

1 **Evaluation of Bayesian Linear Regression Models as a**

2 **Fine Mapping tool**

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10 Abstract

11 Our aim was to evaluate Bayesian Linear Regression (BLR) models with BayesC and BayesR
12 priors as a fine mapping tool and compare them to the state-of-the-art external models:
13 FINEMAP, SuSIE-RSS, SuSIE-Inf and FINEMAP-Inf. Based on extensive simulations, we
14 evaluated the different models based on F_1 classification score. The different models were
15 applied on quantitative and binary UK Biobank (UKB) phenotypes and evaluated based upon
16 predictive accuracy and features of credible sets (CSs). We used over 533K genotyped and 6.6
17 million imputed single nucleotide polymorphisms (SNPs) for simulations and UKB phenotypes
18 respectively, from over 335K UKB White British Unrelated samples. We simulated
19 phenotypes from low (GA1) to moderate (GA2) polygenicity, heritability (h^2) of 10% and
20 30%, causal SNPs (π) of 0.1% and 1% sampled genome-wide, and disease prevalence (PV) of
21 5% and 15%. Single marker summary statistics and in-sample linkage disequilibrium were
22 used to fit models in regions defined by lead SNPs. BayesR improved the F_1 score, averaged
23 across all simulations, between 27.26% and 13.32% relative to the external models. Predictive
24 accuracy quantified as variance explained (R^2), averaged across all the UKB quantitative
25 phenotypes, with BayesR was decreased by 5.32% (SuSIE-Inf) and 3.71% (FINEMAP-Inf),
26 and was increased by 7.93% (SuSIE-RSS) and 8.3% (BayesC). Area under the receiver
27 operating characteristic curve averaged across all the UKB binary phenotypes, with BayesR
28 was increased between 0.40% and 0.05% relative to the external models. SuSIE-RSS and
29 BayesR, demonstrated the highest number of CSs, with BayesC and BayesR exhibiting the
30 smallest average median size CSs in the UKB phenotypes. The BLR models performed similar
31 to the external models. Specifically, BayesR's performance closely aligned with SuSIE-Inf and
32 FINEMAP-Inf models. Collectively, our findings from both simulations and application of the
33 models in the UKB phenotypes support that the BLR models are efficient fine mapping tools.

35 **Introduction**

36 To better understand the genetic make-up of complex quantitative phenotypes and
37 multifactorial diseases, it is essential to be able to identify the set of genetic variants that are
38 most likely causal or linked with the causal genetic variants. Genome-wide association studies
39 (GWAS) often identify too many genetic variants because long-range linkage disequilibrium
40 (LD) complicates statistical inference. Hence, additional work such as statistical fine mapping
41 is often required to refine signal from GWAS to determine the genetic variant (or variants)
42 most likely responsible for complex phenotypes, given verified association of region (or
43 regions) with a phenotype. This task of determining the genetics variants and quantifying the
44 evidence of association is crucial as it is often followed by large-scale replication studies, or
45 laboratory functional studies to gain further biological insights for potential clinical application
46 with drug discovery, drug-repositioning in humans. Fine mapping methods usually assume
47 presence of potential causal genetic variants in the data (1). With the concept of existence of
48 multiple causal genetic variants in a locus various Bayesian fine mapping methods were
49 developed (2). Bayesian fine mapping methods can quantify uncertainty of a potential causal
50 genetic variant through posterior inclusion probability (PIP) in a model. The PIP of a SNP
51 refers to the mean of posterior probability that the SNP is included in a model with non-zero
52 effect, which provides evidence for that the SNP potentially is causative (3). These methods
53 are also able to leverage knowledge about genetic architecture of complex phenotypes through
54 prior distribution of the effects and number of the genetic variants, which helps to improve
55 statistical power for identifying effective genetic variants (4).

56 In the quest for accurate identification of effects of potential causal genetic variants in complex
57 phenotypes, various Bayesian fine mapping methods have been developed with different
58 modeling assumptions. FINEMAP (5) for GWAS summary statistics, uses Shotgun Stochastic

59 Search (SSS) algorithm to search for different possible causal configurations followed by
60 focusing on the configurations with non-negligible probability. The algorithm conducts a pre-
61 defined number of iterations within the space of causal configurations. In each iteration, the
62 neighborhood of the current causal configuration is defined by configurations that result from
63 deleting, changing, or adding a causal SNP from the current configuration. In the next iteration,
64 a new causal configuration is sampled from the neighborhood configurations based on its
65 posterior normalized within the neighborhood. The sum of single effects (SuSIE) (6), a new
66 formulation of Bayesian Variable Selection Regression (BVSR) uses a procedure called as
67 iterative Bayesian stepwise selection approach (IBSS) to fit a model assuming few sparse
68 causal genetic variants in a locus. It estimates the vector of regression coefficients for sparse
69 genetic variants by summing up multiple single-effect vectors that each have one non-zero
70 entry for a potential causal variant. Recently, (7) extended SuSIE by implementing the use of
71 GWAS summary data and introduced it as SuSIE-RSS. SuSIE-Inf and FINEMAP-Inf (4), an
72 infinitesimal model similar to linear mixed models, are extensions of SuSIE and FINEMAP
73 respectively where infinitesimal effects for genetic variants in LD (estimated separately for
74 locus) with those of the sparse components are modeled. As in FINEMAP, FINEMAP-Inf used
75 SSS algorithm, where the posterior inference of the sparse genetic variants is marginalized over
76 the infinitesimal effects and the residuals. As in SuSIE, SuSIE-Inf estimated the sparse causal
77 effects by summing up multiple single-effect vectors, where for the posterior inference of
78 sparse genetic effects the joint distribution of single-effect vectors is marginalized over the
79 infinitesimal effects and residuals.

80 Following a similar pursuit of accurate fine mapping, we were interested in investigating the
81 performance of Bayesian Linear Regression (BLR) models for fine mapping. The BLR models
82 have been applied for mapping genetic variants, prediction (polygenic scores), estimation of
83 genetic parameters and genetic architectures (8). The genetic architecture of a trait

84 encompasses number, frequency and effect size of causal variants (9). The BLR models allow
85 joint estimation of marker effects while accounting for LD among SNPs capturing the amount
86 of variation at a genetic locus whose extent is based on the extent of the colocalization of
87 multiple causal genetic variants. Assumptions of number and effect sizes of genetic variants
88 are based on different prior distributions. Here we focused on the performance of the BLR
89 models with the priors BayesC (10), and BayesR (11). During joint marker estimation,
90 depending on priors, BLR models either shrink non-causal genetic variants' effects or induce
91 both variable selection method and shrinkage helping to obtain accurate estimates of effects of
92 genetic variants. To our knowledge, the BLR models have been investigated numerous times
93 in predictions (12-14) but only few studies have investigated precision and power of these
94 models in fine mapping approach (4, 7).

95 In fine mapping, no single genetic variant is identified as causal due to the complex LD patterns
96 between the genetic variants hence “credible sets” of potentially causal genetic variants are
97 prioritized (1). Credible sets help in variable selection by narrowing down a larger number of
98 variants to a small set of most likely causal variants with certain probability, refining the fine
99 mapping approach. (15) developed a standard Bayesian approach for fine mapping, assumed a
100 single causal locus per genetic region, to prioritize for example 99% credible set (a set whose
101 cumulative sum of PIPs exceeds 0.99 threshold) of potentially causal genetic variants providing
102 a credible set for per region (2). With the main goal of improving fine mapping resolution, the
103 credible sets are meant to contain as few genetic variants as possible while still capturing an
104 effective genetic variant.

105 In our study, we aimed to assess the efficiency of the BLR models (BayesC and BayesR) as a
106 fine mapping tool using GWAS summary statistics. Using simulations, we designed credible
107 sets and investigated precision and power of inclusion of causal variants in the credible sets
108 to calculate F_1 classification score (F_1 score). We also evaluated these models based on the

109 features of credible sets, and the prediction accuracy using fine mapped regions for five
110 binary and five quantitative phenotypes from the UK Biobank (UKB) (16). The results from
111 BayesC and BayesR priors were compared to the state-of-the-art methods such as FINEMAP
112 (5), SuSIE-RSS (7), SuSIE-Inf and FINEMAP-Inf (4). We also aimed to investigate
113 validation of the BLR model through a detailed examination of the outcomes derived from its
114 application to Type two diabetes (T2D) within the UKB phenotypes.

115 **Material and Methods**

116 In our study, efficiency of different models was investigated on simulations and the UKB
117 phenotypes. We explored efficiency of the models on complex nature of phenotypes by
118 simulating phenotypes with low to moderate polygenic background and creating different
119 genetic architectures utilizing different values for heritability (h_{SNP}^2), proportion of causal
120 markers (π) and their effect sizes (9). Efficiency of the models was also investigated using five
121 quantitative and five binary UKB phenotypes available from the UKB. We have discussed the
122 theory behind single marker-linear regression analysis and its extension to summary data
123 followed by the prior assumptions of BayesC and BayesR, used in our study. Marginal marker
124 effects obtained from the single SNP association analysis were adjusted at multiple designed
125 fine-mapping regions using the BLR models and external fine-mapping models. We present
126 the design of credible sets (CSs) and definition of precision and power in terms of CSs to
127 estimate F_1 classification score (F_1 score) on simulations. For the UKB phenotypes, we
128 compared the predictive abilities (coefficient of determination: R^2 for quantitative phenotypes
129 and Area under the receiver operating characteristic curve (AUC) for binary phenotypes) and
130 the features of CSs. Lastly, we explored the biological mechanisms underlying T2D, drawing
131 insights from the outcomes derived by implementing the BLR model.

132 **Data**

133 UKB genotyped and imputed data were used for simulations and analysis of the UKB
134 phenotypes respectively. In our study, we had information about 488,377 participants. To
135 obtain a genetic homogeneous study population we restricted our analyses to unrelated British
136 Caucasians and excluded individuals with more than 5,000 missing markers or individuals with
137 autosomal aneuploidy. Remaining ($n=335,532$) White British unrelated individuals (WBU)
138 were used for analyses. Then, we excluded markers with minor allele frequency < 0.01 , call
139 rate < 0.95 and the markers deviating from Hardy-Weinberg equilibrium (P -value $<$
140 1×10^{-12}). We excluded markers located within the major histocompatibility complex
141 (MHC), having ambiguous allele (i.e., GC or AT), were multi-allelic or an indel (17). This
142 resulted in a total of 533,679 single nucleotide polymorphism (SNP) markers in the simulated
143 data. For the UKB imputed data, firstly the markers with the probability of 70% (–hard-call
144 threshold 0.7) were converted to genotypes followed by retaining markers with imputation
145 INFO score ≥ 0.8 using PLINK 2.0 (18). The same quality control criteria were applied to the
146 imputed markers as for the genotyped data, except that we included MHC in the UKB
147 phenotypes as this region contains many known disease-associated markers. After quality
148 control we retained 6,627,732 SNPs and 335,532 WBU for downstream analysis in the UKB
149 imputed data.

