

A minimal model of T cell avidity may identify subtherapeutic vaccine schedules

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Abstract

T cells protect the body from cancer by recognising tumour-associated antigens. Recognising these antigens depends on multiple factors, one of which is T cell avidity, i.e., the total interaction strength between a T cell and a cancer cell. While both high- and low-avidity T cells can kill cancer cells, durable anti-cancer immune responses require the selection of high-avidity T cells. Previous experimentation with anti-cancer vaccines, however, has shown that most vaccines elicit low-avidity T cells. Optimising vaccine schedules may remedy this by preferentially selecting high-avidity T cells. Here, we use mathematical modelling to develop a simple, phenomenological model of avidity selection that may identify vaccine schedules that disproportionately favour low-avidity T cells. We calibrate our model to our prior, more complex model, and then validate it against several experimental data sets. We find that the sensitivity of the model's parameters change with vaccine dosage, which allows us to use a patient's data and clinical history to screen for suitable vaccine strategies.

1 Introduction

2 T cells maintain anti-tumour immunity by recognising and killing cancer cells. T cells recognise these cancerous cells
3 through a surface protein—the T cell receptor (TCR)—binding to molecules known as peptide major histocompatibility
4 complexes (pMHCs), which reside on the surface of cancer cells (Murphy, 2011). The overall strength of these TCR-
5 pMHC interactions is termed avidity (Abbas et al., 2014).

6 Several studies have shown that the selection of high-avidity T cells may be a requirement for durable tumour eradication
7 in certain cancers such as melanoma (Molldrem et al., 2003; Chung et al., 2014). Low-avidity T cells, by
8 contrast, are weakly-tumour killing (Stuge et al., 2004) and may even temper anti-tumour activity by selectively in-
9 hibiting high-avidity T cells (Chung et al., 2014). Indeed, experimental evidence suggests that certain cancer vaccines
10 may promote the expansion of low-avidity T cells (Stuge et al., 2004; Rezvani et al., 2011), which may explain why
11 these vaccines cannot maintain durable anti-tumour immunity in clinical trials (Schwartzentruber et al., 2011; Sosman
12 et al., 2008).

13 To remedy this, multiple techniques have been proposed. These techniques range from searching through peptide li-
14 braries to identify peptides that will stimulate high-avidity T cells (McMahan et al., 2006), to harnessing the plasticity
15 of naive T cells to promote their differentiation into high-avidity T cells (Kroger and Alexander-Miller, 2007). More
16 recently, evolutionary principles have been used to select for high-avidity T cells (Bassan et al., 2019). Complement-
17 ing these experimental studies are mathematical models that aim to improve the efficacy of cancer vaccines, namely
18 treatment schedules (i.e., vaccine dose and timing), from different perspectives. For example, in Sigal et al. (2019), the
19 authors optimise treatment schedules to maximise the clearance of cancer stem cells by killer T cells. Moreover, in Wei

20 [et al. \(2017\)](#) the authors optimise the injection of helper T cells to enhance cytokine-mediated tumour clearance. More
21 broadly, in [Joshi et al. \(2009\)](#), the authors examine how vaccine schedules can be leveraged to avoid tumour recurrence.
22 Besides these studies, researchers have also sought to optimise vaccine schedules in the context of combination
23 therapies. For example, [Lai and Friedman \(2017\)](#) examine how immune checkpoint blockers can be combined with
24 cancer vaccines for enhanced anti-tumour immunity, while [Wilson and Levy \(2012\)](#) look at how a regulatory-protein
25 inhibitor can be combined with a cancer vaccine to induce anti-tumour immunity. Indeed, our own previous modelling
26 work found that vaccine schedules, when optimised, may elicit high-avidity T cells ([Kumbhari et al., 2020b,a](#)). Our
27 model, however, is complex, and this complexity makes experimental validation difficult. Moreover, this complexity
28 introduces an element of model uncertainty as not all immune pathways and processes are well understood.

29 To address this, we develop a simple phenotypic ordinary differential equation (ODE) model that can reproduce the
30 results of our prior model. We validate our model against in vivo murine data from [Hailemichael et al. \(2013\)](#), ex vivo
31 human data from [Rezvani et al. \(2011\)](#) and in vitro data from [Wu et al. \(2017\)](#) and [Cawthon et al. \(2001\)](#). Notably,
32 the model presented here is a reduction of the model developed in [Kumbhari et al. \(2020a\)](#), obtained not via a formal
33 model reduction, but rather via a conceptual reduction informed by our sensitivity analysis from [Kumbhari et al.](#)
34 ([2020a](#)) and a review of the biological literature. Specifically, our model is based on the experimental observations that
35 (1) mature DCs present antigens at different levels; (2) low DC antigen loads activate only high-avidity T cells, while
36 high DC antigen loads activate both low- and high-avidity T cells; and (3) a history of antigen exposure attenuates
37 T-cell expansion.

38 We find that the sensitivity of the model's parameters, which are abstractions of different biological processes, vary
39 with dosage. We use this sensitivity analysis to eliminate inappropriate vaccine schedules (i.e., a schedule that promotes
40 low-avidity T cells) based on a patient's underlying conditions. This increases the likelihood of electing high-avidity T
41 cells and thus, the likelihood of durable anti-tumour responses. While our study still requires experimental validation,
42 it nevertheless provides a vital proof-of-concept basis for further development of this approach.

43 2 Model

44 In this section, we develop a minimal model of T cell avidity. Our minimal model establishes a framework for sys-
45 tematically incorporating additional complexity, which may help in quantifying the extent to which different pathways
46 impede tumour clearance. Moreover, in the context of optimising vaccine schedules, our model is amenable to more
47 sophisticated optimisation techniques (that are beyond the scope of this study) such as numerical optimal control. Fi-
48 nally, we note that while no model is perfect, by using only well understood phenotypes of avidity selection, we are
49 able to reduce any model uncertainty in our predictions.

50 In developing a minimal model of avidity selection, however, we exclude many aspects of the immune response. For
51 example, for example our model does not account for certain cell populations such as natural killer cells, regulatory T
52 cells and helper T cells. We also omit signalling pathways such as cytokine secretion. Importantly, our goal here is to
53 develop a caricature model with a plausible biological basis, rather than a model that aims to capture all known T cell
54 dynamics.

55 To this end, we assume immature dendritic cells (iDCs) take up antigen and start maturing upon contact with the
56 injected vaccine due to tumour-associated peptides and maturation signals such as vaccine adjuvant, danger signals,
57 or tissue derived immunogenic signals [Coffman et al. \(2010\)](#); [Gardner and Ruffell \(2016\)](#). Maturing DCs migrate to
58 draining lymph nodes, where they present antigens to antigen-specific naive T cells, resulting in their activation to
59 effector T cells ([Murphy, 2011](#); [Abbas et al., 2014](#)). Importantly, different DCs present varying levels of antigen on
60 their surfaces, affecting the avidity of T cells that are activated. For simplicity, we focus on the dynamics of killer T
61 cells that are cytolytic against tumours and are the primary target of anti-cancer vaccines ([Lollini et al., 2006](#); [Chung](#)
62 [et al., 2014](#); [Peng et al., 2019](#)).

63 To model these interactions, we consider several populations: P , the concentration of vaccine peptides; I , the concen-
64 tration of iDCs; M_L and M_H , the concentrations of mature DCs expressing low or high levels of vaccine peptide on
65 their surfaces; and T_L and T_H , the concentrations of killer T cells of low and high avidity. A diagram of the different
66 interactions between these populations is shown in Figure 1. We model the interactions between these populations with
67 an ODE system:

$$\dot{P} = u(t) - d_P P - k_P P (M_L + M_H) - k_{Pi} P I, \quad (1)$$

$$\dot{I} = s_I - d_I I - \alpha \frac{P}{\chi + P} I, \quad (2)$$

$$\dot{M}_L = \alpha [1 - p_{\text{high}}(P)] \frac{P}{\chi + P} I - d_D M_L, \quad (3)$$

$$\dot{M}_H = \alpha p_{\text{high}}(P) \frac{P}{\chi + P} I - d_D M_H, \quad (4)$$

$$\dot{T}_H = k_H \frac{M_L + M_H}{X_0 + M_L + M_H} - d_K \varphi(P) T_H, \quad (5)$$

$$\dot{T}_L = k_L \frac{M_H}{X_0 + M_H} - d_K \varphi(P) T_L, \quad (6)$$

68 where $u(t)$ is the vaccine injection rate.

