

Dissecting causal pathways using Mendelian randomization with summarized genetic data: application to age at menarche and risk of breast cancer

Stephen Burgess,^{*1,2} Deborah J Thompson,³ Jessica MB Rees,²
Felix R Day,⁴ John R Perry,⁴ Ken K Ong.⁴

June 27, 2017

¹ MRC Biostatistics Unit, University of Cambridge, Cambridge, Cambridgeshire, CB2 0SR, UK.

² Cardiovascular Epidemiology Unit, University of Cambridge, Cambridge, Cambridgeshire, CB1 8RN, UK.

³ Cambridge Centre for Genetic Epidemiology, University of Cambridge, Cambridge, Cambridgeshire, CB1 8RN, UK.

⁴ MRC Epidemiology Unit, University of Cambridge, Cambridge, Cambridgeshire, CB2 0QQ, UK.

* Corresponding author. Email: sb452@medsch.cam.ac.uk.

Running title: Dissecting causal pathways using MR.

Keywords: Mendelian randomization, instrumental variable, mediation analysis, direct effect, causal inference.

Abstract

Mendelian randomization is the use of genetic variants as instrumental variables to estimate causal effects of risk factors on outcomes. The total causal effect of a risk factor is the change in the outcome resulting from intervening on the risk factor. This total causal effect may potentially encompass multiple mediating mechanisms. For a proposed mediator, the direct effect of the risk factor is the change in the outcome resulting from a change in the risk factor keeping the mediator constant. A difference between the total effect and the direct effect indicates that the causal pathway from the risk factor to the outcome acts at least in part via the mediator (an indirect effect). Here, we show that Mendelian randomization estimates of total and direct effects can be obtained using summarized data on genetic associations with the risk factor, mediator, and outcome, potentially from different data sources. We perform simulations to test the validity of this approach when there is unmeasured confounding and/or bidirectional effects between the risk factor and mediator. We illustrate this method using the relationship between age at menarche and risk of breast cancer, with body mass index (BMI) as a potential mediator. We show an inverse direct causal effect of age at menarche on risk of breast cancer (independent of BMI) and a positive indirect effect via BMI. In conclusion, multivariable Mendelian randomization using summarized genetic data provides a rapid and accessible analytic strategy that can be undertaken using publicly-available data to better understand causal mechanisms. (250 words)

1 Introduction

2 Mendelian randomization is the use of genetic variants as instrumental variables to assess and
3 estimate the causal effect of a risk factor on an outcome [Davey Smith and Ebrahim, 2003;
4 Burgess and Thompson, 2015b]. A risk factor has a causal effect on an outcome if intervening
5 on the risk factor leads to changes in the outcome. Correlation between a risk factor and an
6 outcome may arise because the risk factor is a cause of the outcome. However, it may also
7 reflect confounding (the risk factor and outcome have common causes) or reverse causation
8 (the outcome is a cause of the risk factor). Instrumental variable analysis represents one way
9 of assessing whether there is a causal effect of the risk factor on the outcome under certain
10 assumptions using observational data.

11 For a genetic variant to be a valid instrumental variable, it must satisfy three assumptions.
12 First, the genetic variant must be associated with the risk factor. Secondly, the genetic
13 variant must not be associated with confounders of the risk factor-to-outcome association.
14 Thirdly, the genetic variant must not affect the outcome except via the risk factor of interest
15 (no direct effect on the outcome) [Greenland, 2000; Lawlor et al., 2008]. Whereas phenotypic
16 variables tend to display widespread correlations with other phenotypes, genetic variants are
17 often more specific in their associations [Davey Smith et al., 2007], meaning that Mendelian
18 randomization investigations are less susceptible to biases from confounding that adversely
19 affect observational studies. Additionally, as the genetic code is fixed at conception, genetic
20 associations are less susceptible to reverse causation or confounding due to environmental
21 factors.

22 The instrumental variable assumptions can be assessed to some extent by testing for as-
23 sociations between the genetic variants and potential measured confounders [Burgess et al.,
24 2015b]. However, it is possible that a covariate associated with a genetic variant is not a
25 confounder, but rather a mediator on the causal pathway from the risk factor to the outcome
26 [Haycock et al., 2016]. This is particularly likely if several variants all have directionally con-
27 cordant associations with the same covariate. Genetic associations with a mediator may not
28 represent pleiotropic effects of the variants, but rather represent downstream consequences
29 of intervening on the risk factor. In such a case, the genetic variants are still valid instru-
30 ments, as the only causal pathway from the variants to the outcome is via the risk factor
31 (and potentially also via the mediator).

32 In many scenarios, it is relevant not only whether the risk factor is a cause of the outcome,
33 but also via what mechanism this causal effect acts. Mediation analysis can be used to dissect
34 the total causal effect of the risk factor on the outcome into an indirect effect of the risk
35 factor on the outcome via the mediator, and a direct effect of the risk factor on the outcome
36 not via the mediator (possibly via other causal pathways or other mediators) [VanderWeele
37 and Vansteelandt, 2009]. This is illustrated in Figure 1. The total effect is defined as the
38 change in the outcome resulting from intervening on the risk factor (say, increasing its value
39 by 1 unit). The direct effect is the change in the outcome resulting from intervening on
40 the risk factor but holding the mediator constant. The indirect effect is the change in the
41 outcome resulting from manipulating the value of the mediator as if we had intervened on
42 the risk factor, but in fact holding the risk factor constant. If all variables are continuous and
43 all relationships between variables are linear, then the total effect is equal to the direct effect
44 plus the indirect effect. Formally, a direct effect defined by intervening on the risk factor and
45 mediator separately is a controlled direct effect, which does not have a counterpart indirect
46 effect. If all relationships are linear, then the controlled direct effect is equal to the natural
47 direct effect, which does have a counterpart, the natural indirect effect. Full details are
48 provided in the Supplementary Material A.1.

49 [Figure 1 should appear about here.]

50 Mendelian randomization analyses using summarized data have recently become widespread
51 due to the increasing public availability of suitable data in large sample sizes from GWAS
52 consortia, and the possibility of ‘two-sample’ Mendelian randomization in which genetic as-
53 sociations with the risk factor and outcome are estimated in different samples [Burgess et al.,
54 2015b]. It has previously been demonstrated that a (univariable) Mendelian randomization
55 estimate can be obtained from summarized data (beta-coefficients and standard errors) by
56 regressing genetic associations with the outcome on genetic associations with the risk factor
57 [Burgess et al., 2016]. This represents the total effect of the risk factor on the outcome. It
58 has also been demonstrated that direct causal effects of related risk factors can be estimated
59 by regressing genetic associations with the outcome on genetic associations with each of the
60 risk factors in a multivariable regression model; this is referred to as multivariable Mendelian
61 randomization [Burgess and Thompson, 2015a].

62 In this report, we demonstrate how the total effect and the direct effect of the risk

63 factor on the outcome can be estimated from summarized data, we consider the assumptions
64 necessary for genetic variants to satisfy for consistent estimation, and we exemplify how
65 these estimates can be used to interrogate causal mechanisms with an applied example of
66 the effect of age at menarche on breast cancer risk, with body mass index (BMI) as a potential
67 mediator.