150 **Genetic architectures for simulations**

151 To simulate genetic architectures from low to high polygenicity, we simulated quantitative
152 phenotypes with heritability (h_{SNP}^2) of 30% and 10%, with two different proportions of causal
153 SNPs (π), 0.1% and 1%, chosen randomly from the genome.

154 We generated two different types of genetic architectures under a multiple regression model.
155 In the first genetic architecture (GA_1), causal SNPs (m_C) effects were sampled from the same
156 normal distribution:

157

$$y_i = \sum_{j=1}^{m_C} w_{ij} b_j + e_i,$$

158 where y_i is the phenotype for i 'th individual, b_j is the estimate of the j 'th SNP effect (normally
159 distributed with mean of 0 and variance given by σ_g^2/m_C). We assumed variance of a phenotype
160 to be 1 such that σ_g^2 is equal to h_{snp}^2 . w_{ij} represents the j 'th centered and scaled genotype of
161 the i 'th individual:

162

$$w_{ij} = \frac{x_{ij} - 2p_j}{\sqrt{2p_j(1 - p_j)}}$$

163 where, x_{ij} is the effect allele count for i 'th individual at the j 'th SNP, p_j is the allele frequency
164 of the j 'th SNP. e_i is the residual that has a normal distribution with mean=0 and variance=
165 $\sigma_g^2(1 / (h_{snp}^2) - 1)$. Residual variance was scaled in a way so that h_{snp}^2 remained 30% (or
166 10%).

167 In the second genetic architecture scenario (GA_2), the effects of causal SNPs are sampled from
168 a mixture of normal distributions.

169

$$y_i = \sum_{j=1}^{m_{C_1}} w_{ij} b_j + \sum_{k=1}^{m_{C_2}} w_{ik} b_k + \sum_{l=1}^{m_{C_3}} w_{il} b_l + e_i,$$

170 where, b_j , b_k , and b_l are the effect of causal SNPs sampled from normal distribution with
171 mean=0 and variance = $(0.6\sigma_g^2)/(0.93m_C)$, $(0.2\sigma_g^2)/(0.05m_C)$, and $(0.2\sigma_g^2)/(0.02m_C)$
172 respectively. In this genetic model, the three normal distributions were designed such that 93%
173 of the causal SNPs would have small effect sizes and the remaining 5% and 2% of the causal

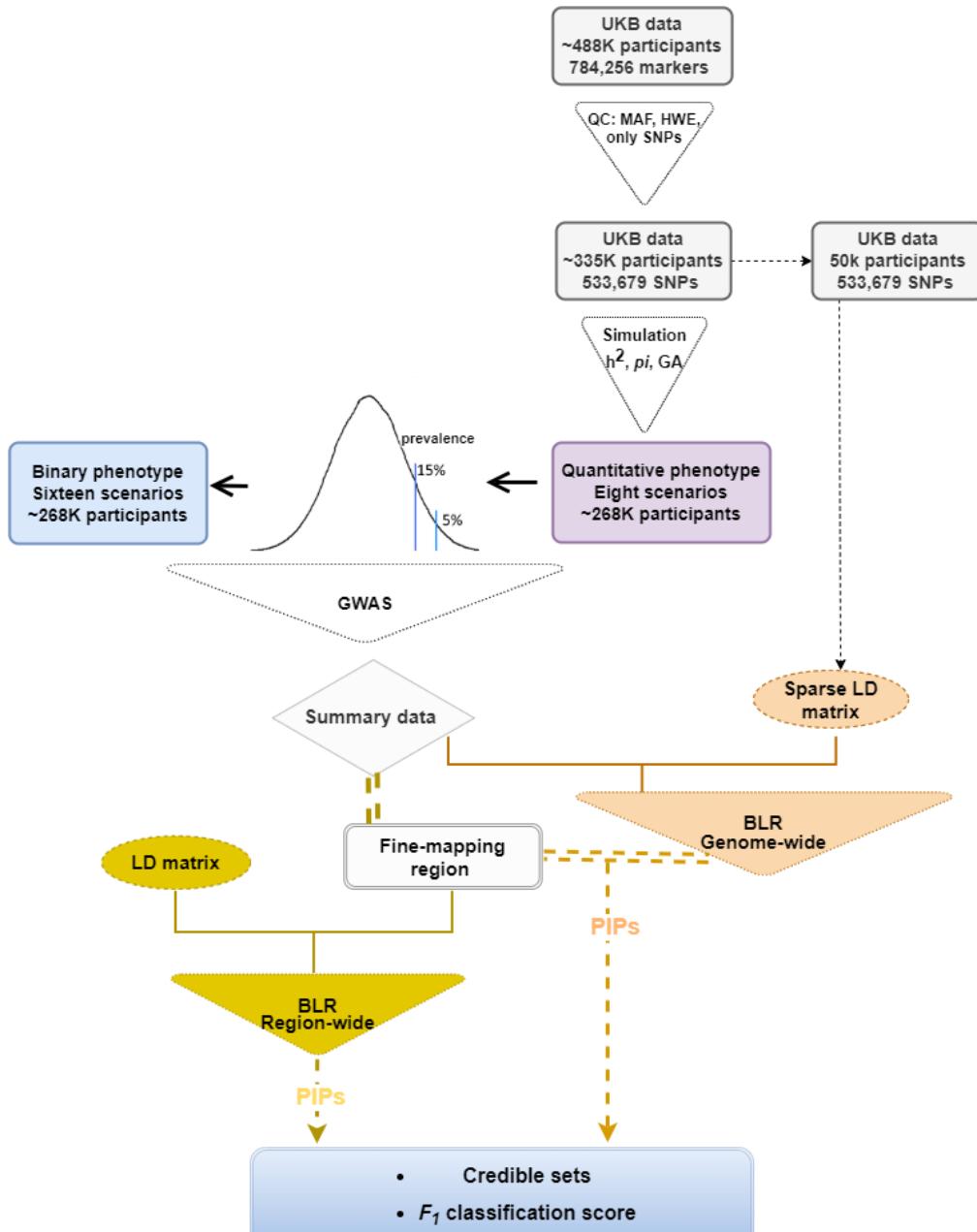
174 SNPs would have moderate and large effect sizes respectively. This genetic architecture was
175 designed in a similar way as designed in the study by (12).

176 All the other parameters in GA_2 are created in a similar way as for the GA_1 .

177 We created ten replicates for each simulation scenario. The total sample of 335,532 were
178 divided into ten replicates. Each replicate contained 80% of the randomly sampled data from
179 the total samples.

180 For the quantitative phenotypes, a total of eight different simulation scenarios were applied:
181 two values of h_{snp}^2 , two different proportions of causal SNPs π and two different genetic
182 architecture scenarios.

183 To simulate binary phenotypes, in addition to the parameters: h_{SNP}^2 , π and genetic architectures,
184 we introduced another parameter “sample disease prevalence” (PV). Two different PV of 5%
185 and 15% were used in our study. We simulated binary phenotypes from quantitative
186 phenotypes. To simulate a binary phenotype, for example with PV 5%, we chose top 5% of
187 individuals with highest simulated quantitative values as cases and the remaining as controls
188 for the total sample in a replicate. Each scenario of a quantitative phenotype gave rise to two
189 different scenarios for binary phenotype. In total we designed 16 different simulation scenarios
190 for the binary phenotypes: two values of h_{snp}^2 , two different proportions of causal SNPs π , two
191 different genetic architecture scenarios, and two prevalence PV . Different scenarios for the
192 quantitative and the binary phenotypes are presented in detail in S1 Table. The flowchart of
193 design of the simulations is presented in Fig 1.



194

195 Fig 1. Flowchart illustrating the design of the simulation scenarios for both quantitative and
196 the phenotypes, followed by fine mapping using Bayesian Linear Regression models. The BLR
197 models were implemented in different ways, and the resulting posterior inclusion probability
198 (PIPs) for SNPs were used to estimate the F_1 classification score based on the credible sets.

199 **Definition of phenotypes from the UKB data**

200 From the UKB we selected five quantitative phenotypes: Body mass index (BMI), Hip
201 circumference (HC), Standing height (Height), Waist circumference (WC) and Waist-to-hip
202 ratio (WHR), and five binary phenotypes: Coronary artery disease (CAD), Hypertension
203 (HTN), Psoriasis (PSO), Rheumatoid arthritis (RA) and Type 2 Diabetes mellitus (T2D). The
204 quantitative phenotypes were identified using specific field codes in the UKB data (see UKB
205 showcase, Table 1a). To obtain WHR, we estimated ratio of the waist circumference to the hip
206 circumference. In the UKB, a phenotype can have multiple instances. We used the first instance
207 because of the least number of non-missing samples in that instance. For the definition of the
208 binary phenotypes, to define individuals as cases for a phenotype of interest we used codes
209 from the data field “Diagnosis-main ICD10” along with codes from the self-reported
210 information (Table 1b). All the individuals missing the appropriate codes for the phenotype of
211 interest were reported as controls. Additional information on age at recruitment (p21022), sex
212 (p31), and the UKB assessment center (p54) were included as covariates in the genetic
213 analyses. Detailed information regarding the number of samples, prevalence for the phenotypes
214 is given in Table 1a and Table 1b.

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222 Table 1a. Details of the data fields along with the total number of non-missing samples, age (mean and standard deviation), number of females
 223 (n_female) and average value for the UKB quantitative phenotypes.

UKB Quantitative phenotypes	UKB data field	Total	Age (Mean [sd])	Sex (n_female)	Phenotype (Mean [sd])
Body Mass Index (BMI)	p21001	334,464	56.87 [7.98]	179,309	27.4 [4.76]
Hip Circumference (HC)	p49	334,949	56.87 [7.98]	179,517	103.44 [9.15]
Standing Height (Height)	p50	334,828	56.87 [7.98]	179,492	168.86 [9.25]
Waist Circumference (WC)	p48	334,983	56.87 [7.98]	179,532	90.37 [13.49]
Waist-Hip Ratio (WHR)	NA*	334,917	56.87 [7.98]	179,503	0.87 [0.09]

224 *NA because the phenotype was calculated in our study.

