

# Commonly used Bayesian diversification-rate models produce biologically meaningful differences on empirical phylogenies

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## Abstract

Identifying along which lineages shifts in diversification rates occur is a central goal of comparative phylogenetics; these shifts may coincide with key evolutionary events such as the development of novel morphological characters, the acquisition of adaptive traits, polyploidization or other structural genomic changes, or dispersal to a new habitat and subsequent increase in environmental niche space. However, while multiple methods now exist to estimate diversification rates and identify shifts using phylogenetic topologies, the appropriate use and accuracy of these methods is hotly debated. Here we test whether five Bayesian methods—Bayesian Analysis of Macroevolutionary Mixtures (BAMM), two implementations of the Lineage-Specific Birth-Death-Shift model (LSBDS and PESTO), the approximate Multi-Type Birth-Death model (MTBD; implemented in BEAST2), and the cladogenetic diversification rate shift model (CLaDS2)—produce comparable results. We apply each of these methods to a set of 65 empirical time-calibrated phylogenies and compare inferences of speciation rate, extinction rate, and net diversification rate. We find that the five methods often infer different speciation, extinction, and net-diversification rates. Consequently, these different estimates may lead to different interpretations of the macroevolutionary dynamics. The different estimates can be attributed to fundamental differences among the compared models. Therefore, the inference of shifts in diversification rates is strongly method-dependent. We advise biologists to apply multiple methods to test the robustness of the conclusions or to carefully select the method based on the validity of the underlying model assumptions to their particular empirical system.

## Lay Summary

Understanding why some groups of organisms have more species than others is key to understanding the origin of biodiversity. Theory and empirical evidence suggest that multiple distinct historical events—such as the evolution of particular morphological features (e.g., the flower, the tetrapod limb) and competition amongst species—can produce this pattern of divergent species richness. Identifying when and where on the tree of life shifts in diversification rates occur is important for explaining the origin of modern-day biodiversity and understanding how disparity among species evolves. Several statistical methods have been developed to infer diversification rates and identify these shifts. While these methods each attempt to make inferences about changes in the tempo of diversification, they differ in their underlying statistical models and assumptions. Here we test if these methods draw similar conclusions using a dataset of 65 time-calibrated phylogenies from across multicellular life. We find that inferences of where rate shifts occur strongly depends on the chosen method. Therefore, biologists should choose the model whose assumptions they believe to be the most valid and justify their model choice *a priori*, or consider using several independent methods to test an evolutionary hypothesis.

**Key Words:** [diversification-rate analyses, Phylogeny, BAMM, RevBayes, BEAST, CLaDS2, macroevolution, time-dependence]

## Introduction

1 Understanding the patterns and processes that shape the  
2 tree of life is one of the central pursuits of biology. How-  
3 ever, inferring the tempo of evolution among lineages—  
4 the patterns of speciation and extinction that gave rise to  
5 our extant biodiversity—remains a difficult problem both  
6 theoretically and computationally (Rabosky, 2010; Moore  
7 et al., 2016; Louca and Pennell, 2020).

8 Several methods estimate diversification rates (speci-  
9 nation and extinction rates, individually) assuming that  
10 rates are constant across the tree (Morlon, 2014). Re-  
11 cently developed methods have built upon constant-rate  
12 models by allowing diversification parameters to vary  
13 depending on the state of a focal character (Maddison  
14 et al., 2007) or, even more recently, among branches of the

15 phylogeny, which allows for lineage-specific diversifica-  
16 tion rate estimates (e.g., Rabosky, 2014; Höhna et al., 2019;  
17 Barido-Sottani et al., 2020; Maliet and Morlon, 2022).

18 Such lineage-specific methods have the potential to of-  
19 fer powerful insights into our understanding of evolu-  
20 tion, such as the potential time-dependency of macroevo-  
21 lutionary diversification (Henao Diaz et al., 2019), the  
22 macroecological and macroevolutionary causes of the  
23 latitudinal diversity gradient (Givnish et al., 2018; Ra-  
24 bosky et al., 2018), and macroevolutionary support of  
25 Darwinian and Simpsonian theories of microevolution  
26 within adaptive zones (Cooney et al., 2017).

27 The application of these methods, however, has been  
28 marred by controversy over their implementation (Moore  
29 et al., 2016; Rabosky et al., 2017; Meyer and Wiens, 2018;  
30 Meyer et al., 2018; Rabosky, 2018) and by theoretical

31 findings that seemingly undermine the general reliability  
32 of inferring diversification parameters from phylogenies  
33 of extant species (Louca and Pennell, 2020; Helmstetter  
34 et al., 2021). These issues are liable to discourage em-  
35 piricists, who may wonder if the disagreements among  
36 model developers and theorists correspond with biologi-  
37 cally relevant inference differences in empirical studies.

38 To address this question, we assess how inferences  
39 under five leading contemporary Bayesian methods—  
40 Bayesian Analysis of Macroevolutionary Mixtures (BAMM;  
41 Rabosky, 2014); the Lineage-Specific Birth-Death-Shift  
42 model (LSBDS; Höhna et al., 2019) and its MCMC-free im-  
43 plementation: Phylogenetic Estimation of Shifts in the  
44 Tempo of Origination (PESTO; Kopperud et al., 2023a);  
45 the approximate Multi-Type Birth-Death model (MTBD;  
46 Barido-Sottani et al., 2020); and the Cladogenetic Diver-  
47 sification Rate Shift model (CLaDS2; Maliet et al., 2019)—  
48 compare to each other.

49 While all five methods aim to estimate lineage-specific  
50 diversification rates, they differ in how and where rate  
51 shifts are allowed to occur.