68 Methods

69 Assumed framework of summarized data and genetic associations

70 We initially assume that all variables are continuous, and relationships between variables
71 (in particular, the genetic associations with the risk factor X , mediator M , and outcome Y ,
72 and the causal effects of the risk factor and mediator on the outcome, and of the risk factor
73 on the mediator) are linear with no effect modification (that is, they are the same for all
74 individuals in the population and do not vary for different values of the independent variable).
75 For each genetic variant G_j ($j = 1, 2, \dots, J$), we assume that we have an estimate $\hat{\beta}_{Xj}$ of
76 the association of the genetic variant with the risk factor obtained from linear regression.
77 Similar association estimates are assumed to be available for the mediator ($\hat{\beta}_{Mj}$) and outcome
78 ($\hat{\beta}_{Yj}$). The standard error of the association estimate with the outcome is $se(\hat{\beta}_{Yj})$. If any
79 of the variables is binary, then these summarized association estimates may be replaced
80 with association estimates from logistic regression; more detail on the binary outcome case
81 is provided later in the paper. The relationships between these variables are illustrated in
82 Figure 2.

83 [Figure 2 should appear about here.]

84 We also assume that all genetic variants are uncorrelated (that is, not in linkage disequilibrium). Although conventional instrumental variable methods for analysing summarized
85 data from correlated variants have been developed [Burgess et al., 2016] and software code
86 for analysing correlated variants is provided in the Supplementary Material, as we shall see
87 later there are problems of identification in the mediation setting that may be accentuated
88 by the use of correlated variants. Although this is a strict assumption, often genetic variants
89 in Mendelian randomization investigations are chosen to be the top hits from different gene
90 regions identified by a genome-wide association study, and so the assumption is naturally

92 satisfied. The method makes no specific requirements for the level of statistical significance of
93 the associations between the genetic variants and the risk factor, but variants with robustly
94 verified associations represent more informative instrumental variables.

95 **Weighted regression for estimation of total and direct effects**

96 If $\hat{\beta}_{Xj}$, $\hat{\beta}_{Mj}$, and $\hat{\beta}_{Yj}$ are the genetic associations of variant G_j ($j = 1, 2, \dots, J$) with the
97 risk factor (X), mediator (M) and outcome (Y), and $\text{se}(\hat{\beta}_{Yj})$ are the standard errors of the
98 genetic associations with the outcome, then the weighted regression:

$$\hat{\beta}_{Yj} = \theta_T \hat{\beta}_{Xj} + \epsilon_{Tj}, \quad \epsilon_{Tj} \sim \mathcal{N}(0, \text{se}(\hat{\beta}_{Yj})^2) \quad (1)$$

99 provides an estimate of the total effect of the risk factor on the outcome θ_T , known as
100 the inverse-variance weighted estimate [Burgess et al., 2013]. This regression model does
101 not take into account uncertainty in the genetic associations with the risk factor; however,
102 these associations are typically more precisely estimated than those with the outcome, and
103 ignoring this uncertainty does not lead to inflated Type 1 error rates in realistic scenarios
104 [Burgess et al., 2013].

105 The inverse-variance weighted estimate can be motivated as the fixed-effect meta-analysis
106 pooled estimate of the variant-specific causal estimates $\frac{\hat{\beta}_{Yj}}{\hat{\beta}_{Xj}}$ with standard errors taken as
107 $\frac{\text{se}(\hat{\beta}_{Yj})}{\hat{\beta}_{Xj}}$ (the leading order term from the delta expansion for the standard error of the ratio of
108 two variables). This meta-analysis estimate can also be obtained by the weighted regression
109 model in equation 1 [Thompson and Sharp, 1999]. The weighted regression model can be
110 expanded by including genetic associations with the mediator:

$$\hat{\beta}_{Yj} = \theta_D \hat{\beta}_{Xj} + \theta_M \hat{\beta}_{Mj} + \epsilon_{Dj}, \quad \epsilon_{Dj} \sim \mathcal{N}(0, \text{se}(\hat{\beta}_{Yj})^2) \quad (2)$$

111 to provide an estimate of the direct effect θ_D . The weighted regression method for calculating
112 the total effect (equation 1) is equivalent to the two-stage least squares (2SLS) method with
113 individual-level data, in which the first stage of the method regresses the risk factor on
114 the genetic variants, and the second stage regresses the outcome on fitted values of risk
115 factor [Burgess et al., 2016]. The weighted regression method for calculating the direct
116 effect (equation 2) is also equivalent to a two-stage regression method, except that the first

117 stage also regresses the mediator on the genetic variants, and the second stage regresses the
118 outcome on fitted values of the risk factor and fitted values of the mediator [Burgess et al.,
119 2015a]. Software code to implement these analyses is provided in Supplementary Material
120 A.2. With a continuous outcome, the indirect effect of the risk factor on the outcome can
121 be calculated as $\theta_I = \theta_T - \theta_D$.

122 For consistent estimation, it is required that all genetic variants used to estimate the
123 total effect of the risk factor on the outcome satisfy the standard assumptions of Mendelian
124 randomization: they are associated with the risk factor, not associated with confounders,
125 and there is no pathway from any genetic variant to the outcome except via the risk factor.
126 All variants used to estimate the direct effect of the risk factor on the outcome must satisfy
127 the assumptions of multivariable Mendelian randomization: they are associated with the
128 risk factor and/or mediator, not associated with confounders, and there is no pathway from
129 any genetic variant to the outcome except via the risk factor and/or the mediator [Burgess
130 and Thompson, 2015a].

131 **Identification of the direct effect**

132 If the genetic associations with the mediator are entirely determined by their associations
133 with the risk factor, then with an infinite sample size (if associations are perfectly linear with
134 no heterogeneity) the direct effect would not be identified, as the genetic associations with
135 the risk factor and mediator would be perfectly correlated. Hence, it is necessary for there to
136 be some heterogeneity in the genetic associations or the relationships between the variables.
137 This may occur for a complex variable such as BMI, where different genetic variants may
138 influence BMI in different ways or via different biological pathways, potentially leading to
139 different magnitudes of causal effect on the mediator and/or outcome. Alternatively, if
140 there are genetic variants that are instrumental variables for the mediator only, then these
141 variants could be included in the multivariable Mendelian randomization analysis. However,
142 such variants are not valid instrumental variables for the risk factor, and so should not be
143 used to estimate the total causal effect of the risk factor on the outcome.

144 Applied example

145 As an illustrative example, we consider the causal effect of age at menarche on breast cancer
146 risk. Numerous genetic variants have been discovered that influence age at menarche. Later
147 puberty reduces the total number of ovulatory cycles and hence the life-time sex-hormone
148 exposure, thus we expect later menarche to be protective for breast cancer. This is in line
149 with observational epidemiological findings [Collaborative Group on Hormonal Factors in
150 Breast Cancer, 2012]. However, later menarche is also associated with lower BMI, and it
151 is known that genetically predicted BMI (and also adolescent BMI) is inversely associated
152 with breast cancer risk [Guo et al., 2016; Baer et al., 2010]. Therefore age at menarche will
153 likely have an indirect effect on breast cancer risk via BMI as well as a direct effect (in the
154 opposite direction) not via BMI.

155 We have taken 375 genetic variants demonstrated to be associated with age at menarche
156 at a genome-wide level of significance [Day et al., 2017]. Genetic associations with age
157 at menarche (measured in years) were obtained from the Reprogen consortium based on
158 329,000 women of European descent. Genetic associations with BMI were obtained from the
159 GIANT consortium, based on 339,000 individuals, 95% of whom are of European descent
160 [Locke et al., 2015]. Genetic associations with breast cancer risk were obtained from the
161 Breast Cancer Association Consortium (BCAC) on 47,000 cases and 43,000 controls (all
162 female) of European descent [Michailidou et al., 2015]. Although genetic associations with
163 BMI were estimated at different timepoints for different studies in the GIANT consortium,
164 as genetic variants typically influence variables across the whole life-course, it is not crucial
165 when these associations are measured, provided that they are measured in individuals before
166 they have disease events (to prevent reverse causation, see Discussion for more detail). A
167 more detailed analysis of these same data (although based on the individual-level data) was
168 previously reported by Day et al. [2017]; further details relating to applied aspects of the
169 analysis are provided in that paper.

