

# 1 Thermodynamics determines the coupling 2 between growth and byproduct production

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10 Keywords: Thermodynamics; Metabolic Engineering; Mixed Integer Linear Programming; Bilevel  
11 Optimization

## 12 Abstract

13 Genetic manipulation of cells to couple byproduct production and growth rate is important in  
14 bioengineering and biotechnology. In this way, we can use growth rate as a selective  
15 pressure, where the mutants with higher growth have higher production capacity.  
16 Computational methods have been proposed to find knockouts that couple growth and  
17 byproduct production. However, none of these methods consider the energetic and  
18 thermodynamic feasibility of such knockout strategies. Furthermore, there is no  
19 computational study of how variations in metabolite concentrations affect the coupling  
20 between growth and byproduct formation. One of the computational methods to find  
21 knockouts that couple growth and byproduct formation is OptKnock. OptKnock is a bi-level  
22 optimization problem. Here, we integrated thermodynamic constraints into the bilevel  
23 formulation of OptKnock to create TOptKnock. We show that the computational efficiency  
24 of TOptKnock is comparable to that of OptKnock. TOptKnock can account for the  
25 thermodynamic viability of the knockouts and examine how variations in metabolite  
26 concentrations affect the coupling. We have shown that the coupling between growth and  
27 byproduct formation can change in response to variations in concentrations. Thus, a knockout  
28 strategy might be optimal for one intracellular condition but suboptimal for another. If  
29 metabolomics data are available, TOptKnock can search for optimal knockout interventions  
30 under the given condition. We also envision that the TOptKnock framework will help  
31 develop strategies for manipulating metabolite concentrations to couple growth and  
32 byproduct formation.

### 33      **Introduction**

34      Redesigning and engineering microorganisms to produce valuable biochemicals is an  
35      important goal of metabolic engineering. Assuming that microorganisms have evolved to  
36      maximize their growth, engineering approaches that genetically couple product formation and  
37      growth are more robust. In addition, because biomass formation is accompanied by product  
38      formation, microorganisms engineered by such approaches will produce more products over  
39      generations by maximizing their growth.

40      Many computational tools have been developed to find optimal strategies to enhance  
41      biochemical production in a host organism. Such methods can suggest strategies for adding  
42      heterologous genes<sup>1-3</sup>, performing gene knockouts<sup>4,5</sup>, and suppressing or activating native  
43      genes<sup>6</sup>. Many of these methods use constraint-based optimization, where an objective  
44      function is optimized, subject to physicochemical constraints.

45      Bilevel optimization, the optimization of two nested problems, is a popular  
46      framework for designing strains with improved product yield. This structure allows searching  
47      for optimal interventions with the outer problem while considering that the organism  
48      optimizes its physiological objective (usually growth rate) with the inner problem. In this  
49      way, we find the interventions that couple product yield with biomass yield. Bilevel problems  
50      are nonlinear; however, some can be reformulated into linear problems, such as Linear  
51      Programming (LP) or Mixed-Integer Linear Programming (MILP). There are two  
52      reformulations for bilevel problems. Two reformulations exist for bilevel problems. One  
53      reformulation uses the strong duality<sup>7</sup>, and the other uses Karush-Kuhn-Tucker conditions<sup>8</sup>.

54      OptKnock is a bilevel method for finding reaction knockout strategies that couple  
55      biochemical production and growth rate<sup>7</sup>. OptKnock has been used to design mutant  
56      *Escherichia coli* to overproduce malonyl-CoA<sup>9</sup> and 1,4-butanediol<sup>10</sup> and mutant  
57      *Saccharomyces cerevisiae* to overproduce 2,3-butanediol<sup>11</sup>. Several other methods have been  
58      derived from OptKnock with variations in intervention strategy, such as reaction suppression  
59      and activation<sup>6</sup>, heterologous reaction addition<sup>12</sup>, and gene deletion<sup>4</sup>.

60      The OptKnock formulation includes constraints that account for (i) reaction removals,  
61      (ii) mass balances, (iii) reaction capacities, (iv) substrate availability, and (v) reaction  
62      reversibilities. The latter is imposed as they are in the Genome-scale Metabolic Models  
63      (GEMs). The reversibility of the reactions in a GEM is determined based on the standard  
64      Gibbs free energies or other available information about the directionality of the reactions<sup>13</sup>.  
65      However, assigning reaction directionality in this way can be inaccurate in some cases

66 because the thermodynamic properties of living organisms differ from the standard condition.  
67 Thermodynamic-based Flux Balance Analysis (TFA) is a formulation for determining  
68 directionalities based on Gibbs free energy under biological conditions<sup>14</sup>.

69 Here, we integrated thermodynamic constraints into the OptKnock formulation to  
70 develop TOptKnock. Then, we recast the nonlinear formulation of TOptKnock as a MILP  
71 and used it to find thermodynamically feasible knockout strategies to couple succinate  
72 production with growth rate in *E. coli* under anaerobic conditions. Finally, we investigated  
73 how the concentration of metabolic cofactors affects the coupling between growth and  
74 product secretion. We showed that the performance of a knockout strategy depends on  
75 metabolite concentrations and that different strategies may be optimal depending on the  
76 abundance of key metabolites. TOptKnock is the first bilevel formulation to consider  
77 thermodynamic feasibility, and its development paves the way for incorporating  
78 thermodynamics into other bilevel formulations.

## 79 Results

### 80 Computational performance

81 TOptKnock has additional variables compared to OptKnock, including metabolite  
82 concentrations, Gibbs free energies, and reaction directionalities. The number of constraints  
83 is also higher due to the thermodynamic constraints. Therefore, solving TOptKnock requires  
84 more computational resources. Table 1 lists the number of variables and constraints in the  
85 reformulated OptKnock and TOptKnock for iJO1366.

86 To increase the computational efficiency, we used a numerical trick. We constrained  
87 the slack variables for the dual problem (see Methods) to be in the range [0,1]. Based on the  
88 formulation, the slack variables are unbounded from above. However, random sampling of  
89 the slack variables in the dual problem showed that these variables are usually much smaller  
90 than 1. We observed that tightening the bounds on these variables significantly affected the  
91 solution time, such that the solution time of TOptKnock was comparable to the original  
92 OptKnock (Figure 1).

