2015). We posit that a systems approach to understand COX-2 mechanism will improve inhibitor
43 design to achieve desired outcomes in clinical settings.

44 COX-2 activity also represents an ideal model system to study the detailed dynamics of a biological
45 MIMO system. As a single enzyme, it is sufficiently simple to allow for the construction, simulation and
46 parameterization of a detailed systems biochemistry model that can capture all of the relevant transitions
47 between intermediates and products. Nonetheless, it is sufficiently complex that it represents a non-trivial
48 example of how multiple inputs lead to multiple outputs in a physiological context. We focus our study
49 on the allosteric regulation network of COX-2 by two important substrates, arachidonic acid (AA) and
50 2-arachidonoylglycerol (2-AG), which generate unusual dynamics in the COX-2 network when both are
51 present (Mitchener et al., 2015). Levels of AA and 2-AG also vary widely *in vivo* (Seibert et al., 1997;
52 Monjazeb, 2006; Sugiura et al., 2006), and it is unclear how such variation would influence COX-2 signal
53 processing.

54 In this work, we analyze the execution mechanism of a biochemical reaction network with multiple
55 inputs. Our work explains how a MIMO system integrates information on the concentration and nature
56 of its substrates to yield potentially different outputs. In previous work, we developed a detailed model
57 of the COX-2 reaction network that comprises all possible biochemical enzyme states dictated by AA
58 and 2-AG occupancy of the allosteric or active sites, and all the kinetic transitions between these states
59 (Mitchener et al., 2015). The reaction rate parameters for the kinetic system were determined using a
60 Bayesian inference methodology (Shockley et al., 2017) to fit the model to experimental data on COX-2
61 kinetics. This Bayesian approach produced an ensemble of model parameters that represent the uncertainty
62 in the kinetic rates given the available data and restricts our analysis to plausible kinetic states of the
63 network (Mitchener et al., 2015; Shockley et al., 2017). To explore the COX-2 MIMO signal processing
64 mechanism, we first employed a graph-theoretic approach to enumerate all possible paths a substrate
65 can take from reactant to product molecule. We found that changing the concentration of the inputs
66 modulates not only the most dominant path that is taken by the system, but also the diversity of the paths
67 the system employs. We also used an information-theoretic approach (Shannon, 1948) to understand the
68 flow of information between network inputs, various intermediates, and the product outputs. This analysis
69 reveals that competition between AA and 2-AG for the allosteric and active site generates highly complex
70 concentration-dependence curves for COX-2 that are context-sensitive. In addition to providing insight
71 into how COX-2 functions as a hub for the processing of inflammatory signals, our work suggests that
72 our systems biochemistry framework provides useful information relevant to the study of other MIMO
73 biological systems. This work also demonstrates that the extreme context-sensitivity of MIMO systems
74 must be considered when attempting to modulate their behavior through targeted interventions.

75 RESULTS

76 A Mathematical Model of COX-2 Allostery and Catalysis

77 We built the COX-2 Reaction Model (CORM) (Fig. 1B) to understand how substrate-dependent allosteric
78 regulation affects COX-2 catalytic rates (Mitchener et al., 2015). Here, we employ this model to study
79 how multiple signals are processed in the context of a complex chemical reaction network, given a
80 range of substrate concentrations and input correlations. Briefly, CORM encodes the reaction kinetics
81 between COX-2 and two of its substrates: the fatty acid arachidonic acid (AA) and the endocannabinoid
82 2-arachidonoylglycerol (2-AG). Both AA and 2-AG can bind at the catalytic and/or allosteric site on
83 COX-2 with different affinities. At the catalytic site, AA is turned over to prostaglandin (PG) while 2-AG
84 produces prostaglandin-glycerol (PG-G). Binding of either molecule at the allosteric site modulates the
85 rate of catalysis (Mitchener et al., 2015). Although CORM includes only two substrates, the MIMO nature
86 of COX-2 kinetics results in a complex network (Fig. 1B). CORM has been calibrated to experimental
87 data using PyDREAM, a Bayesian parameter inference framework, to obtain the probabilistic likelihood