225 Table 1b. Details of the ICD10 and self-reported code used for diagnosis of cases for the UKB binary phenotypes, total number of cases, controls
 226 along with the distribution of age (mean and standard deviation) and number of females (n_female) within cases and controls.

UKB Binary phenotypes	Definition of cases		Cases	Controls	Age (Mean [sd])		Sex (n_female)	
	ICD10 code [p41270]	Self-reported code			Cases	Controls	Cases	Controls
Coronary Artery Disease (CAD)	I21; I22; I23; I24; I25	1075	34,726	300,806	61.24 [6.38]	56.37 [7.99]	10,845	168,989
Hypertension (HTN)	I10	1065	129,580	205,952	59.75 [6.98]	55.07 [8.04]	60,859	118,975
Psoriasis (PSO)	L40	1453	6628	328,904	57.15 [7.88]	56.87 [7.98]	3090	176,744
Rheumatoid Arthritis (RA)	M06	1464	7955	327,577	59.6 [7.04]	56.81 [7.99]	5251	174,583
Type 2 Diabetes (T2D)	E11	1220;1223	25,828	309,704	60.11 [6.9]	56.6 [8.01]	10,072	169,762

227

228 **Statistical model**

229 In the multiple regression model the phenotype is related to the set of genetic markers:

230
$$y = Xb + e,$$

231 where y is the phenotype, X a matrix of genotyped markers, where values are standardized to

232 give the ij th element as: $x_{ij} = (x_{ij} - 2p_j)/\sqrt{2p_j(1 - p_j)}$, with x_{ij} the number of copies of
233 the effect allele (e.g. 0, 1 or 2) for the i th individual at the j th marker and p_j the allele frequency
234 of the effect allele. b are the genetic effects for each marker, and e the residual error. The
235 dimensions of y , X , b and e are dependent upon the number of phenotypes, k , the number of
236 markers, m , and the number of individuals, n . The residuals, e , are a priori assumed to be
237 independently and identically distributed multivariate normal with null mean and covariance
238 matrix $I\sigma_e^2$.

239 **Extensions to summary data**

240 The key parameter of interest in the multiple regression model is the marker effects. These can
241 be obtained by solving an equation system like:

242
$$b = \left(X'X + I \frac{\sigma_e^2}{\sigma_b^2} \right)^{-1} X'y.$$

243 To solve this equation system individual level data (genotypes [X] and phenotypes [y]) are
244 required. If these are not available, it is possible to reconstruct $X'y$ and $X'X$ from a LD
245 correlation matrix B (from a population matched LD reference panel) and data (Llyod-Jones et
246 al. 2019):

247
$$\begin{aligned} X'X &= D^{0.5} B D^{0.5}, \\ X'y &= D b_m, \end{aligned}$$

248 where $D_i = \frac{1}{\sigma_{b_i}^2 + b_i^2/n_i}$ if the markers have been centered to mean 0 or $D_i = n_i$ if the markers
249 have been centered to mean 0 and scaled to unit variance, b_i is the marker effect for the i 'th
250 marker, $\sigma_{b_i}^2$ is the variance of the marginal effects from GWAS. $b_m = D^{-1}X'y$ is the vector of
251 marginal marker effects obtained from a standard GWAS. The LD correlation matrix, B , was
252 computed using squared Pearson's correlation.

253 **Estimation of parameters using BLR models**

254 The BLR models use an iterative algorithm, Markov Chain Monte Carlo (MCMC) gibbs
255 sampling techniques, to estimate joint marker effects which depends on additional model
256 parameters such as a probability of being causal (π), an overall marker variance (σ_b^2), and
257 residual variance (σ_e^2). The posterior density of the model parameters $(b, \sigma_b^2, \sigma_e^2)$ depend on the
258 likelihood of the data given the parameters and a prior probability for the model parameters
259 which is discussed in detail by (19).

260 Ideally the choice of prior for the marker effect should reflect the genetic architecture of the
261 phenotype. Most complex phenotypes and diseases are likely highly polygenic, with hundreds
262 to thousands of causal genetic variants, most frequently of small effect sizes (20). Thus, the
263 prior distribution should account for many small and few large effect genetic variants. Also,
264 marker effects are a priori assumed to be uncorrelated, but markers can be in strong linkage
265 disequilibrium and therefore a high posterior correlation may exist. To accommodate evolving
266 ideas genetic architectures of phenotypes and diseases, many priors for marker effects have
267 been proposed. Each prior gives rise to a method or family of methods, and two of them are
268 described below:

269 **BayesC**

270 In the BayesC approach the marker effects, b , are a priori assumed to be sampled from a
271 mixture with a point mass at zero and univariate normal distribution conditional on common
272 marker effect variance σ_b^2 . This reflects a very common thought that there were not many causal
273 loci. This can be implemented by introducing additional variables δ_i which indicates if the i 'th
274 marker has an effect or not. In turn, these variables δ have a prior Bernoulli distribution with
275 the probability π of being zero. Therefore, the hierarchy of priors is:

276
$$p(b_j | \delta_i, \sigma_{b_i}^2, \pi) = \begin{cases} 0 & \text{with probability } \pi, \\ \sim N(0, \sigma_{b_i}^2) & \text{with probability } 1 - \pi \end{cases}$$

277
$$p(\sigma_{b_i}^2 | v_b, S_b^2) = S_b^2 \chi_{v_b}^{-1},$$

278 where $S_b^2 = \sigma_b^2 v_b$ with $\sigma_b^2 = \frac{\sigma_g^2}{(1-\pi)2 \sum_i p_i (1-p_i)}$ because the variance of a t distribution is $\frac{v_b}{v_b-2}$.

279 **BayesR**

280 In the BayesR (Erbe et al. 2012) approach the marker effects, b , are a priori assumed to be
281 sampled from a mixture with a point mass at zero and univariate normal distributions
282 conditional on common marker effect variance σ_b^2 , and variance scaling factors, γ :

283
$$b_j | \pi, \sigma_b^2 = \begin{cases} 0 & \text{with probability } \pi_1, \\ \sim N(0, \gamma_2 \sigma_b^2) & \text{with probability } \pi_2, \\ \vdots & \\ \sim N(0, \gamma_c \sigma_b^2) & \text{with probability } 1 - \sum_{c=1}^{C-1} \pi_c \end{cases}$$

284 where $\pi = (\pi_1, \pi_2, \dots, \pi_C)$ is a vector of prior probabilities and $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_C)$ is a
285 vector of variance scaling factors for each of C marker variance classes. The γ coefficients are
286 prespecified and constrain how the common marker effect variance σ_b^2 scales within each

287 mixture distribution. Typically, $\gamma = (0,0.01,0.1,1.0)$. and $\pi = (0.95,0.02,0.02,0.01)$ are
288 starting values which can be updated each iteration.

289 The prior distribution for the marker variance σ_b^2 is assumed to be an inverse Chi-square prior
290 distribution, $\chi^{-1}(S_b, v_b)$.

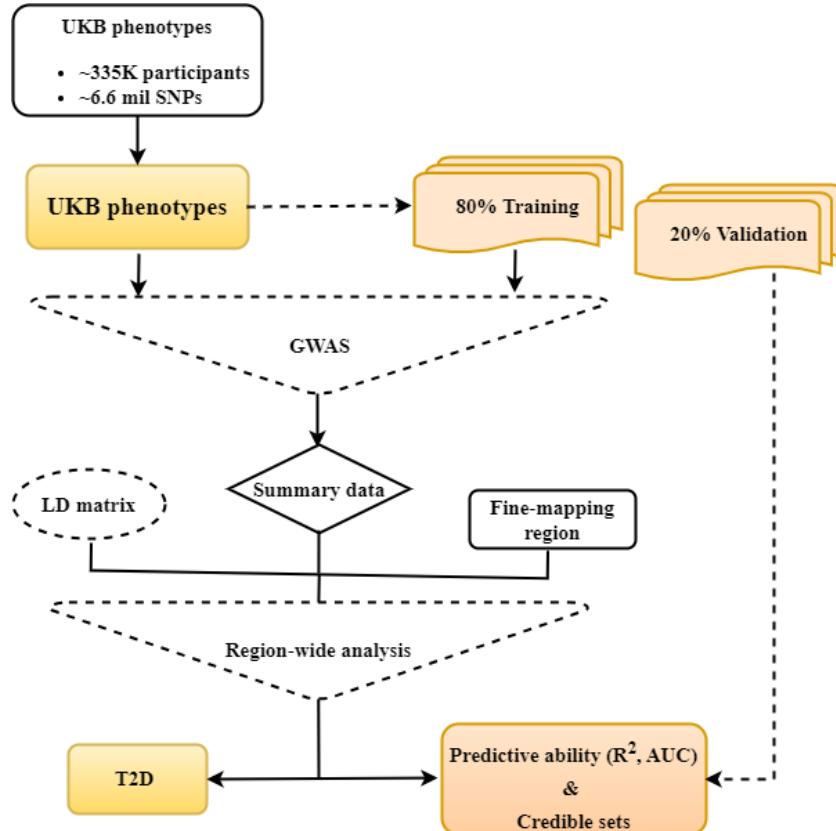
291 The proportion of markers in each mixture class π follows a Dirichlet $(C, c + \alpha)$ distribution,
292 where c is a vector of length C that contains the counts of the number of variants in each
293 variance class and $\alpha = (1,1,1,1)'$ such that the π is updated only using information from the
294 data.

295 Using the concept of data augmentation, an indicator variable $d = (d_1, d_2, \dots, d_{m-1}, d_m)$, is
296 introduced, where d_j indicates whether the j th marker effect is zero or nonzero.

297 **Genome-wide association study (GWAS)**

298 For simulations, we had eight and sixteen simulation scenarios (with ten replicates per
299 scenario) for quantitative and binary phenotypes, respectively. We performed GWAS by fitting
300 a single marker linear regression model using the R package “qgg” (19). No co-variates were
301 used in the model because no co-variates were simulated. For analysis of the UKB phenotypes,
302 the total population (no missing phenotype) was divided into five replicates of training (80%)
303 and validation (20%) populations. The design for the analysis of the UKB phenotypes is
304 presented in Fig 2. GWAS was performed in the training population of the five replicates for
305 all the UKB phenotypes. For T2D, GWAS was also performed in the total population. We
306 performed single marker linear regression using the R package “qgg” (19), and logistic
307 regression analysis using PLINK 1.9 (21) for the quantitative and binary UKB phenotypes,
308 respectively. To account for any cryptic relatedness in the data, we used top ten principal
309 components (PCS) along with age, sex and the UKB assessment center as co-variates in the

310 analysis of the UKB imputed data. We computed PCs for WBU from 100K randomly sampled
311 SNPs from the genotyped data after removing SNPs in the autosomal long-range LD regions
312 (22) with pairwise correlation (r^2)>0.1 in 500Kb region, using PLINK 2.0 (18).