69 In Eq. (1), vaccine peptides are injected at rate $u(t)$, decay at rate d_P , and are consumed by mature DCs at rate k_P
70 and by immature DCs are rate k_{Pi} . In Eq. (2), iDCs are replenished at rate s_I and turnover at rate d_I . The final term
71 in Eq. (2), models the maturation of iDCs due to adjuvant. Because adjuvant is usually not antigen-specific (Garcon
72 and Di Pasquale, 2017), as a simplifying assumption we assume that *all* peptides within the periphery of an iDC are
73 presented at rate α . It follows that if the concentration of non-vaccine proteins is denoted by χ , then the proportion of
74 peptides presented that are vaccine-associated is $P / (\chi + P)$. Together, these equate to a net flux of $\alpha P / (\chi + P)$.

75 In Eq. (3) and Eq. (4), iDCs transition into mature DCs at rate $\alpha \frac{P}{\chi + P} I$ and turnover at rate d_D . The specific probability
76 of transitioning to a mature DC presenting low levels of surface antigens is $1 - p_{\text{high}}(P)$, while the probability of
77 transitioning to a mature DC presenting high levels of surface antigens is $p_{\text{high}}(P)$.

78 Finally, in Eq. (5) and (6), killer T cells activate and proliferate as a function of DC concentration, and decay at rate
79 $d_K \varphi(P)$. Here, $\varphi(P)$ is an increasing function of antigen, P , that models T cell hyporesponsiveness (Hailemichael
80 et al., 2013). As a simplifying assumption, we do not model the activation of naive T cells explicitly but instead use
81 a saturating Hill function with parameters k_L , k_H and X_0 chosen so that we obtain biologically realistic behaviours.
82 Furthermore, a key feature of avidity selection is that low levels of antigen expression on DCs stimulate high-avidity
83 T cells and high-levels of antigen expression on DCs stimulate both low- and high-avidity T cells (Alexander-Miller
84 et al., 1996; Bullock et al., 2001; Kedl et al., 2002; Kroger et al., 2008; Rezvani et al., 2011). As such, the activation
85 rate for low-avidity T cells is dependent only on the concentration of DCs with high levels of antigen presentation, M_H ,
86 whereas the activation rate for high-avidity T cells is dependent on the total concentration of DCs with low and high
87 levels of antigen presentation, $M_L + M_H$.

88 2.1 Parameter estimates

89 A list of parameters used in our simulations is given in Table 1. To obtain estimates, we used experimental values for
90 a peptide vaccine against melanomas in humans, but stress that our model readily generalises to other forms of anti-
91 tumour vaccines. Where possible, we have used experimental data from humans to characterise our model parameters;
92 however, specific phenomenological parameters are fit to the results of our previous model.

93 2.1.1 Vaccine

94 In Eq. (1), we assume that the vaccine is given systemically at a fixed dose of $u_0 \text{ ng mL}^{-1}$ with a dosing interval of $\zeta \text{ d}$
95 equating to a vaccine injection rate of

$$u(t) = u_0 \sum_{a=0}^{\infty} \delta(t - \zeta a).$$

96 The in vitro decay rate of an immunogenic peptide such as HVDGKILFV is estimated to 6.16 d^{-1} , thus, we use a vac-
97 cine decay rate, d_P , of 6.16 d^{-1} (Harndahl et al., 2012). We assume that iDCs have an uptake rate of k_{Pi} , while mature
98 DCs have an uptake rate of k_P . Previously (Kumbhari et al., 2020a), we used human data from the literature (Platt

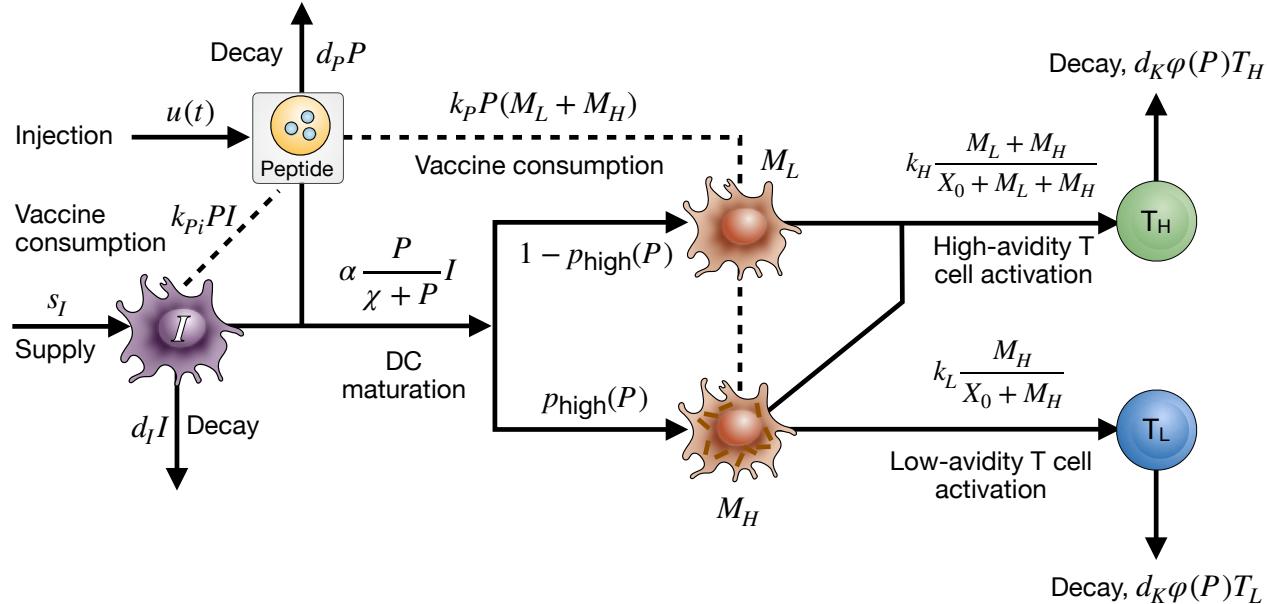


Fig. 1. A diagram of the key interactions between injected vaccine peptides, P ; immature DCs, I ; mature DCs with different levels of antigen expression, M_L (low) and M_H (high); and a population of low- and high-avidity killer T cells, T_L and T_H

Table 1. Table of parameters for the ODE model and estimated values. Estimates that are characterised by human, mice and in vitro data are marked with superscripts $(\cdot)^H$, $(\cdot)^M$, and $(\cdot)^V$. Here, d denotes days, and k denotes 10^3 cells.