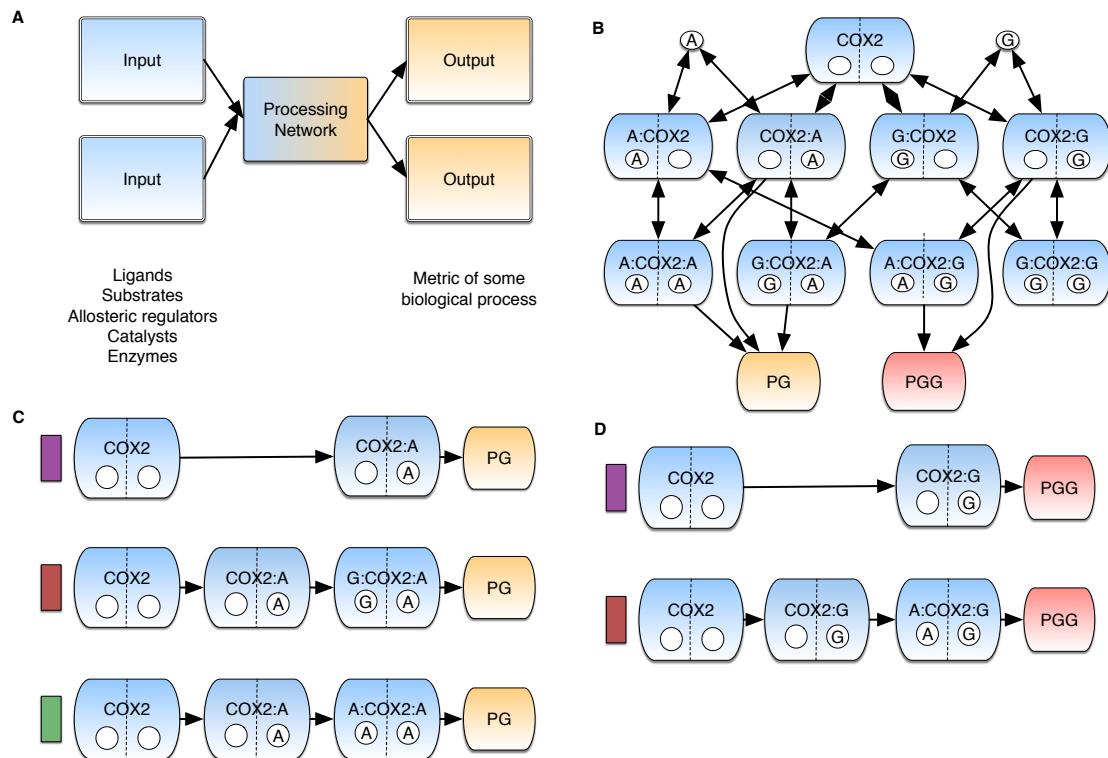


Figure 1. Network interactions within CORM. (A) The Multi-input Multi-output motif in a biological context. (B) The COX-2 Reaction Model (CORM) represents the network of interactions in the COX-2 system. The diagram depicts the possible biochemical states that the COX-2 enzyme (blue lozenges) can adopt through its allosteric (lower left circle) and catalytic (lower right circle) subunits, respectively. AA bound in either site is indicated with A and 2-AG with G within the circle. AA is turned over to produce prostaglandin (PG) and 2-AG is turned over to produce prostaglandin-glycerol (PG-G). Double-headed arrows indicate reversible reactions while single-headed arrows indicate irreversible reactions. Credible intervals for all fitted parameters are included in SI. (C) Dominant PG Production Paths in CORM. Colors correspond to path fluxes in Fig. 2A. (D) Dominant PG-G Production Paths in CORM. Colors correspond to path fluxes in Fig. 2B.

88 of parameters given experimental data, and information about the uncertainty in those parameter values
 89 (Shockley et al., 2017). CORM is encoded in Python using PySB, which provides a flexible tool to query
 90 the mixture of complexes present in the system at any time point given starting concentrations. Many
 91 of these complexes would be costly or impossible to measure experimentally. Employing the Python
 92 environment also facilitated the sophisticated analyses we present in this work (Lopez et al., 2013).

93 Substrate-Dependent Reaction Fluxes in Signal Execution

94 We first explored the net flow of reaction flux through the network using a graph theoretic approach to
 95 calculate all possible paths between the unbound enzyme and each final product. Briefly, we evaluated
 96 the system of ordinary differential equations (ODEs) in CORM at time intervals to extract the integrated
 97 reaction flux at a given time point for each chemical reaction. We then built paths from product to reactant
 98 following the reactions with net forward flux. Finally, we calculated the total chemical flux that passed
 99 through a given path and used this as a measure of the probability of product formation via that path;
 100 a detailed description of this procedure is given in SI Methods and Fig. S1. All fluxes were calculated
 101 for the first ten seconds of catalysis after mixture with the substrates, a time chosen to match previous
 102 experimental work (Mitchener et al., 2015). Path flux distributions were calculated for an ensemble of
 103 calibrated parameter values to quantify path flux uncertainty arising from parameter uncertainty.

104 Our analysis indicates that there are six possible paths to produce PG (Fig. S2) and four possible paths
 105 to produce PG-G (Fig. S3) for all evaluated substrate concentration combinations. However, not all paths

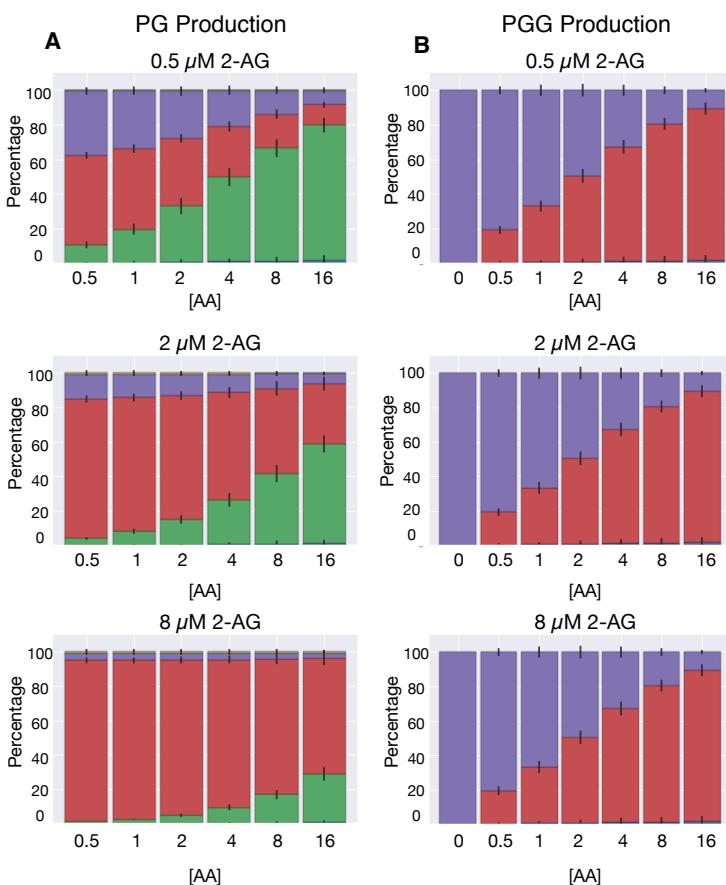


Figure 2. Concentration-dependent PG and PG-G production paths. (A) Dominant Reaction paths for PG Production Vary with AA and 2-AG Concentration. Each individual plot depicts the amount of flux through each path in 1C for a given concentration of 2-AG across varying concentrations of AA. Colors correspond to labeled paths in Fig. 1B. The error bars in each plot indicates the flux variation resulting from inferred kinetic rates. (B) Dominant Mechanisms of PG-G Production Vary with AA Concentration. Each individual plot is at a given concentration of 2-AG. In all plots AA increases from left to right at concentrations of 0.5, 1, 2, 4, 8 and 16 μM in A and 0, 0.5, 1, 2, 4, 8, and 16 μM in B. Colors correspond to labeled paths in Fig. 1C. The error bars in each plot indicates the flux variation from inferred kinetic rates.

