

# 1 Improved estimation of phenotypic correlations using 2 summary association statistics

3 Ting Li<sup>1,†</sup>, Zheng Ning<sup>2,†</sup>, Xia Shen<sup>1,2,3\*</sup>

4 January 13, 2021

5 <sup>1</sup>Biostatistics Group, School of Life Sciences, Sun Yat-sen University, Guangzhou, China

6 <sup>2</sup>Department of Medical Epidemiology and Biostatistics, Karolinska Institutet, Stockholm, Sweden

7 <sup>3</sup>Centre for Global Health Research, Usher Institute, University of Edinburgh, Edinburgh, United Kingdom

8

9 \*Correspondence should be addressed to: xia.shen@ed.ac.uk

10 †These authors contributed equally.

11 **Estimating the phenotypic correlations between complex traits and diseases based on their genome-  
12 wide association summary statistics has been a useful technique in genetic epidemiology and statistical  
13 genetics inference. Two state-of-the-art strategies, Z-score correlation across null-effect SNPs and LD  
14 score regression intercept, were widely applied to estimate phenotypic correlations. Here, we propose  
15 an improved Z-score correlation strategy based on SNPs with low minor allele frequencies (MAFs),  
16 and show how this simple strategy can correct the bias generated by the current methods. Comparing  
17 to LDSC, the low-MAF estimator improves phenotypic correlation estimation thus is beneficial for  
18 methods and applications using phenotypic correlations inferred from summary association statistics.**

## 19 **Introduction**

20 Phenotypic correlation is an essential parameter that helps understand observational correlations between complex  
21 traits and the etiological perspectives underlying complex diseases. Conventionally, estimation of the phenotypic  
22 correlation between a pair of phenotypes, by definition, is straightforward in a sample where both phenotypes are  
23 measured. Depending on the distribution of each phenotype, the estimated phenotypic correlation serves as a sufficient  
24 statistic for many linear statistical models, such as ordinary linear and logistic regressions, allowing us to assess  
25 parameters such as odds ratios of risk factors on disease outcomes.

26 Since a large number of genome-wide association studies (GWAS) were conducted, many GWASed phenotypes had  
27 measurements in an overlapping set of individuals, where many were from more than one participating cohort in GWAS  
28 meta-analysis. In practice, inference of the phenotypic correlations across these phenotypes would be complicated if  
29 estimating using the conventional way, which requires individual-level phenotypic data and subsequent meta-analysis.  
30 Fortunately, the phenotypic correlations can be estimated based on established GWAS summary statistics, especially  
31 when the proportion of sample overlap between two GWASed phenotypes is large. Two state-of-the-art strategies were  
32 proposed:

33 1. *“Z-cut” estimator.* The phenotypic correlation can be estimated by the correlation between the two sets of  
34 GWAS estimated effects or Z-scores, assuming the genetic effect per SNP (single nucleotide polymorphism) is  
35 tiny or even null<sup>1, 2, 3, 4</sup>.

36 2. *LDSC intercept.* The phenotypic correlation can be estimated by the intercept of a bivariate linkage disequilibrium  
37 score regression (LDSC)<sup>5, 6, 7</sup>.

38 Both estimators have reasonable performance in practice, however, bias exists for both strategies. Stephens (2013)<sup>1</sup>  
39 reasoned that the correlation between Z-scores for the two phenotypes under the null is the same as the phenotypic  
40 correlation, thus “a set of putative null SNPs” were selected, by taking SNPs with  $|z| < 2$ . The same idea was also

41 adopted by later studies<sup>2, 4</sup>. The tool metaCCA<sup>3</sup> neglected the null effect requirement, as the genetic effect per variant  
 42 is tiny, and computed the correlation between Z-scores across as many SNPs as possible. However, the Z-cut estimator  
 43 can generate bias due to its constrain on the summary statistics of the SNPs<sup>7</sup>. LDSC intercept performs better thus  
 44 was adopted in statistical methods that requires pre-calculated phenotypic correlations<sup>6, 7</sup>, but the intercept collects  
 45 noise generated by e.g., population substructure, which may also lead to biased estimates of phenotypic correlations<sup>8</sup>.

46 Here, we revisit the correlation between GWAS summary statistics of two phenotypes and propose an alternative  
 47 approach to select variants for the Z-score correlation estimation strategy. We show that selecting SNPs with low  
 48 minor allele frequencies (MAFs) can lead to simple and consistent estimation of phenotypic correlations based on  
 49 multi-SNP Z-score correlations. Via simulations, we show that the “low-MAF” estimator can overcome bias generated  
 50 by the Z-cut estimator and the LDSC intercept. With higher estimation efficiency, when applied to UK Biobank  
 51 GWAS results, the low-MAF estimator could discover 30% more significant phenotypic correlations than using the  
 52 LDSC intercept.