313

314 Fig 2. Flowchart illustrating the design of populations for the analysis of the UK Biobank
315 phenotypes to determine the predictive abilities and features of credible sets across different
316 models.

317

318 Designing genomic region for fine mapping

319 For the simulated phenotypes, we designed fine-mapping regions based on the number of SNPs
320 (at most 1000 SNPs in total). The regions were designed by defining a window of ~500 SNPs
321 to the left and right of the causal SNPs. The number of the fine-mapping regions depended on
322 the type of simulation scenario. We did not consider any overlaps across the regions. For the

323 UKB phenotypes, we designed the fine-mapping regions based on the physical position around
324 each lead SNP. Significant SNPs (p-value < 5 X 10-8) from GWAS were used as the lead SNPs
325 to design the fine-mapping regions. We defined a genomic region of one mega base pair (1MB)
326 (~1000kb on both sides) of the lead SNP. If the regions overlapped by more than 500kb then
327 the regions were merged. This arbitrary number was chosen to limit the size of the regions and
328 assuming that the SNPs added to the region might just increase the size but do not contribute
329 to the analysis.

330 **Methods for fine mapping using summary statistics**

331 We implemented BayesC and BayesR and the following external models: FINEMAP (5),
332 SuSIE-RSS (7), SuSIE-Inf and FINEMAP-Inf (4) for fine mapping.

333 **BLR models**

334 The BLR models BayesC and BayesR, differ based on their assumption of prior variance of
335 the marker effects. Their assumptions have already been discussed in detail above in the section
336 “BLR models”. For the simulations, BayesC and BayesR were implemented region-wide and
337 genome-wide, using the R package “qgg” (19). This implementation is illustrated in Fig 1.

338 To apply these models’ region-wide, summary data from the GWAS for the SNPs in the fine-
339 mapping regions along with the pair-wise linkage disequilibrium (LD) information among all
340 the SNPs were used. The region-wide analysis was performed in different ways (the following
341 three options) depending on estimation of different model parameters as part of an iterative
342 estimation procedure (Gibbs sampling technique) from fully conditional posterior distributions.
343 For the first option, the parameter π was treated as random and estimated in each iteration along
344 with the marker variance and the residual variance. For the second option, π was kept constant.

345 For the third option, only the marker variance and residual variance were estimated. Option 1:
346 $\sigma_b^2, \sigma_g^2, \sigma_e^2$ and π – update Option 2: σ_b^2, σ_g^2 and σ_e^2 – update Option 3: σ_b^2 , and σ_e^2 – update
347 For genome-wide application, we used summary data from the GWAS and sparse LD matrix.
348 We randomly sampled 50,000 out of $n=335,532$ WBU to estimate sparse LD for a group of
349 SNPs in a sliding genomic window containing 2000 SNPs, which slid 1 SNP at a time. Due to
350 computational challenge, for genome-wide analysis, only the “Option 1” was used. The PIPs
351 for SNPs obtained from the genome-wide analysis were used further to design credible sets for
352 the fine-mapped regions. A total of 3000 iterations were used in the analysis with the first 500
353 as burn-in.

354 **External fine mapping tools**

355 *SuSIE-RSS model:*

356 The model was applied using the R package susieR (6). We provided the summary statistics
357 (beta estimates and standard error), the LD information and number of samples for the fine
358 mapping regions. The residual variance was estimated as suggested by the model because in-
359 sample LD was used. We used ten causal SNPs which is the default number in the R package
360 SusieR. We used default parameters in the functions. No priors for the SNPs were provided.

361 *SuSIE-Inf and FINEMAP-Inf models:*

362 To apply these models, we downloaded python package “run_fine_mapping.py” from the link:
363 <https://github.com/FinucaneLab/fine-mapping-inf> (4). We provided the summary statistics
364 (SNP estimates and standard error) along with LD information and number of samples for the
365 fine mapping regions. The number of causal SNPs was assumed to be ten to be consistent with
366 the default number of causal SNPs in susieR. SuSIE-Inf and FINEMAP-Inf models were
367 applied separately. No variance was shared and no priors for the SNPs were provided.

368 *FINEMAP model:*

369 We downloaded FINEMAP software from the link:

370 http://www.christianbenner.com/finemap_v1.4_x86_64.tgz (v1.4) (5). We provided the
371 summary statistics (SNP estimates and standard error) along with minor allele frequency
372 (MAF), LD information, and the number of samples for the fine mapping regions. The
373 number of causal SNPs was assumed to be ten. No priors for the SNPs were provided.

374 **Quality control/convergence for models**

375 The external fine mapping tools FINEMAP, SuSIE-RSS, SuSIE-Inf and FINEMAP-Inf,
376 explicitly mentioned convergence of the models in the output.

377 For the BLR models, we estimated the convergence of the key parameters: σ_b^2 , σ_g^2 , σ_e^2 , and π .
378 To assess the convergence, we used the metric “zscore”. This involved calculating the
379 difference between the average parameter values taken at the start and end of the iterations.
380 This difference served as our metric to gauge the convergence of the desired parameter. The
381 fine mapping regions with the absolute value of the metric “zscore”, for any of the parameters,
382 greater than three was further investigated by thorough evaluation of the trace plots of the
383 parameters, and scatter plots.

384 **Assessment of fine mapping models in simulations**

385 Efficiency of different models were investigated based on the F_1 score, a harmonic mean of
386 precision and power estimated for the credible sets (CS).

387 **Credible sets for simulations**

388 Credible sets (CSs) help to refine association signals. The CS are defined as the minimum set
389 of SNPs that contains all causal SNPs with probability α . When we assume only one causal

390 SNP, α can be the sum of the PIPs for SNPs in a set. The CS in our study was designed
391 according to (15). To design the CS of SNPs with coverage probability (cut-off or threshold)
392 of α , firstly SNPs were ranked according to descending order of their PIPs. A vector of
393 cumulative sum of PIPs was created. We added each element of the vector until it crossed a
394 specified coverage probability of α . All the sets exceeding the given threshold of α in the fine
395 mapping regions were referred as the CSs. A CS can contain multiple SNPs if they cross the
396 given PIP threshold. We use a strict coverage probability of 99% for the CSs.

397 In simulations, with an interest to compare only the core algorithms among different models in
398 our study, we designed the CSs for all the models irrespective of potential of the models to
399 output the CSs. FINEMAP-Inf doesn't give CSs, however we used the PIPs from FINEMAP-
400 Inf to design CSs for the model. In the scenario where multiple SNPs have the same value of
401 PIP, we investigated the list of SNPs, and if one of those SNPs is the simulated causal SNP
402 then we included that SNP in the CS. The same procedure was applied to design CS for all the
403 models. As we used only one causal SNP per fine-mapping regions without considering
404 overlaps across the regions, the concept of one region harboring one causal SNP remained valid
405 and supported our design of the CSs.

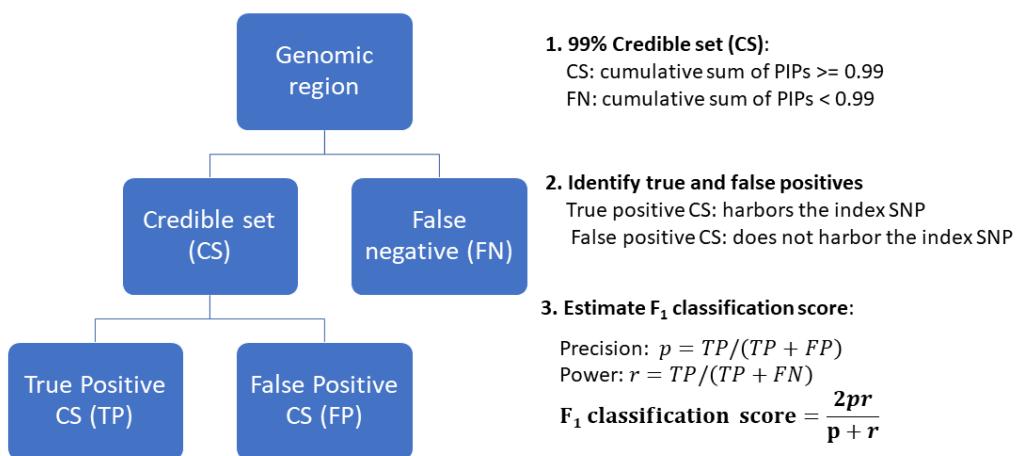
406 **F_1 classification score for simulations**

407 We assessed F_1 score for the fine-mapping regions based on the credible sets. All the fine-
408 mapping regions harbored a simulated causal SNP (index SNP). The F_1 score takes a value
409 between 0 and 1. The value close to 1 refers to the capability of a fine-mapping model to better
410 identify true causal SNPs and reduce false positives.

$$411 \quad F_1 = \frac{2pr}{p + r},$$

412 where, precision, $p = TP/TP + FP$ and recall $r = TP/TP + FN$.

413 F_1 score was calculated for each replicate of a simulation scenario. For one simulation replicate,
414 True positive (TP) was the total number of CS (referred to as true positive CS; TP_CS) which
415 contained the index SNP corresponding to that region. False positive (FP) was the total number
416 of CS which crossed the threshold of alpha but did not contain the index SNP (referred to as
417 false positives CS; FP_CS). False negative (FN) was the total number of genomic regions
418 where the cumulative sum of PIPs did not cross the threshold of alpha, and no CS was detected.
419 In addition to this criterion in our study for the FN, we also considered two additional criteria.
420 We denoted “unconverged” fine-mapping regions for any methods as FN. We also considered
421 “TP_CS” which contained more than ten SNPs as FN because large credible sets add little to
422 no information in search of causal variants in fine-mapping procedure. We investigated the
423 number of SNPs in true positive credible sets (TP_CS) to investigate the efficient model and
424 tried to have the least number of SNPs in a CS as possible. The design of CSs and estimation
425 of F_1 score is represented in Fig 3.



426
427 Fig 3. Design of credible sets with a 0.99 threshold for the cumulative sum of Posterior
428 Inclusion Probabilities (PIPs), and estimation of the F_1 classification score based on the
429 credible sets.

430 **Investigate influence of different factors in simulations**

431 To investigate the influence of each parameters: h_{SNP}^2 , π , genetic architectures (*GA*) and *PV* on
432 the performance of the models, we performed TukeyHSD test in R. To quantify the factors
433 with the greater influence in the simulations, for each model, we also performed ANOVA on
434 the linear model where the F_1 score was regressed on h_{SNP}^2 , π , and *GA* for the quantitative
435 phenotypes (and *PV* for the binary phenotypes).

