

1 A guide to performing Polygenic Risk Score analyses

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7 **The application of polygenic risk scores (PRS) has become routine across genetic**
8 **research. Among a range of applications, PRS are exploited to assess shared aetiology**
9 **between phenotypes, to evaluate the predictive power of genetic data for use in clinical**
10 **settings, and as part of experimental studies in which, for example, experiments are**
11 **performed on individuals, or their biological samples (eg. tissues, cells), at the tails of**
12 **the PRS distribution and contrasted. As GWAS sample sizes increase and PRS become**
13 **more powerful, they are set to play a key role in personalised medicine. However,**
14 **despite the growing application and importance of PRS, there are limited guidelines for**
15 **performing PRS analyses, which can lead to inconsistency between studies and**
16 **misinterpretation of results. Here we provide detailed guidelines for performing**
17 **Polygenic risk score analyses relevant to different methods for their calculation,**
18 **outlining standard quality control steps and offering recommendations for best-**
19 **practice. We also discuss different methods for the calculation of PRS, common**
20 **misconceptions regarding the interpretation of results and future challenges.**

21
22 Genome-wide association studies (GWAS) have identified a large number of genetic variants,
23 typically single nucleotide polymorphisms (SNP), associated with a wide range of complex
24 traits [1–3]. However, the majority of these variants have a small effect and typically
25 correspond to a small fraction of truly associated variants, meaning that they have limited
26 predictive power [4–6]. Using a linear mixed model in the Genome-wide Complex Trait
27 Analysis software (GCTA) [7], Yang et al (2010) demonstrated that much of the heritability of
28 height can be explained by evaluating the effects of all SNPs simultaneously [6]. Subsequently,
29 statistical techniques such as LD score regression (LDSC) [8,9] and the polygenic risk score
30 (PRS) method [4,10] have also aggregated the effects of variants across the genome to
31 estimate heritability, to infer genetic overlap between traits and to predict phenotypes based
32 on genetic profile or that of other phenotypes [4,5,8–10].

33
34 While GCTA, LDSC and PRS can all be exploited to infer heritability and shared aetiology
35 among complex traits, PRS is the only approach that provides an estimate of genetic
36 propensity to a trait at the individual-level. In the standard approach [4,11–13], polygenic risk
37 scores are calculated by computing the sum of risk alleles corresponding to a phenotype of

38 interest in each individual, weighted by the effect size estimate of the most powerful GWAS
39 on the phenotype. Studies have shown that substantially greater predictive power can usually
40 be achieved by using PRS rather than a small number of genome-wide significant SNPs
41 [11,14,15]. As an individual-level genome-wide genetic proxy of a trait, PRS are suitable for a
42 range of applications. For example, as well as identifying shared aetiology among traits, PRS
43 have been used to test for genome-wide G*E and G*G interactions [15,16], to perform
44 Mendelian Randomisation studies to infer causal relationships, and for patient stratification
45 and sub-phenotyping [14,15,17,18]. Thus, while polygenic scores represent individual genetic
46 predictions of phenotypes, prediction is generally not the end objective, rather these
47 predictions are then typically used for interrogating hypotheses via association testing.

48

49 Despite the popularity of PRS analyses, there are minimal guidelines [13] regarding how best
50 to perform PRS analyses, and no existing summaries of the differences and options among
51 the main PRS approaches. Here we provide a guide to performing polygenic risk score
52 analysis, outlining the standard quality control steps required, options for PRS calculation and
53 testing, and interpretation of results. We also outline some of the challenges in PRS analyses
54 and highlight common misconceptions in the interpretation of PRS and their results. We will
55 not perform a comparison of the power of different PRS methods nor provide an overview of
56 PRS applications, since these are available elsewhere [13,19], and instead focus this article
57 on the issues relevant to PRS analyses irrespective of method used or application, so that
58 researchers have a starting point and reference guide for performing polygenic score analyses.

59 **1. Introduction to Polygenic Risk Scores**

60 We define polygenic risk scores, or polygenic scores, as a single value estimate of an
61 individual's propensity to a phenotype, calculated as a sum of their genome-wide genotypes
62 weighted by corresponding genotype effect sizes – potentially scaled or shrunk – from
63 summary statistic GWAS data. The use of summary statistic data for the genotype effect size
64 estimates differentiates polygenic scores from phenotypic prediction approaches that exploit
65 individual-level data only, in which genotype effect sizes are typically estimated in joint models
66 of multiple variants and prediction performed simultaneously, such as via best linear unbiased
67 prediction (BLUP) [20,21] and least absolute shrinkage and selection operator (LASSO)
68 [22,23]. While we note that such methods may offer great promise in performing powerful
69 prediction within large individual-level data sets [22], we limit our focus to polygenic scores
70 specifically, which we believe are likely to have enduring application due to (i) the desire to
71 test specific hypotheses on locally collected small-scale data sets, (ii) data sharing restrictions,
72 (iii) heterogeneity across data sets, (iv) large general population data sets, such as the UK

73 Biobank [24], having relatively few individuals with specific diseases compared to dedicated
74 case/control studies.

75

76 Therefore, PRS analyses can be characterized by the two input data sets that they require: i)
77 base (GWAS) data: summary statistics (e.g. betas, *P*-values) of genotype-phenotype
78 associations at genetic variants (hereafter SNPs) genome-wide, and ii) target data: genotypes
79 and phenotype(s) in individuals of the target sample. If the population-level effects of the SNPs
80 were estimated from the GWAS without error, then the PRS could predict the phenotype of
81 individuals in the target data with variance explained equal to the “chip-heritability” (h_{snp}^2) of
82 the trait [25]. However, due to error in the effect size estimates and inevitable differences in
83 the base and target samples, the predictive power of PRS are typically substantially lower
84 than h_{snp}^2 (see Figure 4a) but tend towards h_{snp}^2 as GWAS sample sizes increase.