## 53 Methods

54 We start by deriving a general mathematical form of the correlation between the summary statistics of two phenotypes  
 55  $\mathbf{y}_1$  and  $\mathbf{y}_2$ , centred at a zero mean. For a single genetic variant in an association analysis, the model is  $\mathbf{y}_i = \mathbf{g}_i \beta_i + \mathbf{e}_i$   
 56 ( $i = 1, 2$ ), where  $\mathbf{g}_i$  is the vector of genotypic values with 0-1-2 coding, and  $\mathbf{e}_i$  are the residuals. Assuming Hardy-  
 57 Weinberg equilibrium (HWE), for SNP  $j$ , we have  $g_{ij} \sim B(2, f_j)$ , where  $f_j$  is the MAF of SNP  $j$ , and  $B(n, p)$  stands  
 58 for the binomial distribution with size  $n$  and success probability  $p$ .  $\mathbf{g}_1$  and  $\mathbf{g}_2$  may differ due to different levels of  
 59 sample overlap between the two phenotypes. At the single SNP  $j$  (omitted the subscripts  $j$  for simplicity),

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \sim \text{dist} \left( \mathbf{0}, \begin{bmatrix} \sigma_{\beta_1}^2 & r_G \sigma_{\beta_1} \sigma_{\beta_2} \\ r_G \sigma_{\beta_1} \sigma_{\beta_2} & \sigma_{\beta_2}^2 \end{bmatrix} \right), \quad (1)$$

60 and

$$\mathbf{e}_i \sim \text{dist} \left( \mathbf{0}, \begin{bmatrix} \sigma_1^2 \mathbf{I}_{N_1 \times N_1} & r_E \sigma_1 \sigma_2 \mathbf{1}_{N_1 \times N_2} \\ r_E \sigma_1 \sigma_2 \mathbf{1}_{N_2 \times N_1} & \sigma_2^2 \mathbf{I}_{N_2 \times N_2} \end{bmatrix} \right), \quad (2)$$

61 where  $r_G$  is the underlying genetic correlation at SNP  $j$ , and  $r_E$  is the residual correlation. In an association study,  
 62  $r_G$  is un-identifiable at a single SNP. The estimated genetic effects are  $\hat{\beta}_i = \mathbf{g}_i' \mathbf{y}_i / \mathbf{g}_i' \mathbf{g}_i$ , then

$$\begin{aligned} \text{var}(\hat{\beta}_i) &= \frac{\text{var}(\mathbf{g}_i' \mathbf{y}_i)}{(\mathbf{g}_i' \mathbf{g}_i)^2} \\ &= \frac{\text{var}(\mathbf{g}_i' \mathbf{g}_i \beta_i + \mathbf{g}_i' \mathbf{e}_i)}{(\mathbf{g}_i' \mathbf{g}_i)^2} \\ &= \sigma_{\beta_i}^2 + \sigma_i^2 (\mathbf{g}_i' \mathbf{g}_i)^{-1}. \end{aligned} \quad (3)$$

63 So that

$$\begin{aligned}
 \text{cor}(\hat{\beta}_1, \hat{\beta}_2) = \text{cor}(z_1, z_2) &= \frac{\text{cov}(\mathbf{g}'_1 \mathbf{y}_1, \mathbf{g}'_2 \mathbf{y}_2)}{\sqrt{(\mathbf{g}'_1 \mathbf{g}_1)^2 \sigma_{\beta_1}^2 + \mathbf{g}'_1 \mathbf{g}_1 \sigma_1^2} \sqrt{(\mathbf{g}'_2 \mathbf{g}_2)^2 \sigma_{\beta_2}^2 + \mathbf{g}'_2 \mathbf{g}_2 \sigma_2^2}} \\
 &= \frac{(\mathbf{g}'_1 \mathbf{g}_1)(\mathbf{g}'_2 \mathbf{g}_2) \text{cov}(\beta_1, \beta_2) + \mathbf{g}'_1 \text{cov}(\mathbf{e}_1, \mathbf{e}_2) \mathbf{g}_2}{\sqrt{(\mathbf{g}'_1 \mathbf{g}_1)^2 \sigma_{\beta_1}^2 + \mathbf{g}'_1 \mathbf{g}_1 \sigma_1^2} \sqrt{(\mathbf{g}'_2 \mathbf{g}_2)^2 \sigma_{\beta_2}^2 + \mathbf{g}'_2 \mathbf{g}_2 \sigma_2^2}} \\
 &= \frac{(\mathbf{g}'_1 \mathbf{g}_1)(\mathbf{g}'_2 \mathbf{g}_2) r_G \sigma_{\beta_1} \sigma_{\beta_2} + \mathbf{g}'_1 \mathbf{g}_2 r_E \sigma_1 \sigma_2}{\sqrt{(\mathbf{g}'_1 \mathbf{g}_1)^2 \sigma_{\beta_1}^2 + \mathbf{g}'_1 \mathbf{g}_1 \sigma_1^2} \sqrt{(\mathbf{g}'_2 \mathbf{g}_2)^2 \sigma_{\beta_2}^2 + \mathbf{g}'_2 \mathbf{g}_2 \sigma_2^2}} \\
 &= \frac{2f(1-f)\sqrt{N_1 N_2} r_G \sigma_{\beta_1} \sigma_{\beta_2} + N_0 / \sqrt{N_1 N_2} r_E \sigma_1 \sigma_2}{\sqrt{2f(1-f)N_1 \sigma_{\beta_1}^2 + \sigma_1^2} \sqrt{2f(1-f)N_2 \sigma_{\beta_2}^2 + \sigma_2^2}}
 \end{aligned} \tag{4}$$