- 52 1. BAMM models diversification rates as varying across  
53 lineages by testing among models that include dif-  
54 ferent numbers of diversification-rate regimes (sets  
55 of speciation and extinction parameters) and differ-  
56 ent placements of those regimes in the tree; however  
57 BAMM does not model rate shifts on extinct (thus un-  
58 observed) branches (Rabosky, 2014).
- 59 2. The LSBDS model, as implemented in RevBayes  
60 (Höhna et al., 2016), samples rate regimes from a  
61 prior distribution discretized into a fixed number of  
62 rate categories; this discretization facilitates compu-  
63 tation and allows the method to model shifts on ex-  
64 tinct branches (Höhna et al., 2019).
- 65 3. PESTO is a new implementation of the LSBDS model  
66 that analytically computes the posterior mean spe-  
67 ciation and extinction rates conditional on a set of  
68 hyperparameters without the need for Monte Carlo  
69 sampling (Kopperud et al., 2023a).
- 70 4. The MTBD method is based on a multitype birth-death  
71 process that infers the number of rate regimes as well  
72 as the transition rate  $\gamma$  between rate regimes (Barido-  
73 Sottani et al., 2020). This approach allows for the  
74 same rate regime to be present in different parts of  
75 the tree. The approximate MTBD, tested here, assumes  
76 that no rate changes occur in the extinct parts of the  
77 tree; this approximation, when applied with a high  
78 transition rate prior, has been found to not substan-  
79 tially differ from the exact MTBD method, which al-  
80 lows rates changes along extinct lineages (Barido-  
81 Sottani et al., 2020).

82 5. Finally, in the CLaDS2 model, diversification rates  
83 only change at speciation events. Descendant lin-  
84 eages inherit the speciation rate via a stochastic pro-  
85 cess that is influenced by the  $\alpha$  parameter, which  
86 represents the long-term trend (i.e., increase or de-  
87 creases) of the speciation rate (Maliet et al., 2019).  
88 This model results in many small and frequent shifts  
89 in diversification rates regimes, unlike the other  
90 methods, which tend to infer a few large shifts in  
91 rate regimes (Maliet et al., 2019; Maliet and Morlon,  
92 2022). Another aspect of CLaDS2 is that extinction  
93 rates are not inferred per branch. Instead, the model  
94 estimates a global turnover parameter ( $\epsilon = \mu_i / \lambda_i$ ).  
95 However, shifts are allowed to occur along extinct  
96 branches.

97 Other methods, not tested here, leverage hidden states  
98 using a maximum likelihood framework (e.g., Vasconcelos  
99 et al., 2022).

100 To assess whether the theoretical and computational  
101 differences among these methods result in biologi-  
102 cally meaningful differences, we reanalyze 65 empirical  
103 datasets, compiled from Henao Diaz et al. (2019), using  
104 each of BAMM, LSBDS, PESTO, MTBD, and CLaDS2.  
105 We address the question: do different analytical methods for  
106 estimating branch-specific diversification rates produce  
107 significantly different results across an array of empirical  
108 datasets?

## Methods

### Empirical Data

111 Our empirical data are derived from the set of  
112 104 chronograms compiled and analyzed with BAMM by  
113 Henao Diaz et al. (2019). From the Henao Diaz et al. set  
114 we excluded trees with fewer than 30 extant taxa in order  
115 to concentrate on more informative datasets, resulting in  
116 our final set of 76 chronograms.

### Model Settings

117 Our goal was to apply each method as a typical dili-  
118 gent user might. For each chronogram, we used the  
119 incomplete-sampling fraction collected from the original  
120 study by Henao Diaz et al. (2019), and applied that sam-  
121 pling fraction when we ran each of the five inference  
122 methods. While the methods differ in their specific pa-  
123 rameterizations of the birth-death process, we attempted  
124 to use comparable settings and priors across methods.

125 For BAMM analyses, we modified the control files  
126 from Henao Diaz et al. (2019). We set lambda to  
127 be time-constant rather than time-variable in order to  
128 more closely match the inferences of other methods and

given concerns about the statistical validity of time-varying diversification analyses (Louca and Pennell, 2020). We set BAMM priors for each phylogeny using the `setBAMMpriors()` function in the `BAMMtools` R package (Rabosky et al., 2014b). This function computes dataset-specific priors by estimating metrics from the dataset such as the root age of the chronogram and then estimating reasonable and broad expectations for shifts and rates. We ran BAMM v. 2.5.0 using the `BAMMtools` priors and control files, which determined the phylogeny-specific number of generations for a single MCMC chain. We removed the first 10% of the MCMC samples as burnin and assessed convergence by computing estimates of effective sample size (ESS) using the R package `Coda` (Plummer et al., 2006). We specifically looked for convergence of the log-likelihood parameter and the 'number of distinct regimes' parameter, as is recommended (Rabosky et al., 2014a). Analyses that did not reach convergence were run for additional generations until they converged.

For LSBDS analyses we used the same set of priors for all phylogenies (except for sampling fraction) with eight categories for speciation and for extinction (64 total rate categories). The number of rate categories was chosen after performing a test on one representative phylogeny, which found that increasing the rate categories above 64 did not result in a significant change in model fit. For each chronogram we ran four MCMC chains for 5,000 generations. Convergence was assessed for each chain by checking that the ESS values for all model parameters in the log files were greater than 200 using the R package `Coda` (Plummer et al., 2006). Chains that did not reach convergence were restarted and run for an additional 5000 generations. We merged the posteriors, retaining the last 4000 generations from the MCMC (10% burnin for non-restarts and 60% burnin for restarts).

We applied PESTO in a three-step fashion. First, we estimated the parameters of a constant-rate birth-death process and treated these as hyperparameters: the speciation rate ( $\lambda$ ) and the extinction rate ( $\mu$ ). Second, we set up a state-dependent speciation-extinction (SSE) model. In this model, we used rate values that correspond to ten quantiles of two lognormal distributions with medians  $\lambda$  and  $\mu$ , and standard deviation 0.587. In the SSE model, we used all pairwise comparisons of these (i.e. 100 rate categories). Further, we estimated the shift rate parameter  $\eta$  conditional on the speciation and extinction rates, using maximum likelihood. Third, we calculated the posterior state probabilities along each branch. Finally, we plotted the posterior mean rates averaged over the time span for each individual branch.