Parameter	Description	Estimate	95% CI	Source
d_P	Vaccine peptide decay rate V	6.16 d^{-1}		Harndahl et al. (2012)
k_P	Mature DC uptake rate H^V	$3 \times 10^{-2} (\text{k}/\mu\text{L})^{-1}/\text{d}$		Sallusto et al. (1995)
k_P	Immature DC uptake rate M^V	$6.84 \times 10^{-2} (\text{k}/\mu\text{L})^{-1}/\text{d}$		Platt et al. (2010); Sallusto et al. (1995)
d_I	Immature DC decay rate H^V	$5 \times 10^{-2} \text{ d}^{-1}$		Kaplan et al. (1987)
M_{total}	Total DC population H	$5.9976 \text{ k}/\mu\text{L}$		Di Girolamo et al. (2008)
d_D	Mature DC turnover rate H	0.33 d^{-1}		Kaplan et al. (1987)
χ	Concentration of non-vaccine peptides H	$7 \times 10^7 \text{ ng mL}^{-1}$		Delamarre et al. (2003)
d_K	Net killer T cell turnover rate M	0.4 d^{-1}		De Boer et al. (2003)
α	DC maturation rate	$3 \times 10^5 \text{ d}^{-1}$	$[2.966 \times 10^5, 3.052 \times 10^5]$	
L	Transition probability parameter	$1 \times 10^6 \text{ ng mL}^{-1}$	$[9.825 \times 10^5, 1.012 \times 10^6]$	
k_H	High-avidity T cell activation rate	$2.884 (\text{k}/\mu\text{L})/\text{d}$	$[0, 10.56]$	
k_L	Low-avidity T cell activation rate	$3.673 (\text{k}/\mu\text{L})/\text{d}$	$[0, 20.78]$	Fit to Kumbhari et al. (2020a)
X_0	T cell saturation constant	$5139 \text{ k}/\mu\text{L}$	$[4826, 5438]$	
φ_0	Antigen saturation constant	$1 \times 10^5 \text{ ng mL}^{-1}$	$[99670, 1.003 \times 10^5]$	
	Minimum number of pMHCs required for a DC to have a high-antigen load	11 pMHCs		

99 et al., 2010; Sallusto et al., 1995) to estimate a mature DC uptake rate of $3 \times 10^{-2} (\text{k}/\mu\text{L})^{-1}/\text{d}$ and an immature DC
100 uptake rate of $6.84 \times 10^{-2} (\text{k}/\mu\text{L})^{-1}/\text{d}$. Notably, data from Sallusto et al. (1995) shows that while the rate of antigen
101 capture by DCs saturates for large antigen concentrations, the saturation constant associated with this response is large.
102 In other words, even though the rate of antigen capture technically saturates, it *effectively* behaves as a linear function.
103 Thus, as a simplifying assumption, we use mass-action kinetics rather than saturation-type kinetics. Finally, because
104 the vaccine is delivered at $t = 0$, we set $P(0) = u(0) = u_0$.

105 2.1.2 Dendritic cells

106 In Eq. (2), the rate at which immature DCs turnover, d_I , is $1/20 \text{ d}^{-1} = 5 \times 10^{-2} \text{ d}^{-1}$ based on human estimates from Ka-
107 plan et al. (1987). To calculate the supply rate, s_I , we force the system to be at steady state when there is no antigen,
108 i.e., $P = 0$, equating to $s_I - d_I I(0) = 0$, or $s_I = d_I I(0)$. The baseline concentration of non-vaccine peptides, χ , is
109 $7 \times 10^7 \text{ ng mL}^{-1}$ in humans (Delamarre et al., 2003). In Eq. (3) and Eq. (4), the mature DC turnover rate, d_D , is
110 estimated to be $1/72 \text{ h}^{-1} = 0.33 \text{ d}^{-1}$ in humans (Kaplan et al., 1987).

111 While directly obtaining measurements of DC antigen loads *over time* is challenging, several indirect techniques exists.
112 One such technique involves measuring the percentage of activated low-avidity T cells, which leverages the fact that
113 low-avidity T cell exclusively require high antigen loads for activation. Because the percentage of low-avidity T cells
114 activated (as measured by cytokine secretion and tetramer staining) exhibits a saturation-type response (Bullock et al.,
115 2003), we phenomenologically model the probability of transitioning to a mature DC presenting high levels of surface
116 antigens, $p_{\text{high}}(P)$, with a first-order Hill function, i.e.,

$$p_{\text{high}}(P) = \frac{P}{L + P}. \quad (7)$$

117 The phenomenological nature of this function means that other sigmoidal functions can be used to model this transition.
118 As the goal here is not to develop a fine-grained model of DC pMHC dynamics (which would be beyond the scope
119 of this paper), we note that a first-order Hill function is sufficiently simple. The model parameter L , along with the
120 maturation rate, α , is fit to the results of our previous model (Kumbhari et al., 2020a). Details of the fitting procedure
121 are provided in Section 2.2.

122 For our initial conditions, we note that the total DC population at steady-state conditions, M_{total} , is reported to be
123 $5.9976 \text{ k}/\mu\text{L}$ in humans (Di Girolamo et al., 2008). As such, we set $I(0) = M_{\text{total}} = 5.9976 \text{ k}/\mu\text{L}$. Additionally, we
124 assume that initially there are no mature DCs presenting vaccine-associated peptides, i.e., $M_{L,H}(0) = 0$.

125 2.1.3 T cells

126 Finally, in Eqs. (5) to (6), both low- and high-avidity T cells decay at rate d_K , which De Boer et al. (2003) estimate to
127 be 0.4 d^{-1} in mice. Motivated by De Boer and Perelson (2013), T cell activation and proliferation is modelled with a
128 saturation function (i.e., a Hill function) with shape parameter $n = 1$. The activation rates k_H and k_L ; and saturation
129 constant X_0 are fit to the results of our previous model.

130 Activation induced cell death (AICD) – also known as “exhaustion”, “senesce”, “adapted” etc. (Blank et al., 2019) –
131 is a phenomenon whereby chronic antigen exposure tempers T cell expansion and is considered a major reason for
132 tumour escape (Hashimoto et al., 2018; June et al., 2018). To model this, we assume our turnover rate, d_K , increases
133 as antigen accumulates. In particular, antigen accumulation, $\varphi(P)$, is modelled with the following function:

$$\varphi(P) = \frac{\int_0^t P(s) ds}{\varphi_0 + \int_0^t P(s) ds}, \quad (8)$$

134 where φ_0 is a saturation constant that is also fitted (details of the fitting procedure are provided in Section 2.2). While
135 the mechanisms behind AICD are unclear (Hashimoto et al., 2018; Blank et al., 2019), it is generally understood that
136 this dysfunctional state occurs due to a history of antigen exposure (Hashimoto et al., 2018). Thus, to account for this
137 history of antigen exposure, we use the integral of P , $\int_0^t P(s) ds$, rather than P alone. Finally, we assume that initially
138 there are no vaccine-associated effector T cells, i.e., $T_{L,H}(0) = 0$.

139 2.2 Parameter fitting

140 To parametrise our model, we first check for structural identifiability (detailed in Section 2.2.1 below). We then
141 simultaneously fit our model to data generated using our previous model (Kumbhari et al., 2020a), which in turn was

142 based on a model was calibrated to ex vivo human data from [Chung et al. \(2014\)](#), and validated against data from
143 [Rezvani et al. \(2011\)](#) and [Hailemichael et al. \(2013\)](#).

144 **2.2.1 Structurally identifiability analysis**

145 A model is structurally identifiable if, given an infinite amount of noiseless data, all model parameters and initial
146 conditions can be uniquely determined from measurements of its output ([Bellman and Astrom, 1970](#)). Moreover,
147 structural identifiability is prerequisite for both prediction ([Villaverde et al., 2016](#); [Heinemann and Raue, 2016](#); [Bandara](#)
148 [et al., 2009](#)), experimental validation ([Villaverde et al., 2016](#); [Walter, 1997](#); [Karr et al., 2015](#)), and importantly practical
149 identifiability (i.e., determining parameter values with noisy data).