### 93 Using TOptKnock to find knockout strategies

94 We used TOptKnock to find knockout strategies to couple the biomass and product  
95 yield. Such knockout strategies are thermodynamically feasible due to the inclusion of  
96 thermodynamic constraints. As a case study, we investigated the overproduction of succinate  
97 in *E. coli* under anaerobic conditions. Wild-type *E. coli* scarcely produces succinate at its  
98 maximum growth rate (Figure 2). We generated mutant *E. coli* strains with single, double,  
99 and triple knockouts.

100 Removal of the fumarase reaction (FUM), which converts fumarate to L-malate in the  
101 TCA cycle, resulted in the only single-knockout mutant with higher succinate production at  
102 maximal growth rate (Figure 2). Increasing the number of reaction knockouts to two and  
103 three resulted in more mutant strains with significantly higher succinate yields. Most of these  
104 strategies focused on interrupting the conversion of phosphoenolpyruvate (PEP) to pyruvate  
105 and diverting PEP towards the reductive branch of the TCA cycle (Figure 2a).

106 In all mutant strains, succinate yield was improved at the expense of a reduced  
107 biomass yield. We defined the following score to find the knockout mutants with a good  
108 trade-off between the biomass and product yields (Figure 3):

$$OH = \mu_{\max} \sqrt{v_{\text{prod}}^{\max} v_{\text{prod}}^{\min}}$$

109 where  $\mu_{\max}$  is the maximum growth rate of the mutant strain. To find  $v_{\text{prod}}^{\max}$  and  $v_{\text{prod}}^{\min}$ , the  
110 growth was fixed at its maximum, i.e.,  $\mu_{\max}$ , and product (succinate) production was  
111 maximized and minimized, respectively. We included  $v_{\text{prod}}^{\min}$  in the definition of OH because  
112 higher values of  $v_{\text{prod}}^{\min}$  indicate that the coupling between the product and biomass production  
113 is forced. In other words, the mutant organism can only increase its growth rate by increasing  
114 the production of the product. The OH score for the single knockout mutant with FUM  
115 removed was 7.40. The double- and triple-knockout mutants had significantly higher OH  
116 scores (Figure 2b). The highest OH for the double-knockout mutants was 11.91, which was  
117 obtained by removing pyruvate kinase (PYK) and fructose-6-phosphate aldolase (F6PA).  
118 However, the highest OH, 131.89, was obtained for a triple knockout strategy in which  
119 succinate production was tightly coupled to biomass production (Figure 2b). This strategy  
120 prevented the flux from being diverted to the production of alternative byproducts. Removal  
121 of pyruvate formate lyase (PFL), D-lactate dehydrogenase (LDH\_D), and alcohol  
122 dehydrogenase (ALCD2x) blocked the production of formate, lactate, and ethanol,  
123 respectively.

#### 124 The impact of metabolite concentrations on the solution space

125 The intracellular concentration of key metabolites such as NADH and acetyl-CoA  
126 (AcCoA) has been shown to influence succinate production<sup>15,16</sup>. In addition, metabolomic  
127 analyses showed that the NAD<sup>+</sup>/NADH and NADP<sup>+</sup>/NADPH ratios differ significantly  
128 between aerobic and anaerobic conditions<sup>17-19</sup>. Mainly due to an inactive electron transfer  
129 chain, an increased level of NADH is observed under the anaerobic condition<sup>20</sup>. On the other  
130 hand, the reduced flux through the oxidative pentose phosphate pathway and the reduced  
131 need for protection against superoxide radicals caused a decrease in the level of NADPH<sup>20</sup>.  
132 AcCoA/CoA, however, remains almost unchanged<sup>20</sup>.

133 Such results show how metabolite concentrations can vary in response to changes in  
134 metabolism. We performed a sensitivity analysis to evaluate the optimality of different  
135 mutants to variations in metabolite concentrations. We generated three mutant strains using  
136 the knockout strategies found by TOptKnock to couple succinate production and growth: (i)  
137 strain  $\alpha$  contains PYK and GLCptspp knockouts, (ii) strain  $\beta$  contains PFL, LDH\_D, and  
138 ALCD2x knockouts, and (iii) strain  $\gamma$  contains PFL, PYK, and GLCptspp knockouts. These  
139 three mutants were the best double knockout (strain  $\alpha$ ), the best triple knockout (strain  $\beta$ ),  
140 and the second-best triple knockout (strain  $\gamma$ ) mutants. We then constrained the relative

141 concentrations of NAD<sup>+</sup>/NADH, AcCoA/CoA, and NADP<sup>+</sup>/NADPH to be within specified  
142 ranges.

143 In total, we considered 1000 different intracellular conditions, each specified by  
144 defining a range for the variation of the key cofactors (Supplementary Table S1). Figure 4  
145 shows the solution space with the highest OH score for each strain and the cofactor ratios for  
146 which this solution space is obtained. We assumed that succinate production and growth were  
147 tightly coupled if  $v_{\text{prod}}^{\text{min}}$  was more than 50% of  $v_{\text{prod}}^{\text{max}}$  under that condition, and  $v_{\text{prod}}^{\text{max}}$  was  
148 more than 50% of the maximum succinate production under all conditions. Strains  $\alpha$ ,  $\beta$ , and  
149  $\gamma$  showed tight coupling between succinate production and growth in 23, 34, and 52  
150 conditions (Supplementary Table S1), respectively, suggesting that the  $\gamma$  strain is more robust  
151 against perturbations in intracellular concentrations.

