

1 Directional Gaussian Mixture Models of the gut
2 microbiome elucidate microbial spatial structure

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9 **Abstract:** The gut microbiome is spatially heterogeneous, with environmental niches con-
10 tributing to the distribution and composition of microbial populations. A recently developed
11 mapping technology, MaPS-seq, aims to characterize the spatial organization of the gut mi-
12 crobiome by providing data about local microbial populations. However, information about
13 the global arrangement of these populations is lost by MaPS-seq. To address this, we pro-
14 pose a class of Gaussian Mixture Models (GMM) with spatial dependencies between mixture
15 components in order to computationally recover the relative spatial arrangement of micro-
16 bial communities. We demonstrate on synthetic data that our spatial models can identify
17 global spatial dynamics, accurately cluster data, and improve parameter inference over a
18 naive GMM. We applied our model to three MaPS-Seq datasets taken from varying regions
19 of the mouse intestine. On cecal and distal colon datasets, we find our model accurately
20 recapitulates known spatial behaviors of the gut microbiome, including compositional dif-
21 ferences between mucus and lumen-associated populations. Our model also seem to capture
22 the role of a pH gradient on microbial populations in the mouse ileum and proposes new
23 behaviors as well.

24 **Importance:** The spatial arrangement of the microbes in the gut microbiome is a defin-
25 ing characteristic of its behavior. Various experimental studies have attempted to provide
26 glimpses into the mechanisms that contribute to microbial arrangements. However, many of
27 these descriptions are qualitative. We developed a computational method that takes micro-
28 bial spatial data and learns many of the experimentally validated spatial factors. We can
29 then use our model to propose previously unknown spatial behaviors. Our results demon-
30 strate that the gut microbiome, while exceptionally large, has predictable spatial patterns
31 that can be used to help us understand its role in health and disease.

32 **Code availability:** github.com/amepas/Spatial_Mbiome

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35 1 Introduction

36 A defining characteristic of the gut microbiome community is its spatial structure. Nu-
37 trients and chemical conditions differ along the gastrointestinal (GI) tract, impacting the
38 distribution of taxa that reside there (1, 2). This spatial arrangement of microbes within
39 the gut microbiome likely contributes to major aspects of its dynamic behavior, including
40 community stability and host-microbe interactions (3, 4).

41 Recently, a novel DNA technology, Metagenomic Plot Sampling by sequencing (MaPS-
42 seq), was developed to offer insights into the spatial organization of the gut microbiome
43 (5). In MaPS-seq, high-resolution segments ($\sim 20\mu\text{m}$ squares) are extracted directly from
44 along the gut. Segments are encapsulated in droplets with barcoded 16S rRNA amplification
45 primers, such that sequencing reads with the same barcode originate from the same segment.
46 Hence, MaPS-seq preserves localized information about the spatial structure of the micro-
47 biome, and is a valuable tool for investigating the biogeography of the gut microbiome. Yet,
48 the assignment of barcodes to droplets is a random process: MaPS-seq does not preserve the
49 global arrangement of droplets along the gut.

50 Known characteristics of the biogeography of the gut microbiome suggest it may be possi-
51 ble to reconstruct the global arrangement of MaPS-seq droplets. For example, antimicrobial
52 peptides, oxygen levels, and acidity vary along the length of the small intestine. Conse-
53 quently, bacterial loads increase along the longitudinal axis of the small intestine and lead to
54 a more microbe-rich ileum (2). In the colon, the density of the mucus layer increases along
55 its longitudinal and cross-sectional axes—creating environmental niches favored by different
56 species (1). In principle, it should be possible to reconstruct some of these global patterns
57 from the high-resolution sampling of MaPS-seq.

58 1.1 Our contribution

59 We developed a class of computational models to recover known characteristics of the bio-
60 geography of the gut microbiome from MaPS-seq data. Our models build upon the classical
61 Gaussian Mixture Model (GMM, Figure 1). In a GMM, observations are mixtures of latent
62 clusters, each of which is modeled as a multidimensional Gaussian random variable, inde-
63 pendent of the others and with its own mean. We expand this framework by introducing
64 spatial dependence between latent clusters. Specifically, clusters are arranged as a line (one-
65 dimensional model) or grid (two-dimensional model) to investigate directional changes along
66 the longitudinal axis only, or, respectively, both the longitudinal and radial axes of the gut.

67 A key question is whether our model can differentiate longitudinal from radial changes
68 in the gut. We demonstrate on synthetic data that our model is capable of discriminating
69 between one-dimensional and two-dimensional models. We apply our model to MaPS-seq
70 mouse ileum, cecal, and distal colon datasets. We provide strong evidence for the presence
71 of spatial structures across all datasets, with distinct regional characteristics. We show that
72 our proposed model recovers known biological behaviors of microbes within the GI tract
73 while also providing new insights into the spatial structure of the gut microbiome.