150 To determine if our model is structurally identifiable, we use DAISY (Differential Algebra for Identifiability of SYs-
151 tems). This software tool checks ODE models with either polynomial or rational nonlinearities for structural identi-
152 fiability ([Bellu et al., 2007](#)). Explicitly, DAISY accepts a set of ODEs describing the state equations (initialised with
153 either known or unknown initial conditions) and uses Ritt's pseudodivision algorithm to generate an input-output map
154 of the system (i.e., a set of polynomial equations involving only the known variables and their time derivatives). DAISY
155 then uses the Grobner basis of this map to determine if our input-output map is finite-to-one, and thus identifiable ([Sac-](#)
156 [comani and Thomaseth, 2018](#); [Meshkat et al., 2009, 2011, 2012](#)).

157 A limitation of DAISY is that it only handles rational polynomial nonlinearities and yet Eq. (8) contains an integral.
158 We reconcile this by replacing $\int_0^t P(s) ds$ with a dummy variable P_I (defined such that $dP_I/dt = P$) and thus leverage
159 the fact that Eqs. (5) and (6) are decoupled from Eqs. (1) to (4). Since we are fitting our model to data generated by
160 our prior model, we assume all state variables are observable.

161 Using DAISY, we determine that our model is globally structurally identifiable. Our model also implements several
162 first- or second-order Hill functions to model various immunological processes. To assess whether or not structural
163 identifiability depends on the order of the Hill function used, we systematically vary the order from 1 to 10. As a
164 simplifying assumption, we limit these orders to integers. We find that structural identifiability is maintained regardless
165 of which integer-order Hill function is used.

166 **2.2.2 Fitting procedure**

167 Structural identifiability establishes that our model can be parametrised via noiseless data. Motivated by this, we
168 calibrate our current model to data from our previous study ([Kumbhari et al., 2020a](#)). While our previous model
169 tracked DCs by the number pMHCs being presented, our current model classifies DC antigen loads as being "high" or
170 "low". To compare the output between the two models, we cluster DC populations as follows. Motivated by reports
171 that as few as four pMHCs suffice to trigger T cell stimulation ([Deeg et al., 2013](#); [Varma et al., 2006](#); [Manz et al.,](#)
172 [2011](#)), we classify DCs presenting between 1 to 10 pMHCs, i.e., on the same order of magnitude, as having a low
173 antigen density. We then classify DCs presenting over ten pMHCs as having a high antigen load. Our prior work also
174 considered 20 avidity classes, with an avidity state of 1 denoting the lowest and 20 the highest avidity state. Thus, to
175 compare this to our current work, we consider T cells with avidity states ranging from 1–10 as low and states ranging
176 from 11–20 as high.

177 We then fit our model to a simulated vaccine dose of $7 \times 10^5 \text{ ng mL}^{-1}$ given fortnightly. This dosage is chosen as it
178 is similar to the protocols of previous clinical trials ([Schwartzentruber et al., 2011](#); [Sosman et al., 2008](#); [Smith et al.,](#)
179 [2003](#); [Rezvani et al., 2011](#)). We generate a time trace for the following four variables: DCs with high antigen loads,
180 M_H ; DCs with low antigen loads, M_L ; high-avidity T cells, T_H ; and low-avidity T cells, T_L . Then, for each variable,
181 we calculate the L^2 -norm of the error between the time trace predicted by our prior work (after being clustered as per
182 the previous paragraph) and the time trace predicted by our current model. Finally, we use MATLAB's optimisation
183 routine "fmincon" to find estimates that minimise this aggregate L^2 -error. 95% confidence intervals were obtained by
184 bootstrapping residuals 1000 times.

185 We estimate $\alpha = 3 \times 10^5 \text{ d}^{-1}$ (95% CI: $[2.966 \times 10^5, 3.052 \times 10^5]$); $L = 1 \times 10^6 \text{ ng mL}^{-1}$ (95% CI: $[9.825 \times 10^5, 1.012 \times 10^6]$);
186 $k_H = 2.884 \text{ (k}/\mu\text{L})/d$ (95% CI: $[0, 10.56]$); $k_L = 3.673 \text{ (k}/\mu\text{L})/d$ (95% CI: $[0, 20.78]$); $X_0 = 5139 \text{ k}/\mu\text{L}$ (95% CI: $[4826, 5438]$);
187 and $\varphi_0 = 1 \times 10^5 \text{ ng mL}^{-1}$ (95% CI: $[99670, 1.003 \times 10^5]$). This suggests that, relative to the parameters α, L, X_0 and
188 φ_0 , k_L and k_H are somewhat poorly identifiable.

189 As Figure 2 shows, our reduced model underestimates the amplitude of the initial peak for T cells and overestimates
190 the amplitude of secondary T cell peaks. This occurs due to the omission of negative feedback mechanisms such as

191 induced regulatory T cells in our model. Moreover, while other high-low set points could be used, we note that using
192 ten pMHCs provides good *qualitative* agreement with our prior results (see Figure 2).

193 3 Results

194 3.1 The model is consistent with experimental data

195 Individualised mathematical models for personalised medicine often necessitate simplicity because of the sparsity of
196 patient data (Andre et al., 2013; Kronik et al., 2010; Gevertz and Wares, 2018). Simple models, however, may be
197 perceived by some to trade mechanistic complexity for abstractions that cannot capture the full scope of experimental
198 data. Here, we validate our reduced model against in vivo murine data from Hailemichael et al. (2013), ex vivo human
199 data from Rezvani et al. (2011) and in vitro data from Wu et al. (2017) and Cawthon et al. (2001).

200 In Hailemichael et al. (2013), the authors show that repeated vaccination with the gp100 vaccine induces T cell hyporesponsiveness, whereby repeated exposure to an antigen inhibits T cell expansion. To emulate this study, we use a dosage
201 identical to that used in Hailemichael et al. (2013), namely, 100 μg in a 100 μL injection every 42 days, or equivalently
202 10⁶ ng mL^{-1} every 42 days. Simulating this protocol we find that our model also predicts T cell hyporesponsiveness
203 (see Figure 3A), but the decrease is predicted by our model (20%) is less dramatic than that reported by Hailemichael
204 et al. (2013) (approximately 50%). Since an implicit goal of this study is to develop a minimal model of T cell avidity,
205 we do not include several cell populations (such as myeloid-derived suppressor cells or pro-tumour macrophages) that
206 inhabit the tumour niche and temper T cell expansion. We expect including these factors will produce better agreement
207 with the data from Hailemichael et al. (2013).

208 Next, in Rezvani et al. (2011), the authors conduct a small-scale clinical trial with a peptide vaccine, and in doing so
209 observe the depletion of high-avidity T cell (quantified by decreasing ratio of high-avidity to-low-avidity T cell). To
210 test if we also observe a similar depletion in our model, we simulate a dosage of 7 \times 10⁵ ng mL^{-1} given every two
211 weeks. We find that after vaccinating at this dosage (see Figure 3B), high-avidity T cells become depleted as observed
212 by Rezvani et al. (2011).

213 Programmed cell death protein 1 (PD1) is a protein that inhibits T cell activity and is overexpressed on T cells in
214 cancer. In our model, this is implicitly modelled via an increased rate of T-cell turnover (see Eq. (8)). To validate this
215 component of our model, we compare the average value of Eq. (8) against data from Wu et al. (2017) (see Figure 3C), in
216 which the authors show that PD1 expression, quantified via mean fluorescence intensity (MFI), increases with vaccine
217 dosages in vitro (Wu et al., 2017). To simulate Wu et al. (2017)'s in vitro set up, we use a 2-hour dosing frequency and a
218 timespan of 4 days. To simulate the doses reported by Wu et al. (2017), we first note that antigen was distributed across
219 a 24-well plate, which assuming a well working volume of 0.475 mL (Sigma-Aldrich, 2020) and a control volume of
220 1 mL, implies that 1 $\mu\text{g mL}^{-1}$ of vaccine in vitro equates to a simulated dose of

$$\frac{1 \mu\text{g mL}^{-1} \times \text{well volume} \times \text{number of wells}}{\text{control volume}} = \frac{1 \mu\text{g mL}^{-1} \times 0.475 \text{ mL} \times 24}{1 \text{ mL}} = 11.4 \mu\text{g mL}^{-1}.$$

221 Given that MFI readouts are instrument specific, to compare PD1 MFI readouts, we normalise Wu et al.'s data so that
222 the maximum MFI is mapped to a value of 100%, and the minimum value is mapped to a value of 0%. We then
223 compared this against the average value of Eq. (8), which we similarly normalise. We find that our reduced model
224 agrees well with data from Wu et al. (2017) (see Figure 3C).