152 We specifically explored the solution space for four intracellular conditions (Table 2)  
153 by fixing the growth at different values and minimizing/maximizing the succinate production  
154 (Figure 5). These four conditions were chosen based on our sensitivity analysis  
155 (Supplementary Table S1) to demonstrate how an optimal solution under one condition might  
156 be suboptimal or non-optimal under another condition. The wild-type organism produces and  
157 secretes formate as the main byproduct. However, formate production is blocked in the  $\beta$  and  
158  $\gamma$  strains because both strains are knocked out for PFL, which in turn causes succinate to be  
159 an essential byproduct of growth under condition A. On the other hand, the  $\alpha$  strain can  
160 produce formate; thus, succinate production is not essential under this condition. We  
161 performed a variability analysis to find the reaction directions affected after imposing the  
162 concentration ranges. Such changes are shown in Supplementary Tables S2-S4.

163 Condition B, however, forced a much lower availability of AcCoA. As a result, only  
164 the reverse direction of the phosphotransacetylase reaction (PTAr) was thermodynamically  
165 feasible due to the positive Gibbs free energy. This adversely affected the ability of the cell to  
166 produce and secrete acetate and formate, even in the  $\alpha$  strain, leaving the cells no choice but  
167 to produce succinate. Thus, succinate production was tightly coupled to growth in all  
168 mutants in condition B.

169 Like condition B, condition C featured a low relative concentration of AcCoA, which  
170 helped couple growth and succinate production. However, condition C also had lower  
171 NAD<sup>+</sup>/NADH than condition B, and it has been previously reported that NADH  
172 accumulation negatively affects the growth rate<sup>16,21</sup>. As a result of the reduced growth rate,  
173 the coupling between growth and succinate production was not tight in the  $\beta$  strain (i.e.,

174  $v_{\text{prod}}^{\min}$  is close to zero). According to the model, the  $\beta$  strain can produce L-alanine from  
175 pyruvate as an alternative byproduct. On the other hand, strains  $\alpha$  and  $\gamma$  cannot produce L-  
176 alanine at maximum growth because the removal of GLCptspp and PYK interrupted the  
177 conversion of PEP to pyruvate in these strains. Variability analysis showed that reaction  
178 directionalities are identical between conditions B and C except for one reaction  
179 (Supplementary Tables S2-S4). The reaction with the changed directionality was 2Fe-2S  
180 regeneration (S2FE2SR) in the  $\alpha$  strain and Octanoate non-lipooylated apo domain ligase  
181 (OCTNLL) in the  $\beta$  and  $\gamma$  strains.

182 In condition D, the  $\text{NAD}^+/\text{NADH}$  ratio was the same as in condition C, but the  
183  $\text{NADP}^+/\text{NADPH}$  ratio was significantly lower. This caused changes in reaction  
184 directionalities such that  $v_{\text{prod}}^{\max}$  decreased strongly. Thus, succinate production and growth  
185 were not coupled in any of the strains under condition D.

186 We also examined the effect of changing the relative concentrations of each cofactor  
187 on the OH score (Figure 6). We observed no significant differences between different strains  
188 in response to the variation in  $\text{NAD}^+/\text{NADH}$  and  $\text{NADP}^+/\text{NADPH}$ . The middle ranges of  
189  $\text{NAD}^+/\text{NADH}$ , i.e.,  $1 \leq \ln \frac{\text{NAD}^+}{\text{NADH}} \leq 5$ , resulted in the highest OH scores. Succinate production  
190 was reduced at the higher ratios, while the organisms failed to grow at the lower ratios. On  
191 the other hand, we observed a switch-like behavior in response to the change in  
192  $\text{NADP}^+/\text{NADPH}$ , where at high ratios, i.e.,  $3 \leq \ln \frac{\text{NADP}^+}{\text{NADPH}}$ , the growth vanished.

193 Below a certain  $\text{AcCoA}/\text{CoA}$ , i.e.,  $\ln \frac{[\text{acetyl-CoA}]}{[\text{CoA}]} \leq -1$ , none of the strains could  
194 grow. At the higher ratios, however, the strains responded differently to the variations in  
195  $\text{AcCoA}/\text{CoA}$ . Strain  $\alpha$  had the highest OH scores for the range  $-1 \leq \ln \frac{[\text{acetyl-CoA}]}{[\text{CoA}]} \leq 5$ .  
196 Then, the OH score decreased by increasing the  $\text{AcCoA}/\text{CoA}$  since formate could be  
197 produced as the alternative byproduct when  $\text{AcCoA}$  was sufficiently available. In strains  $\beta$   
198 and  $\gamma$ , the OH score increased gradually by increasing the  $\text{AcCoA}/\text{CoA}$  due to the increase in  
199 the minimum and maximum succinate production.

200 **Discussion**

201 In this work, we integrated the thermodynamic constraints into the bilevel framework  
202 of OptKnock to create a new formulation called TOptKnock. We then recast the bilevel  
203 formulation of TOptKnock as a MILP that is solvable using conventional solvers with similar  
204 computational resources as the original OptKnock. TOptKnock searches for optimal  
205 knockout interventions that (i) are thermodynamically feasible and (ii) couple byproduct  
206 production and growth. We have shown that variations in the abundance of key metabolites  
207 can significantly affect the coupling between growth and byproduct formation, either by  
208 inhibiting growth or affecting the ability to produce the byproduct. The different behavior of  
209 the knockout strains under different metabolite concentrations indicates the importance of  
210 including thermodynamic constraints in the search for optimal interventions. We observed  
211 that a strategy may be optimal under one condition but suboptimal under another. We also  
212 observed that some knockout mutants are more robust to perturbations in metabolite  
213 concentrations.

214 In the TOptKnock formulation, the reaction directionalities are determined based on  
215 the metabolite concentrations. If metabolomics data are available, TOptKnock can find  
216 appropriate interventions for the current cellular state. Furthermore, the TOptKnock  
217 framework helps to develop strategies to manipulate metabolite concentrations instead of or  
218 in combination with gene knockouts to couple biomass and product yields. Finally, the  
219 integration of thermodynamic constraints into OptKnock paves the way for incorporating  
220 these constraints into other bilevel methods. This incorporation is of greater importance for  
221 methods that involve the addition of novel reactions to a host organism, as the directionality  
222 of such reactions in the host is usually not known<sup>1</sup>.