We ran the MTBD model under default priors (implemented in BEAST2; Bouckaert et al., 2014; Barido-Sottani et al., 2020). We ran three MCMC chains for 100,000,000

generations per phylogeny. We removed the first 25% as burnin and assessed MCMC convergence by checking that ESS values were higher than 200 for all rates.

We ran CLaDS2 using the default priors (as described in Maliet and Morlon, 2022). We ran three MCMC chains for each dataset and took a 25% burnin, as is the default setting for CLaDS2. Convergence was assessed by calculating the Gelman statistic (Gelman et al., 2014) every 1000<sup>th</sup> generation and stopping the analysis once it achieved a Gelman statistic of 1.05, following the standard guidelines for CLaDS2.

## Convergence analysis

In cases where MCMC convergence was difficult, we aimed to determine the potential underlying cause. To assess whether the subset of trees where one or more method failed to converge was substantially different from the subset that did converge, we compared descriptive metrics including phylogeny size, phylogeny age, incomplete sampling fraction, branch length variance, and multidimensional scaling (MDS) via Robinson-Foulds (RF Robinson and Foulds, 1981) and Kuhner-Felsenstein (KF, Kuhner and Felsenstein, 1994) distances.

## Processing Model Output

We obtained estimates of the relevant diversification parameters (e.g., speciation rate, extinction rate, etc.) from each model. BAMM posterior estimates of speciation rate and extinction rate were extracted using the `getMarginalBranchRateMatrix()` function in the `BAMMtools` R package (Rabosky et al., 2014b).

We extracted LSBDS posterior distributions from the stochastic branch rate log file produced by the `mnStochasticBranchRate()` function in LSBDS. In the PESTO analyses, we computed the branch rates averaged across the branch. If  $\lambda_k$  and  $\mu_k$  are the rate values in state  $k$ , and  $P_k(t)$  is the posterior probability of being in state  $k$  at time  $t$ , then the average net-diversification rate along a branch is

$$\frac{1}{t_1 - t_0} \int_{t_0}^{t_1} \left[ \sum_k (\lambda_k - \mu_k) P_k(t) \right] dt, \quad (1)$$

where  $t_0$  is the youngest and  $t_1$  is the oldest end point of the branch.

The posterior distributions of speciation and extinction rates of the MTBD model were obtained from the extended Newick file produced by BEAST2 using a modified `read.beast()` function from the `treeio` package (Wang et al., 2020). As CLaDS2 does not directly infer extinction rates, we calculated extinction rates per branch by multiplying the inferred global turnover value ( $\epsilon$ ) by the branch-specific speciation rates ( $\mu_i = \lambda_i * \epsilon$ ). For all branches and models, we calculated net diversification by

233 subtracting extinction rate from speciation rate ( $\lambda_i - \mu_i$ )  
234 per MCMC generation.

## 235 Comparing Model Inferences

236 To compare inferences among the five models, we (1) visualized rate estimates on individual chronograms, (2)  
237 summarized inferences across all chronograms in the  
238 dataset to reveal systematic differences, (3) identified differences in the location and magnitude of inferred shifts  
239 among methods, and 4) tested for overlap in the 95%  
240 HPD interval of the posterior distributions.

## 243 Visualizing rates on trees

244 The canonical way of presenting the results of branch-  
245 specific diversification-rate analyses is by coloring the  
246 branches of the tree by the estimated rates. For each tree,  
247 we colored each branch by the posterior median estimate  
248 of speciation, extinction, and net diversification to visual-  
249 ize if the methods inferred similar shifts in similar loca-  
250 tions on the tree.

## 251 Comparing rate estimates by method

252 To understand whether the methods displayed any con-  
253 sistent differences across the chronograms, we calculated  
254 six summary statistics for each tree. For each diversifica-  
255 tion rate (*i.e.*, speciation rate, extinction rate, and net di-  
256 versification rate) we calculated the posterior medians for  
257 each branch, and from those posterior medians we calcu-  
258 lated the tree-wide mean and variance in branch rates for  
259 each phylogeny. For each of the six summary statistics  
260 (mean and variance for each of the three rates), we set up  
261 a linear mixed-effect model:

$$262 \log(\text{summary statistic}) = \mathbf{X}\beta + \mathbf{Z}u + r, \quad (2)$$

263 with inference method as a fixed-effect categorical pre-  
264 dictor (effect sizes  $\beta$ ), phylogeny as a random effect cat-  
265 egorical predictor ( $u$ ), and an error term  $r$ .  $\mathbf{X}$  and  $\mathbf{Z}$  are  
266 design matrices for the fixed and random effects. We vi-  
267 sually checked that the residuals ( $r$ ) were normally dis-  
268 tributed and did not suffer from heteroscedasticity; phylo-  
269 genies that violated these assumptions were excluded  
270 from this analysis. For each linear model, we tested if the  
271 least-square means of each pair of methods were statisti-  
272 cally different using Tukey's corrected p-value for multi-  
273 ple comparisons.

## 274 Location and magnitude of rate shifts

275 We additionally tested whether the methods inferred con-  
276 sistent locations and magnitudes of rate shifts, using the  
277 rooted version of the Kuhner-Felsenstein distance (Kuh-  
278 ner and Felsenstein, 1994). To do this, we first replaced

278 branch lengths of each timetree with the posterior me-  
279 dian rate estimate, from a given method, then scaled each  
280 branch by the total tree height. This produces a method-  
281 dependent tree with branch lengths that provide informa-  
282 tion regarding the magnitude and location of rate shifts  
283 but with identical topology. We calculated KF distances  
284 between the rescaled trees from each pair of methods; this  
285 distance is equivalent to the mean square error (MSE)  
286 given that the two trees being compared have the same  
287 topology, as they do in our analyses. For each tree and for  
288 each diversification parameter, we computed the mean  
289 square error among the different methods:

$$290 \text{MSE} = \frac{1}{N} \sum_i^N (\lambda_i - \lambda'_i)^2, \quad (3)$$

291 where  $\lambda_i$  (or similarly  $\mu_i$ , or  $(\lambda_i - \mu_i)$ ) is the diversifica-  
292 tion rate parameter for branch  $i$ .