85

86 Important challenges in the construction of PRS are the selection of SNPs for inclusion in the
87 score and what, if any, shrinkage to apply to the GWAS effect size estimates (see Section
88 3.1). If such parameters are already known, then PRS can be computed directly on the target
89 individual(s). However, when parameters for generating an optimal PRS are unknown, then
90 the target sample can be used for model training, allowing optimisation of model parameters.
91 How to perform this parameter optimisation without producing overfit PRS is discussed in
92 Section 4.4. First, we outline recommended quality control (QC) of the base and target data.
93 In Figure 1, a flow chart summarises the fundamental features of a PRS analysis and reflects
94 the structure of this guide.

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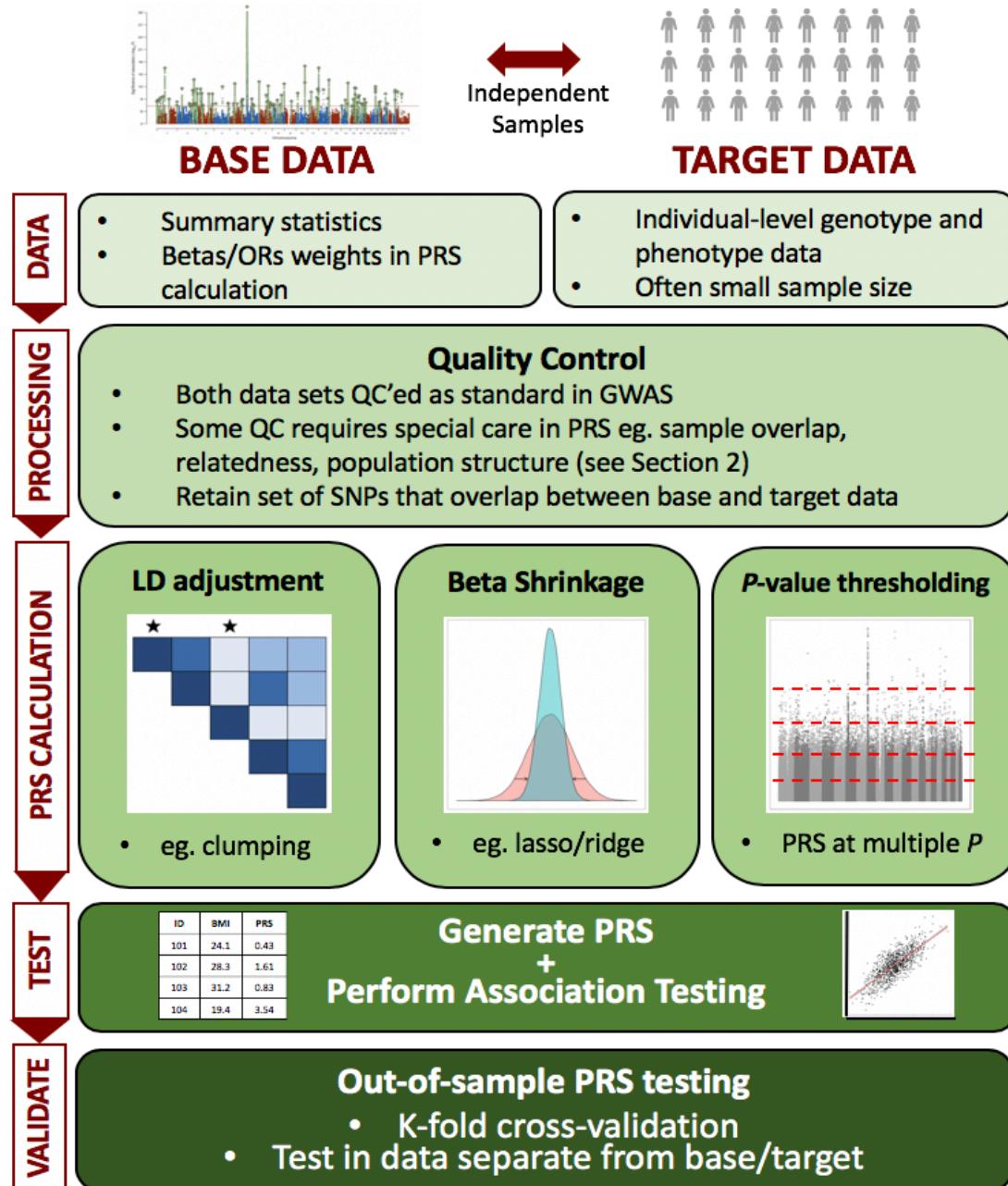
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Figure 1: The Polygenic Risk Score (PRS) analysis process. PRS can be defined by their use of base and target data, as in Section 1. Quality control of both data sets is described in Section 2, while the different approaches to calculating PRS – e.g. LD adjustment via clumping, beta shrinkage using lasso regression, *P*-value thresholding – is summarised in Section 3. Issues relating to exploiting PRS for association analyses to test hypotheses, including interpretation of results and avoidance of overfitting to the data, are detailed in Section 4.

115 **2. Quality Control of Base and Target data**

116 The power and validity of PRS analyses are dependent on the quality of the base and target
117 data. Therefore, both data sets must be quality controlled to the high standards implemented
118 in GWAS studies, e.g. removing SNPs according to low genotyping rate, minor allele
119 frequency or imputation ‘info score’ and individuals with low genotyping rate (see [26–28]).
120 PLINK is a useful software for performing such quality control (QC) [29,30]. Particular care
121 should be taken over these standard QC procedures since any errors that occur may
122 aggregate across SNPs when PRS are computed. In addition to these standard GWAS QC
123 measures, the following QC issues more specific to PRS analyses need special attention and
124 should act as a checklist for PRS analyses:

125

126 **File transfer:** Since most base GWAS data are downloaded online, and base/target data
127 transferred internally, one should ensure that files have not been corrupted during transfer,
128 e.g. using md5sum [31]. PRS calculation errors are often due to corrupt files.