436 **Similarities among model assumption in simulations**

437 We also investigated similarities among the models based on their assumptions of genetic
438 architectures for a complex phenotype. SuSIE, FINEMAP and BayesC assume contribution of
439 sparse genetic variants in the genetic makeup of a complex phenotype. In addition to these
440 sparse genetic variants, SuSIE-Inf and FINEMAP-Inf consider the influence of multiple
441 genetic variants with small effect sizes (infinitesimal models). The BayesR model assume
442 influence of sparse genetic variants with large effect sizes and non-sparse genetic variants with
443 moderate to small effect sizes in the genetic makeup of a complex phenotype. We used total
444 true positive credible sets (TP_CS) determined by each model for only the simulation scenarios
445 for the quantitative phenotype. We investigated the number of overlaps of TP_CS of the
446 BayesC model with SuSIE and FINEMAP, and the overlap of the BayesR with SuSIE-Inf and
447 FIENMAP-Inf.

448 **Assessment of fine mapping models in the UKB phenotypes**

449 Only the fine-mapped regions which converged across the models were used for downstream
450 analysis to estimate predictive ability and features of the CSs.

451 **Predictive Ability**

452 For quantitative phenotypes, the predictive ability was determined by estimating the coefficient
453 of determination, (R^2). For binary phenotypes, the predictive ability was determined by
454 estimating Area under the receiver operating characteristic curve (AUC).

455 Firstly, genomic score (GS) (predicted phenotype) of an individual, also known as a predictive
456 score for a phenotype was calculated for the validation population for each replicate. GS for an
457 individual is the sum of the product of effect alleles weighted by their estimated effect size:

458

$$GS = \sum_{i=1}^m X_i \hat{b}_i.$$

459 where X_i refers to the genotype matrix that contains an allelic count and \hat{b} is the estimated
460 marker effect for the i -th variant, m is the number of SNPs.

461 To quantify the accuracy of the GS for real quantitative phenotypes, co-variates adjusted scaled
462 phenotypes for validation population was regressed on the predicted phenotypes. The
463 coefficient of determination, R^2 , from the regression was used as a metric to assess the
464 predictive ability of the model. To quantify the accuracy of the GS for real binary phenotypes,
465 AUC (23) was reported:

466

$$AUC = \frac{1}{n_d} \left(\bar{r}_d - \frac{n_d}{2} - \frac{1}{2} \right).$$

467 where, n_d : number of controls n_d : number of cases \bar{r}_d : average rank of the cases.

468 Difference in the estimates of R^2 and AUC (averaged across five replicates) among different
469 methods was compared using TukeyHSD test.

470 **Credible sets for the UKB phenotypes**

471 Unlike simulations where a fine-mapping regions were not merged irrespective of overlaps, in
472 the UKB phenotypes, fine-mapping regions were merged if they shared a 500kb overlap of
473 SNPs. This approach increased the likelihood of containing multiple potentially causal variants
474 within a single fine-mapped region. To accommodate this, we designed credible sets (CSs)
475 allowing multiple causal variants to be fine-mapped region within the same genomic region.
476 To design a CS, in addition to the algorithm from (15) we also used information of LD. A
477 flowchart detailing the CS design process is presented in S1 Fig. To identify significant SNP
478 sets that are in LD, we utilized posterior PIPs and LD criteria. For each fine-mapped region,
479 CSs were comprised of SNPs where the cumulative PIP was at least 0.80 ($PIP_{cums-set} \geq 0.80$).
480 When a CS contained multiple SNPs, the LD (r^2) between the SNP with the highest PIP
481 in the CS and the other SNPs was at least 0.5. Detailed steps utilized to explore the presence
482 of multiple CSs within a fine-mapped region are mentioned in S1 Text.

483 We applied this methodology (S1) across all models in our study, aiming to compare the
484 efficiency of different algorithms by using a consistent CS creation approach. This allowed us
485 to focus solely on algorithmic efficiency by eliminating other variables. For each trait, non-
486 converged fine-mapped regions were excluded across all the models. Afterwards, for each
487 model, we determined the average total number of CSs, the average median CS size (SNP
488 counts in a CS), and the average median value for the average correlations ($avg.r^2$) among
489 SNPs in the CS. To estimate $avg.r^2$, we excluded the sets with only one SNP as they were not
490 informative, and we used absolute pair-wise correlations among SNPs in the CS. In case the
491 size of CS exceeded 100, only randomly chosen 100 SNPs were used to obtain $avg.r^2$ for that
492 CS. In a fine-mapped region, SNPs with $PIP_{SNP} \leq 0.001$ was excluded before designing
493 multiple CSs assuming that they would have little to no contribution in meeting the criterion
494 of PIP.

495 **Application of BLR model in T2D**

496 In earlier sections of our study, we examined the efficacy of the BLR models. This section
497 delves into the application of the BLR model to a complex trait, T2D. We aimed to validate
498 the results obtained from implementation of the BLR model.

499 We performed single SNP logistic regression in PLINK 1.9 (21) leveraging the entire UKB
500 cohort for T2D (Table 1), followed by adjustments of the marginal summary statistics with the
501 BayesR model. Fine mapping regions were created as for the UKB phenotypes discussed above
502 in the section “Designing genomic region for fine mapping”. Multiple credible sets (CSs) per
503 fine-mapped region were designed as discussed above in the section “Credible sets for the UKB
504 phenotypes”.

505 To validate the results obtained from BayesR model for T2D, we conducted non-exhaustive
506 comparison of our findings with the external study. Also, using the R package “gact”, we
507 performed a gene set enrichment analysis to identify diseases enriched for T2D-associated
508 genes and tissue-specific expression Quantitative loci (eQTLs) enrichment analysis to identify
509 tissues enriched for T2D.

510 In the initial step, we mapped SNPs from multiple CSs to genes using the Ensembl Gene
511 Annotation database available at
512 https://ftp.ensembl.org/pub/grch37/release109/gtf/homo_sapiens/Homo_sapiens.GRCh37.87.gtf.gz. This mapping targeted SNPs within the open reading frame (ORF) of a gene, including
513 regions 35kb upstream and 10kb downstream of the ORF, due to their potential regulatory role
514 in controlling main ORF translation.

516 **Comparison with large-scale meta-GWAS study**

517 To obtain any overlapping genes in our study with (24), one of the largest and most
518 comprehensive meta-GWAS on T2D. The study consisted of imputed genetic variants from
519 898,130 European-descent individuals (9% cases). Our study limited comparison to genes
520 given by the study in the Supplementary Table 2 which provided information of 243 loci (135
521 newly identified in T2D predisposition) comprising 403 unique genetic signals/associations.

522 **Gene-diseases association enrichment analysis**

523 To determine diseases significantly enriched for the gene set of our interest, we first curated a
524 set of genes with PIP of at least 0.5 (sum of PIP_{SNP}). We then downloaded the disease-gene
525 associations data from the DISEASE database (25). This database contained disease–gene
526 association scores (full and filtered) derived from curated knowledge databases, experiments
527 primarily GWAS catalog, and automated text mining of biomedical literature. The analysis was
528 conducted on the final disease-gene association data where association of a gene to a disease
529 was combined from all the above-mentioned sources. This database includes over 10,000
530 diseases. However, multiple terms in the database were used to refer to the same disease. We
531 investigated enrichment via hypergeometric test (26).

532 **Tissue-specific eQTLs enrichment analysis**

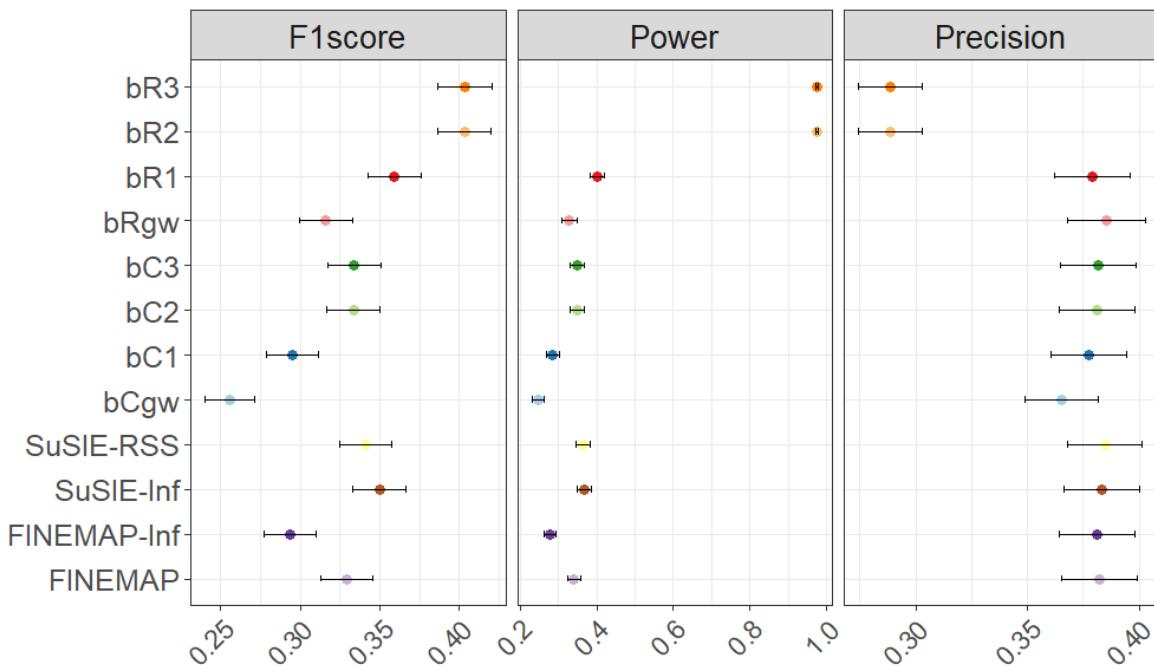
533 To determine tissues enriched for eQTLs associated with T2D, firstly multi-tissue cis-eQTL
534 annotation was obtained from GTEx (Genotype-Tissue Expression) consortium
535 (https://storage.googleapis.com/adult-gtex/bulk-qt/v8/single-tissue-cis-qt/GTEx_Analysis_v8_eQTL.tar) (27). We identified only eQTLs within our fine-mapped
537 regions for each tissue. We then assessed the enrichment of tissue-specific eQTLs using a
538 multiple linear regression model, adjusting for the influence of other tissue-specific eQTLs.
539 The analysis was conducted using absolute beta-estimates from the BayesR model. The

540 regression model allowed us to calculate z-scores (coefficient estimates/standard errors) and p-
541 value for each tissue. Tissue-specific eQTLs with a p-value less than 0.05 were considered
542 significantly enriched.