170 Univariable Mendelian randomization suggested a null effect of age at menarche on breast
171 cancer risk (odds ratio per 1 year later menarche 1.00, 95% confidence interval 0.96, 1.05).
172 However, a multivariable Mendelian randomization analysis adjusting for genetic associations
173 with BMI suggested a protective direct effect of later age at menarche (odds ratio 0.94, 95%
174 confidence interval 0.89, 0.98). This suggests that an intervention to delay menarche would

175 have no net effect on breast cancer risk if it also had the expected consequence of lowering
176 adolescent BMI (or, similarly, if the delay in menarche was achieved by reducing pre-pubertal
177 BMI). However, an intervention which had an effect on post-pubertal sex-hormone exposure
178 equivalent to a later menarche would be likely to have a protective effect on breast cancer
179 risk, as such an intervention could not affect pubertal timing and hence would not alter BMI;
180 hence only the direct effect of age at menarche on breast cancer risk would apply here. We
181 note that the results presented here using the summary statistics method are, to 2 decimal
182 places, identical to those computed using individual-level BCAC data and reported in Day
183 et al. [2017]. As the outcome is binary, we do not provide an estimate of an indirect causal
184 effect (see Discussion).

185 Simulation study

186 To validate the utility of the multivariable Mendelian randomization method for estimating
187 a direct causal effect, we performed a simulation analysis. We generated data on 10 genetic
188 variants, a risk factor (X), mediator (M), and outcome (Y) for 10 000 individuals in a one-
189 sample Mendelian randomization context. Full details of the simulation setup are provided
190 in Supplementary Material A.3. Briefly, we considered eight different sets of values of the
191 parameters θ_1 (the causal effect of X on M), θ_2 (the direct effect of X on Y), and θ_3 (the
192 effect of M on Y) – see Figure 2. The indirect effect of X on Y via M is $\theta_1\theta_3$, and the total
193 effect of X on Y is $\theta_2 + \theta_1\theta_3$. We included scenarios where there is no direct effect, no indirect
194 effect, a direct effect and a directionally concordant indirect effect, and a direct effect and
195 a directionally discordant indirect effect. Parameters were chosen to take realistic values
196 and cover a range of scenarios. 10 000 simulated datasets were generated for each choice
197 of parameter values. Heterogeneity to ensure identification of the model was generated by
198 additionally allowing the genetic variants to affect the mediator directly; these effects were
199 drawn from a normal distribution with mean zero. Although this formally leads to pleiotropy
200 and violation of the instrumental variable assumptions, it has been shown that such ‘balanced
201 pleiotropy’ does not lead to bias in causal estimates [Bowden et al., 2015].

202 For each simulated dataset, we performed univariable Mendelian randomization analyses
203 to estimate the total causal effect of the risk factor on the outcome, and multivariable
204 Mendelian randomization for the direct causal effect not via the mediator. Each analysis was

205 performed by weighted regression using the summarized data only (genetic associations with
206 the risk factor, mediator, and outcome: beta-coefficients plus standard errors). We assumed
207 that all genetic variants were uncorrelated (no linkage disequilibrium); their distributions
208 in the data-generating model were independent. This assumption can be relaxed using
209 generalized weighted linear regression as described elsewhere [Burgess et al., 2016].

210 Table 1 shows mean estimates of the total and direct effects, mean bias and standard
211 deviations of the estimates, and coverage of the 95% confidence interval (the proportion of
212 confidence intervals that include the true value of the parameter). The standard errors for
213 the causal estimates were adjusted for underdispersion (residual standard error in the regres-
214 sion model less than 1) as described in the software code. No correction for overdispersion
215 was applied [Burgess and Thompson, 2017]. The Monte Carlo error (uncertainty due to
216 the limited number of simulations) was around 0.001 for each mean estimate, and 0.2% for
217 the coverage proportion. We see that mean univariable Mendelian randomization estimates
218 are similar to the total causal effect, whereas mean multivariable Mendelian randomization
219 estimates are similar to the direct causal effect in each scenario considered. Bias in the mean
220 estimates is small throughout, and is likely to be due to weak instrument bias arising from
221 the limited strength of the genetic variants [Burgess et al., 2011] (no bias was observed on
222 repeating the simulation study with a sample size of 1 000 000 for a small number of simu-
223 lated datasets). Bias was consistent in direction for the total effect, but varied in direction
224 for the direct effect. Coverage rates were close to nominal levels (95%) throughout, except
225 for when there was substantial weak instrument bias in estimates of the direct effect. There
226 was no noticeable undercoverage resulting from the regression models failing to account for
227 uncertainty in the genetic associations with the risk factor or mediator. Further results in
228 Supplementary Material A.4 indicate that these findings hold even when there are bidirec-
229 tional effects of the risk factor on the mediator and vice versa (as may be the case for age
230 at menarche and BMI).

231 [Table 1 should appear around here]

232 Discussion

233 In this paper, we have demonstrated how summarized data on genetic associations can be
234 used to investigate causal mechanisms, in particular whether the causal effect of a complex

235 risk factor on an outcome acts via a given mediator. Although the assumptions required
236 for a genetic variant to be an instrumental variable are very stringent, in other ways, the
237 requirements necessary to perform this analysis are quite flexible – only summarized data
238 on genetic associations are required. This allows for the leverage of data from large-scale
239 GWAS consortia. As with two-sample Mendelian randomization [Pierce and Burgess, 2013],
240 the summarized data methods described here do not require the genetic associations with
241 the risk factor, mediator and outcome to be measured in the same individuals. For exam-
242 ple, Eppinga et al. used genetic variants to investigate the effect of resting heart rate on
243 mortality in UK Biobank [Eppinga et al., 2016]. As a sensitivity analysis, they adjusted
244 the genetic associations with the outcome for some covariates using individual-level data
245 to assess whether the effect of resting heart rate was mediated via any of those variables.
246 Additionally, they adjusted for genetic associations with lipid fractions using the multivari-
247 able Mendelian randomization approach outlined here, as lipid measurements are currently
248 not available in the dataset. Combining summary statistics from different sources is also
249 important in the example of age at menarche and breast cancer here, as BMI measurements
250 for breast cancer cases were only available post-diagnosis. These measurements would likely
251 be influenced by the disease process, as well as by treatment and lifestyle changes. It is
252 therefore preferable here to estimate the effects of the genetic variants on BMI in a separate
253 dataset.

254 Compatibility of datasets

255 When using genetic associations from multiple datasets in a two-sample Mendelian ran-
256 domization setting, ideally the associations should be estimated on samples from the same
257 underlying population. This is particularly important with regard to ethnicity, as different
258 linkage disequilibrium structures can mean that genetic variants may be associated with the
259 risk factor in one population and not in another, or be valid instruments in one population
260 but not in another. Ideally, genetic associations should not be adjusted for covariates apart
261 from principal components of ancestry, particularly if these covariates may be on causal
262 pathways relating to the risk factor, mediator or outcome. It is also important to ensure
263 that genetic associations with the risk factor and mediator are estimated in individuals who
264 have not had disease events, so that these associations are not influenced by reverse causa-
265 tion. However, even if associations are estimated in different datasets (say, associations with

266 the risk factor are measured in 20-year olds and associations with the mediator in 50-year
267 olds, or vice versa), as genetic variants typically influence variables across the whole life-
268 course, inferences from Mendelian randomization for the causal null hypothesis should still
269 be qualitatively valid, even if the parametric assumptions necessary for causal estimation
270 are not satisfied [Burgess et al., 2016]. In any case, as Mendelian randomization estimates
271 represent the effect of changing people's genetic variants at conception, causal estimates from
272 Mendelian randomization should not be interpreted too literally as the expected impact of
273 intervening on the risk factor in practice [Burgess et al., 2012]. These issues are discussed in
274 greater detail in Burgess et al. [2016] and Bowden et al. [2017].