106 exhibit significant reaction flux during catalysis across all the concentrations. This occurs because paths
 107 in which binding of a species to the allosteric site precedes binding to the catalytic site are kinetically
 108 disfavored in CORM. As shown in Fig. 1C and 1D, three paths dominate PG production and two paths
 109 dominate PG-G production. The dominant PG-producing paths (Fig. 1C) include those with one or
 110 two intermediates, and the allosteric site empty or occupied by AA or 2-AG. Our results show that the
 111 dominant path is highly dependent on the substrate input concentrations. The presence of AA and 2-AG in
 112 the allosteric site enhances the production of PG (Mitchener et al., 2015). The dominant PG-G-producing
 113 paths include one or two intermediates (Fig. 1D) with the allosteric site empty or occupied by AA. The
 114 presence of AA in this site reduces the rate of PG-G production Mitchener et al. (2015). Similar to PG
 115 production, we also found that the flux through each dominant path for PG-G production is dependent on
 116 substrate concentration (Fig. 2).

117 In the absence of 2-AG and at low (0.5 μM) AA, PG is produced without allosteric modulation (Fig.
 118 2A, purple; purple-labeled path in Figure 1C, top); as the concentration of AA increases, the proportion of
 119 PG produced with AA as an allosteric modulator also increases (Fig. 2A, green). When 2-AG is added
 120 to the system, PG production shifts to using 2-AG as an allosteric modulator (Fig. 2A, red), with this
 121 path favored to a greater extent as the concentration of 2-AG increases (Fig. 2A, lower plots). Even in

122 the absence of 2-AG, about 20% of PG is produced by AA-modulated COX-2, and once even a small
123 amount of 2-AG (0.5 μ M) is added to the system, more than half of PG production occurs via a 2-AG or
124 AA allosterically modulated path. In the presence of high concentrations of either modulator, as much as
125 90% of PG is produced via an allosterically modulated path.

126 Because 2-AG and COX-2 display substrate-dependent inhibition (Mitchener et al., 2015), the
127 production of PG-G occurs via fewer paths than are available to PG. In the absence of AA, all PG-G
128 produced is generated in the absence of an allosteric modulator (Fig. 2B, purple), because the intermediate
129 with 2-AG bound in both catalytic and allosteric sites is not turned over. As AA is added to the system,
130 the proportion of PG-G produced by the AA-modulated pathway (Fig. 2B, red) increases. Thus, in the
131 range of tested substrate concentrations, the dominant mechanism of PG-G production depends entirely
132 on the amount of AA present in the system. Compared to PG, a smaller proportion of PG-G produced
133 by the system results from an allosterically regulated pathway because PG-G is only created via the
134 AA-modulated species or the allosterically unbound species. Nevertheless, at high concentrations of AA,
135 again as much as 90% of PG-G is produced by AA-modulated COX-2. For paths containing a species
136 bound in the allosteric site, binding at the catalytic site followed by binding at the allosteric site is the
137 favored mechanism.

138 We note that at any given substrate concentration, the uncertainty arising from the calibrated kinetic
139 parameter distributions never exceeds a 20% change in the percentage of product produced by a given
140 path (Fig. S4-S8). We find that changes in substrate levels and their relative ratios have a much larger
141 effect on the dominant reaction paths than changes in kinetic rates within the calibrated CORM parameter
142 distributions. Overall, these findings suggest that variation of substrate concentrations in physiologically-
143 relevant ranges has a significant impact on COX-2's mechanism of catalysis.

144 **Pathway Entropy is Dynamic Across Input Concentrations**

Calculating the flux through each path allows us to obtain information about the preferred sequences of reactions that the system executes while processing AA and 2-AG. However, these measurements do not provide an estimate of how chemical traffic (i.e. the flow of chemical signals in the network) is distributed throughout the network. To explore the distribution of biochemical network traffic, we introduce the *pathway entropy* to quantify the degree to which COX-2 utilizes multiple paths at different concentrations of substrates. Our definition of entropy, originally introduced by Claude Shannon (Shannon, 1948) provides a measure of the uncertainty in a probability distribution across states as follows:

$$H = - \sum_{x=1}^n P(x_i) \log_2 P(x_i) \quad (1)$$

145 where H is entropy and $P(x_i)$ is the probability of any state x_i . To determine the degree of uncertainty
146 associated with product production (the pathway entropy), we considered each pathway as a state and use
147 the fraction of flux that a given pathway contributes to the product as a measure for the probability of
148 that state. This analysis yields a measure of how evenly distributed production is across possible paths.
149 In general, evenly distributed fluxes across paths in a network would maximize pathway entropy for a
150 multi-path system.

151 Since the dominant paths vary with substrate concentration (Fig. 2), we would expect that pathway
152 entropy would also vary. In Fig. 3 we present the pathway entropy dependence on input concentration for
153 PG (Fig. 3A) and PG-G (Fig. 3B). The pathway entropy for PG production is highest at intermediate levels
154 of AA and low levels of 2-AG, while the pathway entropy is highest for PG-G production at intermediate
155 levels of AA and any level of 2-AG. These maxima correspond to states where the reaction flux is most
156 spread across the possible paths from reactant to product (see Fig. 2A, top plot, center, and Fig. 2B,
157 top plot, center). In contrast, in the lowest entropy states - low AA and high 2-AG for PG (Fig. 2A,
158 bottom plot, far left) and low AA across the entire 2-AG spectrum for PG-G (Fig. 2B, bottom row), flux is
159 concentrated in a single or a few paths. Reaction flow is thus highly distributed in some conditions yet
160 highly concentrated in one path in other conditions. This finding suggests that MIMO networks utilize
161 multiple execution modes across input concentrations. It also suggests that approaches to modulate or
162 inhibit network activity, which focus on disrupting one or more of these paths, may need to be tailored
163 to specific conditions. These behaviors could have physiological relevance. For example, high-entropy
164 conditions with highly redundant path fluxes may require multiple targets for inhibition compared to a
165 condition with low entropy.