225 Finally, T-cell activation is modelled explicitly via a saturation function (see Eqs. (5) and (6)) and implicitly via a DC
226 pMHC transition probability (see Eqs. (3) and (4)). To validate these components of our model, we compare the average
227 net high- and low-avidity activation rates, $k_H(M_L + M_H)/(X_0 + M_L + M_H)$ and $k_L M_H/(X_0 + M_H)$, against in vitro T-cell
228 activation data (quantified via interferon-gamma readouts) from Cawthon et al. (2001). To emulate Cawthon et al.'s
229 in vitro set up, we use a 2-hour dosing frequency and a timespan of 1 day. And finally, as data from Cawthon et al.
230 (2001) is normalised to be between 0% and 100%, we similarly normalise our data. We find that our reduced model
231 agrees well with data from Cawthon et al. (2001) (see Figure 3D). Together, these findings show that our model predicts
232 behaviours consistent with the biological literature.

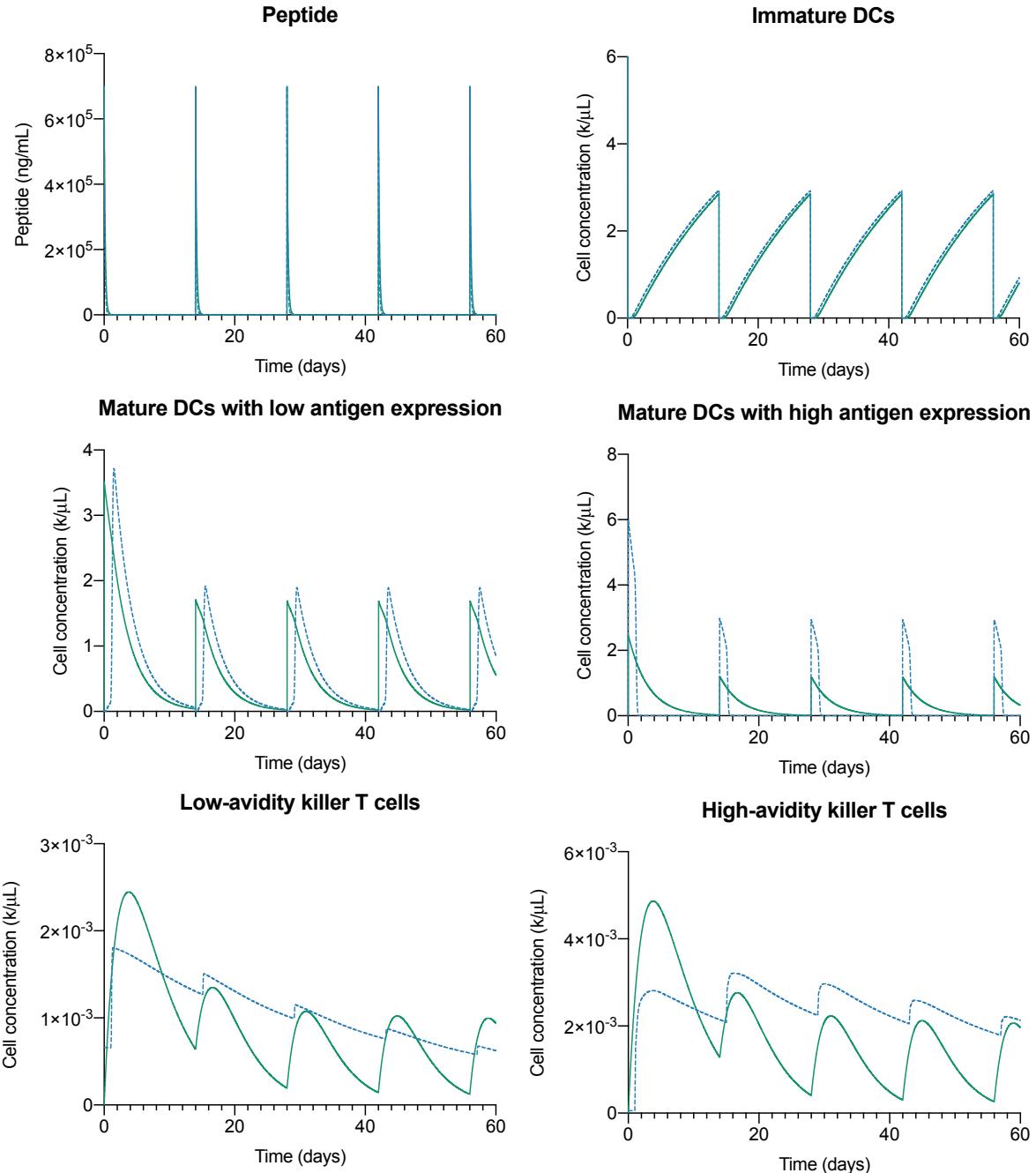


Fig. 2. Comparison between the reduced model and our prior work from Kumbhari et al. (2020a) for a dosage of $7 \times 10^5 \text{ ng mL}^{-1}$ fortnightly. Here, solid lines correspond to predictions made by our current model and dashed lines to predictions made by our previous model. Simulated cell concentrations are in thousands per micro-litre. Here, we classify a DC presenting between 1 to 10 pMHCs as having a low-antigen load, while anything greater than 10 pMHCs as having a high-antigen load.

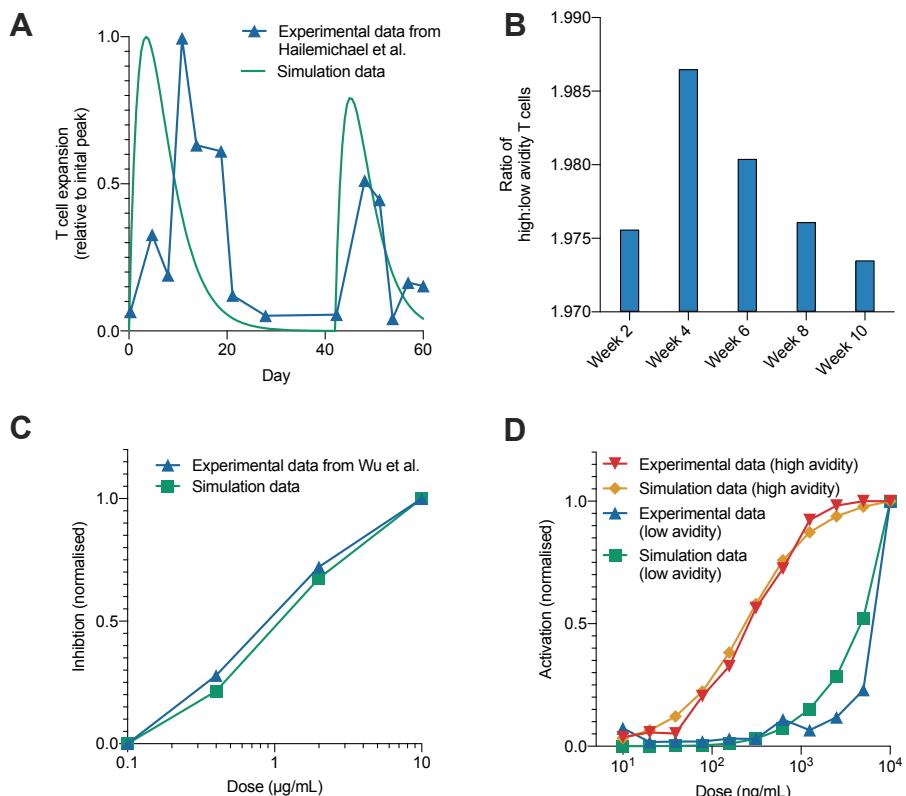


Fig. 3. (A) The model predicts T cell hyporesponsiveness as reported by [Hailemichael et al. \(2013\)](#). (B) The model predicts the depletion of high-avidity T cells as reported by [Rezvani et al. \(2011\)](#). (C) Here, we use PD1 expression as an ad-hoc measure of inhibition, which in our model is governed by Eq. (8). PD1 data from [Wu et al. \(2017\)](#) is normalised by mapping the largest MFI to 100% and the lowest MFI to 0%. The average value of Eq. (8), labelled “simulation data”, is similarly normalised. (D) Comparison of activation rates against normalised activation data, quantified via interferon-gamma expression, from [Cawthon et al. \(2001\)](#).