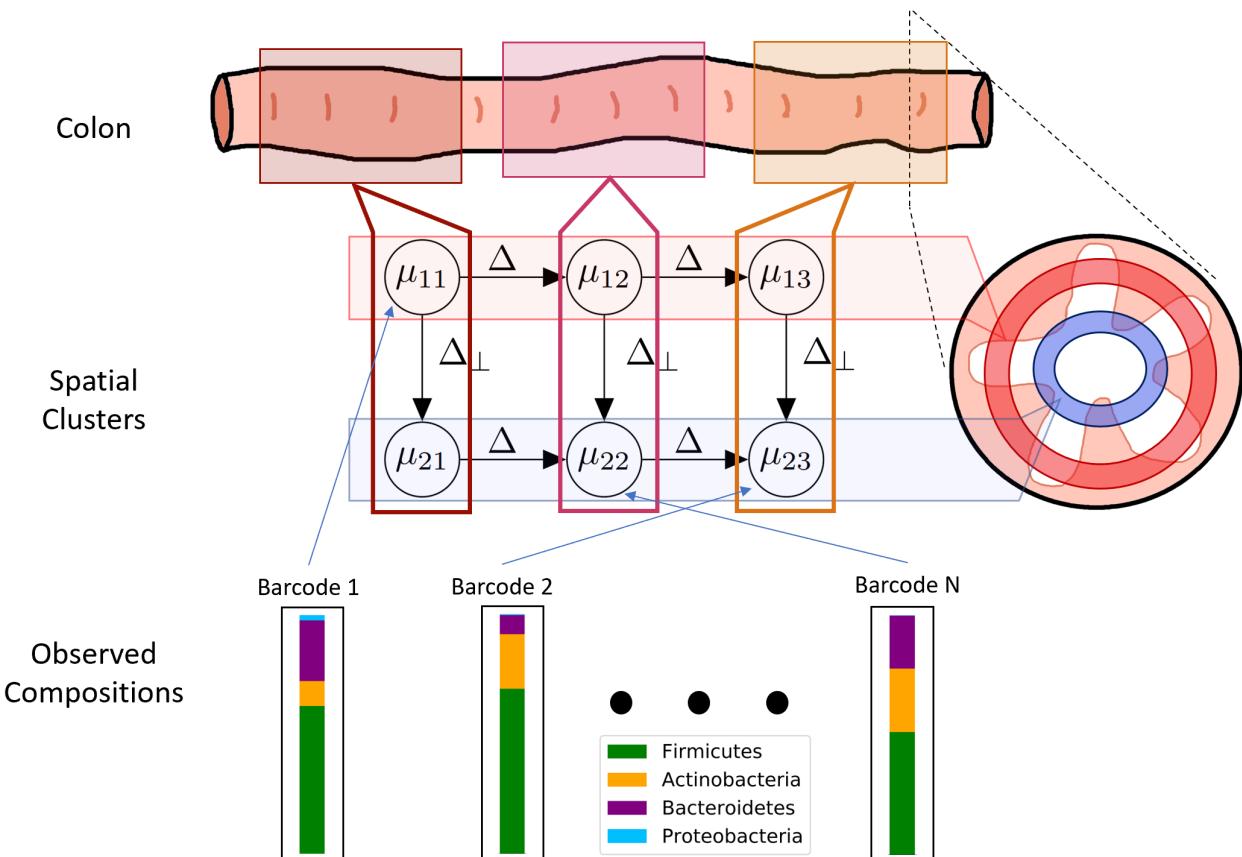


Figure 1: **Schematic Overview of directional Gaussian mixture model.** Given observed compositions from each barcode, the model simultaneously learns the community composition of each latent cluster (μ_i), and the assignment of each barcode to a latent cluster.

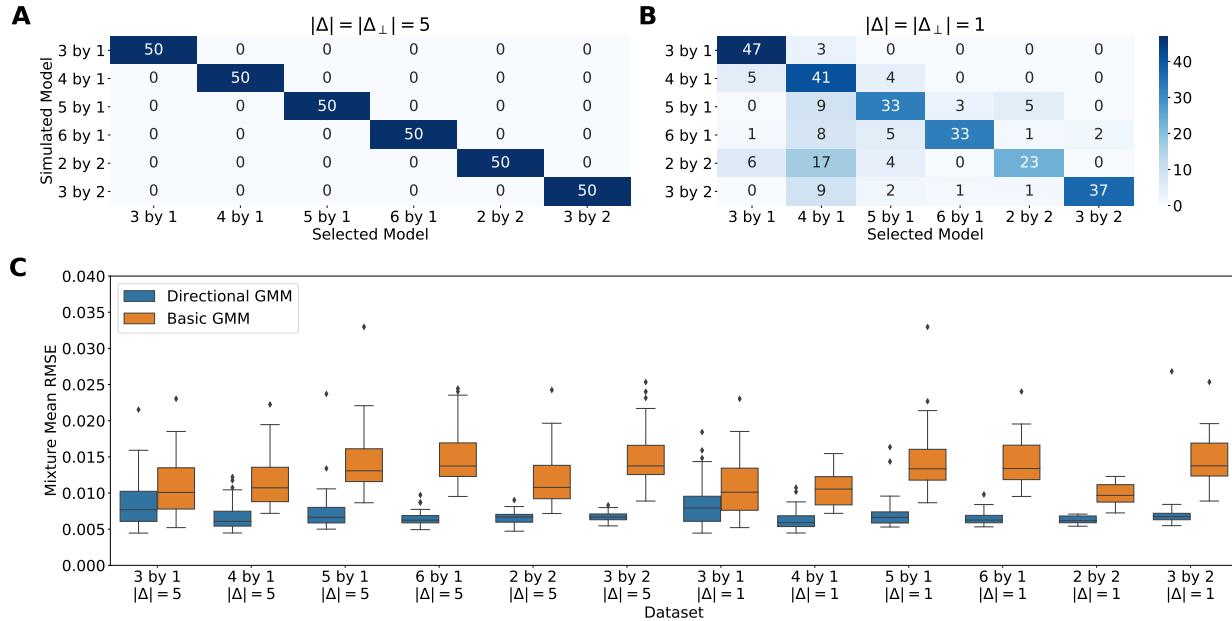


Figure 2: **Directional GMMs accurately select number of latent clusters and infer model parameters.** (A) Heatmap showing the accuracy of the selected model on various simulated datasets with $|\Delta| = |\Delta_{\perp}| = 5$ and within-cluster standard deviation of 1 (B) $|\Delta| = |\Delta_{\perp}| = 1$, keeping the same standard deviation (C) RMSE of learned cluster means on datasets with correct model selection. On all datasets, the directional GMMs are significantly improving parameter inference

74 2 Results

75 2.1 Simulation Results

76 We first evaluated whether our model can differentiate between one- and two-dimensional
77 dynamics using simulated data. We simulated data under the one- and two-dimensional
78 models (Methods 4.4), and asked if we could infer the number of latent clusters and their
79 spatial arrangement. Using the Akaike Information Criterion (AIC), we found that our
80 directional GMM is able to correctly determine the correct number of clusters (Figure 2A:B).
81 Furthermore, the introduction of a dependence between latent clusters in the model also
82 improved parameter inference compared to a naive GMM with no spatial structure (Figure
83 2C). For all dataset forms, the Wilcoxon signed-rank test p-value was less than 0.001.