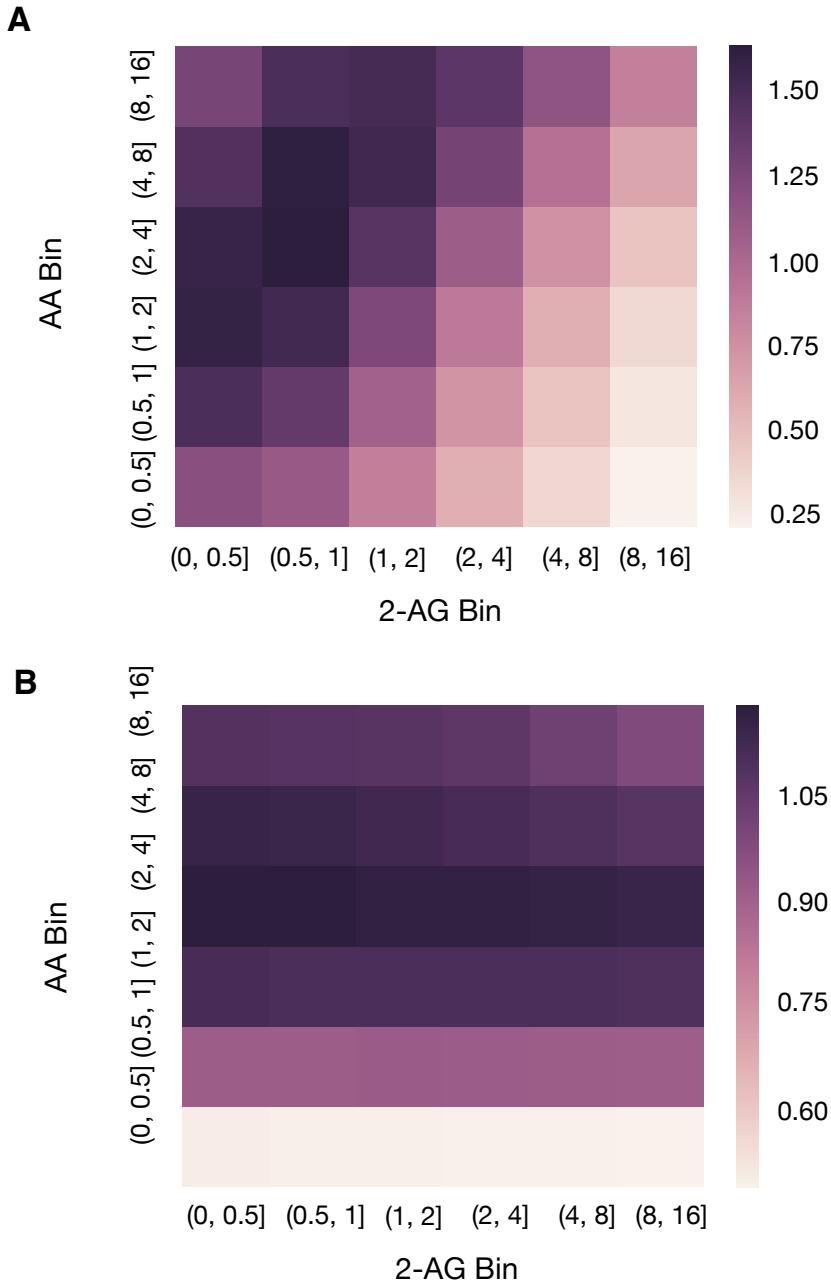


Figure 3. Pathway entropy within CORM. (A) Pathway Entropy for Production of PG. The intensity indicates the pathway entropy in units of bits. (B) Pathway Entropy for Production of PG-G. Units are the same as in A.

166 **Input Output Behavior in CORM**

167 The above findings on pathway entropy suggest a complex relationship between input concentrations,
168 reaction intermediates, and product concentration in CORM. To understand these relationships, we next
169 considered concentration-dependence curves derived from simulations using a fixed set of CORM kinetic
170 parameters in which PG was calculated at increasing AA concentrations in the presence of random
171 quantities of 2-AG (Fig. 4A) or PG-G was calculated at increasing 2-AG concentrations in the presence of
172 random quantities of AA (Fig. 4B). Each data point was taken at steady-state (10 seconds) for consistency
173 with experiments and previous work. Note that the presence of both substrates results in competitive
174 inhibition with suppression of product formation from either one. Thus, the highest levels of output in
175 each case occur when the concentration of the opposing substrate is low. These levels are similar for PG
176 and PG-G because COX-2 utilizes the two substrates with similar catalytic efficiencies when they are
177 present individually. As the concentration of the opposing substrate increases, competitive inhibition
178 is partially balanced by positive allosteric modulation in the case of the conversion of AA to PG, but
179 exacerbated by negative allosteric modulation in the case of the conversion of 2-AG to PG-G. Therefore,
180 the suppression of PG-G formation is greater than that of AA formation as seen in the lower plateau level
181 achieved in (Fig. 4B). In addition, the range of inputs over which the output varies depends significantly on
182 which input/output pair is chosen (note the difference in that range in Fig. 4A,B). Clearly, variation of *both*
183 inputs (e.g. changing AA *in addition* to changing 2-AG in Fig. 4A), results in significant variation in the
184 outputs. Thus, while our simulations are deterministic, introducing uncertainty in the AA concentration
185 generates a type of “extrinsic noise” in the relationship between 2-AG and PG-G (Fig. 4B), and *vice versa*
186 for the impact of 2-AG on the relationship between AA and PG, Fig. 4A). This noise represents allosteric
187 modulation in the network due to varying input concentrations.

188 **Channel Capacity from Substrates to Products**

To better understand how this output variation, combined with the shape of the concentration-dependence curves, influences the COX-2 reaction network, we applied an additional concept from information theory to measure dependence between inputs and outputs, namely the Mutual Information:

$$I(X;Y) = \sum_X \sum_Y P(x,y) \log_2 \frac{P(x,y)}{P(x)P(y)} \quad (2)$$

where X represents a given signal and Y the response to that signal (Shannon, 1948). Mutual information quantifies the degree to which one variable provides information about a second variable. Equivalently, it is a measure of how knowledge about one variable decreases uncertainty in the value of a second variable. For biological systems, quantifying mutual information is challenging because the input distribution is generally unknown. Previous work (Cheong et al., 2011; Selimkhanov et al., 2014; Suderman et al., 2017) has focused on estimating the “channel capacity,” which is the maximum information attainable across all possible input distributions:

$$C = \sup_{p_x(x)} I(X : Y) \quad (3)$$

189 Note that any practical calculation provides a lower bound estimate for the channel capacity C , since only
190 a finite set of input distributions is used to estimate I (Suderman et al., 2017). We calculated channel
191 capacities using the approach and software published in Suderman *et al.* (Suderman et al., 2017), which is
192 similar to that used in Cheong *et al.* (Cheong et al., 2011).