64 When  $\sigma_{\beta_i} = 0$  ( $i = 1, 2$ ), i.e., for any variant with null genetic effect, the above equation simplifies as

$$\text{cor}(\hat{\beta}_1, \hat{\beta}_2) = \text{cor}(z_1, z_2) = \frac{N_0}{\sqrt{N_1 N_2}} r_E = \frac{N_0}{\sqrt{N_1 N_2}} r(\mathbf{y}_1, \mathbf{y}_2) \tag{5}$$

65 where  $r(\mathbf{y}_1, \mathbf{y}_2)$  is the phenotypic correlation based on completely overlapped individual-level data. Particularly, for  
66 perfectly overlap samples, i.e.,  $N_0 = N_1 = N_2$ , we have  $\text{cor}(z_1, z_2) = r(\mathbf{y}_1, \mathbf{y}_2)$ , which is the same as the phenotypic  
67 correlation estimator derived by Zhu et al.<sup>2</sup>.

68 The result suggests that the phenotypic correlation between the two phenotypes  $\mathbf{y}_1$  and  $\mathbf{y}_2$ , subject to a shrinkage  
69 factor corresponding to sample overlap, can be estimated by the sample correlation of the summary statistics across  
70 any sufficient number of null variants. This leads to a commonly adopted strategy of estimating the phenotypic  
71 correlation from summary association statistics by taking a subset with e.g.,  $|z_i| < 2$  ( $i = 1, 2$ ). However, we will show  
72 that such thresholding may introduce bias into the correlation estimate.

73 According to eq. (4), null genetic effect for the variant is a sufficient but not necessary condition for  $\text{cor}(z_1, z_2)$  to  
74 reduce to eq. (5). When  $f = 0$ , eq. (4) also becomes (5). In practice, the phenotypic correlation can be estimated by  
75 the correlation of the summary statistics across a sufficient number of variants with very low minor allele frequencies  
76 (MAFs), *regardless* of whether the genetic effects are null. The thresholding on the MAF does not directly introduce  
77 a threshold on  $\beta_i$  or  $z_i$  so that not prone to bias in the phenotypic correlation estimation.

78 **Simulation settings.** We conducted two sets of simulations to compare the low-MAF estimator with the Z-cut  
79 estimator and LDSC intercept, respectively. For the first simulation, we simulated the genotypes of 5,000 independent  
80 SNPs in 500 individuals, and the MAFs ranged from  $5 \times 10^{-5}$  to 0.5. The genotypes of SNP  $j$  follow HWE. Two  
81 scenarios of phenotypic correlations were evaluated, where in one the phenotypic correlation was set to 0.5 without  
82 genetic correlation; and in the other a genetic correlation of 0.5 and a residual correlation of 0.25 were simulated,  
83 where the genetic effects across the 5,000 SNPs were extracted from a normal distribution with zero mean. In the  
84 simulation, three cutoffs of  $|z| < 2$ ,  $|z| < 1$ , and  $|z| < 0.5$  were evaluated for the Z-cut estimator. Five thresholds of

85 MAFs: 0.5, 0.05,  $5 \times 10^{-3}$ ,  $5 \times 10^{-4}$ , and  $5 \times 10^{-5}$  were evaluated for the low-MAF estimator. The true phenotypic  
86 correlations were computed as the Pearson's correlations of the two vectors of simulated phenotypic values.