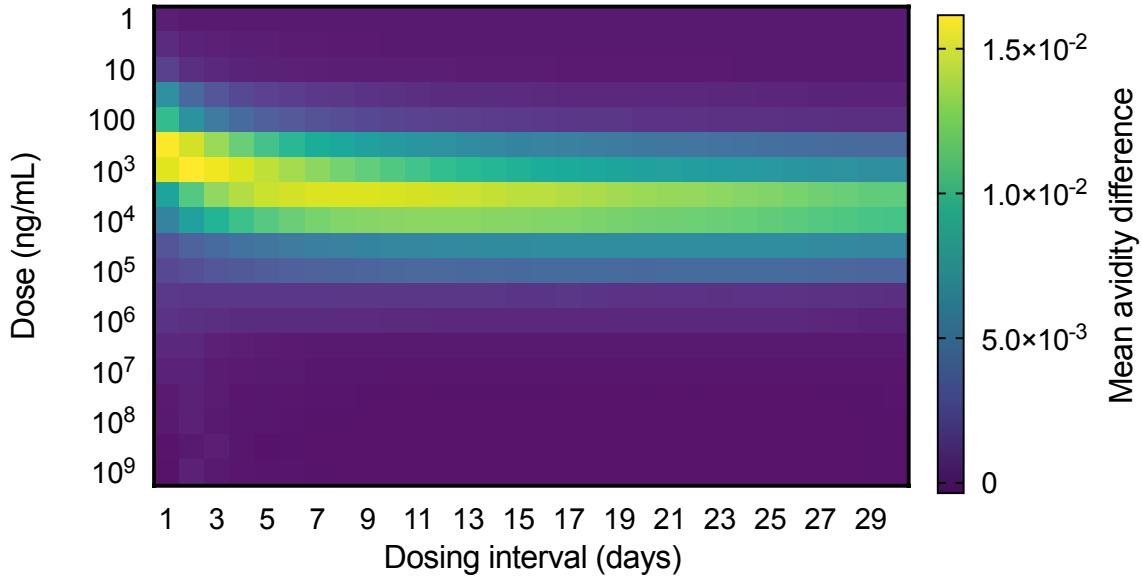


Fig. 4. The model predicts a frequent low-dose strategy maximises the selection of high-avidity T cells, which is consistent with prior work. Here, the selection of high-avidity T cells is quantified via the mean-avidity difference, which though *correlated* with tumour clearance is *not* a direct measure of tumour clearance.

3.2 The selection of high-avidity T cells depends synergistically on the schedule rather than the dose or dosing frequency alone

Since our reduced model reproduces key dynamics from the literature, we can leverage our model to identify vaccine schedules that preferentially select for high-avidity T cells. To quantify the selection of high-avidity T cells, we use the *mean avidity difference*

$$\text{mean avidity difference} = \frac{1}{60} \int_0^{60} [T_H(t) - T_L(t)] dt, \quad (9)$$

which, unlike the ratio of low- to high-avidity T cells, also accounts for the total T cell concentration. We then perform a global dosage sweep, i.e., simulate combinations of doses ranging from 1 ng mL^{-1} to $1 \times 10^9 \text{ ng mL}^{-1}$ with dosing intervals that range from 1 day to 30 days and track the average selection of high-avidity T cells over 60 days (quantified via the mean avidity difference).

We find that a dosage of $1 \times 10^3 \text{ ng mL}^{-1}$ given every two days maximises the mean avidity difference. A more strategic dosage of $5 \times 10^3 \text{ ng mL}^{-1}$ given weekly (see Figure 4) is also comparably effective. Moreover, in Figure 4, for doses between $5 \times 10^3 \text{ ng mL}^{-1}$ to $1 \times 10^4 \text{ ng mL}^{-1}$, we notice the formation of a characteristic “ridge”, along which the selection of high-avidity of T cells is robust to the dosing interval. More generally, our simulations suggest that the selection of high-avidity of T cells, as quantified by the avidity difference, is overall more sensitive to dose than to the dosing interval (see Figure 4). This may explain why previous experiments with the gp100 vaccine, which focused primarily on modulating the dosing interval of a high dose vaccine, were unsuccessful in eliciting high-avidity T cells (Schwartzentruber et al., 2011; Sosman et al., 2008; Smith et al., 2003; Rezvani et al., 2011). However, using a low dose alone is also unlikely to induce a significant high-avidity response as T cell expansion is usually proportional to antigen load (Berzofsky et al., 2001). Together, these results suggest that the selection of high-avidity T cells depends synergistically on the dose and the dosing frequency (or schedule), rather than the dosing frequency or dose alone, which is consistent with our previous work.

3.3 Parameter sensitivity changes with dose

In this section, we perform a global sensitivity analysis on several dosages and find that the selection of high-avidity T cells is sensitive to different parameters for different dosages. These sensitivities can be used to eliminate inappropriate (i.e., those that promote low-avidity T cells) dosages. As in Section 2.2, we consider nine dosages, specifically, doses of 10^3 ng mL^{-1} , $7 \times 10^5 \text{ ng mL}^{-1}$, or 10^8 ng mL^{-1} ; with either weekly, fortnightly, or monthly dosing intervals.

Table 2. Sensitivity of model parameters for different hypothetical dosages.