84 2.2 Spatial Structure of MaPS-Seq Data

85 We applied our directional GMM to three real MaPS-seq datasets from Sheth et al. (5)
86 (Methods 4.5). The provided MaPS-seq data contains samples from 3 regions of a single
87 mouse's GI tract: the cecum ($n = 405$ barcodes), the ileum ($n = 386$ barcodes), and
88 the distal colon ($n = 259$ barcodes). On the Cecum and Distal Colon datasets, the best
89 supported models were two-dimensional (4×2 and 3×2 respectively). On the Ileum dataset,

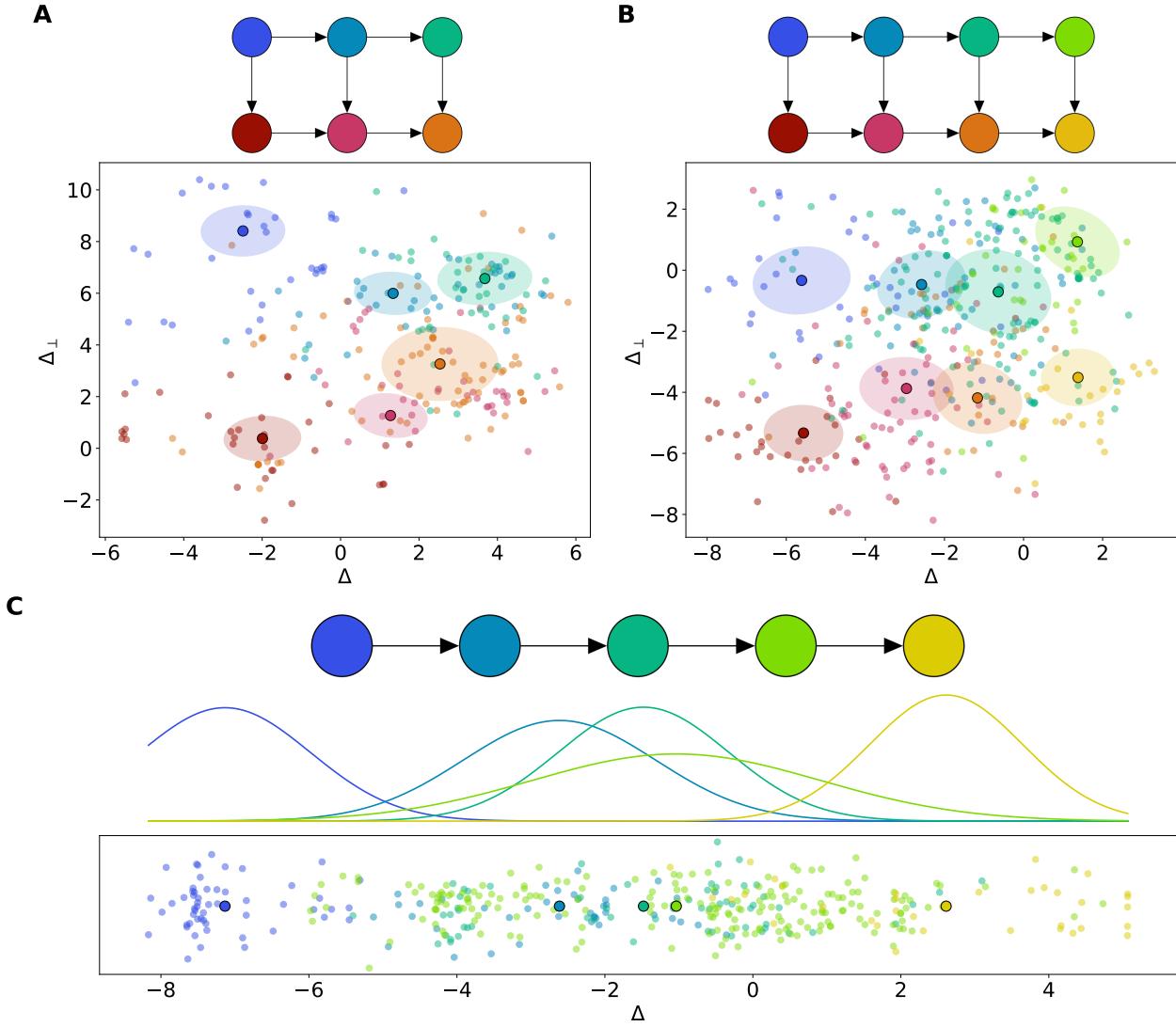


Figure 3: Projections of MaPS-Seq Data. (A) Distal Colon dataset. The selected model and MaPS-Seq data projected along the unit Δ and Δ_{\perp} axes. Colors correspond to samples belonging to a latent cluster. Ellipse radii represent the eigenvectors of the covariance matrix. (B) Cecum dataset. (C) Ileum dataset. Selected model and MaPS-Seq data projected along the unit Δ axis. Normal distribution represent density of covariance matrices around each cluster mean.

