

# 1 Evolution of irreversible differentiation under 2 stage-dependent cell differentiation

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## 15 Abstract

16 The specialization of cells is a hallmark of complex multicellularity. Cell differentiation en-  
17 ables the emergence of specialized cell types that carry out separate functions previously  
18 executed by a multifunctional ancestor cell. One view is that initial cell differentiation oc-  
19 curred randomly, especially for genetically identical cells, exposed to the same life history  
20 environment. How such a change in differentiation probabilities can affect the evolution of

21 differentiation patterns is still unclear. We develop a theoretical model to investigate the ef-  
22 fect of stage-dependent cell differentiation – cells change their developmental trajectories  
23 during a single round of development via cell divisions – on the evolution of optimal differ-  
24 entiation patterns. We found that irreversible differentiation – a cell type gradually losing its  
25 differentiation capability to produce other cell types – is more favored under stage-dependent  
26 than stage-independent cell differentiation in relatively small organisms with limited differ-  
27 entiation probability variations. Furthermore, we discovered that irreversible differentiation  
28 of germ cells, which is the gradual loss of germ cells' ability to differentiate, is a prominent  
29 pattern among irreversible differentiation patterns under stage-dependent cell differentiation.  
30 In addition, large variations in differentiation probabilities prohibit irreversible differentiation  
31 from being the optimal differentiation pattern.

## 32 **Author summary**

33 The differentiation of cells into different branches is a characteristic feature of multicellu-  
34 lar organisms. To understand its origin, the mechanism of division of labour was proposed,  
35 where cells are specialized at distinct tasks. In previous models, a cell type is usually assumed  
36 to produce another cell type with a fixed probability which is referred to as stage-independent  
37 differentiation. However, it has been argued that cell differentiation is a dynamic process in  
38 which cells possess changing differentiation capabilities during the different stages of an  
39 organism's development. Stage-dependent differentiation exhibits more diverse patterns of  
40 development than differentiation with fixed probabilities, thus it can lead to novel targets of  
41 selection. How does stage-dependent differentiation impact the evolution of optimal differ-  
42 entiation patterns compared with stage-independent one? To address this question, we built a  
43 stage-dependent cell differentiation model and classified differentiation patterns based on the  
44 cells' differentiation capability in their last cell division. We investigate how stage-dependent  
45 differentiation probabilities impact the evolution of the optimal differentiation pattern, which  
46 acts on the fitness of an organism. As we take the growth rate as a proxy of an organism's

47 fitness, we seek the “optimal strategy” that leads to the fastest growth. Our numerical results  
48 show that irreversible differentiation which gradually loses its differentiation capability, is fa-  
49 vored over stage-independent differentiation in small organisms. Meanwhile, irreversible dif-  
50 ferentiation won’t be optimal when there are no constraints on the changes of stage-dependent  
51 differentiation probabilities between successive cell divisions.

## 52 **Introduction**

53 The evolution of multicellularity has been viewed as the major evolutionary transition for  
54 the evolution of life on earth Maynard Smith and Szathmáry [1995], Szathmáry and Smith  
55 [1995], Ratcliff et al. [2015], Sebe-Pedros et al. [2017], Márquez-Zacarías et al. [2021b].  
56 One important aspect of this is cell differentiation into different cell types. Cooperation and  
57 division of labor between these cells have been widely investigated in the evolution of multi-  
58 cellularity Ratcliff et al. [2012], Hammerschmidt et al. [2014], West et al. [2015], Gao et al.  
59 [2019], Rose et al. [2020]. Multicellular organisms, especially large ones, possess differ-  
60 ent cell types to perform diverse functions Carroll [2001], McCarthy and Enquist [2005],  
61 Arendt [2008]. It is widely accepted that multicellular life has evolved from unicellular  
62 ancestors Mikhailov et al. [2009], Claessen et al. [2014]. Division of labour in organisms  
63 enables a diversity of cell types, leading unicellular organisms to form increasingly larger  
64 and more complex organizations. Differentiated cells perform distinct functions in varying  
65 conditions and can in this way increase an organism’s reproductive fitness. For example,  
66 cell differentiation occurs under adverse environmental conditions to increase an organism’s  
67 survival chance, such as cyanobacteria differentiating nitrogen-fixing heterocysts to use  $N_2$   
68 when combined-nitrogen is insufficient Gallon [1992], *Saccharomyces cerevisiae* producing  
69 cells with different apoptosis likelihood under gravitational selection Ratcliff et al. [2012] or  
70 *Myxococcus xanthus* producing a new cell type under starvation Claessen et al. [2014].