275 In the context of mediation, potential inconsistencies in genetic association estimates
276 from different sources are more important. In univariable Mendelian randomization, if the
277 genetic associations with the risk factor are misspecified, then the inverse-variance weighted
278 estimate is still a weighted sum of the genetic associations with the outcome, and should
279 differ from zero when the instrumental variable assumptions are satisfied if and only if there
280 is a causal effect of the risk factor on the outcome. However in multivariable Mendelian
281 randomization, if genetic associations with the mediator are misspecified, then adjustment
282 for genetic associations with the mediator may not fully attenuate the coefficient in the
283 weighted regression for the effect of the risk factor even in the case of complete mediation.
284 Multiplying genetic associations by a constant would not affect the significance of coefficients
285 in the weighted regression, hence any differences between populations that would lead to
286 consistent over- or underestimation of genetic associations for all variants should not influence
287 inferences from the methods presented here. However, differences that lead to inconsistent
288 over- or underestimation of genetic associations would adversely affect causal inferences.
289 Therefore, genetic associations should be estimated in as similar populations as possible.

290 **Binary variables and non-linear relationships**

291 It is common for the outcome in a Mendelian randomization investigation to be a binary
292 variable, such as disease status. In this case, typically genetic associations are obtained from
293 logistic regression, and represent log odds ratios. Odds ratios are non-collapsible, meaning
294 that they do not average intuitively, and they depend on the choice of covariate adjust-
295 ment even in the absence of confounding (so conditional odds ratios differ in magnitude to
296 marginal odds ratios) [Greenland et al., 1999]. This means that differences between causal

297 estimates from equations (1) and (2) may arise due to non-collapsibility rather than media-
298 tion. However, these differences are likely to be slight [Burgess, 2017]. In practice, as in the
299 applied example considered in this paper, we would recommend providing estimates of the
300 total and direct effects, but not the indirect effect, as calculation of the indirect effect relies
301 on the linearity of the relationships that cannot occur with a binary outcome. The total and
302 direct effects still have interpretations as population-averaged causal effects (conditional on
303 the mediator for the direct effect), representing the average change in the outcome resulting
304 from intervening on the population distribution of the risk factor (while keeping the medi-
305 ator constant for the direct effect) [Burgess and CHD CRP Genetics Collaboration, 2013].
306 Substantial differences between these estimates would still be informative about the causal
307 pathway from the risk factor to the outcome.

308 Similarly, if there is a non-linear relationship between the risk factor and outcome, the
309 causal effects still have an interpretation as population-averaged causal effects, representing
310 the average change in the outcome resulting from intervening on the population distribution
311 of the risk factor [Burgess et al., 2014]. Again, we would recommend reporting a total effect
312 and a direct effect, but not an indirect effect.

313 In conclusion, we hope that the methods outlined in this manuscript will be used widely
314 in assessing and understanding causal pathways and mechanisms.

Conflicts of interest: None.

Acknowledgements: This study would not have been possible without the contributions of the following: Per Hall (COGS); Douglas F. Easton, Paul Pharoah, Kyriaki Michailidou, Manjeet K. Bolla, Qin Wang (BCAC), Andrew Berchuck (OCAC), Rosalind A. Eeles, Douglas F. Easton, Ali Amin Al Olama, Zsofia Kote-Jarai, Sara Benlloch (PRACTICAL), Georgia Chenevix-Trench, Antonis Antoniou, Lesley McGuffog, Fergus Couch and Ken Offit (CIMBA), Joe Dennis, Alison M. Dunning, Andrew Lee, and Ed Dicks, Craig Luccarini and the staff of the Centre for Genetic Epidemiology Laboratory, Javier Benitez, Anna Gonzalez-Neira and the staff of the CNIO genotyping unit, Jacques Simard and Daniel C. Tessier, Francois Bacot, Daniel Vincent, Sylvie LaBoissière and Frederic Robidoux and the staff of the McGill University and Génome Québec Innovation Centre, Stig E. Bojesen, Sune F. Nielsen, Borge G. Nordestgaard, and the staff of the Copenhagen DNA laboratory, and Julie M. Cunningham, Sharon A. Windebank, Christopher A. Hilker, Jeffrey Meyer and the staff of Mayo Clinic Genotyping Core Facility

Funding for the iCOGS infrastructure came from: the European Community's Seventh Framework Programme under grant agreement number 223175 (HEALTH-F2-2009-223175) (COGS), Cancer Research UK (C1287/A10118, C1287/A 10710, C12292/A11174, C1281/A12014, C5047/A8384, C5047/A15007, C5047/A10692), the National Institutes of Health (CA128978) and Post-Cancer GWAS initiative (1U19 CA148537, 1U19 CA148065 and 1U19 CA148112 - the GAME-ON initiative), the Department of Defence (W81XWH-10-1-0341), the Canadian Institutes of Health Research (CIHR) for the CIHR Team in Familial Risks of Breast Cancer, Komen Foundation for the Cure, the Breast Cancer Research Foundation, and the Ovarian Cancer Research Fund.

Stephen Burgess is supported by Sir Henry Dale Fellowship jointly funded by the Wellcome Trust and the Royal Society (Grant Number 204623/Z/16/Z).

References

Baer, H. J., Tworoger, S. S., Hankinson, S. E., and Willett, W. C. 2010. Body fatness at young ages and risk of breast cancer throughout life. *American Journal of Epidemiology*, 171(11):1183–1194.

Bowden, J., Davey Smith, G., and Burgess, S. 2015. Mendelian randomization with invalid instruments: effect estimation and bias detection through Egger regression. *International Journal of Epidemiology*, 44(2):512–525.

Bowden, J., Del Greco M, F., Minelli, C., Davey Smith, G., Sheehan, N., and Thompson, J. 2017. A framework for the investigation of pleiotropy in two-sample summary data Mendelian randomization. *Statistics in Medicine*, 36(11):1783–1802.

Burgess, S. 2017. Estimating and contextualizing the attenuation of odds ratios due to non-collapsibility. *Communications in Statistics – Theory and Methods*, 46(2):786–804.

Burgess, S., Butterworth, A., Malarstig, A., and Thompson, S. 2012. Use of Mendelian randomisation to assess potential benefit of clinical intervention. *British Medical Journal*, 345:e7325.

Burgess, S., Butterworth, A. S., and Thompson, S. G. 2013. Mendelian randomization analysis with multiple genetic variants using summarized data. *Genetic Epidemiology*, 37(7):658–665.

Burgess, S. and CHD CRP Genetics Collaboration 2013. Identifying the odds ratio estimated by a two-stage instrumental variable analysis with a logistic regression model. *Statistics in Medicine*, 32(27):4726–4747.

Burgess, S., Davies, N., Thompson, S., and EPIC-InterAct Consortium 2014. Instrumental variable analysis with a nonlinear exposure–outcome relationship. *Epidemiology*, 25(6):877–885.