193 We applied this estimate to two different sets of simulations. In the first set of simulations, we
194 considered a case where AA and 2-AG are perfectly correlated with each other; to do this, we sampled
195 the AA concentration from a uniform distribution on $[0, 16 \mu M]$ and set the 2-AG concentration to
196 be exactly the same. In the second set, we *independently* sampled the input AA and 2-AG substrate
197 concentrations from a uniform distribution on the interval $[0, 16 \mu M]$. In each case, we sampled a total of
198 500 distinct input conditions and ran CORM simulations to 10s to agree with experiments and previous
199 work (Mitchener et al., 2015). The channel capacity was then estimated between the two different inputs
200 (either AA or 2-AG) and every possible intermediate and product. The maximum theoretical channel
201 capacity, $\log_2(500) \approx 9$ bits, would be obtained if each of the 500 inputs yielded a distinct response. We
202 repeated the channel capacity calculation for the top 5000 most probable parameter vectors from the
203 calibrated parameter ensemble. This then allowed us to quantify the effect of kinetic parameter variation
204 on channel capacities in the system. In total the analysis required approximately 1.5M CPU hours. An

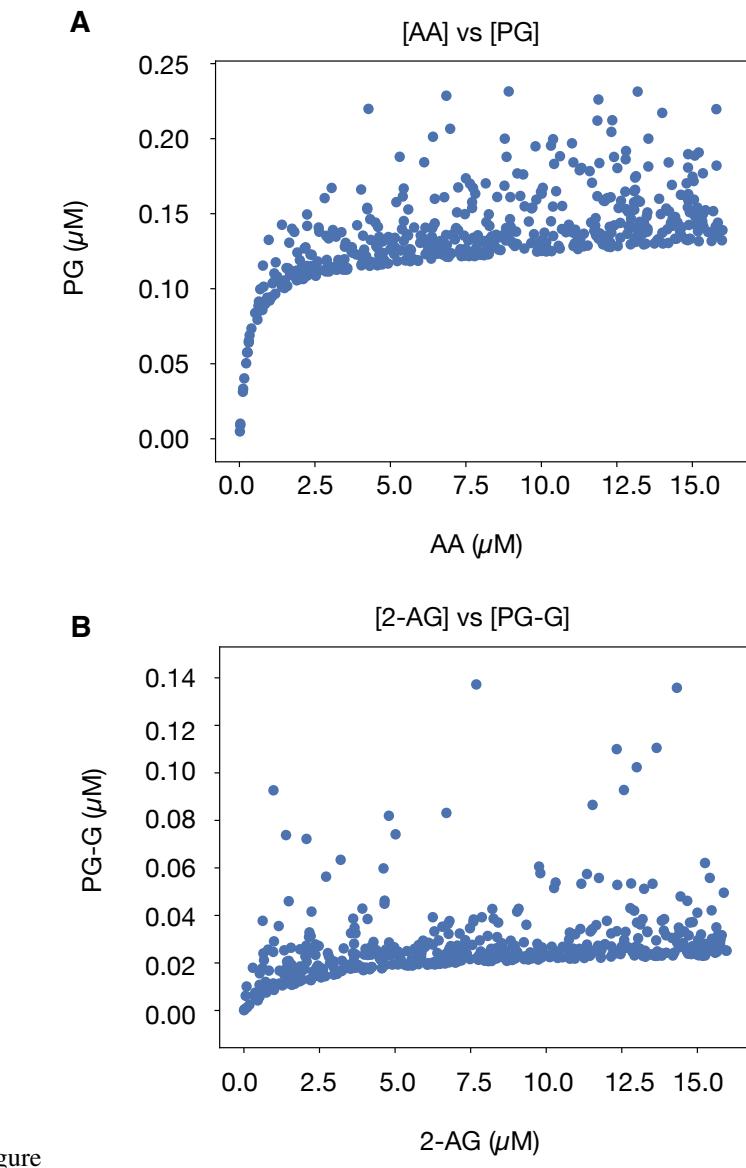


Figure 4. Input vs output plots for substrates and products in CORM. (A) Input vs Output plots for AA to PG. 2-AG varies randomly. All concentrations are measured at steady-state (10 seconds). (B) Input vs Output plots for 2-AG to PG-G. AA varies randomly. All concentrations are measured at steady-state (10 seconds).

205 example of input data used for calculating channel capacities from AA to PG and 2-AG to PG-G for a
206 single parameter set is shown in Fig. 4. Greater detail is provided in the SI Methods.

207 COX-2 Integrates Information from Both AA and 2-AG

208 For ease of visualization, we estimated kernel densities of channel capacities given variation in calibrated
209 kinetic parameters as shown in the violin plots in Fig. 5. In these plots, the data are represented by a
210 central box plot that provides the mean, interquartile range and 95% credible interval, and the surrounding
211 shape depicts the probability distribution, with wider regions indicating a higher probability. Because the
212 input-output relationship in these simulations is deterministic, deviations from the theoretical maximum
213 (≈ 9 bits) arise from the two phenomena described above: either changes in the input do not really lead to
214 significant changes in the output (*i.e.* the “flat” part of the concentration-dependence curves in Fig. 4) or
215 the independent variation in one of the substrates generates variation in the output that is not due to the
216 input being considered (*i.e.* the apparent noise in Fig. 4).

217 From Fig. 5, it is clear that the combination of these effects significantly reduces the observed channel
218 capacities from the theoretical maximum. The highest observed value for any of the input/output pairs
219 (AA to PG, 2-AG to PG-G, etc.) is at most half of the theoretical maximum (less than 4.5 bits). When input
220 values are perfectly correlated ($[AA] = [2\text{-AG}]$), Fig. 5A, the channel capacity between the (correlated)
221 inputs and the outputs is between 3 and 4.5 bits (depending on the parameters), indicating that, while not
222 perfect, the concentration-dependence curves allow for high levels of information flow between inputs
223 and outputs. It is interesting to note that the uncertainty in the kinetic parameters leads to some variation
224 in the calculated channel capacities; since the inputs here are correlated, this variation is due to changes
225 in the shape of the concentration-dependence curves between data sets. Many channel capacities in the
226 correlated case are bimodal, suggesting that two specific concentration-dependent curve shapes are most
227 likely.