129

130 **Genome Build:** Ensure that the base and target data SNPs have genomic positions assigned
131 on the same genome build [32]. LiftOver [33] is an excellent tool for standardizing genome
132 build across different data sets.

133

134 **Effect allele:** Some GWAS results files do not make clear which allele is the effect allele and
135 which the non-effect allele. If the incorrect assumption is made in computing the PRS, then
136 the effect of the PRS in the target data will be in the wrong direction, and so to avoid misleading
137 conclusions it is critical that the effect allele from the base (GWAS) data is known.

138

139 **Ambiguous SNPs:** If the base and target data were generated using different genotyping
140 chips and the chromosome strand (+/-) for either is unknown, then it is not possible to match
141 ambiguous SNPs (i.e. those with complementary alleles, either C/G or A/T) across the data
142 sets, because it will be unknown whether the base and target data are referring to the same
143 allele or not. While allele frequencies can be used to infer which alleles match [34], we
144 recommend removing all ambiguous SNPs since the allele frequencies provided in base
145 GWAS are often those from resources such as the 1000G project, and so aligning alleles
146 according to their frequency could lead to systematic biases in PRS analyses. When there is
147 a non-ambiguous mismatch in allele coding between the data sets, such as A/C in the base
148 and G/T in the target data, then this can be resolved by ‘flipping’ the alleles in the target data
149 to their complementary alleles. Most polygenic score software can perform this flipping
150 automatically.

151 **Duplicate SNPs:** Ensure that there are no duplicated SNPs in either the base or target data
152 since this may cause errors in PRS calculation unless the code/software used specifically
153 checks for duplicated SNPs.

154

155 **Sex-check:** While sex-check procedures are standard in GWAS QC, they are critical in PRS
156 analyses because errors may generate false-positive associations that are due to sex
157 differences in the target phenotype generated by factors other than autosomal genetics. If the
158 aim is to only model autosomal genetics, then all X and Y chromosome SNPs should be
159 removed from the base and target data to eliminate the possibility of confounding by sex.
160 Proper modelling of the sex chromosomes would improve the predictive power of PRS, but a
161 lack of consensus on how best to analyse the sex chromosomes in GWAS has meant that
162 they have, unfortunately, not generally been considered in PRS studies to date.

163

164 **Sample overlap:** Sample overlap between the base and target data can result in substantial
165 inflation of the association between the PRS and trait tested in the target data [35] and so
166 must be eliminated either, (1) directly: either removing overlapping samples from the target
167 data, or if this removes most/all target individuals, then in the base data followed by
168 recalculation of the base GWAS, or (2) indirectly: if, and only if, the overlapping samples
169 correspond to the entire target sample, and the GWAS that contributed to the base data is
170 available for use, then the overlap can be eliminated using the analytic solution described in
171 [36]. We expect a correction in more complex scenarios of sample overlap, when these
172 solutions are unavailable, to be an objective of future methods development.

173

174 **Relatedness:** A high degree of relatedness among individuals between the base and target
175 data can also generate inflation of the association between the PRS and target phenotype.
176 Assuming that the results of the study are intended to reflect those of the general population
177 without close relatedness between the base and target samples, then relatives should be
178 excluded. If genetic data from the relevant base data samples can be accessed, then any
179 closely related individuals (eg. 1st/2nd degree relatives) across base and target samples should
180 be removed. If this is not an option, then every effort should be made to select base and target
181 data that are very unlikely to contain highly related individuals.

182

183 **Heritability check:** A critical factor in the accuracy and predictive power of PRS is the power
184 of the base GWAS data [4], and so to avoid reaching misleading conclusions from the
185 application of PRS we recommend first performing a heritability check of the base GWAS data.
186 We suggest using a software such as LD Score regression [8] or LDAK [37] to estimate chip-
187 heritability from the GWAS summary statistics, and recommend caution in interpretation of

188 PRS analyses that are performed on GWAS with a low chip-heritability estimate (eg. $h_{snp}^2 <$
189 0.05).

190 **3. The Calculation of Polygenic Risk Scores**

191 Once quality control has been performed on the base and target data, and the data files are
192 formatted appropriately, then the next step is to calculate polygenic risk scores for all
193 individuals in the target sample. There are several options in terms of how PRS are calculated.
194 GWAS are performed on finite samples drawn from particular subsets of the human population,
195 and so the SNP effect size estimates are some combination of true effect and stochastic
196 variation – producing ‘winner’s curse’ among the top-ranking associations – and the estimated
197 effects may not generalise well to different populations (Section 3.4). The aggregation of SNP
198 effects across the genome is also complicated by the correlation among SNPs – ‘Linkage
199 Disequilibrium’ (LD). Thus, key factors in the development of methods for calculating PRS are
200 (i) the potential adjustment of GWAS estimated effect sizes via e.g. shrinkage and
201 incorporation of their uncertainty, (ii) the tailoring of PRS to target populations, and (iii) the
202 task of dealing with LD. We discuss these issues below, and also those relating to the units
203 that PRS values take, the prediction of traits different from the base trait, and multi-trait PRS
204 approaches. Each of these issues should be considered when calculating PRS – though
205 several are automated within specific PRS software – irrespective of application or whether
206 the PRS will be subsequently used for prediction as an end point or for association testing of
207 hypotheses.

208 **3.1 Shrinkage of GWAS effect size estimates**

209 Given that SNP effects are estimated with uncertainty and since not all SNPs influence the
210 trait under study, the use of unadjusted effect size estimates of all SNPs could generate poorly
211 estimated PRS with high standard error. To address this, two broad shrinkage strategies have
212 been adopted: i) shrinkage of the effect estimates of all SNPs via standard or tailored statistical
213 techniques, and ii) use of *P*-value selection thresholds as inclusion criteria for SNPs into the
214 score.