| AIC Selections | | | | | |
|----------------|---------------|---------------|---------------|---------------|---------------------------------|
| Dataset | One-Dimension | | Two-Dimension | | $AIC_{naive} - AIC_{structure}$ |
| | Score | Mixture Model | Score | Mixture Model | |
| Ileum | -37493 | 5×1 | -37340 | 3×2 | 954 |
| Cecum | 32714 | 8×1 | 31930 | 4×2 | 3832 |
| Distal Colon | 6636 | 6×1 | 5588 | 3×2 | 1168 |

Table 1: **AIC shows strong evidence for spatial structure across the GI tract.** (A) Best directional mixture model and its corresponding AIC score. Scores in bold indicate selected model. Comparison of directional GMMs to naive GMM by AIC metric show introduction of dependence between latent clusters significantly improves model fit. Full relative likelihoods calculated using AIC scores between models are also shown (Methods 4.3).

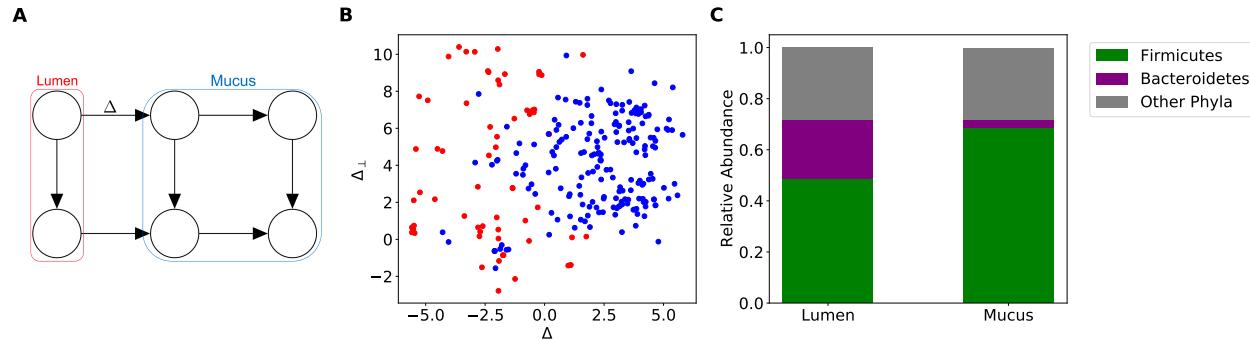


Figure 4: **Directional GMM recovers spatial dynamics in the distal colon** (A) Selected model and corresponding locations of clusters in the distal colon (B) Scatter plot of projected MaPS-seq samples assigned to lumen-associated clusters (red) and mucus-associated clusters (blue). (C) Clusters associated with the mucus are enriched in *Firmicutes* and those associated with the lumen display larger levels of *Bacteroidetes*.

90 the best supported model was a one-dimensional model with 5 clusters (5×1). Using the
 91 model parameters from the best supported model on each dataset, we created one- and
 92 two-dimensional visualizations depicting the directions learned by our model (Figure 3).
 93 Qualitatively, our model appeared to segregate barcodes into distinct clusters along the gut.

94 We also compared the support the selected directional model GMM to a naive GMM with
 95 no spatial structure. To compare models, we computed the AIC scores of our directional
 96 models to a naive GMM with the same number of latent clusters (Table 1). The naive
 97 GMMs have much larger AIC scores than the directional GMMs. Conventionally, models
 98 with scores that are larger by 10 or more are considered to have little support (6).

99 2.3 Recovery of GI Tract Biogeography

100 We also investigated learned model parameters for correspondence to some of the known
 101 spatial dynamics of the gut microbiome. Figure 4 illustrates the recovered dynamics on the
 102 Distal Colon dataset. Under the partition presented in Figure 4, we observe large differences
 103 in the average compositions of *Firmicutes* and *Bacteroidetes* between lumen- and mucus-