Burgess, S., Dudbridge, F., and Thompson, S. G. 2015a. Re: “Multivariable Mendelian randomization: the use of pleiotropic genetic variants to estimate causal effects”. *American Journal of Epidemiology*, 181(4):290–291.

Burgess, S., Dudbridge, F., and Thompson, S. G. 2016. Combining information on multiple instrumental variables in Mendelian randomization: comparison of allele score and summarized data methods. *Statistics in Medicine*, 35(11):1880–1906.

Burgess, S., Scott, R., Timpson, N., Davey Smith, G., Thompson, S. G., and EPIC-InterAct Consortium 2015b. Using published data in Mendelian randomization: a blueprint for efficient identification of causal risk factors. *European Journal of Epidemiology*, 30(7):543–552.

Burgess, S. and Thompson, S. 2015a. Multivariable Mendelian randomization: the use of pleiotropic genetic variants to estimate causal effects. *American Journal of Epidemiology*, 181(4):251–260.

Burgess, S. and Thompson, S. G. 2015b. *Mendelian randomization: methods for using genetic variants in causal estimation*. Chapman & Hall.

Burgess, S. and Thompson, S. G. 2017. Interpreting findings from Mendelian randomization using the MR-Egger method. *European Journal of Epidemiology*. Available online before print. doi: 10.1007/s10654-017-0255-x.

Burgess, S., Thompson, S. G., and CRP CHD Genetics Collaboration 2011. Avoiding bias from weak instruments in Mendelian randomization studies. *International Journal of Epidemiology*, 40(3):755–764.

Collaborative Group on Hormonal Factors in Breast Cancer 2012. Menarche, menopause, and breast cancer risk: individual participant meta-analysis, including 118 964 women with breast cancer from 117 epidemiological studies. *The Lancet Oncology*, 13(11):1141–1151.

Davey Smith, G. and Ebrahim, S. 2003. ‘Mendelian randomization’: can genetic epidemiology contribute to understanding environmental determinants of disease? *International Journal of Epidemiology*, 32(1):1–22.

Davey Smith, G., Lawlor, D., Harbord, R., Timpson, N., Day, I., and Ebrahim, S. 2007. Clustered environments and randomized genes: a fundamental distinction between conventional and genetic epidemiology. *PLoS Medicine*, 4(12):e352.

Day, F., Thompson, D., Helgason, H., et al. 2017. Genomic analyses identify hundreds of variants associated with age at menarche and support a role for puberty timing in cancer risk. *Nature Gen*, 49:834–841.

Eppinga, R. N., Hagemeijer, Y., Burgess, S., et al. 2016. Identification of genomic loci associated with resting heart rate and shared genetic predictors with all-cause mortality. *Nature Genetics*, 48:1557–1563.

Greenland, S. 2000. An introduction to instrumental variables for epidemiologists. *International Journal of Epidemiology*, 29(4):722–729.

Greenland, S., Robins, J., and Pearl, J. 1999. Confounding and collapsibility in causal inference. *Statistical Science*, 14(1):29–46.

Guo, Y., Andersen, S. W., Shu, X.-O., et al. 2016. Genetically predicted body mass index and breast cancer risk: Mendelian randomization analyses of data from 145,000 women of European descent. *PLOS Medicine*, 13(8):e1002105.

Haycock, P. C., Burgess, S., Wade, K. H., Bowden, J., Relton, C., and Davey Smith, G. 2016. Best (but oft-forgotten) practices: the design, analysis, and interpretation of Mendelian randomization studies. *The American Journal of Clinical Nutrition*, 103(4):965–978.

Lawlor, D., Harbord, R., Sterne, J., Timpson, N., and Davey Smith, G. 2008. Mendelian randomization: using genes as instruments for making causal inferences in epidemiology. *Statistics in Medicine*, 27(8):1133–1163.

Locke, A. E., Kahali, B., Berndt, S. I., et al. 2015. Genetic studies of body mass index yield new insights for obesity biology. *Nature*, 518(7538):197–206.

Michailidou, K., Beesley, J., Lindstrom, S., et al. 2015. Genome-wide association analysis of more than 120,000 individuals identifies 15 new susceptibility loci for breast cancer. *Nature Genetics*, 47(4):373–380.

Pierce, B. and Burgess, S. 2013. Efficient design for Mendelian randomization studies: sub-sample and two-sample instrumental variable estimators. *American Journal of Epidemiology*, 178(7):1177–1184.

Thompson, S. and Sharp, S. 1999. Explaining heterogeneity in meta-analysis: a comparison of methods. *Statistics in Medicine*, 18(20):2693–2708.

VanderWeele, T. and Vansteelandt, S. 2009. Conceptual issues concerning mediation, interventions and composition. *Statistics and its Interface*, 2(4):457–468.

Supplementary Material

A.1 Technical discussion about estimation of indirect and direct effects

There are several versions of direct and indirect effects. We present definitions using counterfactual terminology, using potential values of the outcome $Y(x, m)$, representing the outcome which would be observed if X were set (by intervention) to x and M were set to m , and potential values of the mediator $M(x)$, the value taken by the mediator if X were set to x . All effects are given on the difference scale; with a binary outcome, effects on a relative risk or odds ratio scale can also be defined, but the decomposition is more complex [VanderWeele and Vansteelandt, 2010; Kaufman, 2010]. This text is adapted from Burgess et al. [2015].

A total effect is defined as the effect of a change in the exposure from, say, $X = x$ to $X = x + 1$. It comprises the effects of the change in the exposure, and the change in the mediator as a result of the change in the exposure:

$$TE(x, x + 1) = Y(x + 1, M(x + 1)) - Y(x, M(x)) \quad (A1)$$

A controlled direct effect is defined as the effect of a change in the exposure keeping the mediator fixed at a given level, say $M = m$ [Robins and Greenland, 1992; Pearl, 2001]. The controlled direct effect may depend on the choice of m :

$$CDE(m; x, x + 1) = Y(x + 1, m) - Y(x, m) \quad (A2)$$

A natural direct effect is defined as the effect of a change in the exposure with the mediator fixed at the level it would naturally take if the exposure were fixed at a given level, say $X = x$:

$$NDE(x; x, x + 1) = Y(x + 1, M(x)) - Y(x, M(x)) \quad (A3)$$

A natural indirect effect is defined as the effect of a change in the mediator from the value it would naturally take if the exposure were unchanged to the level it would take if the exposure were changed. The exposure itself is kept fixed at a given level, say $X = x + 1$:

$$NIE(x + 1; x, x + 1) = Y(x + 1, M(x + 1)) - Y(x + 1, M(x)) \quad (A4)$$

In the linear case, the natural direct and indirect effects represent a decomposition of the total effect, in that $TE(x, x + 1) = NDE(x; x, x + 1) + NIE(x + 1; x, x + 1)$ (or alternatively $TE(x, x + 1) = NDE(x + 1; x, x + 1) + NIE(x; x, x + 1)$). Under the condition:

$$Y(x + 1, m_1) - Y(x, m_1) = Y(x + 1, m_2) - Y(x, m_2) \quad (\text{A5})$$

for all values of $M = m_1, m_2$, and for all individuals, the controlled direct effect is equal to the natural direct effect [Robins and Greenland, 1992]. The natural direct effect has a clearer intuitive interpretation as a measure of mediation than the controlled direct effect. However, it is not possible to conceive of an experiment which would produce the natural direct effect, as the quantity requires the outcome if the exposure were set at two different levels (for example, in $NDE(x; x, x + 1)$, $Y(x + 1, M(x))$ requires $X = x + 1$ for Y , but $X = x$ for M). This is known as a “cross-world” quantity, as setting the exposure to two different values is only possible in two different worlds [Richardson and Robins, 2013].