228 When the inputs are varied independently, channel capacity values decrease even further (Fig. 5B
229 and C). The channel capacity between AA and PG or PG-G is generally less than 2 bits, and the channel
230 capacity between 2-AG and those outputs is generally less than 1.5 bits. This could occur for two reasons.
231 First, a lack of correlation could result in less entropy in the response (*i.e.* less uncertainty in the value of
232 the product). Since the mutual information is limited by the response entropy (eq. 2, (Shannon, 1948;
233 Cheong et al., 2011; Suderman et al., 2017)), this would cause a decrease in the mutual information.
234 However, if the response entropy remains constant when there is no correlation between inputs, then
235 mutual information can only decrease if information transfer through the network is less efficient. As
236 shown in Fig. S9, the response entropy does not differ between the independent and correlated cases,
237 indicating that independent variation in one of the inputs while the other input is known has a large effect
238 on the output. In other words, COX-2 is truly an integrator of these signals, since accurate determination
239 of the substrate concentrations given the output is considerably more difficult if the two substrates are
240 independently varied.

241 Since perfect correlation and complete independence represent only the two extremes of the rela-
242 tionship between AA and 2-AG concentration, we also investigated the behavior of the system when
243 the inputs exhibit moderate correlation (Pearson correlation coefficient = 0.5), and when the inputs are
244 consistently present in a 2-to-1 AA-to-2-AG ratio (Fig. S10 and Fig. S11). The behavior when input
245 ratios were fixed was similar to that for the correlated values (when the input levels were fixed equal to
246 each other); channel capacities were again higher than in the independent case and the effect of kinetic
247 parameter variation on channel capacity was higher. When the inputs are moderately correlated, the
248 system is still able to obtain high channel capacities for some kinetic parameter sets, although the overall
249 distribution of channel capacities shifts to lower values compared to when input correlation is perfect,
250 further confirming COX-2 input integration.

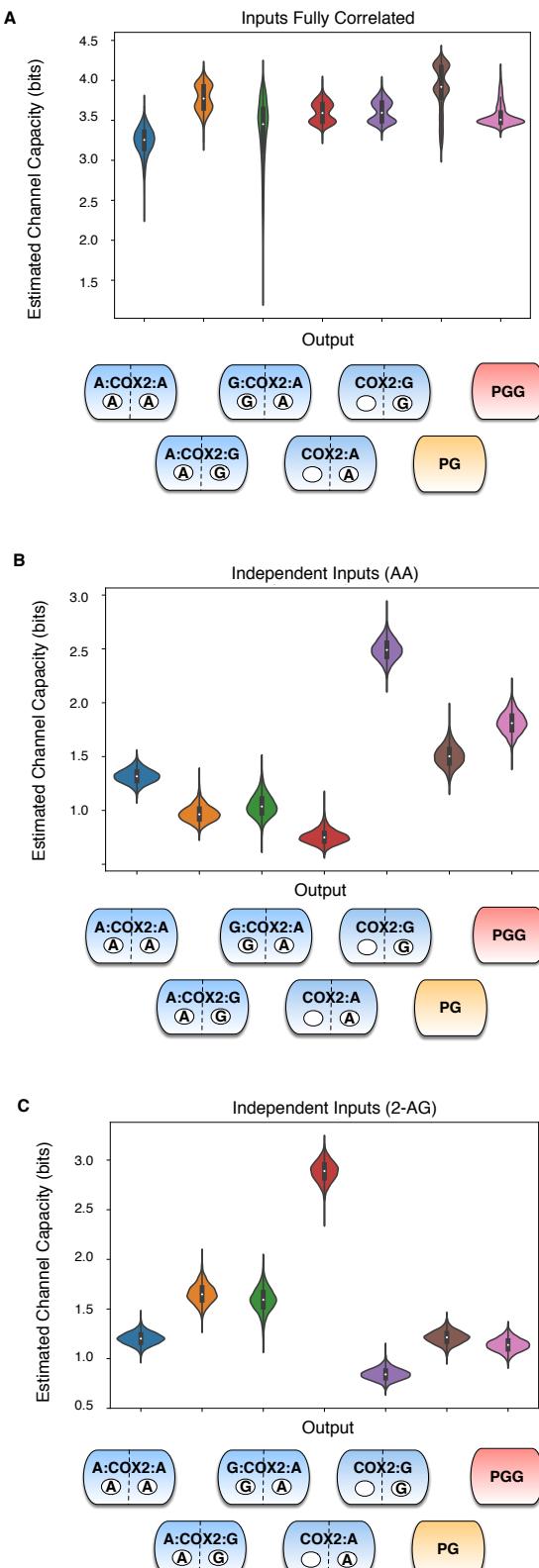


Figure 5. Estimated channel capacities from substrates to intermediates or products in CORM. (A) Estimated Channel Capacities from Input to intermediates and final products within CORM when levels of AA and 2-AG are strongly correlated (Pearson correlation coefficient = 1). Distributions in the channel capacities arise from uncertainty in the kinetic parameter values after model calibration. (B) Estimated Channel Capacities from AA to intermediates and final products within CORM when AA and 2-AG are varied independently. Distributions in the channel capacities arise from uncertainty in the kinetic parameter values after model calibration. (C) Estimated Channel Capacities from 2-AG to intermediates and final products within CORM when AA and 2-AG are varied independently. Distributions in the channel capacities arise from uncertainty in the kinetic parameter values after model calibration.