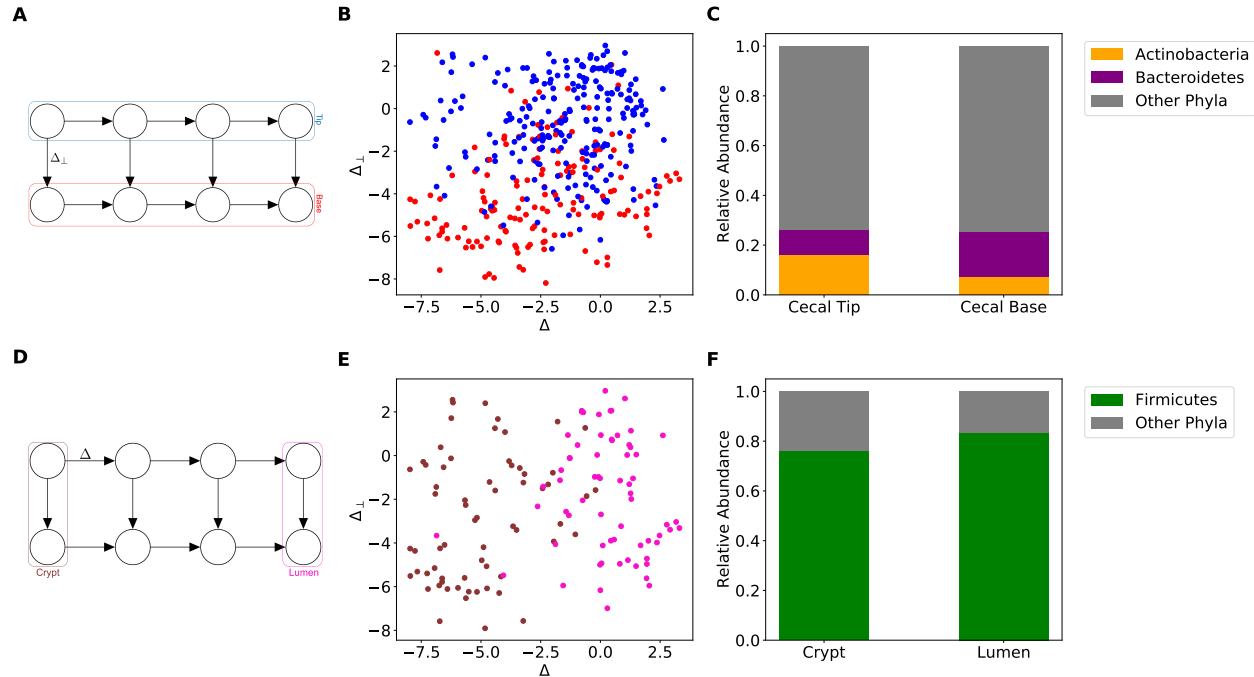


Figure 5: Directional GMM recovers spatial dynamics in the cecum (A) Selected model and corresponding locations of latent clusters in the cecum. (B) Scatter plot of projected MaPS-seq samples assigned to cecal tip-associated clusters (blue) and cecal base-associated clusters (red). (C) Clusters associated with the cecal tip have lower relative abundances of *Bacteroidetes* and higher relative abundances of *Actinobacteria* than the cecal base. (D) Selected model and corresponding locations of mixtures in the cecum. (E) Scatter plot of projected MaPS-seq samples assigned to cecal crypt-associated clusters (brown) and cecal lumen-associated clusters (pink). (F) *Firmicutes* are enriched in the lumen clusters compared to the crypt clusters.

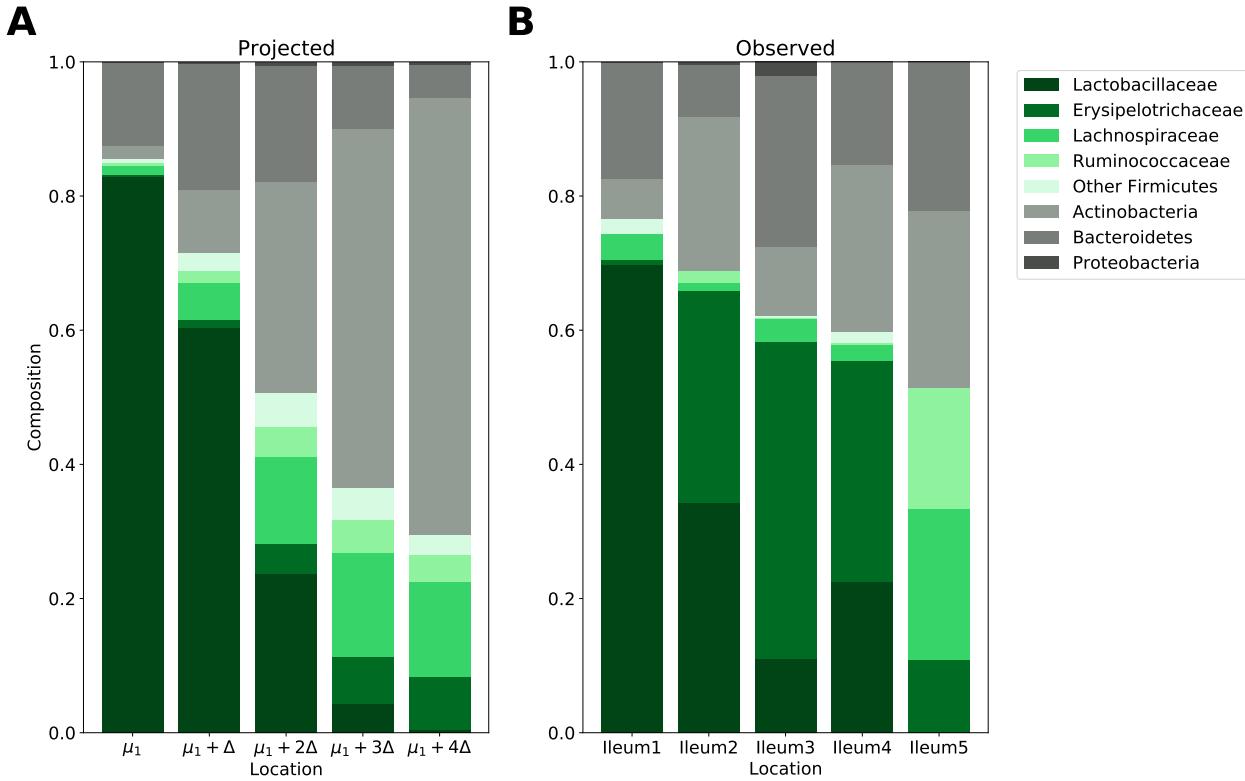


Figure 6: **Projected and observed Ileum dynamics** (A) Projected compositions moving along Δ axis (B) Observed compositions in learned model clusters. Green bars correspond to observed families in the *Firmicutes* phylum and gray correspond to other observed phyla

104 associated clusters.

105 On the Cecum dataset, we observed compositional differences along both axes. Figure
 106 4A:C shows a cecal tip and base partition that has a noticeable compositional difference in
 107 the abundances of *Actinobacteria* and *Bacteroidetes*. The clusters on the two ends of the
 108 model have differences in the abundances of *Firmicutes* that correspond to the cecal crypt
 109 and lumen (Figure 5D : F).