215

216 (i) PRS methods that perform shrinkage of all SNPs [19,38] generally exploit
217 commonly used statistical shrinkage/regularisation techniques, such as LASSO or
218 ridge regression [19], or Bayesian approaches that perform shrinkage via prior
219 distribution specification [38]. Under different approaches or parameter settings,
220 varying degrees of shrinkage can be achieved: some force most effect estimates

221 to zero or close to zero, some mostly shrink small effects, while others shrink the
222 largest effects most. The most appropriate shrinkage to apply is dependent on the
223 underlying mixture of null and true effect size distributions, which are likely a
224 complex mixture of distributions that vary by trait. Since the optimal shrinkage
225 parameters are unknown *a priori*, PRS prediction is typically optimised across a
226 range of (tuning) parameters (for overfitting issues relating to this, see Section 4.4),
227 which in the case of LDpred, for example, includes a parameter for the fraction of
228 causal variant [38].
229

230 (ii) In the *P*-value selection threshold approach, only those SNPs with a GWAS
231 association *P*-value below a certain threshold (eg. $P < 1 \times 10^{-5}$) are included in the
232 calculation of the PRS, while all other SNPs are excluded. This approach
233 effectively shrinks all excluded SNPs to an effect size estimate of zero and
234 performs no shrinkage on the effect size estimates of those SNPs included. Since
235 the optimal *P*-value threshold is unknown *a priori*, PRS are calculated over a range
236 of thresholds, association with the target trait tested for each, and the prediction
237 optimised accordingly (see Section 4.4). This process is analogous to tuning
238 parameter optimisation in the formal shrinkage methods. An alternative way to view
239 this approach is as a parsimonious variable selection method, effectively
240 performing forward selection ordered by GWAS *P*-value, involving block-updates
241 of variables (SNPs), with size dependent on the increment between *P*-value
242 thresholds. Thus the ‘optimal threshold’ selected is defined as such only within the
243 context of this forward selection process; a PRS computed from another subset of
244 the SNPs could be more predictive of the target trait, but the number of subsets of
245 SNPs that could be selected is too large to feasibly test given that GWAS are based
246 on millions of SNPs.

247 **3.2 Controlling for Linkage Disequilibrium**

248 The association tests in GWAS are typically performed one-SNP-at-a-time, which, combined
249 with the strong correlation structure across the genome, makes identifying the independent
250 genetic effects (or best proxies of these if not genotyped/imputed) extremely challenging.
251 While the power of GWAS can be increased by conditioning on the effects of multiple SNPs
252 simultaneously [39], this requires access to raw data on all samples, so researchers generally
253 need to exploit standard GWAS (one-SNP-at-a-time) summary statistics to compute polygenic
254 scores. There are two main options for approximating the PRS that would have been
255 generated from full conditional GWAS: (i) SNPs are *clumped* so that the retained SNPs are

256 largely independent of each other and thus their effects can be summed, assuming additivity,
257 (ii) all SNPs are included and the linkage disequilibrium (LD) between them is accounted for.
258 Usually option (i) is chosen in the ‘standard approach’ to polygenic scoring, involving *P*-value
259 thresholding, while option (ii) is generally favoured in methods that implement traditional
260 shrinkage methods [19,38] (see Table 1). In relation to (i), some researchers, however, prefer
261 to perform the *P*-value thresholding approach without clumping, meaning that the effects of
262 correlated SNPs are summed as though they were independent. While breaking this
263 assumption may lead to minimal losses in some scenarios [19], we recommend performing
264 clumping [13] when non-shrunk effect sizes estimates from GWAS are used because the non-
265 uniform nature of LD across the genome is likely to generate some bias in estimates. The
266 reason why the standard approach, though simple, appears to perform comparably to more
267 sophisticated approaches [19,38] may be due to the clumping process capturing conditionally
268 independent effects well; note that, (i) clumping does not merely thin SNPs by LD at random
269 (like *pruning*) but preferentially selects SNPs most associated with the trait under study, (ii)
270 clumping can retain multiple independent effects in the same genomic region if they exist (it
271 does not simply retain only the most associated SNP in a region). A criticism of clumping,
272 however, is that researchers typically select an arbitrarily chosen correlation threshold [35] for
273 the removal of SNPs in LD, and so while no strategy is without arbitrary features, this may be
274 an area for future development of the classical approach.

275

276 Table 1. Comparison of different approaches for performing Polygenic Risk Score analyses

Shrinkage strategy	<i>P</i> -value	Standard approach: thresholding w/o clumping	Penalised Regression	Bayesian Shrinkage
	<i>P</i> -value threshold	Clumping + thresholding (C+T)	LASSO, Elastic Net, penalty parameters	Prior distribution, e.g. fraction of causal SNPs
		Clumping		
Handling Linkage Disequilibrium	N/A		LD matrix is integral to algorithm	Shrink effect sizes with respect to LD
Example software	PLINK	PRSice [12]	Lassosum [19]	LDpred [38]

277

278 **3.3 PRS units**

279 When calculating PRS, the units of the GWAS effect sizes determine the units of the PRS;
280 e.g. if calculating a height PRS using effect sizes from a height GWAS that are reported in
281 centimetres (cm), then the resulting PRS will also be in units of cm. PRS may then be
282 standardised, dividing by the number of SNPs to ensure a similar scale irrespective of number
283 of SNPs included, or standardised to a standard normal distribution. However, the latter
284 discards information that may wish to be retained, since the absolute values of the PRS may
285 be useful in detecting problems with the calculation of the PRS or the sample, identifying
286 outliers, comparing or combining PRS across different samples, or even detecting the effects
287 of natural selection. Negative selection against effect alleles could result in a PRS with a mean
288 negative value due to effect alleles being at lower frequency than non-effect alleles on average,
289 and the opposite for traits under positive selection.

290

291 In calculating PRS on a binary (case/control) phenotype, the effect sizes used as weights are
292 typically reported as log Odds Ratios (log(ORs)). Assuming that relative risks on a disease
293 accumulate on a multiplicative rather than additive scale [40], then PRS should be computed
294 as a summation of log(OR)-weighted genotypes. It is important for subsequent interpretation
295 to know which logarithmic scale was used since the PRS will take the same units and will be
296 needed to transform back to an OR scale.