543 Results

544 Application in simulations

545 For simulations, we presented the results of the F_1 score based on the credible sets to show the
546 overall performance of the models across all simulation scenarios in Fig 4. Then to investigate
547 the influence of each parameter considered while designing simulation scenarios, we present
548 the results of the F_1 score in each simulation scenario for the quantitative (S2 Fig) and the
549 binary phenotypes (S3 Fig).



550
551 Fig 4. F_1 classification score (F_1 score), power and precision, averaged across all twenty-four
552 simulation scenarios, for the BLR fine mapping models: BayesR region-wide models (bR3,
553 bR2 and bR1), BayesR genome-wide model (bRgw), BayesC region-wide models (bC3, bC2

554 and bC1), BayesC genome-wide model (bCgw) and external models. The black solid line
555 represents standard error for the average estimate.

556

557 **Most efficient model**

558 The *bR3* model (option 3) improved the $F_{1avg.sim}$ score (average across all the simulation
559 scenarios) by 21.64%, 11% and 0.06% relative to the BayesR genome-wide analysis (*bRgw*),
560 and *bR1* and *bR2*. We observed similar results for BayesC. The BayesC region-wide model
561 (option 3, *bC3*) improved the $F_{1avg.sim}$ score by 23.41%, 11.72% and 0.10% relative to *bCgw*,
562 *bC1*, and *bC2*.

563 Highest $F_{1avg.sim}$ score, averaged across all the twenty-four simulation (binary and
564 quantitative) scenarios was observed for the BayesR region-wide model (*bR3*) [$F_{1avg.sim}$ score:
565 0.4] followed by SuSIE-Inf [$F_{1avg.sim}$ score: 0.35] and SuSIE-RSS [$F_{1avg.sim}$ score: 0.34] (Fig
566 1). The *bR3* improved the $F_{1avg.sim}$ score by 27.26%, 26.96%, 18.40%, 15.42%, and 13.32%
567 relative to FINEMAP-Inf, *bC3*, FINEMAP, SUSIE-RSS and SUSIE-Inf. The precision
568 ($Prec_{avg.sim}$) and power ($Pow_{avg.sim}$), averaged across all the simulation scenarios ranged
569 between 0.29 to 0.39, and 0.25 to 0.98 respectively. The *bR3* model also improved the
570 $Pow_{avg.sim}$ by 58% to 72% relative to other models. However, this model decreased the
571 $Prec_{avg.sim}$ by 26% to 33% relative to other models. Similar patterns were observed when the
572 models were compared only within the quantitative phenotypes and within the binary
573 phenotypes. In the following we only compared *bR3* and *bC3* with the external methods.

574 **Influence of parameters in simulations**

575 All the models performed the best (highest F_1 score) for the scenario with moderate h_{SNP}^2 of
576 0.3, π of 0.001, and GA_1 for the quantitative phenotypes (S2 Fig), and PV of 15% for the binary
577 phenotypes (S3 Fig).

578 Pairwise comparison of the $F_{1avg,rep}$ score (averaged across the replicates in a scenario)
579 between all the scenarios for both the quantitative and the binary phenotypes showed
580 significant differences between all scenarios (for all the models) as none of the intervals
581 harbored a value of zero. S4 Fig illustrated the results for $bR3$ for the quantitative simulated
582 phenotypes. ANOVA on the results where F_1 scores were regressed on h_{SNP}^2 , π , and GA (and
583 PV for the binary phenotypes) quantified higher influence of π and least influence of GA .

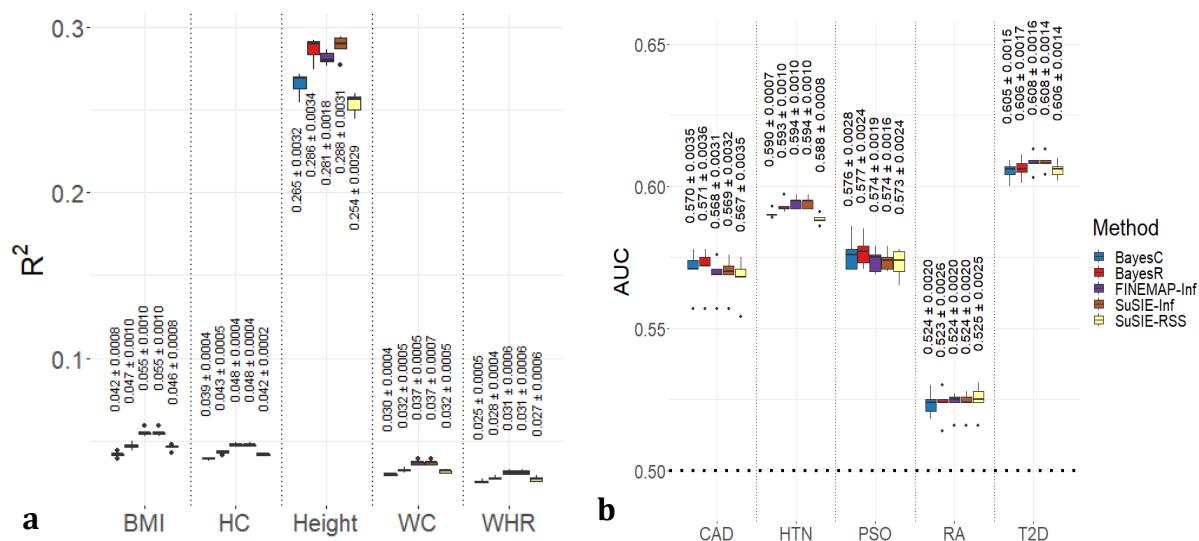
584 **Similarities among methods assumptions in simulations**

585 We observed that at least 50% of the true positive credible sets (TP_CS) were shared among
586 BayesC, SuSIE-RSS and FINEMAP (S5 Fig). We observed similar results for the models
587 BayesR, SuSIE-Inf and FINEMAP-Inf. $bCgw$ identified the fewest number of total TP_CS
588 summed across all the scenarios followed by FINEMAP-Inf. $bCgw$ shared 80% of the total
589 TP_CS with SUSIE-RSS, FINEMAP and BayesC region-wide model ($bClw$ or $bC3$). The
590 $bCgw$ shared ~91% of the total TP_CS with $bClw$. Similarly, FINEMAP-Inf shared 80% of
591 the total TP_CS with SuSIE-Inf, $bRgw$, and $bRlw$ of $bR3$. $bRgw$ shared 85.1% of the total
592 TP_CS with $bRlw$ or $bR3$.

593 Application to UKB phenotypes

594 Predictive ability

595 We observed a significant decrease in the $R_{avg.rep}^2$ (averaged across all the binary phenotypes)
596 of BayesC and BayesR relative to SuSIE-Inf and FINEMAP-Inf for the phenotypes BMI, WC,
597 HC and WHR (Fig 5a). No significant difference in the $R_{avg.rep}^2$ was observed between BayesR
598 compared to SuSIE-Inf and FINEMAP-Inf for Height, whereas a significant decrease was
599 observed for BayesC compared to these models. We observed significant improvement in the
600 $R_{avg.rep}^2$ of BayesR relative to SuSIE-RSS for Height. All the methods could predict Height
601 better compared to other quantitative phenotypes. Prediction $AUC_{avg.bin}$ (averaged across all
602 the binary phenotypes) with BayesR increased by 0.40%, 0.16%, 0.08%, 0.05% compared to
603 SUSIE-RSS, BayesC, FINEMAP-Inf and SuSIE-Inf, respectively (Fig 5b). We didn't observe
604 any significant differences between the $AUC_{avg.rep}$ (averaged across all the replicates) of
605 models compared pairwise for any binary phenotypes except for HTN. For HTN, BayesR
606 improved the $AUC_{avg.rep}$ significantly compared to SuSIE-RSS. The highest estimate of the
607 $AUC_{avg.rep}$ was observed for T2D followed by HTN for all the models. The lowest estimate of
608 the $AUC_{avg.rep}$ was observed for RA. Prediction $R_{avg.qt}^2$ (averaged across all the quantitative
609 phenotypes) with BayesR decreased by 5.32% and 3.71% compared to SuSIE-Inf and
610 FINEMAP-Inf, whereas increased by 7.93% and 8.3% compared to SuSIE-RSS and BayesC.
611 BayesR model improved the $R_{avg.rep}^2$ (averaged across all the replicates) significantly
612 compared to BayesC model for all the quantitative phenotypes except for WHR.



613

614 Fig 5. Prediction accuracies estimated from fine mapped regions. **a.** Bar plot of prediction
615 accuracy, represented by the coefficient of determination (R^2), averaged across five replicates
616 for the UKB quantitative phenotypes: body mass index (BMI), hip circumference (HC),
617 standing height (Height), waist circumference (WC), and waist-hip ratio (WHR). **b.** Bar plot
618 of prediction accuracy, represented by the Area under the Curve (AUC), averaged across five
619 replicates for the UKB binary phenotypes: coronary artery disease (CAD), hypertension
620 (HTN), psoriasis (PSO), rheumatoid arthritis (RA), and type 2 diabetes (T2D). The models
621 used in the fine mapping can be identified by the colors in the legend associated with each
622 model. For each method within a trait, corresponding mean of R^2 or AUC across five replicates
623 and standard error is written on the top of the box plot.

624 **Credible sets**

625 The average total number of fine-mapped regions across five replicates for the quantitative
626 phenotypes ranged from 135.2 for WHR to 461 for Height and for the binary phenotypes ranged
627 from 4 for RA to 137.4 for HTN (Table 1). The highest averaged non-converged regions were
628 observed for RA (55%) followed by PSO (29.62%). For other phenotypes, the non-converged
629 regions ranged from 1.22% to 6.42%.

630 BayesR determined the highest average number of CSs for Height, CAD, HTN and T2D,
631 whereas SuSIE-RSS determined the highest average number of CSs for BMI, HC, WC and
632 WHR (Table 1). For the above-mentioned phenotypes, FINEMAP-Inf determined the smallest
633 average number of CSs. All the models obtained a similar average number of CSs for PSO (9
634 to 12) and RA (1.8 to 2.2).