110 On the Ileum dataset, we compared microbial population relative abundances across
 111 each latent cluster. Along the length of the ileum, we observed a general decreasing trend in
 112 *Lactobacillaceae* and increases in both *Ruminococcaceae* and *Lachnospiraceae* (Figure 6B).
 113 Our model's choice of Δ seems to capture some of these dynamics. Moving along the Δ axis
 114 shows decreases in *Lactobacillaceae* and increases in *Lachnospiraceae* (Figure 6A). Some
 115 discrepancy is observed, most noticeably with the behavior of *Actinobacteria*.

116 3 Discussion

117 Novel experimental methods focused on the gut microbiome's spatial organization have pro-
 118 vided new datasets for computational analysis. Here, we developed directional GMMs with
 119 dependent mixtures to infer spatial behaviors of phyla within the gut microbiome. We
 120 demonstrated the accuracy of the proposed directional GMMs on simulated data in terms of

121 ability to infer model parameters, and to differentiate one-dimensional from two-dimensional
122 spatial structure. On MaPS-seq data, we demonstrated the presence of spatial structure in
123 distinct regions of the mouse GI tract. Encouragingly, our model recapitulated well known
124 spatial phenomena on the Distal Colon and Cecum datasets.

125 In the distal colon, it has been shown that *Bacteroidetes* is enriched in the lumen, while
126 *Firmicutes* are enriched in the mucus layer and crypts (2, 1). We observe these compositional
127 differences, suggesting that our model is recovering the radial dynamics of the distal colon.
128 The presence of four distinct clusters representing the mucus layer is not surprising because
129 mucosal communities vary significantly over lengths as small as 1cm (7).

130 In the Cecum dataset, correspondence with other *in vivo* experiments suggest that we
131 recover dynamics in both the radial and longitudinal directions (Figures 5A:F). Zaborin et
132 al. (8) suggested that in the mouse cecum, *Bacteroidetes* increases in relative abundance
133 from the cecal tip to base. It should be noted that in their experiment, this trend did not
134 reach statistical significance. Our model seems to identify this compositional difference, in
135 addition to a distinction in the relative abundances of *Actinobacteria* (Figure 5A:C). Zaborin
136 et al. (8) did find a statistically significant difference between the levels of *Firmicutes* in the
137 lumen compared to cecal crypts. We observe a similar difference at the two ends of our
138 model (Figure 5D:F).

139 Within the ileum, we select a model with only a single direction of change. There is evi-
140 dence that our choice of model is biologically accurate: unlike in the cecum and distal colon,
141 the small intestine mucus layer is largely uninhabited due to the presence of antimicrobial
142 peptides (9). However, along the length of the small intestine, oxygen concentrations and pH
143 gradients vary (2). Among the learned clusters, we observe a stark decrease in the relative
144 abundance of *Lactobacillaceae* (6). Along the flow of the digesta, the ileum becomes more
145 alkaline. Because *Lactobacillaceae* are known to contribute to highly acidic environments, it
146 is unsurprising that we observe this compositional differences along the length of the ileum.
147 The pH gradient seems to be embedded in the Δ our model learns: cluster means along
148 the Δ axis show a decrease in *Lactobacillaceae* similar to the observed compositions. The
149 presence of discrepancies on phyla like *Actinobacteria* suggest that there potentially exist
150 other sources of microbial dynamics in the ileum as well. To our knowledge, there are not
151 any experimental studies that describe the microbiome's spatial dynamics within the ileum.
152 This demonstrates the utility of our model: not only can we computationally confirm known
153 aspects of the gut biogeography, but we can also propose new microbial spatial behaviors.

154 A limitation of the present approach is the resolution of the resulting clusters. Our direc-
155 tional GMM was able to capture global spatial patterns in the gut microbiome. Specifically,
156 given that MaPS-seq samples are approximately 20 μm apart, the clusters from the best
157 supported models on the Ileum, Distal Colon, and Cecum datasets correspond to approx-
158 imately 1 cm regions. It would be interesting to investigate if a finer resolution change
159 be achieved. Future work should focus on investigating this possibility of high-resolution
160 mapping of MaPS-seq samples.

161 A valuable next step would be designing MaPS-seq experiments with ground truth labels
162 denoting spatial locations. With coarse-grained labels from various adjacent segments of the
163 GI tract, we could better confirm our model's ability to identify the microbiome's spatial
164 structure, and also the spatial scale recovered by the model. Nonetheless, the present work
165 provides strong evidence that global spatial patterns can be reconstructed from MaPS-seq

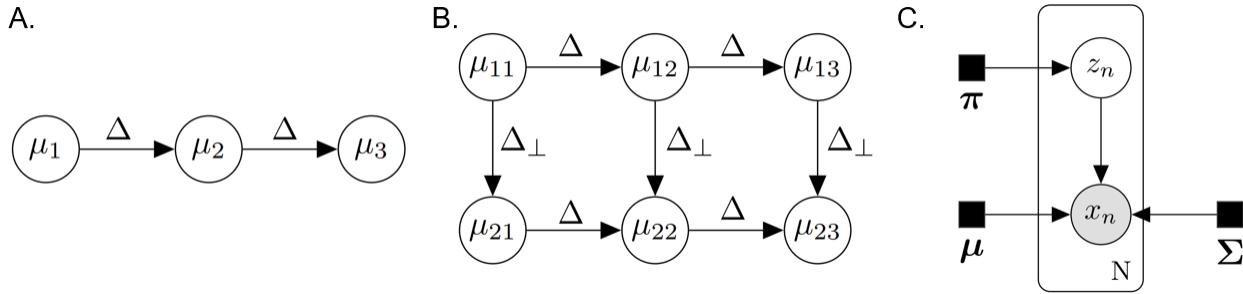


Figure 7: **A directional Gaussian mixture model** (A) Graphical depiction of relationships between latent clusters in a one-dimensional model. (B) Relationships in a two-dimensional model, where changes from left-to-right are described by Δ and perpendicular changes are described by Δ_{\perp} . (C) GMM used to model sampling noise of observed samples x_n . μ_i represent latent clusters with relationships given in (A) and (B).