As we argue in Burgess et al. [2015], we would regard the controlled direct effect as the quantity that is targeted by mediation analysis with instrumental variables, as this is what would be obtained if we were to intervene separately on the risk factor and mediator. As we assume that all relationships between variables are linear and there is no effect heterogeneity, the natural and controlled direct effects are equal, and hence we refer to a ‘direct effect’ throughout this manuscript without further qualification.

A.2 Software code

We provide R code to implement the methods discussed in this paper. The associations of the genetic variants with the risk factor are denoted **betaXG** with standard errors **sebetaXG**. The associations of the genetic variants with the mediator are denoted **betaMG** with standard errors **sebetaMG**. The associations of the genetic variants with the outcome are denoted **betaYG** with standard errors **sebetaYG**. When variables are continuous, these associations are typically estimated using linear regression.

Estimation of the total causal effect using summarized data:

```
total.effect = lm(betaYG ~ betaXG - 1, weights = sebetaYG^-2)$coef[1]
resid.std.error = summary(lm(betaYG ~ betaXG - 1, weights = sebetaYG^-2))$sigma
se.total.effect = summary(lm(betaYG ~ betaXG - 1, weights = sebetaYG^-2))$coef[1,2]
ci.upper.total = max(total.effect + qnorm(0.975) * se.total.effect / resid.std.error,
                      total.effect + qt(0.975, df=length(betaXG)-1) * se.total.effect)
ci.lower.total = min(total.effect - qnorm(0.975) * se.total.effect / resid.std.error,
                      total.effect - qt(0.975, df=length(betaXG)-1) * se.total.effect)
```

The weighted regression model for estimating the total effect is equivalent to a meta-analysis of the variant-specific causal estimates. Setting the residual standard error as 1 is equivalent to a fixed-effect assumption in the meta-analysis formula [Thompson and Sharp, 1999]. If there is no heterogeneity between the causal estimates identified by the individual variants, then the residual standard error should tend to 1 asymptotically. If the estimate of the residual standard error is greater than 1 (overdispersion), then we do not correct for this; this is equivalent to a (multiplicative) random-effects meta-analysis [Burgess and Thompson, 2017]. This would occur if different genetic variants identify different causal estimates (say, different variants influence the risk factor via different mechanisms). However, there is no biological rationale for underdispersion (residual standard error estimate is less than 1). Hence, we correct for underdispersion by dividing the standard error for the total effect by the residual standard error.

The multiplicative random-effects analysis fits the following model, with ϕ representing the residual standard error:

$$\hat{\beta}_{Yj} = \theta_T \hat{\beta}_{Xj} + \epsilon_{Tj}, \quad \epsilon_{Tj} \sim \mathcal{N}(0, \phi^2 \text{se}(\hat{\beta}_{Yj})^2). \quad (\text{A6})$$

For a fixed-effect analysis, the residual standard error is assumed to be known; hence it is appropriate to use a normal distribution for inferences. For a random-effect analysis,

as the residual standard error (the overdispersion parameter ϕ) is estimated rather than known, a t-distribution should be used for making inferences. In the confidence intervals, we take the upper bound to be the maximum of the bounds based on the fixed-effect and random-effect analyses; similarly for the lower bound as the minimum. This ensures that confidence intervals are no wider than they would be from a fixed-effect analysis, but that under-precision is not doubly penalized (by setting the residual standard error to be 1, and then using a t-distribution for inferences).

Estimation of the direct causal effect using summarized data:

```
direct.effect = lm(betaYG ~ betaXG + betaMG - 1, weights = sebetaYG^-2)$coef[1]
se.direct.effect = summary(lm(betaYG ~ betaXG + betaMG - 1, weights = sebetaYG^-2))$coef[1,2]/
  min(summary(lm(betaYG ~ betaXG + betaMG - 1, weights = sebetaYG^-2))$sigma, 1)
ci.upper.direct = max(direct.effect + qnorm(0.975) * se.direct.effect / resid.std.error,
  direct.effect + qt(0.975, df=length(betaXG)-1) * se.direct.effect)
ci.lower.direct = min(direct.effect - qnorm(0.975) * se.direct.effect / resid.std.error,
  direct.effect - qt(0.975, df=length(betaXG)-1) * se.direct.effect)
```

As the additional term in the regression analysis for the estimate of the direct effect lowers the residual standard error, we take the estimated residual standard error from the regression model for the total causal effect. This is because we want this term to represent overdispersion in the genetic associations with the outcome, not the residual associations after adjustment. Hence the t-distribution for making inferences is still on $J - 1$ degrees of freedom.

If the outcome is binary, then genetic associations with the outcome are typically estimated using logistic regression. Beta-coefficients from logistic regression can be used in the estimation of direct and indirect effects, but the precise magnitude of effect estimates should not be over-interpreted, as odds ratios suffer from non-collapsibility when the rare disease assumption is not applicable (instrumental variable estimates represent population-averaged causal effects, which are not the same as subject-specific causal effects on the odds ratio scale, hence the indirect and direct effects may not precisely sum to give the total effect). Therefore in the applied example in this paper, we do not report an indirect effect.

With correlated variants, this correlation can be accounted for by generalized weighted linear regression [Burgess et al., 2016]. We assume that `rho` is the matrix of correlations between genetic variants:

```
Omega = sebetaYG%o%sebetaYG*rho
total.effect.correl = solve(t(betaXG)/*%solve(Omega)/*%betaXG)*t(betaXG)/*%solve(Omega)/*%betaYG
```

```
se.total.effect.fixed  = sqrt(solve(t(betaXG) %*% solve(Omega) %*% betaXG))
resid.total           = betaYG-total.effect.correl*betaXG
se.total.effect.random = sqrt(solve(t(betaXG) %*% solve(Omega) %*% betaXG))*
                         max(sqrt(t(resid.total) %*% solve(Omega) %*% resid.total/(length(betaXG)-1)),1)
direct.effect.correl  = solve(t(cbind(betaXG, betaMG)) %*% solve(Omega) %*%
                         cbind(betaXG, betaMG)) %*% t(cbind(betaXG, betaMG)) %*% solve(Omega) %*% betaYG
se.direct.effect.fixed = sqrt(solve(t(cbind(betaXG, betaMG)) %*% solve(Omega) %*% cbind(betaXG, betaMG))[1,1])
resid.direct           = betaYG-direct.effect.correl[1]*betaXG-direct.effect.correl[2]*betaMG
se.direct.effect.random = sqrt(solve(t(cbind(betaXG, betaMG)) %*% solve(Omega) %*% cbind(betaXG, betaMG))[1,1])*
                           max(sqrt(t(resid.direct) %*% solve(Omega) %*% resid.direct/(length(betaXG)-2)),1)
```

Standard errors are given corresponding to both fixed-effect and random-effects assumptions.