293 A large MSE tells us that the two methods being com-  
294 pared infer different rate magnitudes and/or rate shifts  
295 in different locations. A small MSE, however, indicates  
296 that the two methods give us similar results.

## 297 Computation

298 We ran all diversification analyses either locally, on the  
299 Savio HPC at UC Berkeley, or using the CIPRES Science  
300 Gateway V. 3.3 (Miller et al., 2010).

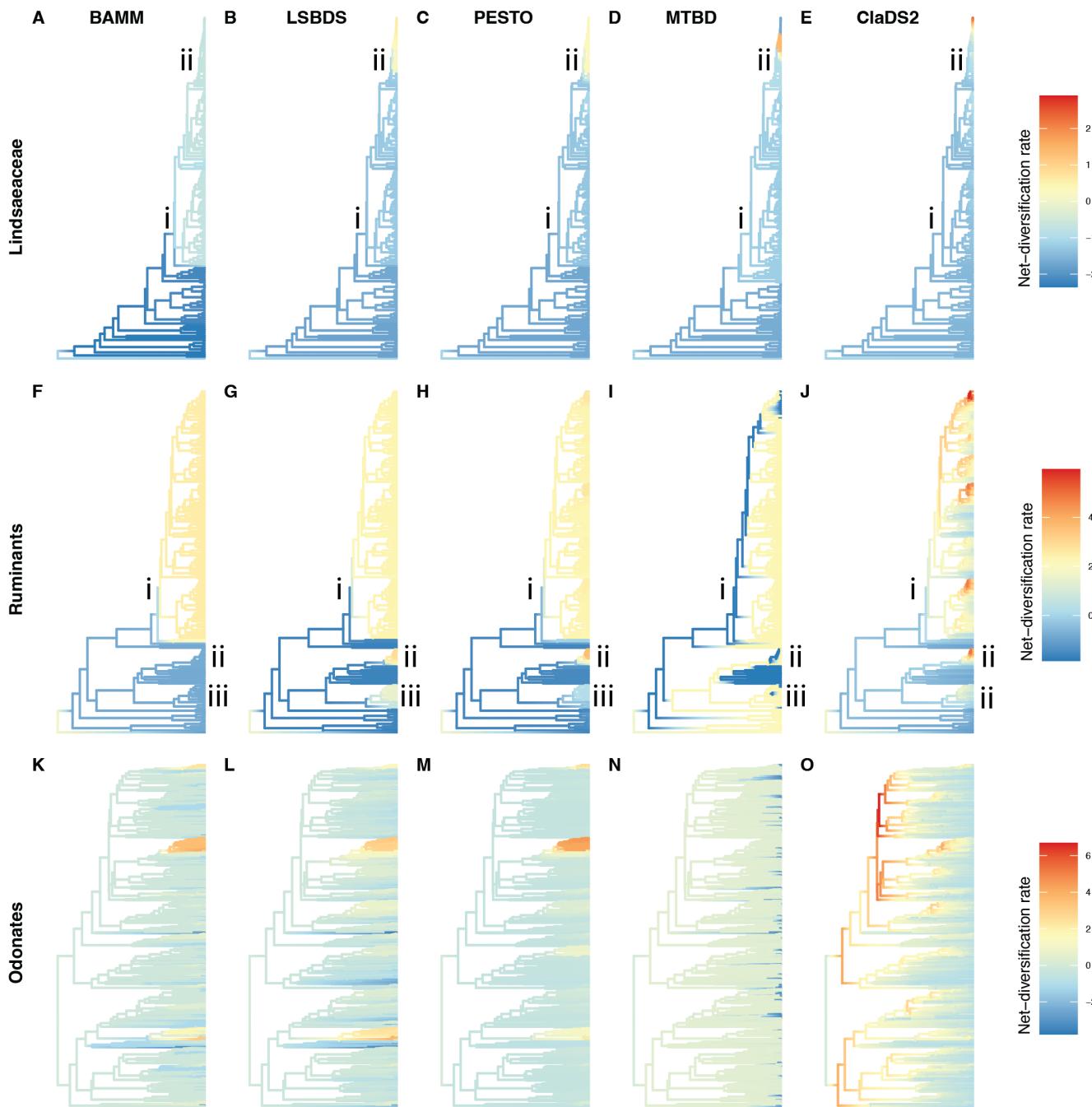
301 We performed all comparison analyses in R version  
302 3.6.0 (R Core Team, 2013). We performed data manip-  
303 ulation with the R packages *phytools*, (Revell, 2012),  
304 *tidyverse* (Wickham, 2017), *reshape2* (Wickham, 2012),  
305 *readr* (Wickham and Hester, 2020), *plyr* (Wickham,  
306 2011b), and *coda* (Plummer et al., 2006). We gener-  
307 ated plots with R packages *see* (Lüdecke et al., 2021),  
308 *ggplot2* (Wickham, 2011a), *ggpubr* (Kassambara, 2018),  
309 *ggtree* (Yu et al., 2018), *ggsignif* (Ahlmann-Eltze, 2017),  
310 *ggExtra* (Attali and Baker, 2016), *cowplot* (Wilke, 2016)  
311 and *pdftools* (Ooms, 2020). We fit linear mixed models  
312 using the R package *lmer* (Bates et al., 2015) and obtain  
313 emeans estimates using the R package *emmeans* (Lenth,  
314 2020). We additionally used *smacof* (Mair et al., 2022) and  
315 *phangorn* (Schliep, 2011) to perform MDS and to calculate  
316 RF and KF distances. Citations for R packages were gen-  
317 erated with *RefManageR* (McLean, 2014).