₁₆₆ data that will only be improved with more detailed collection.

₁₆₇ 4 Methods

₁₆₈ 4.1 Directional Gaussian Mixture Models

₁₆₉ Our approach uses a Bayesian network to describe the relationship of spatially arranged
₁₇₀ clusters in the gut (Figure 7A-B). In detail, given a spatial configuration, the goal is to
₁₇₁ simultaneously learn community states for each latent cluster and assign barcoded MaPS-
₁₇₂ seq droplets to a cluster (Figure 1). The nodes of the Bayesian network, $\{\mu_s | s \in \mathcal{S}\}$, represent
₁₇₃ composition vectors of archetypal communities in respective clusters.

₁₇₄ In the present work, we are interested in changes along one- or two-dimensions. Studies
₁₇₅ suggest the presence of two natural directions in the gut microbiome (1). One dimension
₁₇₆ moves along the flow of the digesta, while the other moves orthogonally along the radial axis
₁₇₇ (inward out).

₁₇₈ This motivates the following definition for our model. We define a one dimensional model
₁₇₉ where $\mathcal{S} = \{i | 1 \leq i \leq K\}$ for K latent clusters (Figure 7A). Let Δ represent directional
₁₈₀ changes between adjacent community compositions.

₁₈₁ We can define

$$f(\mu_1) = \mathcal{N}(\mu_1 | \bar{\mu}_0, Q_0)$$

$$f(\mu_i | \mu_{i-1}) = \mathcal{N}(\mu_i | \mu_{i-1} + \Delta, Q) \quad \text{for } i = 2..K$$

₁₈₂ We also define a two-dimensional model where $\mathcal{S} = \{(i, j) | 1 \leq i \leq K, 1 \leq j \leq 2\}$ for $2K$
₁₈₃ latent clusters (Figure 7B). Let Δ_{\perp} represent the direction along the second dimension, such

184 that $\Delta \cdot \Delta_{\perp} = 0$. We define

$$\begin{aligned} f(\mu_{11}) &= \mathcal{N}(\mu_{11} | \bar{\mu}_0, Q_0) \\ f(\mu_{1i} | \mu_{1(i-1)}) &= \mathcal{N}(\mu_{1i} | \mu_{1(i-1)} + \Delta, Q) \quad \text{for } i = 2..K \\ f(\mu_{21}) &= \mathcal{N}(\mu_{21} | \mu_{11} + \Delta_{\perp}, Q) \\ f(\mu_{2i} | \mu_{1i}, \mu_{2(i-1)}) &= \mathcal{N}\left(\mu_{2i} \left| \frac{1}{2} (\mu_{1i} + \mu_{2(i-1)} + \Delta + \Delta_{\perp}), Q \right. \right) \quad \text{for } i = 2..K \end{aligned}$$

185 MaPS-seq outputs read counts for each of the operational taxonomic units (OTUs) in each
 186 barcoded droplet. However, the total number of reads is independent of overall community
 187 size. Therefore, the sequencing counts only provide information about the proportions of
 188 each OTU in the community. Recent work has advocated using such compositional data
 189 transformations to model microbiome data (10). We transformed read counts to relative
 190 abundances, and then applied the PhILR transformation: an isometric log-transform (ILR)
 191 with a phylogenetically derived basis (11). Each coordinate for the PhILR transformed data
 192 measures the relative proportions of two clades in a phylogeny. Phylogenetic trees were
 193 generated using QIIME (12), and provided as input to the PhILR R package. Given D taxa,
 194 the latent community states are $D - 1$ dimensional vectors $\mu_s \in \mathbb{R}^{D-1}$. Zeros are handled
 195 using multiplicative replacement with $\delta = 1/D^2$ for D -taxa (13).

196 The reads in a particular barcoded droplet provide noisy observations from a latent cluster
 197 (Figure 7C). Thus, we can think of the data generation process as first selecting a latent
 198 community state per barcode, then generating a noisy observation from that community
 199 state. Let \mathcal{B} index the set of barcodes, $x_b \in \mathbb{R}^{D-1}$ for $b \in \mathcal{B}$ be PhILR computed from
 200 the observed sequencing reads for that barcode, and $\pi_{\mathcal{S}} = (\pi_s)_{s \in \mathcal{S}}$ be the probability that a
 201 barcode originated from each cluster $s \in \mathcal{S}$. Let ρ_s be the set of direct ancestors of μ_s . We
 202 have

$$\begin{aligned} p(z_b) &= \text{Categorical}(z_b | \pi_{\mathcal{S}}) \\ p(x_b | z_b, \mu_{z_b}) &= \mathcal{N}(x_b | \mu_{z_b}, \Sigma_{z_b}) = \prod_{s \in \mathcal{S}} [\mathcal{N}(x_b | \mu_s, \Sigma_s)]^{\mathbb{1}(z_b=s)} \end{aligned}$$

203 Altogether, the complete likelihood of the model can be written

$$\begin{aligned} p(\mu_{\mathcal{S}}, z_{\mathcal{B}}, x_{\mathcal{B}}) &= \prod_{s \in \mathcal{S}} f(\mu_s | \mu_{\rho_s}) \prod_{b \in \mathcal{B}} p(x_b | z_b, \mu_{z_b}) p(z_b) \\ &= \prod_{s \in \mathcal{S}} f(\mu_s | \mu_{\rho_s}) \prod_{b \in \mathcal{B}} \prod_{s \in \mathcal{S}} [p(x_b | z_b = s, \mu_s) p(z_b = s)]^{\mathbb{1}(z_b=s)} \end{aligned}$$