87 For the second simulation, in order to compare with LDSC, we used the real UK Biobank (UKB) genotypes  
88 for 336,000 genomic British individuals across the 1,029,876 quality-controlled HapMap3 SNPs selected by the high-  
89 definition likelihood (HDL) software<sup>9</sup>. We draw the genetic effects across 10% of the SNPs from a normal distribution  
90 with zero mean, so that the phenotypic, genetic, and residuals correlations all had a true value of 0.5. 70,042 SNPs with  
91 MAF  $< 5 \times 10^{-4}$  were selected for the low-MAF estimator. Two reference panels were evaluated for LDSC, including  
92 the ldsc software inbuilt 1000 Genomes reference and the UKB reference based on the HDL software reference data.

## 93 Results

94 **The low-MAF estimator corrects the bias of the Z-cut estimator.** In the first simulation setting, when no  
95 genetic effect was present, namely, every SNP had a null effect, the Z-score correlation estimator based on all the  
96 SNPs satisfied eq. (5), resulted in unbiased estimates of the phenotypic correlations. However, constraining the Z-cut  
97 estimator on SNPs filtered by Z-score cutoffs generated downward-biased estimates. On the other hand, constraining  
98 the low-MAF estimator did not generate bias, regardless of the MAF cutoff (**Fig. 1a**). When genetic correlation  
99 was present, the Z-score correlation estimator based on all the SNPs produced inflated estimates, as the common  
100 SNPs with large MAFs substantially contributed to the genetic correlation. Same as in the previous scenario, the  
101 Z-cut estimator generated downward-biased estimates. With sufficiently low MAF cutoffs, the Z-score correlation  
102 maintained as a consistent estimator of the phenotypic correlation (**Fig. 1b**). Also, the estimation efficiency of the  
103 low-MAF estimator attained that of the estimator based on observed phenotypic values (**Table 1**).

104 **The low-MAF estimator corrects the bias of LDSC intercept.** For the second simulation, we observed  
105 downward bias in the LDSC intercept when the default 1000 Genomes reference was applied (**Fig. 2**). Such a bias  
106 was overcome by the UKB reference, nevertheless, the estimates were slightly inflated possibly due to the population  
107 substructure in the UKB genomic British individuals<sup>8</sup>. These biases were all absent when applying the low-MAF  
108 estimator for the phenotypic correlation. Furthermore, the low-MAF estimator had a substantially higher estimation  
109 efficiency than the LDSC intercept, as if the sample size was 10 times larger (**Table 2**).

110 **Example.** We selected the same 30 GWASed phenotypes used by Ning et al.'s in genetic correlation estimation<sup>9</sup>,  
111 as a real data example to compare the low-MAF estimator to LDSC intercept in the estimation of the phenotypic  
112 correlations (**Fig. 3**). The low-MAF estimates were based on 70,042 SNPs with MAF  $< 5 \times 10^{-4}$ , and the LD scores  
113 were calculated based on the 1000 Genomes reference panel (default). At a 5% Bonferroni-corrected p-value threshold  
114 for 435 pairs of traits, the low-MAF method discovered 223 significant phenotypic correlations, and LDSC intercept  
115 discovered 171. Among these, 61 phenotypic correlations were only significant in the low-MAF method, versus 9 only

116 significant using the LDSC intercept.

## 117 Discussion

118 We have proposed the low-MAF estimator of phenotypic correlations based on GWAS summary statistics, as an  
119 improvement of the Z-score correlation strategy based on all SNPs or SNPs that pass a particular Z-score cutoff. The  
120 estimator overcomes the bias generated when thresholding on summary association statistics and even that generated  
121 in the bivariate LDSC intercept. We suggest the use of the low-MAF phenotypic correlation estimator in future  
122 practice. The more consistent and efficient estimation can improve our understanding of connections across human  
123 complex traits and diseases.

124 Although the low-MAF method also introduces a filter on the tested SNPs, it is a threshold-free technique for the  
125 genetic effect parameter. Thus, the low-MAF estimator does not constrain the estimated genetic effects of selected  
126 SNPs, equivalent to sampling a set of null effect SNPs from the genome. This explains why “putative null effect”  
127 SNPs with e.g.,  $|z| < 2$  generate bias whereas the low-MAF estimator does not. Even if all the SNPs are null, some  
128 of them will generate z-score with  $|z| > 2$  due to randomness. Removing them would lead to bias.

129 As the low-MAF estimator is equivalent to sampling a set of null effect SNPs from the genome, the resulted  
130 phenotypic correlation estimates are close to those estimated using individual-level phenotypic data. In the real  
131 UKB genotype data simulation, we showed that the LDSC intercept could not produce consistent estimates of the  
132 phenotypic correlation due to population substructure. Such a complication in LDSC was overcome by the low-  
133 MAF estimator, because although the GWAS summary statistics were used, the estimator approximates observed  
134 phenotypic correlation and is irrelevant to genetic data structure. Namely, the genotypic data are treated as nuisance  
135 in the low-MAF estimator.