297 **3.4 Population structure and global heterogeneity**

298 Population structure is the principal source of confounding in GWAS (post-QC), and thus risk
299 of false-positive findings. Briefly, structure in mating patterns in a population generates
300 structure in genetic variation, correlated most strongly with geographic location, and
301 environmental risk factors can be similarly structured; this creates the potential for
302 associations between many genetic variants and the tested trait that are confounded by e.g.
303 location [41,42]. While this problem is typically addressed in GWAS via adjustment by principal
304 components (PCs) [41] or the use of mixed models [43], population structure poses a
305 potentially greater problem in PRS analyses, because a large number of null variants are
306 typically included in the calculation of PRS and their estimated effects are aggregated. If allele
307 frequencies differ systematically between the base and target data, which may derive from
308 genetic drift or the ascertainment of genotyped variants [44], and if the distributions of
309 environmental risk factors for the trait also differ between the two – both highly likely in most
310 PRS studies – then there is a danger that an association between the PRS and target trait can
311 be generated by differences at null SNPs. Confounding is potentially reintroduced even if the
312 GWAS had controlled for population structure perfectly, because this does not account for

313 correlated differences in allele frequencies and risk factors between the base and target data.
314 When the base and target samples are drawn from the same or genetically similar populations,
315 stringent control for structure in the PRS analysis itself (e.g. including a large number of PCs)
316 should suffice to avoid false-positive findings, but we recommend in general that extreme
317 caution is taken given dramatic differences in PRS distributions observed between populations
318 [44–46]. While these observations do not imply large differences in aetiology across
319 populations – although genuine differences due to variation in the environment, culture and
320 selection pressures are likely to contribute – they do question the reliability of PRS analyses
321 using base and target data from different populations that do not rigorously address the issue
322 of potential confounding from geographic stratification [45]. It is also important to be wary of
323 the fact that highly significant results can be observed due to subtle confounding when
324 exploiting large sample sizes. Note that we use the term ‘population’ here in a statistical sense:
325 problems of population structure are just as relevant within-country given differences in the
326 genetics and environment between individuals in the base and target samples. We expect the
327 issue of the generalisability of PRS across populations to be an active area of methods
328 development in the coming years [46,47].

329 **3.5 Predicting Different Traits and exploiting multiple PRS**

330 While PRS are often analysed in scenarios in which the base and target phenotype are the
331 same, many published studies involve a target phenotype different from that on which the PRS
332 is based. These analyses fall into three main categories: (i) optimising target trait prediction
333 using a different but similar (or ‘proxy’) base trait: if there is no large GWAS on the target trait,
334 or it is underpowered compared to a similar trait, then prediction may be improved using a
335 different base trait (e.g. education years to predict cognitive performance [48,49]), (ii)
336 optimising target trait prediction by exploiting multiple PRS based on a range of different traits
337 in a joint model, (iii) testing for evidence of shared aetiology between base and target trait [50].
338 Applications (i) and (ii) are straightforward in their aetiology-agnostic aim of optimising
339 prediction, achieved by exploiting the fact that a PRS based on one trait is predictive of
340 genetically correlated traits, and that a PRS computed from any base trait is sub-optimal due
341 to the finite size of any GWAS. Application (iii) is inherently more complex because there are
342 different ways of defining and assessing ‘shared aetiology’. Shared aetiology may be due to
343 so-called horizontal pleiotropy (separate direct effects) or vertical pleiotropy (downstream
344 effect) [51] and it is unclear what quantity should be estimated to assess evidence – genetic
345 correlation [9], genetic contribution to phenotypic covariance (co-heritability) [52,53], or a trait-
346 specific measure (eg. where the denominator relates only to the genetic risk of one trait).

347 While there is active method development in these areas [54–56] at present, the majority of
348 PRS studies use exactly the same approach to PRS analysis whether or not the base and
349 target phenotypes differ [50,56]. However, this is rather unsatisfactory because of the non-
350 uniform genetic sharing between different traits. In PRS analysis, the effect sizes and *P*-values
351 are estimated using the base phenotype, independent of the target phenotype. Thus, a SNP
352 with high effect size and significance in the base GWAS may have no effect on the target
353 phenotype. The standard approach could be adapted so that SNPs are prioritized for inclusion
354 in the PRS according to joint effects on the base and target traits [57], but this has yet to be
355 implemented in any standard software. Other more sophisticated solutions are presently being
356 investigated [55] and other approaches will likely be developed in future, each tailored to
357 specific scientific questions.

358 **4. Interpretation and Presentation of Results**

359 If performing individual prediction is the end objective – for example, to make clinical decisions
360 about individual patients – then the most predictive polygenic score method (known at the
361 time) should be applied to the most powerful base sample available on the relevant trait, in
362 order to optimise accuracy of the individual PRS. Little interpretation or presentation of results
363 are required in this setting, and thus Section 4 is devoted to the primary use of PRS in
364 association testing of scientific hypotheses. Once PRS have been calculated, selecting from
365 the options described in Section 3, typically a regression is then performed in the target sample,
366 with the PRS as a predictor of the target phenotype, and covariates included as appropriate.