223 **Methods**

224 **Integration of thermodynamic constraints into the bilevel problem**

225 To determine the directionality of the reactions based on their corrected Gibbs free energy to  
 226 the biological condition, we integrated thermodynamic constraints to the OptKnock  
 227 formulation to construct TOptKnock:

$$\max_{y_j} (v_{prod}^+ - v_{prod}^-)$$

subject to:

$$\max_{v_j} (v_{growth}^+ - v_{growth}^-)$$

subject to:

$$\begin{aligned} \sum_j S_{i,j} (v_j^+ - v_j^-) &= 0 & \forall i \in \text{Met} \\ 0 \leq v_j^- \leq Mb_j^- & & \forall j \in \text{Rxn} \\ 0 \leq v_j^+ \leq Mb_j^+ & & \forall j \in \text{Rxn} \\ v_j^+ \leq M(1 - y_j) & & \forall j \in \text{Rxn} \\ v_j^- \leq M(1 - y_j) & & \forall j \in \text{Rxn} \\ v_{ATPM}^+ \geq V_{ATPM} & & \\ v_{Subs}^- \leq V_{upt} & & \\ -\Delta_r G'_j + RT \sum_{i=1}^m \eta_{ij} C_i & & \forall j \in \text{Rxn}_G \\ + \Delta_r G_j^{\circ} = 0 & & \\ \Delta_r G_j^{\circ} \leq M(1 - b_j^+) & & \forall j \in \{j | \text{UB}_j \\ & & = M\} \\ -\Delta_r G_j^{\circ} \leq M(1 - b_j^-) & & \forall j \in \{j | \text{LB}_j \\ & & = M\} \\ C_i^{LB} \leq C_i \leq C_i^{UB} & & \forall i \in \text{Met}_G \\ \Delta_r G_j^{\circ LB} \leq \Delta_r G_j^{\circ} \leq \Delta_r G_j^{\circ UB} & & \forall j \in \text{Rxn}_G \\ b_j^+, b_j^- \in \{0, 1\} & & \\ y_j \in \{0, 1\}, v_j^+, v_j^- \geq 0 & & \\ \sum_j y_j = K & & \end{aligned}$$

228 where  $C_i$  is the logarithmic concentration of metabolite  $i$ ,  $\Delta_r G_j^{\circ}$  is the Gibbs free energy of  
 229 reaction  $j$ . To account for the forward and backward directions, respectively, each flux is  
 230 represented by two non-negative variables  $v_j^+$  and  $v_j^-$ . Also, two binary variables  $b_j^+$  and  $b_j^-$   
 231 are added to ensure that only one direction is active.  $\text{Met}_G$  is the set of metabolites with

232 known Gibbs free energy of formation, and  $\text{Rxn}_G$  represents the reactions for which  
 233 thermodynamic constraint is applied.

234 **Reformulation of TOptKnock**

235 Like OptKnock, we used the strong duality theorem and added the dual constraints and  
 236 variables of the inner problem to recast ThermoOptKnock as a MILP. However, the  
 237 reformulation of TOptKnock is not as straightforward as OptKnock since the fluxes are split  
 238 into forward and backward directions, and additional binary variables are integrated to  
 239 determine the active directionality. We assumed that  $b_j^+$  and  $b_j^-$  are variables for the outer  
 240 problem but parameters for the inner problem. A similar assumption was made for reaction  
 241 knockout variables  $y_j$  in OptKnock<sup>7,22</sup>. The following is the reformulated TOptKnock:

$$\max_{y_j} (v_{prod}^+ - v_{prod}^-)$$

subject to:

$$\sum_j S_{i,j} (v_j^+ - v_j^-) = 0 \quad \forall i \in \text{Met} \quad (1)$$

$$v_j^+ \leq M(1 - y_j) \quad \forall j \in \text{Rxn} \quad (2)$$

$$v_j^- \leq M(1 - y_j) \quad \forall j \in \text{Rxn} \quad (3)$$

$$0 \leq v_j^+ \leq M b_j^+ \quad \forall j \in \{j | \text{UB}_j = M\} \quad (4)$$

$$0 \leq v_j^- \leq M b_j^- \quad \forall j \in \{j | \text{LB}_j = M\} \quad (5)$$

$$v_{\text{Subs}} \geq V_{\text{upt}}, v_{\text{ATPM}} \geq V_{\text{ATPM}} \quad (6)$$

$$\begin{aligned} v_{\text{growth}}^+ - v_{\text{growth}}^- &= V_{\text{upt}} \mu_{\text{Subs}}^{\text{UB}-} + \\ &V_{\text{ATPM}} \mu_{\text{ATPM}}^{\text{UB}+} - V_{\text{ATPM}} \mu_{\text{ATPM}}^{\text{LB}+} \end{aligned} \quad (7)$$

$$\sum_i S_{i,j} \lambda_i + \mu_j^{\text{UB}+} - \mu_j^{\text{LB}+} = 0 \quad \forall j \in \text{Rxn} \setminus \{\text{growth}\} \quad (8)$$

$$\sum_i S_{i,\text{growth}} \lambda_i - \mu_{\text{growth}}^{\text{LB}+} = 1 \quad (9)$$

$$\sum_i -S_{i,j} \lambda_i + \mu_j^{\text{UB}-} - \mu_j^{\text{LB}-} = 0 \quad \forall j \in \text{Rxn} \setminus \{\text{growth}\} \quad (10)$$

$$\sum_i -S_{i,\text{growth}} \lambda_i - \mu_{\text{growth}}^{\text{LB}-} = -1 \quad (11)$$

$$\mu_j^{\text{LB}+} \leq \mu_j^{\text{LB}+, \text{max}} (1 - b_j^+) \quad \forall j \in \{j | \text{UB}_j = M\} \quad (12)$$

$$\mu_j^{\text{LB}-} \leq \mu_j^{\text{LB}-, \text{max}} (1 - b_j^-) \quad \forall j \in \{j | \text{LB}_j = M\} \quad (13)$$