## 318 Results and Discussion

### 319 Convergence

320 Our full dataset contains 76 chronograms from multicel-  
321 lular organisms, with 31 – 4161 extant tips, root ages of 4.9  
322 – 349.8 MYA, and 0.014% – 100% of extant species sam-  
323 pled (Fig. S1A and Table S1). All methods converged for  
324 43 trees (the “complete subset”; Fig. S1B). Of the methods  
325

Comparing diversification rate methods



**Figure 1:** Three representative phylogenies with Z-transformed (mean centering and scaling to unit variance) posterior median estimate of net diversification painted on the branches. Columns show estimates from BAMM (A,F,K), LSBDS (B,G,L), PESTO (C,H,M), MTBD (D,I,N) and ClaDS2 (E,J,O). (A-E) Phylogeny of Lindsaeaceae (necklace ferns; Testo and Sundue, 2016), (F-J) Phylogeny of Ruminants (tetrapod; Toljagić et al., 2018), and (K-O) Phylogeny of Odonates (dragonflies and damselflies; Waller and Svensson, 2017). The rate values are in units of events per lineage per million years.

324 tested, LSBDS had the most difficulty achieving MCMC  
 325 convergence (it converged for 46 trees). All methods ex-  
 326 cept LSBDS converged in 65 trees (the “partial subset”;  
 327 Fig. S1C); PESTO directly computes the posterior mean  
 328 and thus “convergence” does not apply. Trees that did

329 not converge have poorer taxon sampling (i.e. the  
 330 ratio of sampled species to total species richness; P-value  
 331 = .039), older root ages (P-value = 0.0001), and greater  
 332 branch length variance (P-value = .00006) than the con-  
 333 verged trees, but sample size (number of tips) was not an

important factor (P-value = .076; Fig. S2C–F). The branch length variance is consistent with the degree of spread between the KF and RF MDS analyses (Fig. S2A–B); the KF MDS—which accounts for branch lengths as well as topology—has a larger spread than the RF MDS. Overall these results fit with our intuitive understanding of the challenges in inferring shifts in diversification rates. We expect that older trees and trees with greater variation in branch lengths should undergo more rate shifts than younger trees and those with less variation in branch lengths. Thus inferring the diversification rates of these trees should be generally more challenging. These results suggest that users should be particularly attentive to MCMC convergence if their chronogram(s) are poorly sampled, old, or have a lot of branch-length variation, and especially so if they are using LSBDS. In these cases, even more so than usual, it is important to run each MCMC multiple times independently, to assess both stationarity and convergence.

## Comparison of Methods

### Visualizing rates on trees

None of the 43 phylogenies in our “complete subset” had concordant estimates among all methods given our evaluation criteria (Fig. 1). For some phylogenies, the methods inferred similar shifts in net diversification (e.g., Fig. 1A–E), whereas for others the inferred shifts differed slightly (e.g., Fig. 1F–J) or strongly (e.g., Fig. 1K–O). We would expect some differences when comparing different modeling approaches, as there are patterns to the differences in our results that can be attributed to the fundamental differences between the models. We illustrate these patterns using a few example phylogenies, which are representative of the patterns one will find when perusing the full set of trees in the supplemental materials (Supplemental Section S5).

Occasionally two methods generally identified similar patterns. For example, BAMM and LSBDS identified a similar shift of about the same magnitude in speciation rates for some clades, e.g., the Lindsaeaceae (necklace ferns, clade i; Fig. 1A,B). Nonetheless, there are still differences between the two methods, e.g., a second nested rate shift in the LSBDS and PESTO estimates (clade ii).

In the ruminants (tetrapods) phylogeny (Fig. 1 F–J), we find that even for results that overall appear similar between methods, there are meaningful differences between their estimates. For example, BAMM, LSBDS, and PESTO inferred a shift around the ancestor of clade i, but LSBDS and PESTO also find approximately two more shifts (Fig. 1 G, clades ii and iii). Likewise, MTBD differs from the latter two as it infers several shifts in the largest clade and low net diversification rates on the backbone of that lineage (Fig. 1 I). Similarly, CLaDS2 infers a slightly dif-

ferent history from all of them, including multiple slowdowns as well as an increase in net-diversification within clade i. BAMM, LSBDS, and PESTO identify a shift in approximately the same node (indicated by i) while MTBD infers many replicated increases in rate within clade i. LSBDS and PESTO infer the same shift, which is expected as they are based on the same underlying model and assumptions.

Multiple diversification shifts across a phylogeny is common to many of the MTBD trees (Fig. 1I,N; Barido-Sottani et al., 2020). This pattern is caused by the bimodal posterior distribution commonly inferred by this method (Barido-Sottani et al., 2020). Point-estimate summary statistics (e.g., posterior median) of these types of distributions are susceptible to small variation between the ancestor-descendant branches, which causes point estimates to switch between the two optima producing the rapid switching pattern (Fig. 1I,N).

Likewise, CLaDS2 is the only time-dependent model in our analysis and thus is capable of detecting time-varying diversification patterns. Furthermore, the inherited speciation rate ( $\alpha$ ) only changes at cladogenic events, which results in many small changes at cladogenetic events, rather than the few large changes that characterize the other methods (Maliet et al., 2019; Maliet and Morlon, 2022). When  $\alpha < 1$  evolutionary slowdowns occur where the ancestral lineages have higher net diversification rates than the descendant lineages, a pattern observed in our data (Fig. 1O; Moen and Morlon, 2014).

On the other hand, BAMM, LSBDS, and PESTO are similar models and therefore we may expect them to infer similar diversification rates and shifts (Ronquist et al., 2021). While this is sometimes true (Figure 1 K,L), other times there are pronounced differences (Figure 1 F,G). This may be due to the well-known differences between these two models, namely assuming either rate shifts can (LSBDS and PESTO) or cannot (BAMM) occur on extinct lineages or unsampled lineages (Moore et al., 2016).

### Comparisons of rate estimates by method

To gain a global perspective of the differences between these models, we calculated two tree-wide summary statistics and distance metrics in order to compare these methods across the entire dataset (Fig. 2, S3).