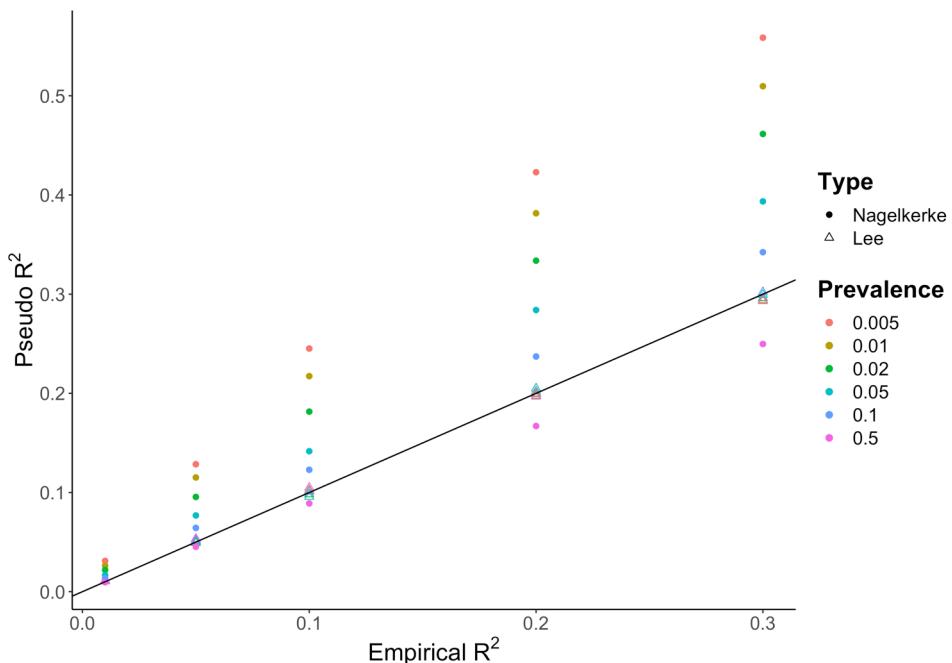
367 **4.1 Association and goodness-of-fit metrics**

368 A typical PRS study involves testing evidence for an association between a PRS and a trait
369 or measuring the extent of the association in the entire or specific strata of the sample. The
370 standard ways of measuring associations in epidemiology, and their related issues, apply here.
371 The association between PRS and outcome can be measured with the standard association
372 or goodness-of-fit metrics, such as the effect size estimate (beta or OR), phenotypic variance
373 explained (R^2), area under the curve (AUC), and *P*-value corresponding to a null hypothesis
374 of no association. The PRS for many traits are such weak proxies of overall genetic burden
375 (presently) that the phenotypic variance that they explain is often very small ($R^2 < 0.01$),
376 although this is not important if the aim is only to establish whether an association exists.
377 However, present evidence on the ubiquity of pleiotropy across the genome [51] indicates that
378 there may be shared aetiology between the vast majority of phenotypes, detectable with
379 sufficient sample size. Thus, establishing the relative extent of associations among a range of

380 traits may be more worthwhile [36,58], and act as a step towards identifying the causal
381 mechanisms underlying these genetic associations [59].

382
383 While variance explained (R^2) is a well-defined concept for continuous trait outcomes, only
384 conceptual proxies of this measure (“pseudo- R^2 ”) are available for case/control outcomes. A
385 range of pseudo- R^2 metrics are used in epidemiology [60,61], with Nagelkerke R^2 perhaps the
386 most popular. However, Nagelkerke R^2 suffers from particular bias when the case/control
387 proportion is not reflective of the case population prevalence [60], and so in the context of
388 estimating the genetic contribution to a polygenic disease it may be preferable to estimate the
389 phenotypic variance explained on the liability scale. Intuitively, the R^2 on the liability scale here
390 estimates the proportion of variance explained by the PRS of a hypothetical normally
391 distributed latent variable that underlies and causes case/control status [60,62]. Heritability is
392 typically estimated on the liability scale for case/control phenotypes [13,60,62]. Lee et al [60]
393 developed a pseudo- R^2 metric that accounts for an ascertained case/control ratio and is
394 measured on the liability scale. We show that, under simulation, this metric indeed controls
395 for case/control ratios that do not reflect disease prevalence, while Nagelkerke R^2 does not
396 (Figure 2).

397



398
399 Figure 2. Results from a simulation study comparing Nagelkerke pseudo- R^2 with the pseudo- R^2 proposed by Lee
400 et al [59] that incorporates adjustment for the sample case:control ratio. In the simulation, 2,000,000 samples were
401 simulated to have a normally distributed phenotype, generated by a normally distributed predictor (eg. a PRS)
402 explaining a varying fraction of phenotypic variance and a residual error term to model all other effects. Case/control
403 status was then simulated under the liability threshold model according to a specified prevalence. 5,000 cases and
404 5,000 controls were then randomly selected from the population, and the R^2 of the original continuous data,

405 estimated by linear regression (Empirical R^2), was compared to both the Nagelkerke R^2 (discs) and the Lee R^2
406 (triangles) based on the equivalent case/control data by logistic regression.

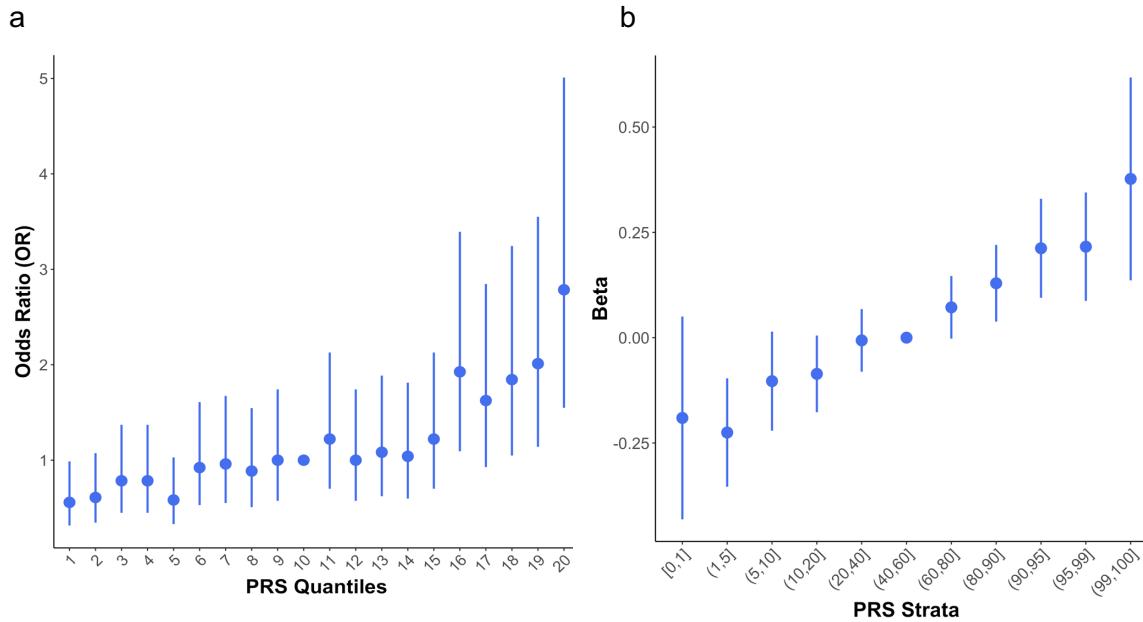
407 **4.2 Graphical representations of results: bar and quantile plots**