251 **Information Flow is Dictated by Substrate Concentration**

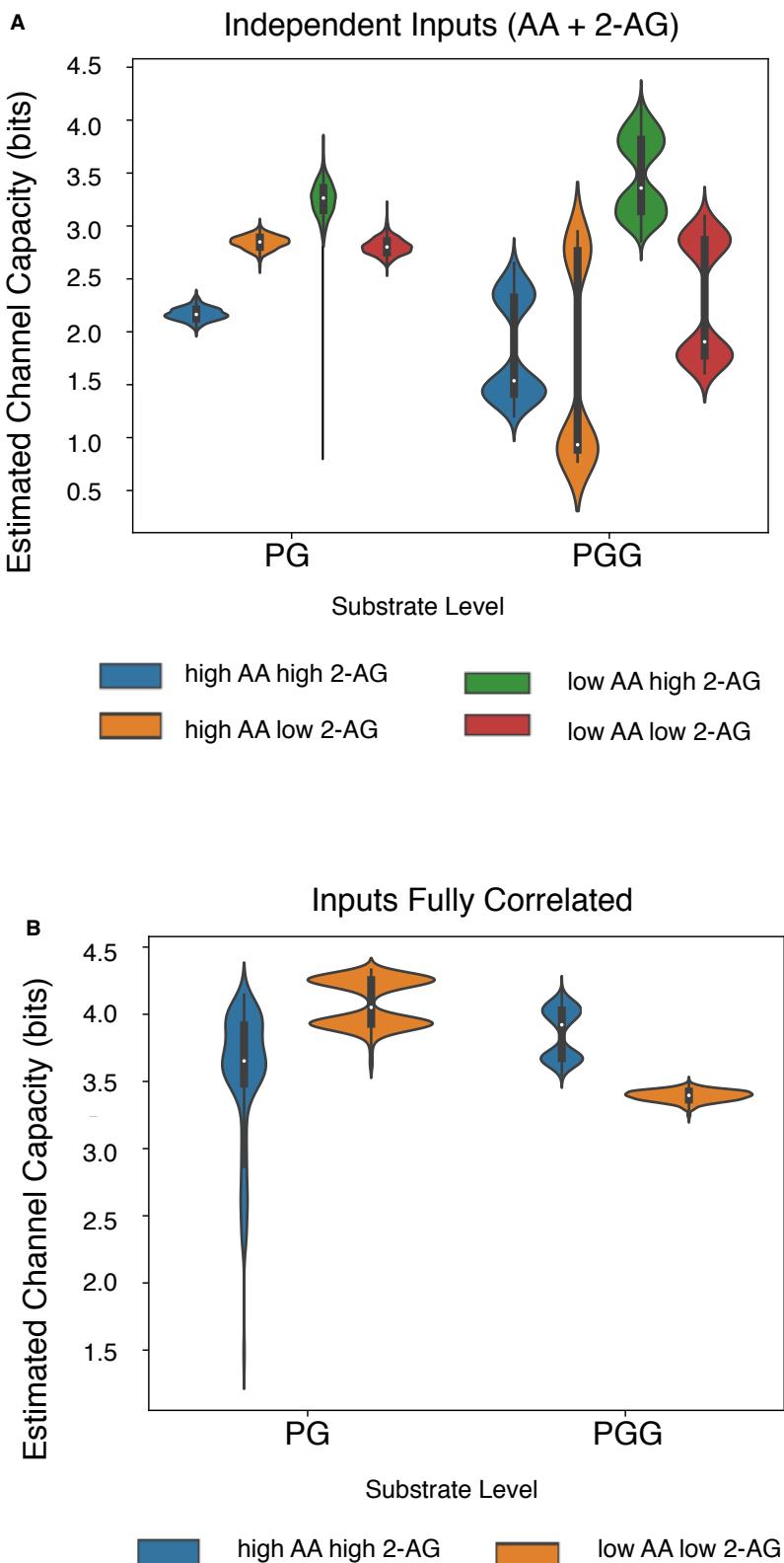


Figure 6. Effect of substrate level on estimated channel capacities between substrates and products in CORM. (A) Total Estimated Channel Capacity from AA and 2-AG combined to products across regions of substrate space. Distributions in the channel capacities arise from uncertainty in the kinetic parameter values after model calibration. (B) Estimated Channel Capacities from input to products when levels of AA and 2-AG are perfectly correlated across regions of substrate space. Distributions in the channel capacities arise from uncertainty in the kinetic parameter values after model calibration.

252 We next tested whether the channel capacity between substrates and products varies with substrate level.
253 We binned the input data into four quadrants (high or low values of either substrate) and calculated the
254 channel capacity between inputs and outputs independently for each quadrant; input ranges were otherwise
255 identical to those used for the calculations described above. Low substrate values spanned 0-8 μ M and
256 high substrate values 8-16 μ M. Both independently varied inputs (Fig. 6A) and correlated inputs (Fig. 6B)
257 yielded estimated channel capacities that were significantly different between the different regions of
258 input space. In addition to differences in PG and PG-G channel capacity, we found that the distribution of
259 information that passed through different intermediates changed with substrate concentration (Fig. S13
260 and Fig. S14); certain paths to product had greater information transfer capacity at particular levels of
261 substrates. This echoes findings from our pathway analysis (Figs. 2 and 3), indicating that changes in
262 substrate concentration result in significant changes in how the enzyme executes its catalytic mechanism.
263 Interestingly, we found no detectable correlation between the flux through a pathway and the mutual
264 information between an input and an intermediate in that path (Fig. S15 and Fig. S16). We leave further
265 investigation of the relationship between information transfer and actual physical reaction fluxes for future
266 work.

267 Splitting the input space into different quadrants also revealed significant variation between different
268 parameter sets, with most distributions showing significant bimodality across parameters (Fig. 6). This
269 suggests that both the shape of the concentration-dependence curves, and the impact of “extrinsic noise”
270 due to variation of one substrate independent of another, varies across parameter sets. Since all of these
271 parameter sets are equally consistent with experimental data (Mitchener et al., 2015), this suggests that
272 multiple modes of information flow are available to the COX-2 reaction network without significant
273 changes to the core functionality of the enzyme.