We ran all comparisons on the complete subset (the 43 trees that converged for all methods) and on the partial subset (the 65 trees that converged for all methods except LSBDS). Comparisons between these two subsets reveal only one small difference (compare BAMM vs. MTBD – Fig. 2C vs. Supplemental Fig. S4C) and the most significant differences did not change. Given the large number of datasets that did not converge for LSBDS but converged for all other methods (unconverged datasets = 22) as well as the theoretical similarities between PESTO and LSBDS,

## Comparing diversification rate methods

**Table 1:** Post-hoc pairwise comparisons of inference methods using the tree-wide average of summary statistics: speciation, extinction, and net-diversification rates. Columns contain the summary statistics, contrasts of inference methods, the ratios of geometric means, standard errors, degrees of freedom, t-ratios, Tukey-adjusted p-values, significances, and the percent variances explained by the random effect.

Summary Statistic	Contrasts	Means Ratio	SE	DF	T-Ratio	Adj. P-Value	Sig.	% Var.
Speciation	BAMM / ClaDS2	1.0114	0.0336	189	0.3402	0.986	N.S.	94.57
	BAMM / PESTO	0.8833	0.0294	189	-3.7326	0.001	**	
	BAMM / MTBD	1.0352	0.0344	189	1.041	0.726	N.S.	
	ClaDS2 / PESTO	0.8733	0.029	189	-4.0728	0.000	***	
	ClaDS2 / MTBD	1.0236	0.034	189	0.7009	0.897	N.S.	
	PESTO / MTBD	1.172	0.039	189	4.7736	0.000	***	
Extinction	BAMM / ClaDS2	1.2616	0.3006	189	0.9752	0.764	N.S.	58.65
	BAMM / PESTO	3.0505	0.7268	189	4.6808	0.000	***	
	BAMM / MTBD	1.119	0.2666	189	0.4718	0.965	N.S.	
	ClaDS2 / PESTO	2.418	0.5761	189	3.7056	0.002	**	
	ClaDS2 / MTBD	0.887	0.2113	189	-0.5034	0.958	N.S.	
	PESTO / MTBD	0.3668	0.0874	189	-4.209	0.000	***	
Net Diversification	BAMM / ClaDS2	0.6467	0.0406	188.067	-6.9392	0.000	***	81.83
	BAMM / PESTO	0.6666	0.0419	188.067	-6.4555	0.000	***	
	BAMM / MTBD	0.8278	0.052	188.067	-3.0073	0.016	*	
	ClaDS2 / PESTO	1.0309	0.0644	188.0003	0.4863	0.962	N.S.	
	ClaDS2 / MTBD	1.2802	0.08	188.0003	3.9523	0.001	***	
	PESTO / MTBD	1.2419	0.0776	188.0003	3.466	0.004	**	

439 we report the following results for the partial dataset (see  
 440 Fig. S4A–F for summaries from the complete dataset).

441 We recover consistent differences in rate estimates  
 442 among methods, particularly between PESTO (which is  
 443 additionally standing-in for LSBDS in these comparisons,  
 444 given that those two approaches share the same under-  
 445 lying model) and all other models; CLaDS2 also was an  
 446 outlier, albeit to a lesser extent (Table 1). In contrast, BAMM  
 447 and MTBD tended to infer similar speciation and extinction  
 448 rates. We find that tree-wide average speciation and ex-  
 449 tinction estimates of PESTO are statistically different from  
 450 all other methods (Fig. 2A–B).

451 While PESTO inferred higher tree-wide average specia-  
 452 tion values, the magnitude of the differences is small (ra-  
 453 tio of means < 1.2 for all significant contrasts; Table 1).  
 454 Conversely, PESTO inferred lower tree-wide averages of  
 455 extinction rates with larger magnitude changes (ratio of  
 456 geometric means > 1.2; Table 1). The significant difference  
 457 between PESTO and other methods holds for tree-wide av-  
 458 erage net-diversification as well, except for the compari-  
 459 son between PESTO and CLaDS2 (Fig. 2C), which is not  
 460 significant.

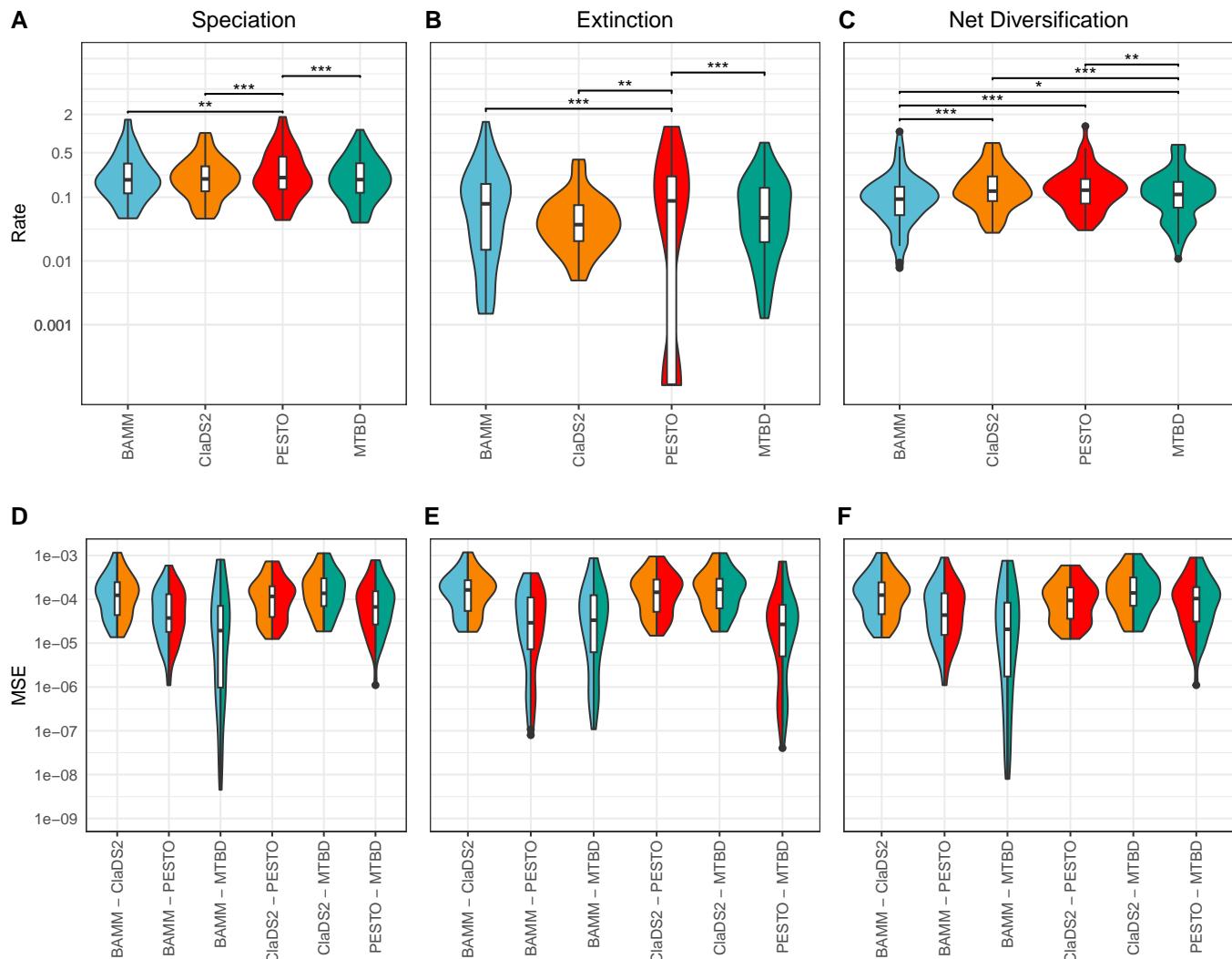
461 Additionally, CLaDS2 tree-wide average net-  
 462 diversification estimates are significantly different  
 463 from BAMM and MTBD (Fig. 2C). A significant difference  
 464 in net-diversification could be driven by the CLaDS2  
 465 parameterization of extinction: extinction is not directly  
 466 estimated in CLaDS2. Therefore the net diversification