136 For binary phenotypes, an advantage of summary-statistics-based estimators, such as the low-MAF estimator, is  
137 that it estimates the underlying phenotypic correlations on the liability scale. The liabilities follow an unobservable  
138 logistic distribution therefore the estimates are not the same as the observed phenotypic correlations directly computed  
139 using the 0-1 outcome data. The phenotypic correlation estimates on the liability scale is mathematically easier to  
140 interpret and can be transformed into odds ratios from logistic regressions.

## 141 Data availability

142 The individual-level genotype and phenotype data are available by application from the UKBB (<http://www.ukbiobank.ac.uk/>). The UKBB GWAS summary statistics by the Neale laboratory can be obtained from <http://www.nealelab.is/uk-biobank/>.

145 **Code availability**

146 HDL: <https://github.com/zhenin/HDL>; ldsc: <https://github.com/bulik/ldsc>.

147 **Acknowledgements**

148 X.S. was in receipt of a Swedish Research Council (VR) Starting Grant (No. 2017-02543). We thank Edinburgh  
149 Compute and Data Facility at the University of Edinburgh and National Supercomputer Center at Sun Yat-sen  
150 University for providing high-performance computing resources.

151 **Author contributions**

152 X.S. initiated and coordinated the study. T.L. and Z.N. contributed to data analysis. X.S. drafted the manuscript.  
153 All authors contributed to manuscript writing and gave final approval to publish.

154 **Ethics declarations**

155 The authors declare no competing interests.

156 **References**

157 [1] Stephens, M. A unified framework for association analysis with multiple related phenotypes. *PLOS ONE* **8**(7), e65245  
158 (2013).

159 [2] Zhu, X., Feng, T., Tayo, B. O., Liang, J., Young, J. H., Franceschini, N., Smith, J. A., Yanek, L. R., Sun, Y. V., Edwards,  
160 T. L., Chen, W., Nalls, M., Fox, E., Sale, M., Bottinger, E., Rotimi, C., Consortium, C. B. P., Liu, Y., McKnight, B., Liu,  
161 K., Arnett, D. K., Chakravati, A., Cooper, R. S., and Redline, S. Meta-analysis of correlated traits via summary statistics  
162 from GWASs with an application in hypertension. *American journal of human genetics* **96**(1), 21–36 (2015).

163 [3] Cichonska, A., Rousu, J., Marttinen, P., Kangas, A. J., Soininen, P., Lehtimäki, T., Raitakari, O. T., Järvelin, M.-R.,  
164 Salomaa, V., Ala-Korpela, M., Ripatti, S., and Pirinen, M. metaCCA: summary statistics-based multivariate meta-analysis  
165 of genome-wide association studies using canonical correlation analysis. *Bioinformatics* **32**(13), 1981–1989, jul (2016).

166 [4] Shen, X., Klarić, L., Sharapov, S., Mangino, M., Ning, Z., Wu, D., Trbojević-Akmačić, I., Pučić-Baković, M., Rudan,  
167 I., Polašek, O., Hayward, C., Spector, T. D., Wilson, J. F., Lauc, G., and Aulchenko, Y. S. Multivariate discovery and  
168 replication of five novel loci associated with Immunoglobulin G N-glycosylation. *Nature Communications* **8**(1), 447, dec  
169 (2017).

170 [5] Bulik-Sullivan, B., Finucane, H. K., Anttila, V., Gusev, A., Day, F. R., Loh, P.-R., Consortium, R., Consortium, P. G., 3,  
171 G. C. f. A. N. o. t. W. T. C. C. C., Duncan, L., Perry, J. R. B., Patterson, N., Robinson, E. B., Daly, M. J., Price, A. L., and  
172 Neale, B. M. An atlas of genetic correlations across human diseases and traits. *Nature genetics* **47**(11), 1236–1241 (2015).

173 [6] Turley, P., Walters, R. K., Maghzian, O., Okbay, A., Lee, J. J., Fontana, M. A., Nguyen-Viet, T. A., Wedow, R., Zacher, M.,  
174 Furlotte, N. A., Magnusson, P., Oskarsson, S., Johannesson, M., Visscher, P. M., Laibson, D., Cesarini, D., Neale, B. M.,  
175 and Benjamin, D. J. Multi-trait analysis of genome-wide association summary statistics using MTAG. *Nature Genetics*  
176 **50**(2), 229–237, feb (2018).