635 The BLR models showed the smallest average median CS size across all the phenotypes
636 compared to the external fine-mapping models (Table 2). BayesR showed the smallest average
637 median size of CS for BMI, Height, WC, WHR, PSO, RA and T2D. BayesC showed the
638 smallest average median size of CS for CAD. Both BayesC and BayesR showed the same
639 average median size for HC and HTN. The highest average median CS size was shown by
640 SuSIE-Inf for BMI, Height, WC, CAD, PSO, RA and T2D. For other phenotypes, SuSIE-RSS
641 showed the highest value for the median CS size.

642 The average median for $avg.r^2$ for the BLR models were smaller compared to the external
643 models. BayesC showed the largest average median value compared to BayesR across all the
644 phenotypes.

645

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Table 2. Average for the total number of fine mapped regions and non-converged regions for the UKB phenotypes along with the total number of credible sets (CSs), median size of the CSs, and median of the average correlations (r^2) of the CSs for all the models.

UKB Phenotypes	Avg. Total FMR	Avg. Non-converged FMR	BayesC			BayesR			FINEMAP			FINEMAP-Inf			SUSIE-Inf			SUSIE-RSS		
			Avg. Total CSs	Avg. Med CS size	Avg. Med r^2	Avg. Total CSs	Avg. Med CS size	Avg. Med r^2	Avg. Total CSs	Avg. Med CS size	Avg. Med r^2	Avg. Total CSs	Avg. Med CS size	Avg. Med r^2	Avg. Total CSs	Avg. Med CS size	Avg. Med r^2	Avg. Total CSs	Avg. Med CS size	Avg. Med r^2
Body Mass Index (BMI)	219.8	4.8	310.6	3.4	0.93	389.8	2	0.86	410.6	5	0.96	89.8	8.6	0.97	204.4	20.6	0.90	447.2	15.6	0.95
Hip Cirumference (HC)	203	5	289.4	1	0.85	359.4	1	0.80	348.4	2	0.98	95.6	3	0.98	200.6	4.4	0.97	410.4	6.8	0.97
Standing Height (Height)	461	29.6	1500	3.4	0.94	1846.8	2	0.87	1668	7.1	0.97	513.6	8.3	0.98	721.2	17.5	0.93	1696	14.6	0.96
Waist Circumference (WC)	164.4	2	229.6	3.6	0.94	276.8	2	0.88	299.4	6	0.97	73.2	7.7	0.98	157.4	20.8	0.92	331.6	15.6	0.96
Waist-Hip Ratio (WHR)	135.2	3.4	184.8	2.8	0.94	224.6	2	0.88	238.2	5.2	0.97	77	6.6	0.98	142.6	11.4	0.93	255.8	14	0.96
Coronary Artery Disease (CAD)	29.4	0.4	38.8	1.9	0.94	54.2	2.3	0.81	35.8	6.3	0.97	28	5.7	0.97	35.8	8.7	0.96	44	10.3	0.97
Hypertension (HTN)	137.4	6.2	211	2.2	0.90	277	2.2	0.80	239.8	4.2	0.97	96.8	5.2	0.98	159.4	8.8	0.96	246.8	9.8	0.96
Psoriasis (PSO)	10.8	3.2	10.2	3.4	0.92	9.6	1	0.89	9.8	5.3	0.98	9	5.7	0.97	12	10.5	0.96	11	6.4	0.98

Rheumatoid Arthritis (RA)	4	2.2	1.8	1.3	NA	1.8	1.2	NA	5.4	1.5	1.00	1.8	2.6	0.95	2.2	3.9	0.93	2.2	2.8	0.98
Type 2 Diabetes (T2D)	49.6	1.2	62.6	1.8	0.93	81.2	1.6	0.79	70.4	4.9	0.97	47.6	6.4	0.97	60	8.5	0.96	73.2	8.1	0.97

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657 **Application of BLR model in T2D**

658 We identified a total of 117 CSs for T2D across 69 fine-mapped regions with a median CS size
659 of 2 (range: 1 to 297), and the median of $avg.r^2$ was 0.80 (range: 0.49 to 1). We identified 53
660 CSs of size 1 (1 SNP counts), 47 CSs of size between 2 to 50, and the remaining 17 CSs of size
661 more than 50 SNPs.

662 **Comparison with large-scale meta-GWAS study**

663 We found 53 of the 181 genes identified from our study, listed in Table S2, overlapped with
664 genes from the study Mahajan et al. (2018) (S2 Table). Among 53 overlapped genes, 10 genes
665 (*DTNB*, *RBM6*, *MBNL1*, *SLCO6A1*, *PDE3B*, *CELF1*, *MAP2K7*, *ZC3H4*, *EYA2*, and *ZBTB46*)
666 were categorized as novel associations in the study by (24).

667 Additionally, our study identified multiple SNPs at *TCF7L2* in addition to rs7903146 (PIP:
668 0.9996). This includes rs34855922 (PIP: 0.3844), rs11196234 (PIP: 0.3512) and rs7912600
669 (PIP: 0.086) within a CS ($avg.r^2$: 0.70), as well as rs145034729 (PIP: 0.992) linked to
670 *TCF7L2* locus.

671 **Gene-Diseases association enrichment**

672 The top 30 significant diseases (p -value < 0.05) enriched for our T2D-related gene set and their
673 corresponding p -values are detailed in S3 Table. The list includes disease terms such as Type
674 2 Diabetes Mellitus, Diabetes Mellitus, ICD10:E11 code for T2D, as used in the UKB database.
675 Additionally, we discovered associations with various forms of diabetes, such as several types
676 of maturity-onset diabetes of the young (MODY), prediabetes syndrome, gestational diabetes,
677 both permanent and transient neonatal diabetes, ICD10-E14 (unspecified T2D), and ICD10-
678 O24 (diabetes in pregnancy). The list also encompassed other conditions, including
679 Rheumatoid Arthritis (RA) with corresponding ICD10 codes: M0, M05, M06 and M069,

680 Wolfram syndrome, hyperglycemia, hyperinsulinism, glucose intolerance, pancreatic agenesis,
681 pancreatic cystadenoma, and insulinoma.

682 **Tissue-specific eQTLs enrichment**

683 Among 49 different tissues, significant enrichment (p -value < 0.05) of T2D-related eQTLs
684 were identified in the 13 tissues (S6 Fig): Brain cerebellar hemisphere (n=419), Cells cultured
685 fibroblasts (n=718), Brain cerebellum (n=467), Pituitary (n=379), Esophagus muscularis
686 (n=615), Brain nucleus accumbens basal ganglia (n=308), Lung (n=624), Skin (not sun
687 exposed suprapubic) (n=678), Artery tibial (n=647), Adipose subcutaneous tissue (n=695),
688 Muscle skeletal tissue (n=639), Thyroid (n=810), and Nerve Tibial (n=804).

689 **Discussion**

690 Here we aimed to assess the efficiency of BayesC and BayesR as a fine mapping tool. We
691 applied these models in simulations and the real UKB data using summary statistics. In
692 simulations, the efficiency was investigated based on F_1 score. For the UKB phenotypes the
693 models efficiency was based on polygenic scores and credible sets. BayesC and BayesR
694 models' efficiency were compared to the state-of-the-art methods such as FINEMAP (5),
695 SuSIE-RSS (7), SuSIE-Inf and FINEMAP-Inf (4). All the models used in our study serve the
696 same purpose of identifying true effects of causal variants. However, they differ in the details
697 in the algorithm and their implementation which applied together can have different impact on
698 the overall performance.

699 **BayesC and BayesR**

700 BayesC and BayesR applied genome-wide and region-wide have the same assumptions of prior
701 variance of marker effects, but they differed in their implementation in our study. To our

702 knowledge this is the first study comparing implementation of BayesC and BayesR in such
703 manner. We implemented the models, genome-wide where the posterior distributions of the
704 model parameters were estimated based on taking SNPs genome-wide whereas region-wide
705 implementation were limited to the fine-mapped regions designed based on the simulated
706 causal SNPs. In the simulations, better performance of both priors when implemented region-
707 wide compared to genome-wide based on the $F_{1avg.sim}$ score for both BayesC and BayesR.
708 However, the genome-wide models showed better $Prec_{avg.sim}$ but less $Pow_{avg.sim}$ (fewer true
709 positive CSs) than region-wide models. High percentage of overlaps of the CSs for the genome-
710 wide models with the region-wide models suggests that there is a potential in the genome-wide
711 models. It would be interesting to investigate further the common CSs determined by the
712 genome-wide and the region-wide models.

713 BayesR showed significant improvement in prediction accuracy, $Prec_{avg.rep}$ for four out of
714 the five quantitative phenotypes relative to BayesC. Our prediction accuracies are consistent
715 with previous studies (28, 29). (28) showed an increase of prediction ability, averaged across
716 various economical phenotypes in cattle, using BayesR compared to BayesC. (29) showed
717 similar results across simulation scenarios for phenotypes with high heritability. BayesR
718 identified a large number of CSs, and small sized CSs relative to BayesC. Our results suggest
719 that BayesR assumption about genetic architecture suits better for polygenic phenotypes
720 predictions where many different effect sizes are observed, relative to BayesC

721 **Comparison of the BLR models to external models**

722 To our knowledge this is the first study comparing BayesR model to the state-of-the-art models:
723 FINEMAP, SuSIE-RSS, SuSIE-Inf and FINEMAP-Inf. Across different simulation scenarios,
724 BayesR had higher $F_{1avg.sim}$ score relative to the external models with high power but with
725 less precision. The average prediction accuracy ($R_{avg.rep}^2$) for the quantitative phenotypes was

726 significantly lower for BayesR and BayesC compared to SuSIE-Inf and FINEMAP-Inf for
727 BMI, HC, WC and WHR. We observed highest estimates for the predictive ability of the
728 infinitesimal models, which might have been because we used SNP effects from both sparse
729 and infinitesimal components for SuSIE-Inf and FINEMAP-Inf for predictions, unlike in the
730 study (4) where only sparse components were used to compare prediction accuracies between
731 SuSIE, FINEMAP, SuSIE-Inf and FINEMAP-Inf. Since infinitesimal components do not
732 neglect any SNP effects, this may also explain high prediction accuracy for the models
733 including infinitesimal effects. Our results showed that the performance of BayesR is closer to
734 the infinitesimal models.

735 (30) compared the performance of BayesC to various methods including SuSIE and FINEMAP
736 in fine mapping, where BayesC performed similar to SuSIE but better than FINEMAP in power
737 and false discovery rate determination, for different simulation scenarios. We showed that
738 BayesC had improved power relative to FINEMAP whereas the power was decreased relative
739 to SuSIE-RSS. This difference in results might be due to differences in implementation of these
740 models as this study applied the models in whole-genome scale using local regression approach
741 where we applied the models only in specific regions defined by simulated causal SNPs, that
742 not necessarily included whole genome. Our study compared SuSIE-RSS (which is an
743 extension of SuSIE that uses summary statistics) and BayesC, FINEMAP using summary
744 statistics with in-sample LD among other models, whereas this study used individual levels
745 data for BayesC and SuSIE, and summary statistics with in-sample LD for FINEMAP.