408 When the standard approach (C+T) is used, the results of the PRS association testing are
409 often displayed as a bar plot, where each bar corresponds to the result from testing a PRS
410 comprising SNPs with GWAS P -value exceeding a given threshold. Typically, a small number
411 of bars are shown, reflecting results at round-figure P -value thresholds (5e-8, 1e-5, 1e-3, 0.01,
412 0.05, 0.1, 0.2, 0.3 etc), and if 'high-resolution' scoring [12] is performed then a bar representing
413 the most predictive PRS is included. Usually the Y-axis corresponds to the phenotypic
414 variance explained by the PRS (R^2 or pseudo- R^2) and the value over each bar (or its colour)
415 provides the P -value of the association between the PRS and target trait. It is important to
416 note that the P -value threshold of the most predictive PRS is a function of the effect size
417 distribution, the power of the base (GWAS) and target data, the genetic architecture of the
418 trait, and the fraction of causal variants, and so should not be merely interpreted as reflecting
419 the fraction of causal variants. For example, if the GWAS data are relatively underpowered
420 then the optimal threshold is more likely to be $P = 1$ (all SNPs) even if a small fraction of SNPs
421 is causal (see [4] for details).

422

423 While goodness-of-fit measures, such as R^2 , provide a sample-wide summary of the predictive
424 power of a PRS, it can be useful to inspect how trait values vary with increasing PRS or to
425 gauge the elevated disease risk that specific strata of the population may be at according to
426 their PRS. This can be easily visualized using a quantile plot (Figure 3a). Quantile plots in
427 PRS studies are usually constructed as follows [2,63]. The target sample is first separated into
428 strata of increasing PRS. For instance, 20 equally sized quantiles, each comprising 5% of the
429 PRS sample distribution (Figure 3a). The phenotype values of each quantile are then
430 compared to those of the reference quantile (usually the median quantile) one-by-one, with
431 quantile status as predictor of target phenotype (reference quantile coded 0, test quantile
432 coded 1) in a regression. Points on the plot depict the beta or OR (Y-axis), along with bars for
433 their standard errors, corresponding to the regression coefficients of these quantile-status
434 predictors. If covariates are controlled for in the main analysis, then a regression can be
435 performed with target trait as outcome and the covariates as predictors, and the residual trait
436 on the Y-axis instead. Stratification may be performed on unequal strata of PRS (eg. Fig. 3b),
437 in which case these are strata rather than quantile plots. Individuals with high PRS may have
438 low trait values or vice versa, particularly if PRS explain minimal phenotypic variance, and
439 thus the quantiles/strata are not necessarily monotonically increasing.

440



441 Figure 3. Examples of quantile/strata plots. (a) shows the odds ratios (Y-axis) of a target trait across twenty equal-sized strata of increasing PRS (X-axis) in relation to the 10th strata, while (b) shows a strata plot with eleven unequal strata that highlight the increased or decreased risk among individuals in the top and bottom percentiles of PRS, relative to individuals with PRS in the middle of the distribution (here from 40%-60%).

445

446 4.3 PRS distribution

447 Quantile plots corresponding to the same normally distributed phenotype in base and target, should reflect the S-shape of the probit function, and likewise for a binary trait underlain by a normally distributed liability, characterised by the liability threshold model [64]. Thus, inflections of risk at the tails of the PRS distribution [65], or at the top/bottom quantiles, should be interpreted according to this expectation. As for the PRS distribution itself, without respect to association with a target trait, the central limit theorem dictates that if the PRS is based on a sum of independent variables (here SNPs) with identical distributions, then the PRS of a sample should approximate the normal (Gaussian) distribution. Strong violations of these assumptions, such as the use of many correlated SNPs or a sample of heterogenous ancestry (thus SNPs with non-identical genotype distributions), can lead to non-normal PRS distributions. Samples of individuals from disparate worldwide populations may lead to highly non-normal PRS distributions (see Section 3.4), thus inspection of PRS distributions may be informative for problems of population stratification in the target sample not adequately controlled for.

461

462

463 **4.4 Overfitting in PRS association testing**

464 A common concern in PRS studies that adopt the standard (C+T) approach is whether the
465 use of the most predictive PRS – based on testing at many *P*-value thresholds – overfits to
466 the target data and thus produces inflated results and false conclusions. While such caution
467 is to be encouraged in general, potential overfitting is a normal part of prediction modelling,
468 relevant to the other PRS approaches (Table 1), and there are well-established strategies for
469 optimising power while avoiding overfitting. One strategy that we do not recommend is to
470 perform no optimisation of parameters – e.g. selecting a single arbitrary *P*-value threshold
471 (such as $P < x10^{-8}$ or $P = 1$) – because this may lead to serious *underfitting*, which itself can
472 lead to false conclusions.