Dosing interval	Dose of 10^3 ng mL^{-1}			Dose of $7 \times 10^5 \text{ ng mL}^{-1}$			Dose of 10^8 ng mL^{-1}		
	Parameter	SRCC	<i>p</i> -value	Parameter	SRCC	<i>p</i> -value	Parameter	SRCC	<i>p</i> -value
Weekly	α	0.4068	$<10^{-12}$	k_H	0.4318	$<10^{-12}$	k_H	0.6452	$<10^{-12}$
	k_H	0.3593	$<10^{-12}$	M_{total}	0.4247	$<10^{-12}$	L	0.2074	3.0485×10^{-6}
	M_{total}	0.1357	0.0024	α	0.1714	1.1948×10^{-4}	k_{Pi}	0.0615	0.1695
	k_p	0.0242	0.5884	φ_0	0.1263	0.0047	χ	0.0090	0.8415
	L	0.0012	0.9794	d_P	0.1177	0.0084	M_{total}	0.0083	0.8537
	k_L	-0.0039	0.9304	L	0.0311	0.4872	d_P	0.0047	0.9170
	d_K	-0.0082	0.8543	k_p	-0.0042	0.9248	φ_0	-0.0032	0.9424
	φ_0	-0.0385	0.3899	k_{Pi}	-0.0203	0.6506	d_D	-0.0101	0.8209
	d_P	-0.0428	0.3389	d_I	-0.0402	0.3699	X_0	-0.0169	0.7055
	d_I	-0.0478	0.2863	k_L	-0.0447	0.3183	d_K	-0.0338	0.4511
	k_{Pi}	-0.2461	2.7460×10^{-8}	d_K	-0.1006	0.0245	k_p	-0.0338	0.4510
Fortnightly	d_D	-0.3788	$<10^{-12}$	χ	-0.1143	0.0106	d_I	-0.0424	0.3434
	X_0	-0.3907	$<10^{-12}$	d_D	-0.3933	$<10^{-12}$	α	-0.0514	0.2508
	χ	-0.4017	$<10^{-12}$	X_0	-0.3994	$<10^{-12}$	k_L	-0.5424	$<10^{-12}$
	α	0.4070	$<10^{-12}$	k_H	0.4437	$<10^{-12}$	k_H	0.6441	$<10^{-12}$
	k_H	0.3592	$<10^{-12}$	M_{total}	0.4064	$<10^{-12}$	L	0.2074	3.0610×10^{-6}
	M_{total}	0.1357	0.0024	α	0.1819	4.3965×10^{-5}	k_{Pi}	0.0613	0.1707
	k_p	0.0244	0.5859	d_P	0.1047	0.0193	χ	0.0090	0.8413
	L	0.0013	0.9776	φ_0	0.0982	0.0282	M_{total}	0.0071	0.8737
	k_L	-0.0040	0.9293	L	0.0266	0.5522	d_P	0.0070	0.8762
	d_K	-0.0080	0.8580	k_p	-0.0126	0.7789	φ_0	-0.0015	0.9737
	φ_0	-0.0392	0.3817	k_{Pi}	-0.0312	0.4869	d_D	-0.0095	0.8316
Monthly	d_P	-0.0429	0.3386	k_L	-0.0484	0.2803	X_0	-0.0173	0.6988
	d_I	-0.0475	0.2888	d_I	-0.0581	0.1943	d_K	-0.0326	0.4665
	k_{Pi}	-0.2464	2.6398×10^{-8}	d_K	-0.0731	0.1026	k_p	-0.0353	0.4311
	d_D	-0.3787	$<10^{-12}$	χ	-0.1183	0.0081	d_I	-0.0432	0.3351
	X_0	-0.3906	$<10^{-12}$	d_D	-0.4031	$<10^{-12}$	α	-0.0503	0.2619
	χ	-0.4015	$<10^{-12}$	X_0	-0.4097	$<10^{-12}$	k_L	-0.5418	$<10^{-12}$
	α	0.4069	$<10^{-12}$	k_H	0.4495	$<10^{-12}$	k_H	0.6430	$<10^{-12}$
	k_H	0.3591	$<10^{-12}$	M_{total}	0.3952	$<10^{-12}$	L	0.2070	3.1920×10^{-6}
	M_{total}	0.1356	0.0024	α	0.1868	2.7250×10^{-5}	k_{Pi}	0.0617	0.1684
	k_p	0.0244	0.5867	d_P	0.0953	0.0332	χ	0.0096	0.8306
	L	0.0010	0.9819	φ_0	0.0823	0.0658	d_P	0.0091	0.8383
	k_L	-0.0040	0.9297	L	0.0239	0.5930	M_{total}	0.0063	0.8877
260	d_K	-0.0080	0.8588	k_p	-0.0181	0.6870	φ_0	-8.2176×10^{-5}	0.9985
	φ_0	-0.0393	0.3798	k_{Pi}	-0.0369	0.4097	d_D	-0.0092	0.8376
	d_P	-0.0428	0.3394	k_L	-0.0496	0.2684	X_0	-0.0174	0.6977
	d_I	-0.0475	0.2889	d_K	-0.0586	0.1911	d_K	-0.0312	0.4866
	k_{Pi}	-0.2464	2.6587×10^{-8}	d_I	-0.0681	0.1281	k_p	-0.0363	0.4178
	d_D	-0.3789	$<10^{-12}$	χ	-0.1225	0.0061	d_I	-0.0423	0.3452
	X_0	-0.3906	$<10^{-12}$	d_D	-0.4100	$<10^{-12}$	α	-0.0511	0.2543
	χ	-0.4015	$<10^{-12}$	X_0	-0.4134	$<10^{-12}$	k_L	-0.5412	$<10^{-12}$
	α	0.4069	$<10^{-12}$	k_H	0.4495	$<10^{-12}$	L	0.2070	3.1920×10^{-6}
	k_H	0.3591	$<10^{-12}$	M_{total}	0.3952	$<10^{-12}$	k_{Pi}	0.0617	0.1684
	M_{total}	0.1356	0.0024	α	0.1868	2.7250×10^{-5}	χ	0.0096	0.8306
	k_p	0.0244	0.5867	d_P	0.0953	0.0332	d_P	0.0091	0.8383
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	d_K	-0.0080	0.8588	k_p	-0.0181	0.6870	d_D	-0.0092	0.8376
	φ_0	-0.0393	0.3798	k_{Pi}	-0.0369	0.4097	X_0	-0.0174	0.6977
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	d_I	-0.0475	0.2889	d_K	-0.0586	0.1911	k_p	-0.0363	0.4178
	k_{Pi}	-0.2464	2.6587×10^{-8}	d_I	-0.0681	0.1281	d_I	-0.0423	0.3452
	d_D	-0.3789	$<10^{-12}$	χ	-0.1225	0.0061	α	-0.0511	0.2543
	X_0	-0.3906	$<10^{-12}$	d_D	-0.4100	$<10^{-12}$	k_L	-0.5412	$<10^{-12}$
	χ	-0.4015	$<10^{-12}$	X_0	-0.4134	$<10^{-12}$	α	0.4069	$<10^{-12}$
	α	0.4069	$<10^{-12}$	k_H	0.4495	$<10^{-12}$	k_H	0.6430	$<10^{-12}$
	k_H	0.3591	$<10^{-12}$	M_{total}	0.3952	$<10^{-12}$	L	0.2070	3.1920×10^{-6}
	M_{total}	0.1356	0.0024	α	0.1868	2.7250×10^{-5}	k_{Pi}	0.0617	0.1684
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	L	0.0010	0.9819	φ_0	0.0823	0.0658	d_P	0.0091	0.8383
	k_L	-0.0040	0.9297	L	0.0239	0.5930	M_{total}	0.0063	0.8877
	d_K	-0.0080	0.8588	k_p	-0.0181	0.6870	φ_0	-8.2176×10^{-5}	0.9985
	φ_0	-0.0393	0.3798	k_{Pi}	-0.0369	0.4097	d_D	-0.0092	0.8376
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	d_I	-0.0475	0.2889	d_K	-0.0586	0.1911	d_K	-0.0312	0.4866
	k_{Pi}	-0.2464	2.6587×10^{-8}	d_I	-0.0681	0.1281	k_p	-0.0363	0.4178
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	χ	-0.4015	$<10^{-12}$	X_0	-0.4134	$<10^{-12}$	k_L	-0.5412	$<10^{-12}$
	α	0.4069	$<10^{-12}$	k_H	0.4495	$<10^{-12}$	α	0.4069	$<10^{-12}$
	k_H	0.3591	$<10^{-12}$	M_{total}	0.3952	$<10^{-12}$	k_H	0.6430	$<10^{-12}$
	M_{total}	0.1356	0.0024	α	0.1868	2.7250×10^{-5}	L	0.2070	3.1920×10^{-6}
	k_p	0.0244	0.5867	d_P	0.0953	0.0332	k_{Pi}	0.0617	0.1684
	L	0.0010	0.9819	φ_0	0.0823	0.0658	χ	0.0096	0.8306
	k_L	-0.0040	0.9297	L	0.0239	0.5930	d_P	0.0091	0.8383
	d_K	-0.0080	0.8588	k_p	-0.0181	0.6870	M_{total}	0.0063	0.8877
	φ_0	-0.0393	0.3798	k_{Pi}	-0.0369	0.4097	φ_0	-8.2176×10^{-5}	0.9985
	d_P	-0.0428	0.3394	k_L	-0.0496	0.2684	d_D	-0.0092	0.8376
	d_I	-0.0475	0.2889	d_K	-0.0586	0.1911	X_0	-0.0174	0.6977
	k_{Pi}	-0.2464	2.6587×10^{-8}	d_I	-0.0681	0.1281	d_K	-0.0312	0.4866
	d_D	-0.3789	$<10^{-12}$	χ	-0.1225	0.0061	k_p	-0.0363	0.4178
	X_0	-0.3906	$<10^{-12}$	d_D	-0.4100	$<10^{-12}$	d_I	-0.0423	0.3452
	χ	-0.4015	$<10^{-12}$	X_0	-0.4134	$<10^{-12}$	α	-0.0511	0.2543
	α	0.4069	$<10^{-12}$	k_H	0.4495	$<10^{-12}$	k_H	0.6430	$<10^{-12}$
	k_H	0.3591	$<10^{-12}$	M_{total}	0.3952	$<10^{-12}$	L	0.2070	3.1920×10^{-6}
	M_{total}	0.1356	0.0024	α	0.1868	2.7250×10^{-5}	k_{Pi}	0.0617	0.1684
	k_p	0.0244	0.5867	d_P	0.0953	0.0332	χ	0.0096	0.8306
	L	0.0010	0.9819	φ_0	0.0823	0.0658	d_P	0.0091	0.8383
	k_L	-0.0040	0.9297	L	0.0239	0.5930	$M_{\text{total$		