177 [7] Zheng, J., Richardson, T. G., Millard, L. A. C., Hemani, G., Elsworth, B. L., Raistrick, C. A., Vilhjalmsson, B., Neale, B. M.,  
178 Haycock, P. C., Smith, G. D., and Gaunt, T. R. PhenoSpD: an integrated toolkit for phenotypic correlation estimation and  
179 multiple testing correction using GWAS summary statistics. *GigaScience* **7**(8), 1–10, aug (2018).

180 [8] Yengo, L., Yang, J., and Visscher, P. M. Expectation of the intercept from bivariate LD score regression in the presence of  
181 population stratification. *bioRxiv* **0**(0), 0–0 (2018).

182 [9] Ning, Z., Pawitan, Y., and Shen, X. High-definition likelihood inference of genetic correlations across human complex traits.  
183 *Nature Genetics* **52**(8), 859–864, aug (2020).

184 **Table 1: Comparison of the phenotypic correlation estimates by the low-MAF and Z-cut estimators.**

185 The results are means and standard deviations (in brackets) summarised from 100 replicates, where in each replicate,  
 186 5,000 SNPs were simulated for 500 individuals, and the minor allele frequencies (MAFs) ranged from 5e-5 to 0.5.  
 187 The true (phenotypic) correlations ( $r_P$ ) were computed as the Pearson's correlations of the two vectors of simulated  
 188 phenotypic values. Scenario 1: The two phenotypes had no genetic correlation and a (residual) phenotypic correlation  
 of 0.5; Scenario 2: The two phenotypes had a genetic correlation of 0.5 and a residual correlation of 0.25.

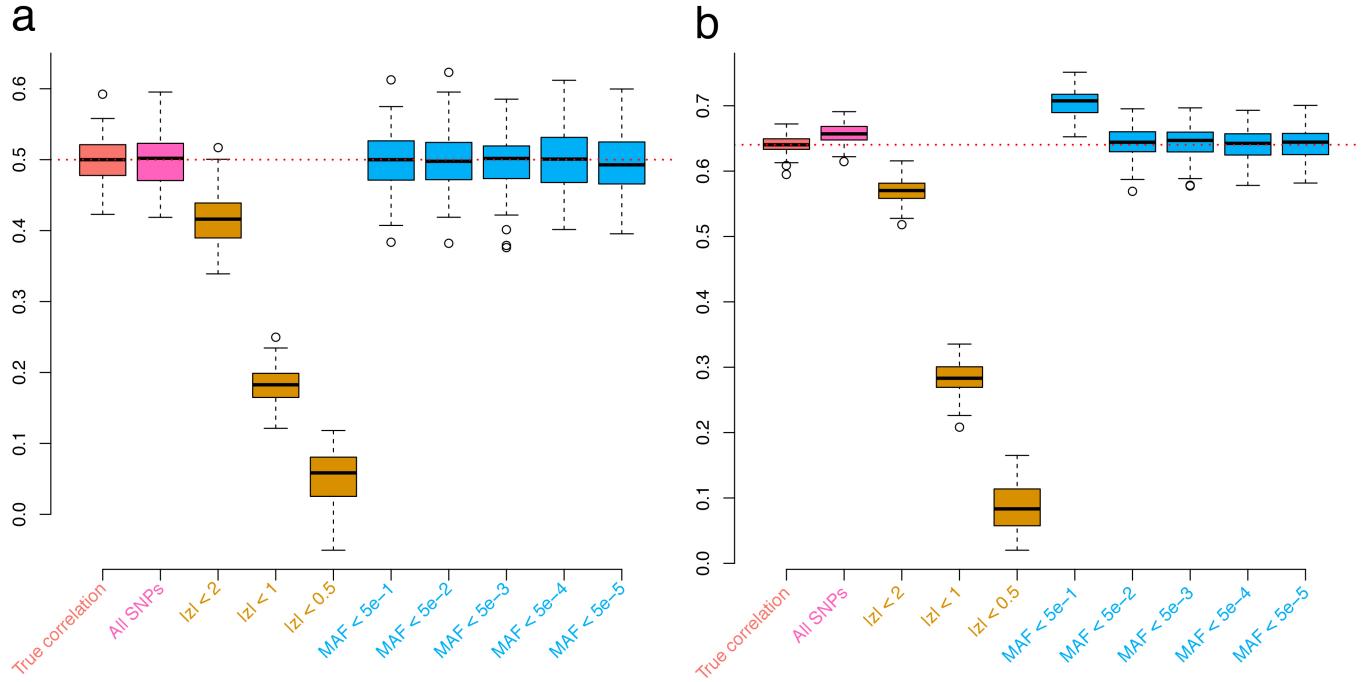
Scenario	True $r_P$	All-SNP	$ z  < 2$	$ z  < 1$	$ z  < 0.5$	MAF < 0.5	< 0.05	< 5e-3	< 5e-4	< 5e-5
1	0.50	0.50	0.41	0.18	0.05	0.50	0.50	0.49	0.50	0.50
	(0.03)	(0.04)	(0.03)	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)
2	0.65	0.67	0.58	0.29	0.10	0.71	0.66	0.65	0.65	0.65
	(0.01)	(0.01)	(0.01)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)