473

474 The gold-standard strategy for guarding against generating overfit prediction models and
475 results is to perform out-of-sample prediction. First, parameters are optimised using a training
476 sample and then the optimised model is tested in a test or validation data set to assess
477 performance. In the PRS setting involving a base and target data, it would be a misconception
478 to believe that out-of-sample prediction has already been performed because polygenic
479 scoring involves two different data sets, when in fact the training is performed on the target
480 data set, meaning that a third data set is required for out-of-sample prediction. In the absence
481 of an independent data set, the target sample can be subdivided into training and validation
482 data sets, and this process can be repeated with different partitions of the sample, e.g.
483 performing 10-fold cross-validation [56,66,67], to obtain more robust model estimates.
484 However, a true out-of-sample, and thus not overfit, assessment of performance can only be
485 achieved via final testing on a sample entirely separate from data used in training.

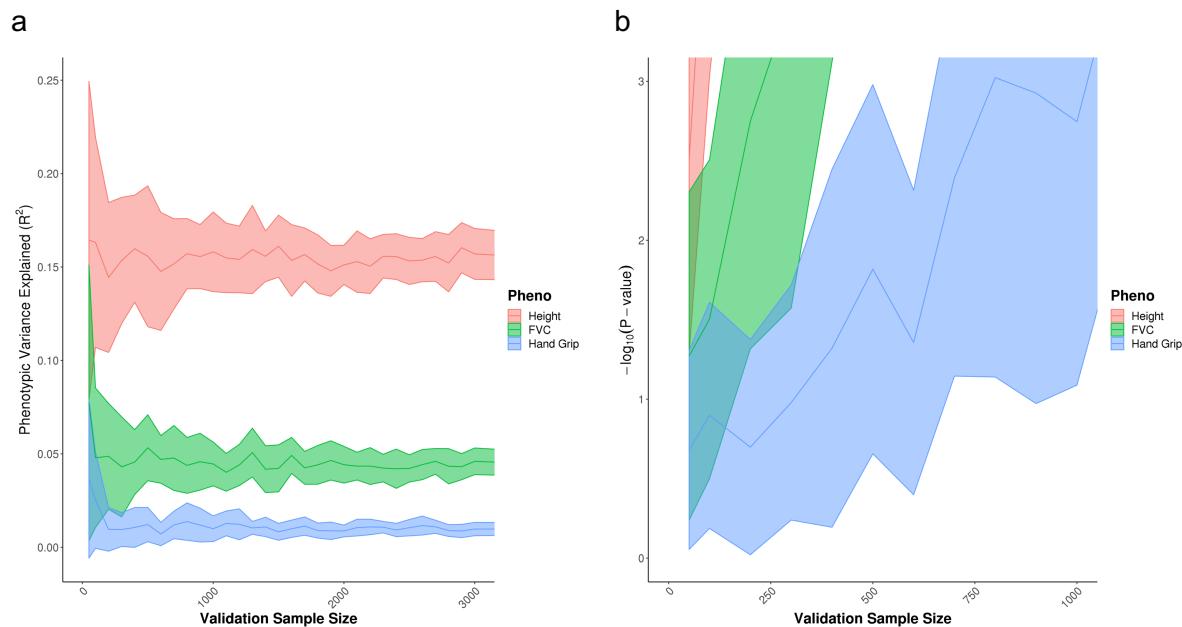
486

487 Without validation data or when the size of the target data makes cross-validation
488 underpowered, an alternative is to generate empirical *P*-values corresponding to the optimised
489 PRS prediction of the target trait, via permutation [12]. While the PRS itself may be overfit, if
490 the objective of the PRS study is association testing of a hypothesis – e.g. H_0 : schizophrenia
491 and rheumatoid arthritis have shared genetic aetiology – rather than for prediction *per se*, then
492 generating empirical *P*-values offers a powerful way to achieve this while maintaining
493 appropriate type 1 error [12]. It is also even possible to generate optimised parameters for
494 PRS when no target data are available [19].

495 **4.5 Power and accuracy of PRS: target sample sizes required**

496 In one of the key PRS papers published to date, Dudbridge 2013 [4] provided estimates of the
497 power and predictive accuracy of PRS in different scenarios of data availability and
498 phenotypes. To complement this work, we performed PRS analyses across three traits in the
499 UK Biobank with high (height), medium (Forced Volume Capacity; FVC) and low (hand grip
500 strength) heritability to provide a guide to the approximate performance of PRS association
501 testing on real data with different heritability and different validation sample sizes, when
502 exploiting a large (100k) base GWAS (Figure 4). While this provides only a very limited
503 indication of the performance of PRS analyses, in our experience, researchers in the field
504 often wish to obtain some idea of whether their own (target/validation) data are likely to be
505 sufficiently powered for future analyses or if they need to acquire more data.

506



507 Figure 4. Examples of performance of PRS analyses on real data by validation sample size, according to (a)
508 phenotypic variance explained (R^2), (b) association P -value. UK Biobank data on height (estimated heritability h^2
509 = 0.49 [8]), Forced Volume Capacity (FVC) (estimated heritability $h^2 = 0.23$ [8]), Hand Grip (estimated heritability
510 $h^2 = 0.11$ [8]), were randomly split into two sets of 100,000 individuals and used as base and target data, while the
511 remaining sample was used as validation data of varying sample sizes, from 50 individuals to 3000 individuals.
512 Each analysis was repeated 5 times with independently selected validation samples. While these results
513 correspond to performance in validation data, the association P -values should reflect empirical P -values estimated
514 from target data (as described in Section 4.4).

515

516

517 **Conclusions**

518 As GWAS sample sizes increase, the application of Polygenic Risk Scores is likely to play a
519 central role in the future of biomedical studies and personalised medicine. However, the
520 efficacy of their use will depend on the continued development of methods that exploit them,
521 their proper analysis and appropriate interpretation, and an understanding of their strengths
522 and limitations.

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535 **Author contributions**

536 SWC and PFO conceived and prepared the manuscript. All authors discussed its contents.
537 SWC performed all statistical analyses. PFO drafted the manuscript, with critical feedback
538 from SWC and TM.

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