275 activation dynamics drive the promotion of high-avidity T cells.

276 By estimating the parameter sensitivity of these candidate dosages, we can use a patient's history and clinical presentation
277 to screen for suitable dosages. For example, a common co-morbidity for melanoma patients is diabetes (Lee et al.,
278 2015). Diabetes is known to decrease phagocytosis by immune cells (Geerlings and Hoepelman, 1999), which in our
279 model would correspond to a decreased rates of antigen uptake, k_p and k_{Pi} . The goal here is to identify dosages where
280 the selection of high-avidity T cells is negatively correlated with both k_p and k_{Pi} . Referring to Table 2, we identify
281 $7 \times 10^5 \text{ ng mL}^{-1}$ given either weekly, fortnightly or monthly as suitable candidate dosages.

282 As an additional example, consider a patient presenting with a history of heart disease another common co-morbidity in
283 skin cancer patients. Coronary artery disease results in a have a lower number of circulating DCs in patients (Van Vre
284 et al., 2011). In our model, this corresponds to a decreased DC concentration, M_{total} . The goal here is to identify
285 dosages where the selection of high-avidity T cells is weakly correlated with M_{total} . Referring to Table 2, we identify
286 $1 \times 10^8 \text{ ng mL}^{-1}$ given either weekly, fortnightly or monthly possible candidate dosages. Together, these examples il-
287 lustrate how our model can be leveraged to personalise dosages based on a patient's history and other conditions.

288 4 Discussion

289 Personalising treatment schedules to induce high-avidity T cells is a promising new approach to maintaining immunity
290 against certain cancers such as melanoma. Here we develop a simple ODE model of T cell avidity that is validated
291 against several experimental datasets. We then use our model to suggest therapy schedules based on a table of sensi-
292 tivities. This method involves first using a patient's history and clinical presentation to determine which parameters
293 are expected to have changed, and then referring to a table of parameter sensitivities to eliminate inappropriate vaccine
294 schedules. Importantly, our study is a proof-of-concept study and still requires experimental validation.

295 Since this study aims to develop a minimal model of avidity selection, our model makes several simplifying biological
296 assumptions. For example, we do not account for certain immunological processes such as the induction of regulatory
297 T cells, lymphocyte trafficking, and cytokine secretion. These processes were, however, modelled in our prior work,
298 against which we calibrated our model. Additionally, we only considered two avidity states, low or high, despite
299 avidity likely existing on a continuous spectrum. Since most experimental studies only report on low- and high-avidity
300 populations, using a system of ODEs (rather than a similar system of PDEs) makes the model more amenable to
301 experimental validation.

302 As an additional simplification, we assumed the probability of an immature DC transitioning to a mature DC pre-
303 presenting high levels of surface antigens, $p_{\text{high}}(P)$, was dependent only on the concentration of antigen. Biologically,
304 this probability depends on additional, more dynamic factors, such as co-signalling pathways (Chen and Flies, 2013).
305 Importantly, our model is not specific to peptide vaccines, and we expect that our model can also apply to newer,
306 neo-antigen-based T cell vaccines.

307 As an alternative to parametrising and then optimising a model to patient data (which may be difficult), we propose
308 using a table of sensitivities to screen to suitable dosages. This table of sensitivities involves performing a sensitivity
309 analysis on model parameters for a set of different simulated dosages. Of these dosages, some, when clinically trialled,
310 were found to promote the low-avidity T cells over high-avidity T cells (Hailemichael et al., 2013; Rezvani et al., 2011;
311 Schwartzentruber et al., 2011). Nonetheless, we included these dosages as we could not rule out the possibility that
312 these dosages are optimised to elicit anti-tumour immunity by additional, non-avidity-based mechanisms not consid-
313 ered in our model. Indeed, we argue that, under the right conditions (i.e., those identified via our table of sensitivities),
314 these dosages may enhance the selection of high-avidity T cells. To ensure our schedules are also practical, we limit
315 our simulated dosing intervals to weekly, fortnightly or monthly intervals. Consequently, we did not include the op-
316 timal dosage of $1 \times 10^3 \text{ ng mL}^{-1}$ given every two days in our table of candidate schedules. Moreover, while these
317 dosing intervals were chosen for their practicality, an alternative approach not explored in this study involves using
318 control theory to identify an optimal vaccine strategy. Indeed, using a control-theoretic approach may identify dosages
319 that maximise the selection of high-avidity T cells beyond what we identified. Finally, while our findings still require
320 preclinical validation, we anticipate that our simulated dosages are safe. This is based on studies in which limited
321 adverse side effects are reported (Hailemichael et al., 2013; Rezvani et al., 2011; Schwartzentruber et al., 2011). As
322 such, we predict that under the right circumstances (such as those suggested by Table 2), these dosages can safely elicit
323 high-avidity T cells.

324 Notably, the schedules identified here all aim to maximise the mean avidity difference over 60 days. We used the

325 avidity difference rather than the ratio of high- to low-avidity T cells, as it not only penalises the selection of low-
326 avidity T cells but also accounts for the total concentration of T cells induced, which is an important predictor of
327 treatment efficacy (Kittlesen et al., 1998). However, as a metric for the selection of high-avidity T cells, the mean
328 avidity difference has limitations. For example, optimising the mean avidity difference results in all-or-nothing control,
329 whereby a response that elicits a low total T cell count has a higher payoff than one that promotes low-avidity T cells
330 at a higher concentration. While this allows us to account implicitly for the inhibition of high-avidity T cells by low-
331 avidity T cells (Chung et al., 2014), this also results in parameter sensitivities that suggest T cell hyporesponsiveness is
332 preferable over the stimulation of low-avidity T cells. This could be addressed by using a metric that penalises both the
333 selection of low-avidity T cells and low total T cell concentrations. Vaccine protocols also need to account for factors
334 other than T cell avidity, such as toxicity constraints and off-target reactions (Tigue et al., 2007), which are factors that
335 our model does not include. Developing a selection metric that accounts for these factors will be the subject of future
336 investigations.

337 Overall, our findings still require substantial experimental validation, which is a priority for future work. Nonethe-
338 less, they provide a vital proof-of-concept link between a phenotypic model of avidity selection and identifying and
339 eliminating sub-therapeutic vaccine schedules, which may help in inducing durable anti-tumour immunity.

340 Conflict of interest

341 The authors declare that they have no conflict of interest.

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