$$-\Delta_r G'_j + RT \sum_{i=1}^m \eta_{ij} C_i + \Delta_r G_j^{\circ} = 0 \quad \forall j \in \text{Rxn}_G \quad (14)$$

$$\Delta_r G'_j \leq M(1 - b_j^+) \quad \forall j \in \text{Rxn}_G \quad (15)$$

$$-\Delta_r G'_j \leq M(1 - b_j^-) \quad \forall j \in \text{Rxn}_G \quad (16)$$

$$0 \leq \mu_j^{\text{UB}+} \leq \mu_j^{\text{UB}+, \text{max}} y_j \quad \forall j \in \{j | \text{UB}_j = M\} \quad (17)$$

$$0 \leq \mu_j^{\text{UB}-} \leq \mu_j^{\text{UB}-, \text{max}} y_j \quad \forall j \in \{j | \text{LB}_j = M\} \quad (18)$$

$$\sum_j y_j = K \quad (19)$$

$$C_i^{\text{LB}} \leq C_i \leq C_i^{\text{UB}} \quad \forall i \in \text{Met}_G$$

$$\Delta_r G_j^{\circ \text{LB}} \leq \Delta_r G_j^{\circ} \leq \Delta_r G_j^{\circ \text{UB}} \quad \forall j \in \text{Rxn}_G$$

$$b_j^+, b_j^- \in \{0, 1\} \quad \forall j \in \text{Rxn}$$

$$\lambda_i \in \mathbb{R} \quad \forall i \in \text{Met}$$

$$0 \leq \mu_j^{\text{LB}+}, \mu_j^{\text{LB}-} \quad \forall j \in \text{Rxn}$$

$$y_j \in \{0, 1\} \quad \forall j \in \text{Rxn}$$

242 where Equations (1-6 are the primal constraints, Equation (7 enforces the equality of primal  
 243 and dual objectives, Equations (8-13 are the dual constraints, and Equations (14-19 are the  
 244 constraints of the outer problem. To increase the computational efficiency, we constrained  
 245  $\mu_j^{\text{LB}+}$ ,  $\mu_j^{\text{LB}-}$ ,  $\mu_j^{\text{UB}+}$ , and  $\mu_j^{\text{UB}-}$  to be in the range [0, 1], which highly reduced the searching  
 246 space without impacting the optimal solutions.

## 247 Setting up the model for the simulations

248 The latest version of iJO1366 was obtained from the BiGG database<sup>23</sup>. Uptake of all carbon  
 249 sources except glucose was blocked. The glucose uptake was constrained to be at most 100  
 250 mmol h<sup>-1</sup> gDW<sup>-1</sup>. The uptake of oxygen was blocked to simulate the anaerobic condition. The  
 251 lower bound of growth was set to 10% of the maximum growth rate to prevent lethal  
 252 knockout strategies. All simulations were performed in python 3.7 using the commercial  
 253 solver CPLEX.

## 254 Acknowledgements

255 We would like to thank *Dr. Ljubisa Miskovic* for his comments on improving the manuscript.  
 256 This project was funded by the Swiss National Science Foundation (SNSF): grant  
 257 200021\_188623, the European Union's Horizon 2020 research and innovation programme  
 258 under grant agreement No 814408, and the École Polytechnique Fédérale de Lausanne.

259 Table 1: Number of constraints and variables in OptKnock and TOptKnock

	Number of continuous variables	Number of binary variables	Number of constraints
OptKnock	12137	2583	12760
TOptKnock	18420	7749	29569

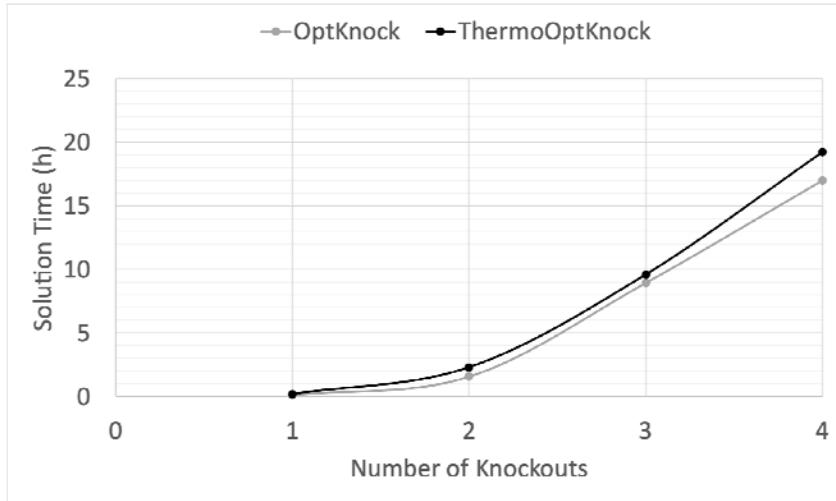
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261

262 Table 2: the four intracellular conditions defined by setting bounds on the relative concentrations of key metabolites