204 4.2 Parameter Inference

205 In both models, we seek to optimize $p(\mu_{\mathcal{S}}, z_{\mathcal{B}}, x_{\mathcal{B}} | \theta)$ where $\theta = (\pi_{\mathcal{S}}, \Sigma_{\mathcal{S}}, \Delta_{*}, Q, Q_0, \mu_0)$. This
 206 optimization is performed through an Expectation-Maximization (EM) algorithm. Under
 207 this algorithm, parameters are inferred by alternating between two steps:

208 • **E step:** Given the current estimates of community states $\mu_{\mathcal{S}}^t$, model parameters θ^t ,
 209 compute the posterior expectation of each cluster assignment: $\mathbb{E}[\mathbb{1}(z_b = s) | \mu_{\mathcal{S}}^t, \theta^t]$

210 • **M step:** Maximize the expected complete log-likelihood $\log p(\mu_{\mathcal{S}}, z_{\mathcal{B}}, x_{\mathcal{B}})$:

$$(\mu_{\mathcal{S}}^{t+1}, \theta^{t+1}) = \arg \max_{(\mu_{\mathcal{S}}, \theta)} \sum_{s \in \mathcal{S}} \log f(\mu_s | \mu_{\rho_s}) \\ + \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}} \mathbb{E}[\mathbb{1}(z_b = s) | \mu_{\mathcal{S}}^t, \theta^t] [\log p(x_b | z_b = s, \mu_s) + \log p(z_b = s)]$$

211 Thus we take maximum a posteriori estimates of $\mu_{\mathcal{S}}$ and maximum likelihood estimates
 212 of the remaining parameters. Model parameters are initialized using a basic GMM with
 213 independent clusters trained on the same data. On simulated data, 20 initializations are
 214 used. On real data, 200 are used. Inference terminates following 5 consecutive steps where
 215 the expected complete log-likelihood increases by $< 10^{-4}$ of the previous step.

216 4.3 Model Selection

217 The Akaike Information Criterion (AIC) was used to evaluate models:

$$\text{AIC}(k) = -2 \ln(\hat{L}) + 2p_k$$

218 where \hat{L} denotes the likelihood of the data under the fitted model and p_k is the number
 219 of parameters for the model k . We used the complete log likelihood as a surrogate for the
 220 log likelihood of the data since it is a lower bound. When comparing models with different
 221 numbers of latent clusters, we choose the model with the *minimum* AIC score (6).

222 In the case of models with the same number of latent clusters (i.e. 4 clusters arranged in a
 223 line vs. clusters arranged in a 2 by 2 grid), we can directly compare the complete likelihoods
 224 $p(\mu_{\mathcal{S}}, z_{\mathcal{B}}, x_{\mathcal{B}})$ of either model.

225 On both simulated and real data, we test up to 8 clusters for the one and two dimensional
 226 models, or until the average community state size is 50 samples, whichever comes first. For
 227 the one dimensional model, clusters were arranged in a line from a 2×1 model up to an
 228 8×1 model. For the two dimensional model, clusters were arranged in a grid from a 2×2
 229 model up to a 4×2 model.

230 4.4 Simulation Analysis

231 This is an unsupervised learning problem, so we first evaluated our model on simulated
 232 data. To this end, we create simulated datasets under the two proposed models. First we
 233 sample two clusters means from a Pareto distribution with $\alpha = 1$, normalize to the relative
 234 abundance space with $D = 47$ taxa, sort taxa in decreasing order, and then transform to the
 235 ILR space. The difference in the ILR space between these two means is defined to be Δ . The
 236 Δ parameter is then scaled to our desired magnitude. Our method for sampling Δ allows
 237 for larger dynamics to be observed on more abundant taxa. For two-dimensional models, we
 238 sample Δ_{\perp} from a standard multivariate normal distribution and then orthogonalize relative
 239 to Δ . The remaining clusters means are arranged around one of the two original cluster
 240 means as per the two models (arranged in the ILR space in a line or in a grid).

241 Then, we randomly sample cluster covariance matrices Σ from an Inverse-Wishart dis-
242 tribution with $\nu = D + 1$ and $\Psi = \frac{I_{D-1}}{D-1}$. Finally, a total of 360 artificial MaPS-seq samples
243 are drawn evenly and independently from each cluster.

244 We analyzed two aspects of model performance on simulated data: 1) selection of the
245 correct number of latent clusters, and 2) parameter estimation accuracy. In order to evaluate
246 our model selection framework, we train both the one and two directional models with varying
247 amounts of latent clusters on simulated data. We used the aforementioned model selection
248 criteria to determine the optimal model.

249 Next, the accuracy of our parameter inference is determined by calculating the average
250 RMSE of the learned cluster means. Although our proposed model assigns labels to clusters
251 to reflect their spatial arrangements, other unsupervised clustering algorithms assign arbi-
252 trary labels. Therefore, to compare RMSE of model parameters, we look at our proposed
253 model's RMSE and the best RMSE of all label permutations of a naive GMM.

254 4.5 MaPS-seq data analysis

255 We used the publicly available data from Sheth et al. The Cecum, Ileum, and Distal Colon
256 datasets were each extracted from 3cm segments of their respective regions. MaPS-seq clus-
257 ters are the same size in all datasets (20 μm). Each sample is a vector of the relative
258 abundances of all OTUs. We focused on the most abundant taxa that constitute 95% of all
259 relative abundance across the three datasets. This corresponded to 47 taxa. Relative abun-
260 dances were then renormalized. Using the provided fasta files, we generated phylogenetic
261 trees in QIIME (12). Data is then transformed using the PhILR R package.

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