189

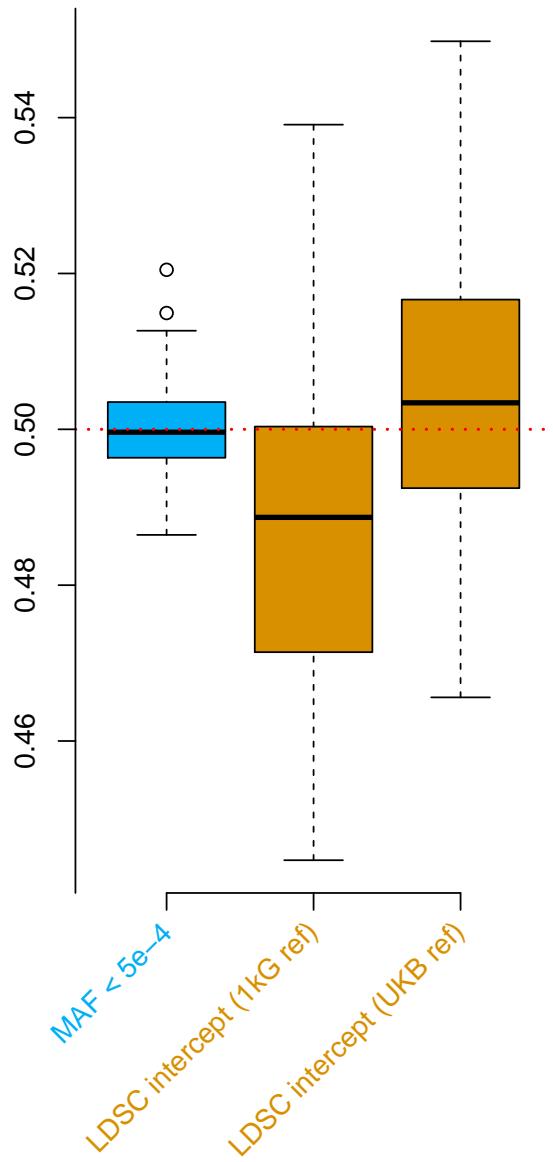
190 **Table 2: Comparison of the phenotypic correlation estimates by the low-MAF estimator and LDSC  
 191 intercept.** The results were summarised from 100 replicates, where in each replicate, two phenotypes were simulated  
 192 for 336,000 genomic British individuals. The true phenotypic, genetic, and residual correlations were all set to 0.5.  
 193 The low-MAF estimates were based on 70,042 SNPs with MAF < 5e-4. 1kG ref: LD scores calculated based on the  
 1000 Genomes reference panel; UKB ref: LD scores calculated based on the UK Biobank reference panel.

	Low-MAF	LDSC (1kG)	LDSC (UKB)
Minimum	0.4865	0.4447	0.4656
25% Quantile	0.4964	0.4719	0.4927
Median	0.4996	0.4887	0.5034
Mean	0.5001	0.4871	0.5040
75% Quantile	0.5035	0.4997	0.5166
Maximum	0.5205	0.5391	0.5498
Variance	3.603e-05	3.626e-04	3.022e-04