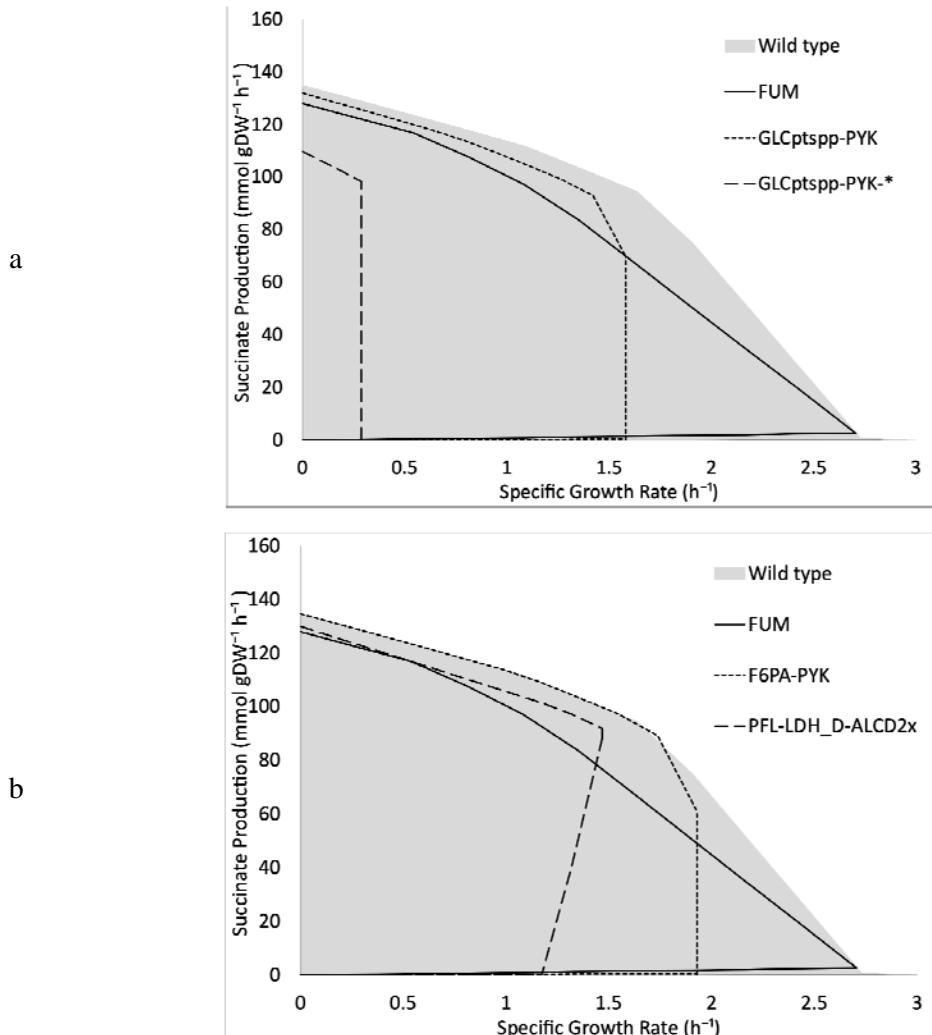
	$x = \ln \frac{[\text{AcCoA}]}{[\text{CoA}]}$	$y = \ln \frac{[\text{NAD}^+]}{[\text{NADH}]}$	$z = \ln \frac{[\text{NADP}^+]}{[\text{NADPH}]}$
Condition A	$7 \leq x \leq 9$	$3 \leq y \leq 5$	$-3 \leq z \leq -1$
Condition B	$1 \leq x \leq 3$	$3 \leq y \leq 5$	$-3 \leq z \leq -1$
Condition C	$1 \leq x \leq 3$	$1 \leq y \leq 3$	$-3 \leq z \leq -1$
Condition D	$7 \leq x \leq 9$	$1 \leq y \leq 3$	$1 \leq z \leq 3$

263

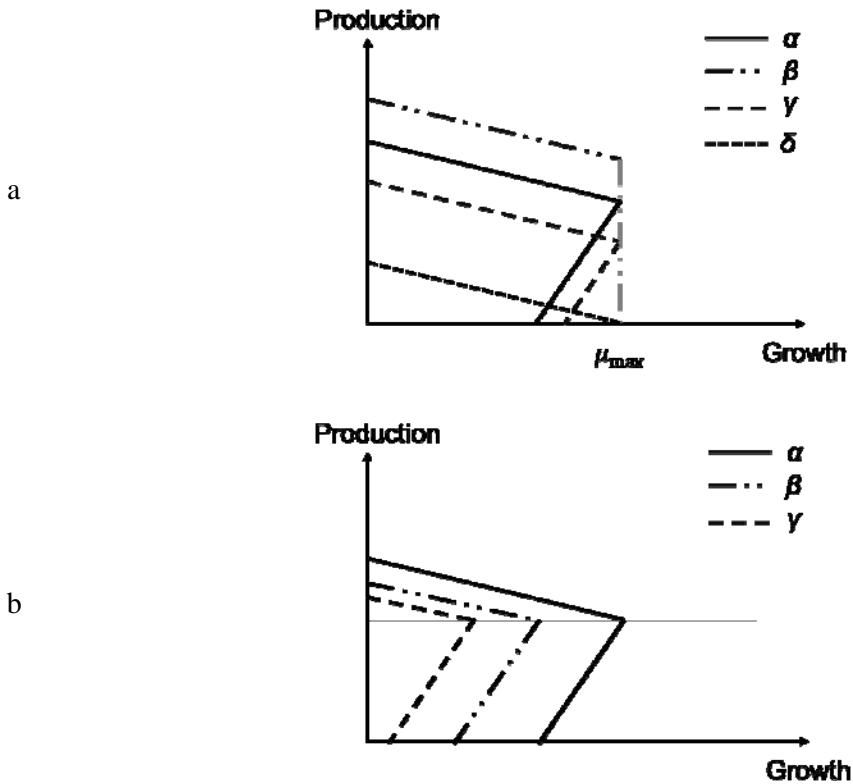


264

265 Figure 1: Comparison of the computational performance of OptKnock and TOptKnock. Despite having more constraints and  
266 variables, the solution time of TOptKnock was on par with the original OptKnock after tightening the upper bounds of slack  
267 variables in the dual problem.