274 DISCUSSION

275 *In vivo*, COX-2, AA, and 2-AG concentrations vary across cells in different tissues (Seibert et al.,
276 1997; Monjazeb, 2006; Sugiura et al., 2006). In most tissues, AA processed by COX-2 is released
277 from membrane phospholipids, predominantly through the action of cytosolic phospholipase A2 (Leslie,
278 2015). In some tissues, (particularly the brain) a major source of AA is hydrolysis of 2-AG (Ignatowska-
279 Jankowska et al., 2014; Long et al., 2008). In turn, 2-AG is also sourced from membrane phospholipids;
280 through the sequential action of phospholipase C, which forms diacylglycerol (DAG), followed by
281 conversion of DAG to 2-AG by DAG lipase (Fezza et al., 2014). Both DAG lipase and cytosolic
282 phospholipase A2 are stimulated by increases in intracellular Ca^{2+} (Leslie, 2015; Bisogno et al., 2003).
283 Thus, many stimuli (such as zymosan phagocytosis by macrophages (Rouzer and Marnett, 2005)) promote
284 the release of AA and 2-AG simultaneously, with concentrations of AA typically higher than those of
285 2-AG. Considering the precursor-product relationship between 2-AG and AA, however, it is conceivable
286 that in some cells, the levels of the two substrates may change inversely to one another, or that the level of
287 one may change while the other remains constant. These considerations suggest that the system features
288 we find that vary with AA and 2-AG level (pathway entropy and information transfer capacity) are states
289 accessible by the true biological system with the attendant repercussions for information transfer within
290 that system. In addition, the postulated link between diet and the substrates available for COX-2 turnover
291 (Chen, 2010) suggests that the information transfer properties of the system could be modulated by fatty
292 acid intake.

293 COX-2 has significant regulatory flexibility: it is an allosteric protein, with multiple substrates and
294 multiple allosteric regulators, all of which can influence how COX-2 operates on its substrates *in vivo*.
295 The pathway analysis (Fig. 1B and 1C) suggests that COX-2 functions by first binding a substrate at
296 the catalytic site, followed by binding of an allosteric regulator. Allostery can be viewed as a shift
297 in the conformational free-energy landscape sampled by COX-2 through preferential binding of the
298 allosteric regulator to particular conformations (Lechtenberg et al., 2012; Nussinov and Tsai, 2013). From
299 this perspective, modulating the concentrations of allosteric regulators in the COX-2 system shifts the
300 conformational ensemble towards conformations favored by particular regulators. In the case of PG,
301 these conformations are more easily turned over to product than the unmodulated enzyme, while for
302 PG-G, the allosteric influence makes catalysis less energetically favorable (shifts the ensemble towards
303 conformations that are less active). This allows COX-2 to manage the balance between PG and PG-G
304 production in a more complex (and potentially farther-reaching) fashion than that provided by simple
305 competition between substrates. This added complexity suggests a physiological reason why the COX-2

306 system would integrate information from multiple inputs: by adding a second competitive input, the
307 system can access different responses than with a single input. Furthermore, the response dynamics
308 of COX-2 gain even greater complexity because its inputs act as allosteric modulators in addition to
309 substrates. The situation *in vivo* is likely far more complicated (and flexible) than considered here, as
310 COX-2 has potential substrates in addition to AA and 2-AG (Rouzer and Marnett, 2009), and some
311 nonsubstrate fatty acids that act as allosteric regulators (Dong et al., 2016b; Yuan et al., 2009; Dong et al.,
312 2011, 2013, 2016a). In addition, many of the non-steroidal anti-inflammatory drugs that target COX-2
313 also may bind at either the catalytic or allosteric site.

314 One advantage of this complexity may be the significant robustness of this system to variation in
315 the kinetic parameters. The 5000 parameter sets we considered here all fit experimental data on PG
316 and PG-G production equally well, despite variation of over three orders of magnitude in some of the
317 parameter values (Mitchener et al., 2015). Our results on both pathway flux (Fig. 2) and information
318 flow (Fig. 5) indicate that different parameter sets favor different distributions of paths from substrate to
319 product, and transfer information through the network in different ways. Yet the overall function of the
320 enzyme is the same despite all of this variation. *In vivo*, a change in the kinetic rates could correspond to
321 a mutation or a change in the level of molecular crowding for the reaction. The availability of multiple
322 “modes of execution” in this complex enzyme thus allow the system to be highly robust to such changes.
323 This complex architecture could also allow the system to be highly evolveable through a mechanism of
324 facilitated variation (Gerhart and Kirschner, 2007). We expect that future work on parameter variation
325 will reveal major insights into the evolution of robustness in enzymes like COX-2.

326 In this work we applied a systems biochemistry framework to understand chemical reaction flux,
327 pathway entropy, and information flow in the COX-2 system and investigate how these adjust to dynamic
328 input concentrations and correlations. The methods and approach utilized here could be applied to
329 further probe the COX-2 system by including more inputs (its other substrates, allosteric regulators, and
330 inhibitors), or transferred to a larger, more complex network. Given the complexity present in even the
331 simple network considered here, we predict that a systems biochemistry approach to larger networks
332 would provide non-intuitive insights into the dynamics of the system as a whole.

333 METHODS

334 Modeling and Model Calibration

335 CORM was encoded as a PySB (Lopez et al., 2013) model containing 13 distinct biochemical species and
336 29 chemical reactions. It was calibrated to experimental data consisting of PG and PG-G concentrations
337 at steady state across a range of substrate concentrations (Mitchener et al., 2015; Shockley et al., 2017).

338 Calculating Path Fluxes and Channel Capacities

339 The method for determining paths of production and the total flux through a path is described in detail in
340 SI Methods and Fig. S1. Channel capacities were calculated using the method from (Cheong et al., 2011)
341 and the software of (Suderman et al., 2017). Extended detail is available in SI Methods.

342 The datasets generated during and/or analysed during the current study are available from the corre-
343 sponding author on reasonable request.

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349 COMPETING INTERESTS

350 The Authors declare no competing financial or non-financial interests.

351 AUTHOR CONTRIBUTIONS

352 Conceptualization: EMS, CFL; Methodology: EMS, CFL, EJD; Investigation: EMS, CFL, EJD, CAR,
353 LJM; Resources: CFL, LJM; Analysis: CFL, EMS, EJD, CAR; Writing original draft: EMS, CFL;

354 Writing Review and Editing: EMS, CFL, EJD, CAR, LJM; Supervision: CFL; Funding Acquisition: CFL,
355 LJM

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