467 rates of CLaDS2 are scaled speciation rates. Alternatively,  
 468 the differences between methods could be due to the  
 469 wider variance of net diversification estimates that both  
 470 BAMM and MTBD have compared to CLaDS2 (Fig. S3A–C).  
 471 However, similar to tree-wide average speciation, the  
 472 magnitude of difference between the contrast is not large  
 473 (Table 1). There is also a weakly significant difference  
 474 between speciation rates of BAMM and MTBD in our partial  
 475 subset that was not found in the smaller complete subset.

476 All methods generally had comparable tree-wide aver-  
 477 age extinction-rate estimates with the exception of PESTO,  
 478 which may infer much lower extinction rates for some  
 479 trees than the other methods (though, on average, it in-  
 480 fers higher extinction rates). The inference of extinction  
 481 rate has been the subject of substantial debate, partic-  
 482 ularly in how failures to account for diversification shifts  
 483 along extinct branches can impact the likelihood function  
 484 (Moore et al., 2016; Rabosky et al., 2017). Regardless of  
 485 the theoretical importance of correctly inferring extinc-  
 486 tion rates, we demonstrate that differences between ex-  
 487 tinction and speciation rates manifest in statistically dif-  
 488 ferent estimates of net diversification in empirical studies.  
 489 Therefore, our results indicate that method-dependent  
 490 tree-wide bias in diversification parameter inference may  
 491 influence the interpretation of evolutionary shifts in di-  
 492 versification rates.

493 We find discrepancies between results derived from  
 494 tree-wide summary statistics and our visual inspection

## Comparing diversification rate methods



**Figure 2:** Comparison of tree-wide summary statistics across methods for the partial subset (n=65). (A–C) Tree-wide mean of posterior median estimates of the branch-specific rate parameters, plotted on a log scale. Asterisks correspond to the p-value of linear mixed model, calculated on the natural log of the rates (\*: 0.05 > P-value > 0.01; \*\*: 0.01 > P-value > 0.001; \*\*\*: 0.001 > P-value). (D–F) Pairwise mean squared error (MSE) between inference methods of phylogenies with branch lengths scaled by rates (speciation, extinction, and net diversification), plotted on a log scale. Split colors correspond to inference method color in A–C. Distributions closer to zero indicate that the inference methods produced more similar rate estimates, whereas higher values indicate greater dissimilarity. (D) MSE of speciation-scaled phylogenies; (E) MSE of extinction-scaled phylogenies; (F) MSE of net-diversification-scaled phylogenies.

of trees (see section “Visualizing rates on trees”). For example we find that CLaDS2 and PESTO show no statistical difference in average net diversification (Fig. 2C). However, visual inspection of many trees suggests that CLaDS2 and PESTO often differ greatly in the number and position of inferred rate shifts (e.g., Fig. 1). Conversely, BAMM and LSBDS often look very similar when we assess individual phylogenies and yet significantly differ when we compare speciation, extinction, and diversification averages S3A–C). This discrepancy reveals the difficulty of summarizing diversification rate estimates across phylogenies to reveal general patterns, and motivates

the topology-informed rate comparisons, discussed in the following section.

### Location and magnitude of rate shifts

We also test whether the models recover similar locations and magnitudes of rate shifts by comparing the mean squared error (MSE) of branch rates; this metric bridges the discrepancies between the global metrics and the observed patterns across the trees (both described above; Fig. 2).

When quantifying differences in the location and magnitude of shifts in speciation and net diversification rates,

518 CLaDS2 differs the most (larger MSE), compared with the  
519 other methods (Fig. 2D,F) and it is by far the biggest out-  
520 lier across the models when visually inspecting the trees  
521 (Fig. 1E,J,O). This result indicates that CLaDS2 estimates  
522 differ strongly from those of the other methods in the de-  
523 gree of the shifts in inferences, and in their location. This  
524 result is in contrast to the tree-wide averages presented  
525 above (see also Fig. 2A–C), where CLaDS2 is unexcep-  
526 tional.

527 These results are also corroborated by analyses that  
528 take into account uncertainty in rate estimates (see Sup-  
529 plemental Section S4).

## 530 Tools for Assessing Methods

531 Inferred rates generally differ depending on the analysis  
532 method; how then should an empirical biologist choose  
533 which method to use?