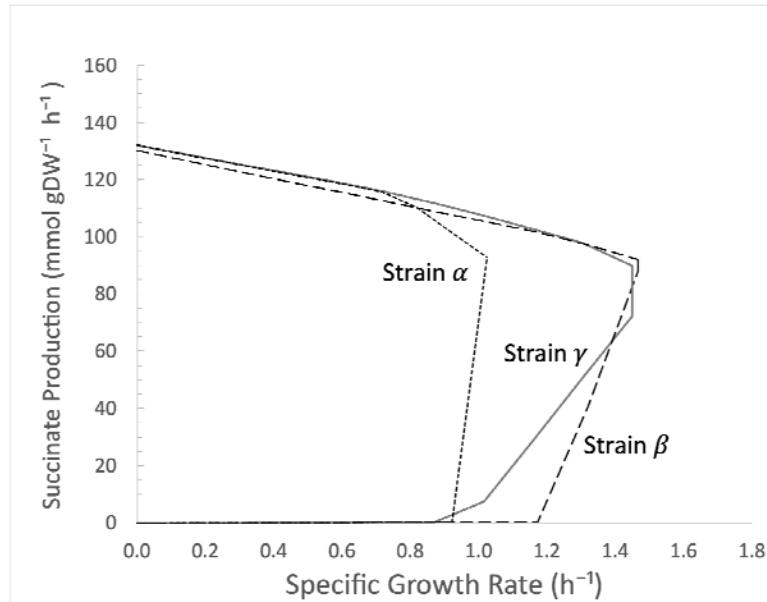


268  
269 Figure 2: The solution space of different mutant strains. (a) The mutants with the highest succinate production at the  
270 maximum growth are compared with the wild-type organism. The only single-knockout mutant with improved succinate  
271 production was obtained by removing fumarase (FUM). The strain with removed glucose transport through pyruvate  
272 phosphotransferase (GLCptspp) and pyruvate kinase (PYK) had the highest succinate production among the double-  
273 knockout mutants. A triple-knockout mutant with removed GLCptspp and PYK (GLCptspp-PYK-\*) in addition to either  
274 dihydroxyacetone phosphotransferase (DHAPT) or fructose 6-phosphate aldolase (F6PA) had the highest rate of succinate  
275 production. (b) The solution space of the mutant strains with the highest OH scores is compared with the wild-type  
276 organism. The strain with removed fructose 6-phosphate aldolase (F6PA) and pyruvate kinase (PYK) had the highest OH  
277 score (~11.91) among the double-knockout mutants. The highest OH score (~131.89) was obtained by the removal of  
pyruvate formate lyase (PFL), D-lactate dehydrogenase (LDH\_D), and alcohol dehydrogenase (ALCD2x).



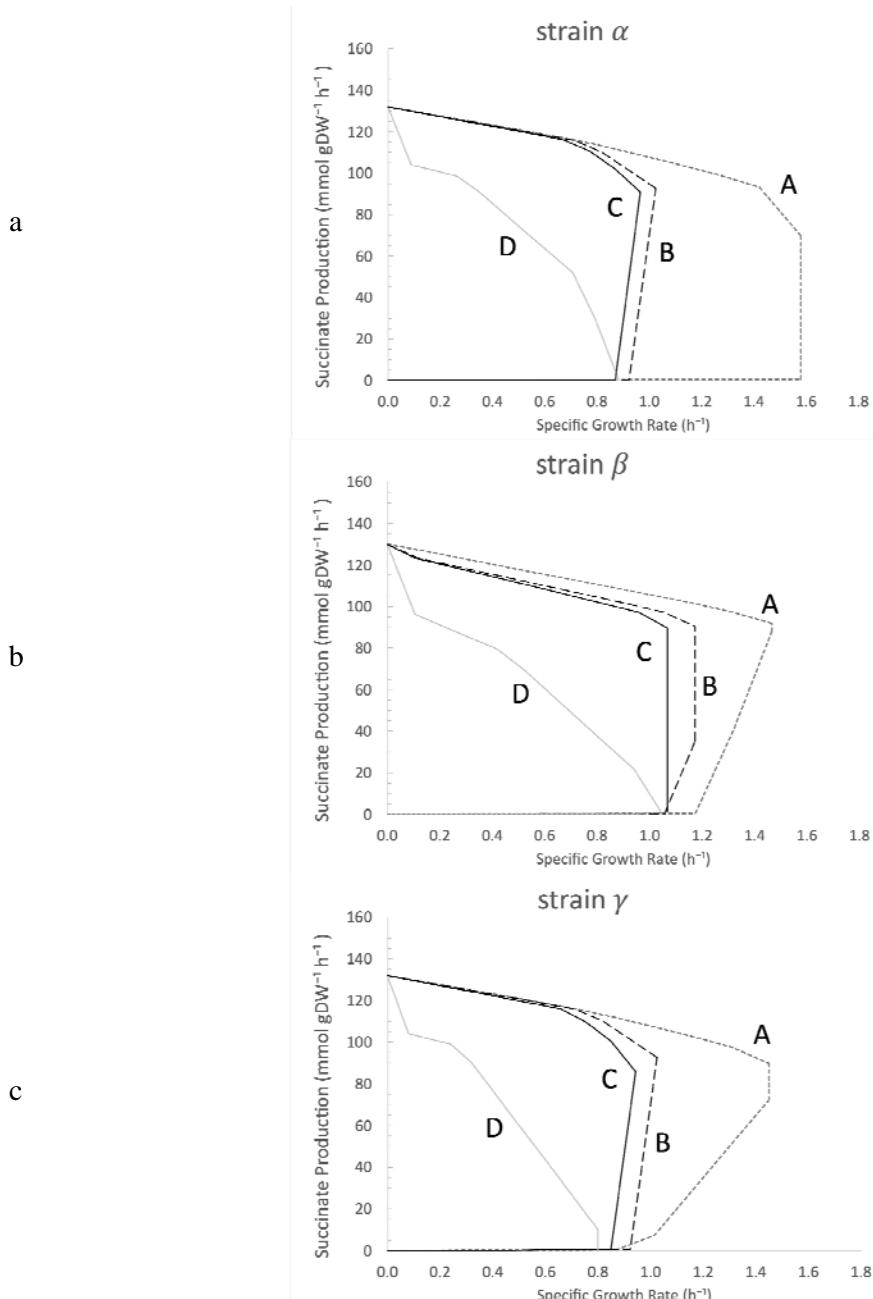
278

279 Figure 3: Schematic representation of the OH score. The OH score is defined to rank the solutions, where higher OH scores  
280 are preferred. The OH score can increase by increasing the maximum/minimum production rate or the  
281 maximum growth (  ). (a)    is the same, but the mutant  $\alpha$  has the highest OH score due to the higher   .  
282 The mutant  $\beta$  shows a higher maximum but lower minimum production rate; in the mutant  $\gamma$ , both minimum and maximum  
283 are lower than the mutant  $\alpha$ . The mutant  $\delta$  has the lowest OH score. (b)    is identical, but the mutant  $\alpha$  has the  
284 highest OH score due to the higher   . Similarly, the OH score for the mutant  $\beta$  is more than the mutant  $\gamma$ .