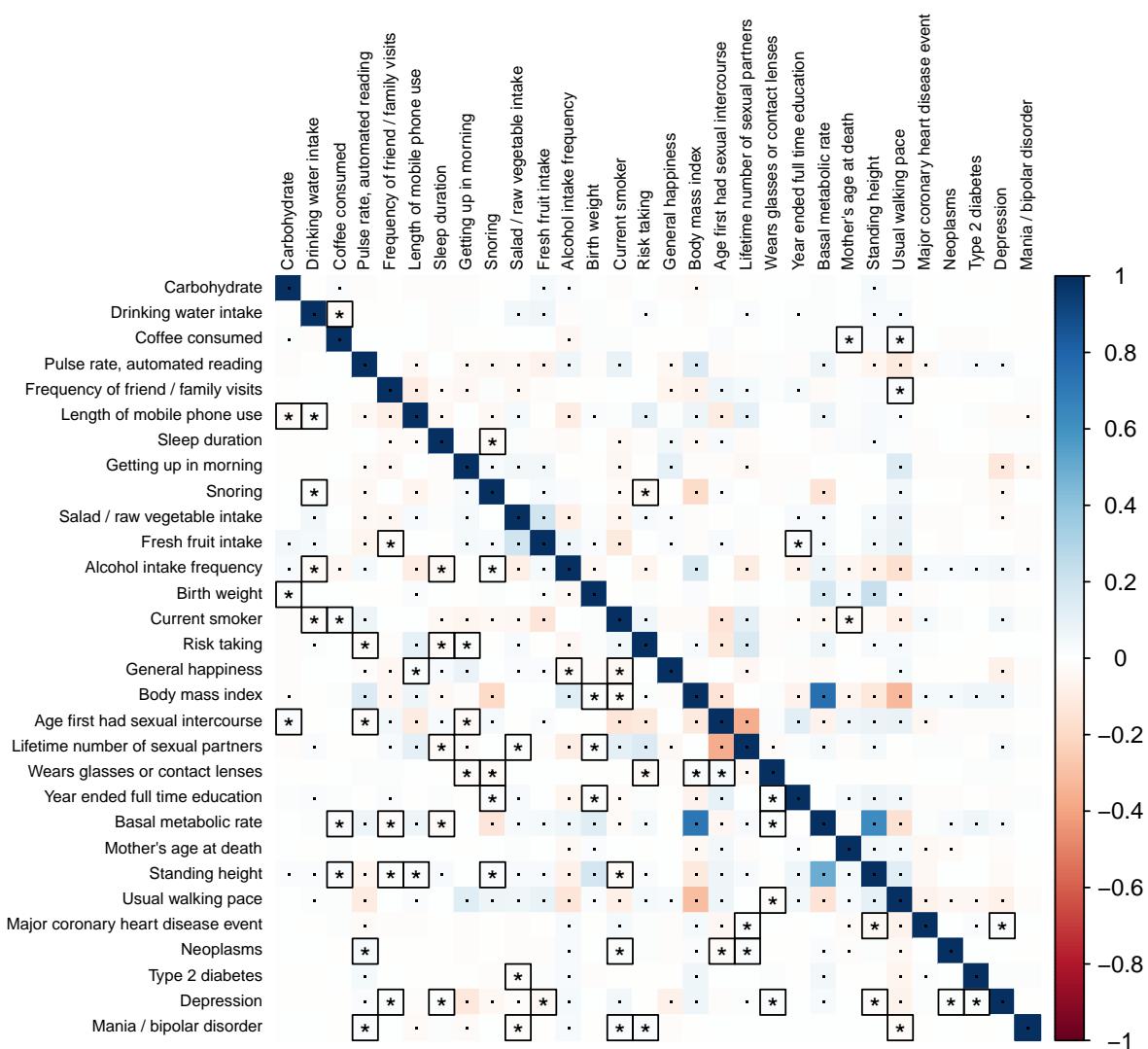
194



195 **Figure 1: Simulations comparing the Z-cut and low-MAF estimators for phenotypic correlation.** The  
196 boxplots show the results from 100 replicates, where in each replicate, 5,000 SNPs were simulated for 500 individuals,  
197 and the minor allele frequencies (MAFs) ranged from 5e-5 to 0.5. The true (phenotypic) correlations were computed  
198 as the Pearson's correlations of the two vectors of simulated phenotypic values. **a.** The two phenotypes had no genetic  
199 correlation and a (residual) phenotypic correlation of 0.5; **b.** The two phenotypes had a genetic correlation of 0.5 and  
200 a residual correlation of 0.25.



201 **Figure 2: Simulations comparing the low-MAF estimator and LD score regression (LDSC) intercept**  
202 **using the UK Biobank genotype data.** The boxplots show the results from 100 replicates, where in each replicate,  
203 two phenotypes were simulated for 336,000 genomic British individuals. The true phenotypic, genetic, and residual  
204 correlations were all set to 0.5. The low-MAF estimates were based on 70,042 SNPs with  $MAF < 5 \times 10^{-4}$ . 1kG ref:  
205 LD scores calculated based on the 1000 Genomes reference panel; UKB ref: LD scores calculated based on the UK  
206 Biobank reference panel.



207 **Figure 3: Phenotypic correlations across 30 UK Biobank traits using the low-MAF estimator (lower**  
 208 **triangle) and LD score regression (LDSC) intercept (upper triangle).** The default 1000 Genomes reference  
 209 panel was used in LDSC. Bonferroni-corrected significant correlations with  $P < 0.05/435$  are marked with asterisks  
 210 or dots, where those correlations that are only significant using one of the two methods are marked with asterisks and  
 211 squares.