A.3 Additional details of simulation study

For the simulation study in the paper, the risk factor X was generated as:

$$X_i = \sum_{j=1}^{10} \alpha_j G_{ij} + U_i + \epsilon_{X_i}$$

where G_{ij} is the number of variant alleles for genetic variant j , U is a confounder, ϵ_{X_i} is an independent error term. The number of variant alleles for each variant was drawn from a binomial distribution with 2 trials and probability 0.3, representing a single nucleotide polymorphism with minor allele frequency 0.3. The genetic effects on the risk factor α_j were generated from a normal distribution with mean 0.2 and variance 0.1². The variants in total explained on average 5.1% of the variance in the risk factor, corresponding to an average F statistic of 53.5 with a sample size of 10 000. The confounder U and all error terms ($\epsilon_X, \epsilon_M, \epsilon_Y$) were drawn from independent standard normal distributions. The mediator M was generated as:

$$M_i = \theta_1 X_i + U_i + \epsilon_{Mi} + \sum_{j=1}^{10} \phi_j G_{ij}$$

where θ_1 is the causal effect of X on M , and ϕ_j are direct effects of the genetic variants on the mediator. These effects are included in the simulation model to ensure that the direct effect is identified, as otherwise genetic associations with the risk factor and mediator would be perfectly correlated for large sample sizes, leading to unstable estimates of the direct effect. The ϕ_j parameters were generated from a normal distribution with mean zero and variance 0.1². The outcome Y was generated as:

$$Y_i = \theta_2 X_i + \theta_3 M_i + U_i + \epsilon_{Y_i}$$

where θ_2 is the direct effect of X on Y , and θ_3 is the effect of M on Y . The indirect effect of X on Y via M is $\theta_1 \theta_3$, and the total effect of X on Y is $\theta_2 + \theta_1 \theta_3$. In total, 10 000 simulated datasets were generated for each choice of parameter values.

We experimented with different values of the variance of the ϕ_j parameters in the data-generating model. Results are shown in Supplementary Table A1. When there was low heterogeneity, estimates were more variable and bias from weak instruments was more pronounced. This is expected, as the associations with the risk factor and mediator are increas-

ingly collinear as the heterogeneity decreases. To demonstrate that the bias is an artifact of limited sample size (so called ‘weak instrument bias’), we repeated the simulation with 100 000 participants (100 iterations per scenario only). As expected, bias did not decrease when there was no heterogeneity, as the collinearity problem does not disappear with increasing sample sizes in this case. However, in all other cases, increasing the sample size decreased bias sharply.

Sample size: 10 000						
θ_1	θ_2	θ_3	$\text{var}(\phi) = 0$	$\text{var}(\phi) = 0.05^2$	$\text{var}(\phi) = 0.1^2$	$\text{var}(\phi) = 0.2^2$
0.3	0.2	1	0.054 (0.110)	0.165 (0.077)	0.196 (0.055)	0.203 (0.050)
0.3	0.2	-1	0.052 (0.115)	0.165 (0.076)	0.195 (0.056)	0.204 (0.050)
0.3	-0.2	1	-0.343 (0.113)	-0.235 (0.071)	-0.205 (0.059)	-0.194 (0.050)
-0.3	-0.2	1	-0.049 (0.101)	-0.153 (0.073)	-0.181 (0.058)	-0.187 (0.052)
0.0	0.2	1	0.205 (0.041)	0.207 (0.048)	0.208 (0.048)	0.207 (0.048)
0.3	0.2	0	0.053 (0.106)	0.168 (0.074)	0.196 (0.058)	0.203 (0.053)
0.3	0.0	1	-0.146 (0.113)	-0.035 (0.071)	-0.004 (0.059)	0.003 (0.050)
-0.2	0.2	1	0.302 (0.076)	0.235 (0.057)	0.213 (0.050)	0.210 (0.048)

Sample size: 100 000						
θ_1	θ_2	θ_3	$\text{var}(\phi) = 0$	$\text{var}(\phi) = 0.05^2$	$\text{var}(\phi) = 0.1^2$	$\text{var}(\phi) = 0.2^2$
0.3	0.2	1	0.053 (0.092)	0.191 (0.027)	0.200 (0.019)	0.201 (0.016)
0.3	0.2	-1	0.051 (0.114)	0.194 (0.030)	0.195 (0.019)	0.202 (0.016)
0.3	-0.2	1	-0.341 (0.098)	-0.206 (0.028)	-0.197 (0.016)	-0.198 (0.015)
-0.3	-0.2	1	-0.049 (0.087)	-0.191 (0.027)	-0.197 (0.020)	-0.202 (0.017)
0.0	0.2	1	0.199 (0.012)	0.202 (0.016)	0.199 (0.016)	0.200 (0.016)
0.3	0.2	0	0.055 (0.106)	0.196 (0.027)	0.199 (0.019)	0.200 (0.017)
0.3	0.0	1	-0.136 (0.099)	-0.005 (0.027)	0.003 (0.018)	0.000 (0.017)
-0.2	0.2	1	0.296 (0.072)	0.206 (0.018)	0.200 (0.018)	0.200 (0.016)

Supplementary Table A1: Mean (standard deviation) of multivariable Mendelian randomization estimates of the direct effect θ_2 across 10 000 simulated datasets (100 datasets for larger sample size) for different values of the variance of the heterogeneity parameters ϕ .

A.4 Additional simulation scenario: bidirectional causal effects between risk factor and mediator

In the applied example, it may be that as well as the risk factor having a causal effect on the mediator, that the mediator also has a causal effect on the risk factor. To consider this scenario, we simulate causal effects in both directions and consider Mendelian randomization and multivariable Mendelian randomization estimates. The data-generating model is:

$$X_{0i} = \sum_{j=1}^{10} \alpha_j G_{ij} + U_i + \epsilon_{X_i}$$

$$M_i = \theta_1 X_{0i} + U_i + \epsilon_{Mi} + \sum_{j=1}^{10} \phi_j G_{ij}$$

$$X_{1i} = X_{0i} \pm M_i$$

$$Y_i = \theta_2 X_{1i} + \theta_3 M_i + U_i + \epsilon_{Y_i}$$

This is the same as the previous data-generating model, except that we first generate X_{0i} and then generate a second risk factor variable X_{1i} that has a causal effect from the mediator. These could be thought of as values of the risk factor at different time points. We consider cases where the mediator has a positive and a negative effect on the risk factor. All other aspects of this simulation are the same as the original.

Results are shown in Supplementary Table A2. The total effect varies depending on whether the effect of the mediator on the risk factor is positive or negative, and is not simply an estimate of $\theta_2 + \theta_1\theta_3$ (as there are additional components of the total effect via the effect of the mediator on the risk factor). However, the direct effect as estimated by multivariable Mendelian randomization is invariant to any bidirectional effect. Therefore the direct effect of age at menarche on breast cancer risk not via BMI can be estimated using multivariable Mendelian randomization whether or not there is a bidirectional relationship between age at menarche and BMI.

θ_1	θ_2	θ_3	Positive effect		Negative effect	
			Univariable	Multivariable	Univariable	Multivariable
0.3	0.2	1	0.525	0.195	0.222	0.195
0.3	0.2	-1	-0.103	0.194	0.173	0.194
0.3	-0.2	1	0.123	-0.204	-0.169	-0.204
-0.3	-0.2	1	-0.195	-0.180	-0.504	-0.180
0.0	0.2	1	0.381	0.208	0.045	0.208
0.3	0.2	0	0.209	0.195	0.197	0.195
0.3	0.0	1	0.323	-0.005	0.017	-0.005
-0.2	0.2	1	0.273	0.217	-0.060	0.217

Supplementary Table A2: Mean of univariable and multivariable Mendelian randomization estimates across 10 000 simulated datasets for different mediation scenarios with positive and negative bidirectional effect of the mediator on the risk factor.

Supplementary References

Burgess, S., Daniel, R., Butterworth, A., Thompson, S., and EPIC-InterAct Consortium 2015. Network Mendelian randomization: extending instrumental variable techniques. *International Journal of Epidemiology*, 44(2):484–495.