534 Our advice for empirical users is to take one of two  
535 paths. The first path is to carefully select a method  
536 based on the model assumptions. The methods presented  
537 in this analysis have theoretical differences in their ap-  
538 proach, which appear to produce corresponding differ-  
539 ences in results. For example, methods differ in whether  
540 shifts in diversification rates are allowed on extinct or  
541 unsampled lineages (LSBDS, PESTO, and the “exact MTBD”  
542 not tested here), whether diversification rates of each  
543 regime are drawn from a continuous distribution (BAMM,  
544 MTBD, and CLaDS2) or from a set of discrete rate cate-  
545 gories (LSBDS and PESTO), and if shifts occur at clado-  
546 genetic events (CLaDS2) or along lineages (BAMM, MTBD,  
547 LSBDS, and PESTO). The models make additional assump-  
548 tions, such as whether shifts in diversification rates af-  
549 fect the process-intrinsic parameters (the speciation and  
550 extinction rates) or transformations thereof (e.g., the net  
551 diversification or turnover rate) and whether shifts affect  
552 single parameters or combinations of parameters. These  
553 assumptions lead to notably different interpretations of  
554 how values change through time. Choice of method can  
555 be supported by taxon-specific data such as species distri-  
556 bution, fossil record, or phenotypic data (Morlon, 2014).  
557 Thus, users should also familiarize themselves with how  
558 these models parameterize and estimate diversification  
559 rates and ensure that these modeling choices reflect the  
560 user’s assumptions about biological processes.

561 The second path is to critically compare multiple meth-  
562 ods when performing diversification analyses. We have  
563 shown that—despite the difference in models—in some  
564 cases multiple methods produce results with similar bi-  
565 ological interpretations. To facilitate the adoption of this  
566 practice, we provide R code to easily visualize the results  
567 of multiple diversification-rate models across the same  
568 phylogeny: [https://github.com/Jesusthebotanist/CompDiv\\_processing\\_and\\_plotting](https://github.com/Jesusthebotanist/CompDiv_processing_and_plotting).

## 569 The Future of Diversification Analyses

570 The rise of methods aiming to identify shifts in diversifi-  
571 cation speaks to the importance of these analyses for un-  
572 derstanding the drivers and impacts of important evolu-  
573 tionary events. However, we advocate for caution, for  
574 two reasons described below.

575 First, taking a cautious approach is especially im-  
576 portant in light of the many potential problems with  
577 these methods, including the controversy surrounding  
578 the identifiability of birth-death models (Louca and Pen-  
579 nell, 2020, but see also Helmstetter et al. 2021; Legried and  
580 Terhorst 2022; Morlon et al. 2022; Kopperud et al. 2023b,  
581 among others).

582 Louca and Pennell (2020) presented a class of birth-  
583 death models that are unidentifiable if the rate func-  
584 tions are time-varying (but homogeneous across lineages)  
585 and allowed to take any continuous shape. Nonetheless,  
586 hypothesis-driven approaches are not allowed to take  
587 any arbitrary shape. Since the rate shapes are designed to  
588 test diversification scenarios, defined *a priori*, it has been  
589 argued that this approach is less prone to the identifiability  
590 issue (Morlon et al., 2022). Even time-varying models  
591 that are more agnostic about prior hypotheses are not typ-  
592 ically allowed to take any continuous rate shape. Among  
593 the “agnostic” models, the piecewise-constant model is  
594 the most eminent (Stadler, 2011; Magee et al., 2020), and  
595 this model has been proven to be asymptotically identifiable  
596 provided there are not too many pieces (Legried and  
597 Terhorst, 2022).

598 However, in spite of the non-identifiability, inferences  
599 of rapidly changing speciation and extinction rates are  
600 still typically robust (Kopperud et al., 2023b). The issue of  
601 non-identifiability remains to be investigated thoroughly  
602 in lineage-heterogeneous models. These models are more  
603 parameter-rich than their homogeneous cousins, and so  
604 we do not expect the issue of non-identifiability to be any  
605 simpler here.

606 Second, we caution against relying too heavily on the  
607 estimates from a single method without justifying the as-  
608 sumptions encoded into the model’s choices regarding  
609 parameterization and estimation, as we describe in detail  
610 in “Tools for Assessing Methods”.

611 The methods investigated in this paper vary in their  
612 underlying model and assumptions, but are theoretically  
613 related (Ronquist et al., 2021). Due to these model dif-  
614 ferences, we expect differences in inferences which, in  
615 turn, could translate into different biological interpreta-  
616 tions. Using a set of empirically derived phylogenies,  
617 we show that this is true (Fig. 1): no two methods in-  
618 ferred the same shifts for any phylogeny. In some cases,  
619 methods generally agreed upon the location and timing  
620 of inferred shifts, but in other cases methods strongly  
621 disagreed. Method-dependent differences of individual  
622 trees were corroborated by tree-wide summary statistics,

which indicated small but significant differences between methods (Fig. 2; Table 1). While these results hold up when we take into account the uncertainty in rate estimates, we also urge caution in relying too heavily on summary statistics and encourage users to carefully examine their posterior distributions, as 95% HPD intervals vary among methods and distributions may be bimodal, which may mislead common summary statistics (Fig. S6, see also Barido-Sottani et al., 2020).

633 Regardless, it is clear there will be a continued interest  
634 in using diversification analysis with a renewed apprecia-  
635 tion for the complexities of these methods and the details  
636 of how rates are parameterized and estimated.

## 637 Author Contributions

638 JM-G, MJS, and CMT designed and performed the analyses, JM-G, MJS, CMT, SH, BTK, WAF, CDS, and CJR wrote  
639 the manuscript.  
640

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## Data accessibility

All scripts, data and outputs can be found at on Dryad at (doi:10.6078/D18Q68) upon publication. A set of R functions to help user analyze outputs of studied Baysian methods can be found at ([https://github.com/Jesusthebotanist/CompDiv\\_processing\\_and\\_plotting](https://github.com/Jesusthebotanist/CompDiv_processing_and_plotting))

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