746 BayesR and SuSIE-RSS identified a greater number of CSs when applied to the UKB
747 phenotypes. However, BayesR showed the smallest average median CS size. We constructed
748 multiple credible sets for all the models based on our algorithm where we applied the cut-off
749 thresholds of 0.80 for a set to be a CS. We are aware that FINEMAP, SuSIE-RSS and SuSIE-
750 Inf also determine CS where multiple CS can be determined based on multiple causal variants

751 in the fine mapped region. Such procedure of determining CSs might alter the features of the
752 CSs. However, the main objective of our study was to compare the efficiency of the algorithms
753 of these models. Introducing a comparison based on the CSs they determine would introduce
754 additional complexity and divert us far from our objective. Hence, we determined multiple CSs
755 using the same algorithm for all the models. However, it would be interesting to compare CSs
756 developed by the external models and evaluate their efficiency.

757 **Influence of parameters in simulations**

758 We observed a significant difference in F_1 score between the different simulated parameters
759 h_{SNP}^2 (30% and 10%), π (0.001 and 0.1) and GA (GA_1 and GA_2), and PV (5% and 15%). The
760 pairwise comparison of $F_{1avg.rep}$ score, within a scenario, among different simulation
761 scenarios for each model showed significant differences among scenarios and significant
762 contribution of each parameter. However, the large value of the F-statistic obtained from
763 ANOVA on the results of the regression ($F_1 = h_{SNP}^2 + \pi + GA$) was seen for the parameter π
764 suggesting greater influence of this parameter in performance of the model. In our simulation,
765 a smaller number of causal SNPs for a given genetic variance would be sampled from a larger
766 marker effect variance compared to a higher number of causal SNPs. This large effect SNPs
767 must have high PIPs such that the credible sets determined by the models harbored the true
768 causal SNPs. The F_1 score was based on the detection of a true simulated causal variant in a
769 credible set. In addition to the threshold for a cumulative sum of PIPs [0.99] that a set needs to
770 cross to be a credible set, we also set a limit on the size of CS (not more than 10). The main
771 motive of the CS was to refine the resolution of the fine mapping region and a CS with large
772 number of SNPs even if it harbored a true causal variant would not be informative. We used a
773 strict cut-off threshold of 0.99 for cumulative sum of PIPs and maximum size of 10 for CS.
774 The results might differ with lenient thresholds for the cumulative sum, and size of CS. As per

775 our expectation, all the models performed significantly better (high $F_{1avg.rep}$ score) for the
776 phenotypes with moderate h_{SNP}^2 [30%], a smaller number of causal SNPs [π : 0.001], and the
777 phenotypes simulated with few SNPs with large effect size [GA_1] for the quantitative
778 phenotypes, and the worst performance was observed for the phenotypes with low h_{SNP}^2 [10%]
779 and a larger number of causal SNPs [π : 0.01].

780 **F_1 classification score – power and precision of the BLR models**

781 F_1 score is a harmonic mean of precision and power/recall and is a well-known performance
782 metric used for model comparison especially under class imbalance. It penalizes the
783 performance even when only one of either precision or power is low. In our study, both
784 precision and power are given equal importance for the performance of a model. We observed
785 higher $Pow_{avg.sim}$ for BayesR compared to other models. Highest power of BayesR referred
786 to the scenario where majority of CSs obtained from BayesR had small size CSs. We used in-
787 sample LD, while using external summary statistics in-sample LD is not always available as
788 also mentioned by (30). Hence, the power may decrease while using an external reference LD
789 panel. We observed low $Prec_{avg.sim}$ of BayesR. This referred to as substantial amount of CSs
790 were false positives. The range of $Prec_{avg.sim}$ across all the models is not vast suggesting that
791 all the models showed similar performance for precision.

792 **The UKB phenotypes, accuracy and fine mapping, credible sets**

793 The predictive accuracies for the UKB phenotypes were smaller compared to other studies.
794 $R_{avg.rep}^2$ for BMI, Height, HC, WHR, and $AUC_{avg.rep}$ for T2D for the UKB data presented by
795 (12) using SbayesR and around 1.1 million SNPs were larger compared to the values estimated
796 using BayesR in our study. In our study, accuracies were derived from imputed SNPs limited
797 only to the fine mapped regions. For polygenic phenotypes for example in Height and BMI,

798 (31) suggested enrichment of heritability from rare genetic variants (MAF < 0.01). In our study,
799 we discarded rare SNPs with MAF < 0.01 and focused only on common SNP effects. In our
800 study, non-converged regions for a model were excluded from analysis for prediction
801 accuracies (also credible sets) which might also have impacted the estimated accuracies.

802 **Validation of BLR model**

803 Compared to the recent meta-GWAS on T2D (24), we identified 10 genes to overlap with the
804 53 genes, which were categorized as novel loci in (24). This finding demonstrates the
805 effectiveness of BayesR model combined with credible sets in identifying potential causal
806 variants, even in studies with comparatively smaller size. This limited number (53) of
807 overlapping genes could be attributed to our study's smaller scale (25,828 cases and 309,704
808 controls compared to 74,124 cases and 824,006 controls in (24)), which could limit ability to
809 detect especially rare variants, and the exclusion of rare variants (excluding SNPs with < 1%
810 MAF in our study). Additionally, the discrepancies in how SNPs were mapped to a gene
811 between our study and that of (24) might also contribute to this limited overlap.

812 *TCF7L2* (Transcription Factor 7-like 2) explained the highest genetic variance (0.035) in our
813 study. This gene plays a crucial role in Wnt signaling pathway, which regulates pancreatic islet
814 cell proliferation and survival (32). In *TCF7L2*, rs7903146 is the largest-effect common variant
815 signal for T2D in Europeans (24). Observation of multiple signals for T2D at *TCF7L2* in
816 addition to rs7903146 in (24) was the first evidence according to this study. In addition to the
817 rs7903146, we also identified SNP rs34855922 associated to T2D similar with (24), which
818 again demonstrates the effectiveness of BayesR model combined with CSs. The rs7903146 and
819 rs34855922 are two of the eight SNPs that mark regulatory elements within *TCF7L2* locus
820 (33). The rs7903146 coordinate regulation of *TCF7L2* expression, and overlaps histone
821 modification marks and an annotated enhancer in the pancreas (33). Our study also identified

822 an intronic variant (rs145034729) at the *TCF7L2* locus. The effect of this intronic SNP is
823 uncertain. However, it may function as an enhancer element, modulating the expression of
824 distal genes without necessarily affecting the function of *TCF7L2* itself. The discovery of
825 multiple variants within the *TCF7L2* locus is interesting, as (33) suggests that it acts as a
826 regulatory hub for genes implicated in the etiology of T2D. Identifying these variants in this
827 locus offers valuable insights into the biological mechanisms underlying T2D.

828 The gene set enrichment analysis for diseases provided further support for the efficacy of
829 BayesR model in T2D. This analysis revealed significant enrichment of our gene set for
830 diseases such as T2D, hyperglycemia (diabetes-like symptoms), hyperinsulinism (one of the
831 processes leading to hyperglycemia (34). Significant enrichment to other types of diabetes and
832 diseases may reflect shared genetic factors (via pleiotropic genes or common pathways)
833 influencing the etiology of diverse conditions (diseases) through different mechanisms. For
834 instance, (35), noted an increased risk of diabetes mellitus incidence in individuals with RA,
835 highlighting the potential role of inflammatory pathways in the T2D pathogenesis.

836 For tissue enrichment analysis, our findings indicate that T2D related eQTLs exhibit tissue-
837 specific effects on gene expression. The implications of our results can be viewed from multiple
838 perspectives. Our results may suggest a complex interplay of regulatory regions in significantly
839 enriched tissues leading to T2D predisposition. Our results may also suggest individuals with
840 T2D might experience adverse effects in these tissues, potentially leading to a range of
841 complications. For instance, (36) explored the association of significantly enriched tissue
842 specific T2D associated eQTLs with different T2D complications. Here we delve into the
843 cerebellar hemisphere region of the brain, the most significant enriched tissue. This region, part
844 of the cerebellum (another significant tissue in our study), has been linked to cognitive
845 impairments when abnormal. (37) highlighted significant cognitive impairments in T2D
846 individuals, correlating these deficits with considerable loss in gray matter volume in brain

847 regions associated with these functions. The decline in insulin transport and resistance in the
848 cerebral cortex, an area dense with high insulin receptor, may impair regional glucose
849 metabolisms, leading to gray matter volume changes potentially leading to structural and
850 functional changes in brain in T2D individuals.

851 No association with pancreatic tissue was found, likely due to the GTEx database's limitations.
852 The pancreatic tissue in GTEx represents mostly (97%) exocrine cells that mask islets signals
853 (38). Pancreatic islets are clusters of specialized endocrine cells that are essential to maintain
854 glucose homeostasis and play a central role in etiology of T2D.

855 Our study was confined to the cis-eQTLs database from GTEx consortium. (39) have shown
856 that trans-eQTLs contribute significantly to T2D heritability, suggesting that further
857 exploration of trans-eQTLs could enhance the understanding of gene expression and cellular
858 functions across different tissues.

859 In conclusion, we observed that the performance of the BLR models was comparable to the
860 state-of-the-art external models. The performance of BayesR prior was closely aligned with
861 SuSIE-Inf and FINEMAP-Inf models. Results from both simulations and application of the
862 models in the UKB phenotypes suggest that the BLR models are efficient fine mapping tools.

863 **Data availability statement**

864 The genetic and phenotypic data utilized in our study were obtained from the UK Biobank
865 Resource (ID 96479).

866 **Ethics statement**

867 Human studies in the UK Biobank project have received approval from the Ethics and
868 Governance Framework (EGF), which ensures data and sample usage adheres to scientific and
869 ethical standards. The consent to participation will apply throughout the lifetime of the UK
870 Biobank, unless participants withdraw, and involves the collection and storage of biological
871 samples (blood, saliva, urine) and electronic health records (GP, hospitals, dental,
872 prescriptions). Individual data is anonymized, with each research project receiving its own
873 anonymized dataset. The ethics committee waived the need for written informed consent.

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879 **Author Approvals**

880 All authors have seen and approved the manuscript, and it hasn't been accepted or published
881 elsewhere.

882 **Competing interests**

883 There are no competing interests.

884

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