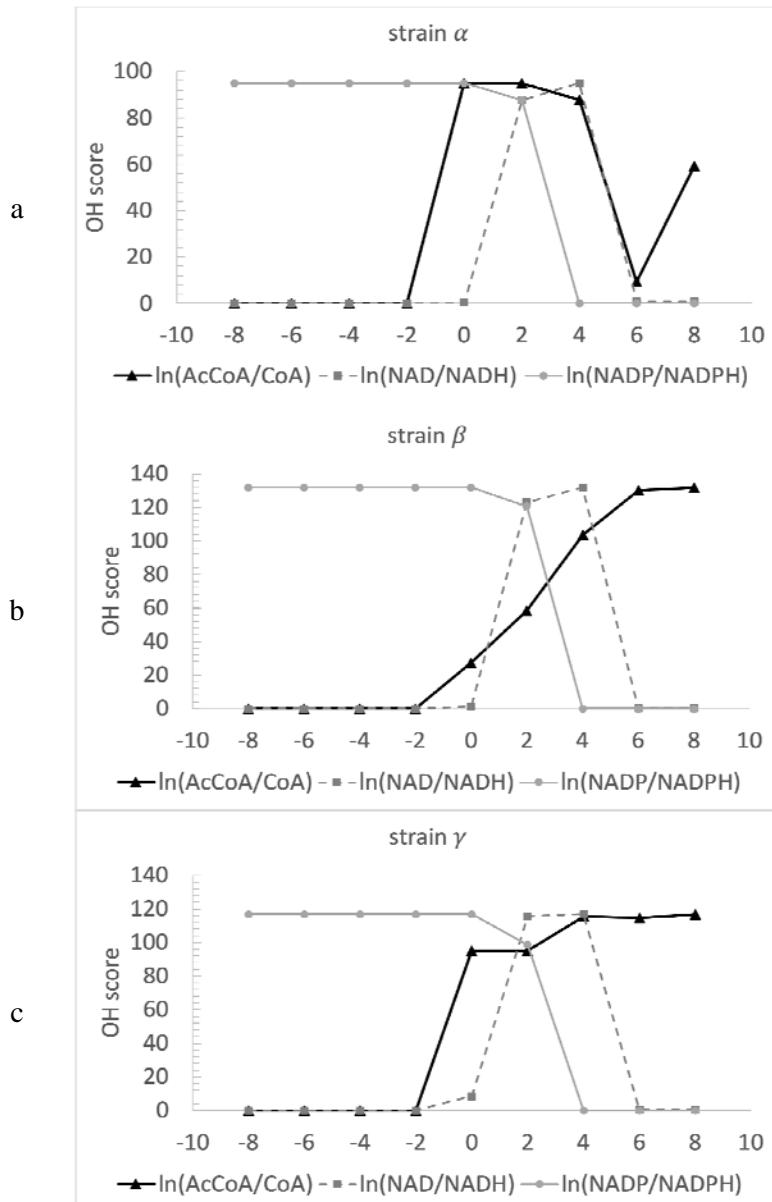


285

286 Figure 4: The optimal solution with the highest OH score for each strain. The highest OH score for the strain was obtained  
287 when 3 —, -1 —, and — . The highest OH score for the strain was obtained when  
288 3 —, 5 —, and — . The highest OH score for the strain was obtained when 1 —,  
289 3 —, and — .



290 Figure 5: The solution space of the mutant strains under different cellular conditions. (a) The strain was generated by  
 291 removing GLCptspp and PYK, which interrupted the PEP conversion to pyruvate. Since this strain could produce formate,  
 292 the succinate production was not tightly coupled to the growth under condition A. In conditions B and D, the AcCoA was  
 293 less available, adversely impacting the acetate and formate production. Therefore, succinate production was an essential  
 294 byproduct of the growth in these two conditions. (b) The strain was knocked out for PFL, LDH\_D, and ALCD2x. Under  
 295 conditions C and D, the NAD<sup>+</sup>/NADH was low, which reduced the maximum growth. NADP<sup>+</sup>/NADPH was high under  
 296 condition C, which diminished the capacity of the cell to produce succinate. The cell could produce succinate under  
 297 condition D due to the lower NADP<sup>+</sup>/NADPH. However, as the maximum growth rate was lower than the other conditions,  
 298 other byproducts could be secreted, and succinate production was not essential. (c) The strain was generated by removing  
 299 PFL, GLCptspp, and PYK. The removal of PFL blocked the formate production, and succinate was an essential byproduct of  
 300 growth under conditions A and B. The removal of GLCptspp and PYK interrupted the PEP conversion to pyruvate, which in  
 301 turn removed the capacity of the cell to produce other byproducts despite the reduced growth under condition D.



302 Figure 6: The variation in OH score in response to the changes in AcCoA/CoA, NAD<sup>+</sup>/NADH, and NADP<sup>+</sup>/NADPH ratios.  
303 (a) The strain showed a switch-like response to the changes in NADP<sup>+</sup>/NADPH. In response to the variation in  
304 NAD<sup>+</sup>/NADH, the OH had a peak for the middle ratios, i.e., 1 . The strain requires lower AcCoA  
305 availability to tightly couple the growth and succinate production since this strain is not knocked out for formate production.  
306 (b) The strain responded similarly to the strain to the variation in NAD<sup>+</sup>/NADH and NADP<sup>+</sup>/NADPH. Since the strain  
307 is knocked out for PFL, this strain cannot produce formate as an alternative byproduct. As a result, even at high AcCoA/CoA  
308 ratios, succinate production is tightly coupled to growth. (c) The strain showed a similar trend to the strain.

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