Burgess, S., Dudbridge, F., and Thompson, S. G. 2016. Combining information on multiple instrumental variables in Mendelian randomization: comparison of allele score and summarized data methods. *Statistics in Medicine*, 35(11):1880–1906.

Burgess, S. and Thompson, S. G. 2017. Interpreting findings from Mendelian randomization using the MR-Egger method. *European Journal of Epidemiology*. Available online before print. doi: 10.1007/s10654-017-0255-x.

Kaufman, J. S. 2010. Invited commentary: Decomposing with a lot of supposing. *American Journal of Epidemiology*, 172(12):1349–1351.

Pearl, J. 2001. Direct and indirect effects. In *Proceedings of the Seventeenth Conference on Uncertainty in Artificial Intelligence*, pages 411–420.

Richardson, T. and Robins, J. 2013. Single World Intervention Graphs (SWIGs): A unification of the counterfactual and graphical approaches to causality. Technical Report 128, Center for Statistics and the Social Sciences, University of Washington.

Robins, J. and Greenland, S. 1992. Identifiability and exchangeability for direct and indirect effects. *Epidemiology*, 3(2):143–155.

Thompson, S. and Sharp, S. 1999. Explaining heterogeneity in meta-analysis: a comparison of methods. *Statistics in Medicine*, 18(20):2693–2708.

VanderWeele, T. and Vansteelandt, S. 2010. Odds ratios for mediation analysis for a dichotomous outcome. *American Journal of Epidemiology*, 172(12):1339–1348.

Figure titles and legends

Figure 1. Title: Graphical representation of mediation scenario. Caption: Total effect of risk factor on outcome comprises an indirect effect (hollow arrows) via mediator, and a direct effect (solid arrow) via other pathways.

Figure 2. Title: Graphical representation of relationships between variables. Caption: Graphical diagram of relationships between risk factor (X), mediator (M), outcome (Y) and genetic variant (G_j). Causal relationships between variables are indicated by solid lines. Associations of the genetic variant are indicated by dashed lines. The direct effect $\theta_D = \theta_2$. The indirect effect $\theta_I = \theta_1\theta_3$. The total effect $\theta_T = \theta_D + \theta_I = \theta_2 + \theta_1\theta_3$.

Table title and legend

Table 1. Title: Simulation study results. Caption: Mean, bias, standard deviation (SD), and coverage of 95% confidence interval (%) of univariable and multivariable Mendelian randomization estimates across 10 000 simulated datasets for different mediation scenarios (X = risk factor, M = mediator, Y = outcome).

θ_1 ($X \rightarrow M$)	θ_2 ($X \rightarrow Y$)	θ_3 ($M \rightarrow Y$)	Total effect	Direct effect	Univariable (total effect)				Multivariable (direct effect)			
			$(\theta_2 + \theta_1\theta_3)$	(θ_2)	Mean	Bias	SD	Coverage	Mean	Bias	SD	Coverage
0.3	0.2	1	0.5	0.2	0.518	0.018	0.166	94.5	0.194	-0.006	0.059	94.4
0.3	0.2	-1	-0.1	0.2	-0.098	0.012	0.154	94.6	0.195	-0.005	0.058	94.6
0.3	-0.2	1	0.1	-0.2	0.114	0.014	0.165	94.7	-0.206	-0.006	0.057	95.0
-0.3	-0.2	1	-0.5	-0.2	-0.480	0.020	0.167	94.8	-0.179	0.021	0.057	93.2
0.0	0.2	1	0.2	0.2	0.217	0.017	0.167	94.6	0.208	0.008	0.047	94.2
0.3	0.2	0	0.2	0.2	0.208	0.008	0.045	94.4	0.195	-0.005	0.057	97.3
0.3	0.0	1	0.3	0.0	0.318	0.018	0.167	94.6	-0.005	-0.005	0.058	95.1
-0.2	0.2	1	0.0	0.2	0.015	0.015	0.166	94.8	0.216	0.016	0.051	93.7

Table 1: Mean, bias, standard deviation (SD), and coverage of 95% confidence interval (%) of univariable and multivariable Mendelian randomization estimates across 10 000 simulated datasets for different mediation scenarios (X = risk factor, M = mediator, Y = outcome).

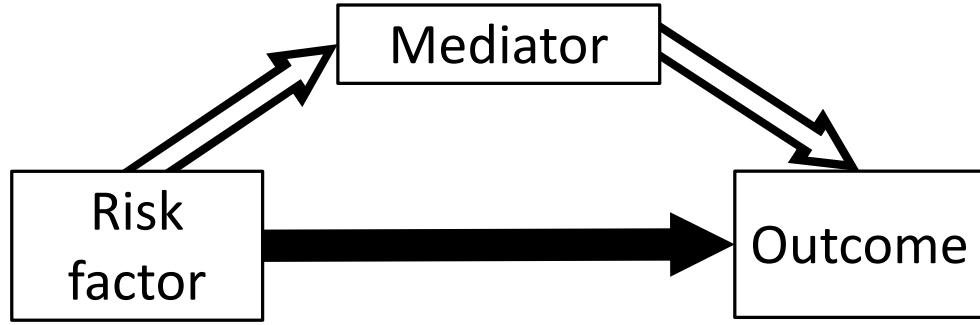


Figure 1: Total effect of risk factor on outcome comprises an indirect effect (hollow arrows) via mediator, and a direct effect (solid arrow) via other pathways.

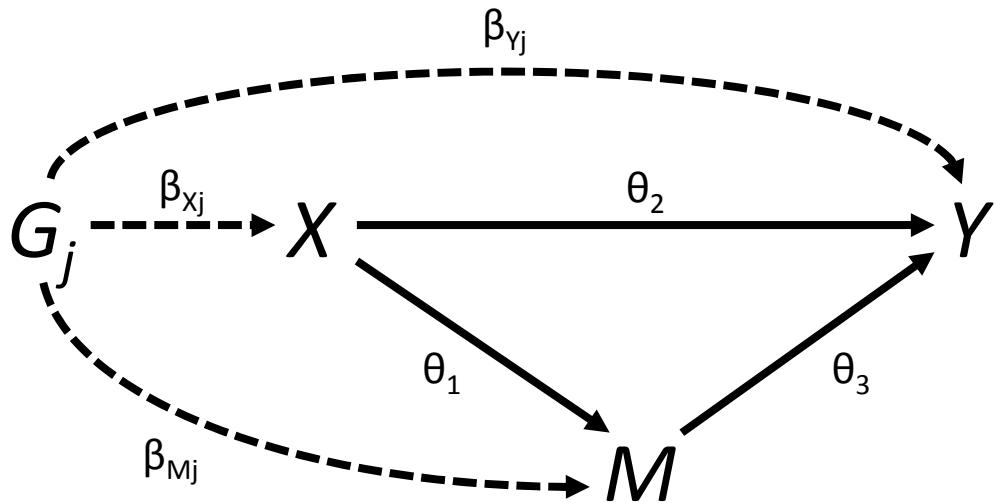


Figure 2: Graphical diagram of relationships between risk factor (X), mediator (M), outcome (Y) and genetic variant (G_j). Causal relationships between variables are indicated by solid lines. Associations of the genetic variant are indicated by dashed lines. The direct effect $\theta_D = \theta_2$. The indirect effect $\theta_I = \theta_1\theta_3$. The total effect $\theta_T = \theta_D + \theta_I = \theta_2 + \theta_1\theta_3$.