71 Several mechanisms have been proposed to understand cell differentiation and phenotypic  
72 variation, from the perspective of gene expression, mutations, epigenetics, and the environ-

73     ment Extavour and Akam [2003], Arendt [2008], Mikhailov et al. [2009], West and Cooper  
74     [2016], Arendt et al. [2016], Brunet and King [2017], Márquez-Zacarías et al. [2021b], Huang  
75     et al. [2024]. These mechanisms are complementary, and thus, more than one mechanism  
76     could act during the evolution of cell differentiation West and Cooper [2016], Brunet and  
77     King [2017]. These mechanisms usually assume that multifunctional and unicellular ances-  
78     tors differentiate into specialized cells to carry out segregated functions, even when cells are  
79     genetically identical and have been exposed to an identical environment. It has been shown  
80     that cells differentiate depending on the development states of an organism. For example,  
81     one out of successive 10 to 15 vegetative cells differentiate into a new cell type, heterocyst,  
82     in filamentous cyanobacteria *Anabaena* sp.PCC 7120 Flores and Herrero [2010]; *Volvox* dif-  
83     ferentiates into two cell types at its 6th round of division in its whole 11 ~ 12 rounds of cell  
84     divisions Matt and Umen [2016]. Moreover, in closed related species of *Volvox* family, it  
85     has been found that the observed stable differentiation patterns are highly likely the evolu-  
86     tion consequences of originally randomly happened cell differentiation. For instance, smaller  
87     species *Gonium* have identical cells, whereas intermediate-sized species *Volvox aureus* and  
88     *Volvox gigas* have partial germ-soma differentiation, whereas *Volvox carteri* and *Volvox ob-*  
89     *versus* have complete germ-soma differentiation Matt and Umen [2016]. How originally  
90     occurred state-dependent cell differentiation in an organism shapes the evolution of cell dif-  
91     ferentiation patterns is still unclear.

92         Studies of cell differentiation have mainly focused on the optimal condition, where ma-  
93         ture cells of an organism allocate their resources to different tasks Michod [2007], Willens-  
94         dorfer [2009], Gavrillets [2010], Rossetti et al. [2010], Rueffler et al. [2012], Ispolatov et al.  
95         [2012], Solari et al. [2013], Goldsby et al. [2014], Cooper and West [2018], Liu et al. [2021],  
96         Cooper et al. [2021, 2022]. Essentially, they are focused on the proportion of each cell type  
97         in an organism, instead of the stochastic developmental process of each cell type during an  
98         organism's growth. Cells capable of switching to another cell type have not been in the focus  
99         yet. Some authors considered cell differentiation abilities, but only in one cell type while  
100        other cell types were terminally differentiated types (without division ability) Willensdorfer

101 [2009], Rossetti et al. [2010], Solari et al. [2013]. Rodrigues et al. considered cell differenti-  
102 ation ability as an evolving trait, but the trait was coupled with varying cell division rates and  
103 organisms were constrained to filament form Rodrigues et al. [2012]. More recently, Cooper  
104 et al. introduced a random specialization model, but the random process only impacts the final  
105 fractions of different cell types rather than the internal organization of task allocation of cells  
106 during an organism’s growth Cooper et al. [2022]. Gavrillets and Gao et al. considered cell  
107 differentiation between cell types, but the differentiation probabilities are assumed to be fixed  
108 rather than stochastic Gavrillets [2010], Gao et al. [2021]. So far, little is known about the ef-  
109 fects of stage-dependent differentiation probabilities on the evolution of cell differentiation  
110 patterns, such as irreversible or reversible.

111 In this study, based on our previous work Gao et al. [2021], we develop a theoretical  
112 model to investigate the effect of cell differentiation with stage-dependent differentiation  
113 probabilities on the evolution of optimal differentiation patterns. Stage-dependent cell dif-  
114 ferentiation refers to the capability of cells having different cell differentiation probabilities  
115 between any two successive cell divisions. Comparatively, stage-independent differentiation  
116 only allows a cell type to have a fixed cell differentiation probability across cell divisions  
117 Gao et al. [2021]. Inspired by the cells’ division of labour of *Volvox*, where germ cells are  
118 responsible for reproduction and somatic cells are responsible for viability Matt and Umen  
119 [2016], we consider two cell types in an organism: germ-like cells and soma-like cells. We  
120 use the expected offspring number of an organism i.e. growth rate as a proxy of an organism’s  
121 fitness because it is the simplest direct criterion Parker and Smith [1990]. We assume that  
122 an organism grows by cell divisions which further depends on the fraction of soma-like cells  
123 and transition probabilities between cell types. Different stage-dependent strategies compete  
124 to maximize the organism’s fitness. We numerically calculate organisms’ growth rates un-  
125 der different parameters and compare the evolutionary differences of optimal strategies under  
126 stage-dependent and stage-independent cell differentiation. Intuitively, reversible differen-  
127 tiation instead of irreversible differentiation under stage-dependent differentiation will be  
128 selected especially when cost is low, because reversible differentiation can “recycle” soma-

129 type cells for reproduction. However, we found that stage-dependent differentiation favors  
130 irreversible differentiation more than stage-independent differentiation even without costs in  
131 small organisms.

## 132 Model and methods

133 We designed a life cycle model for organisms with stage-dependent cell differentiation com-  
134 pared with previous work investigated under stage-independent cell differentiation **Gao et al.**  
135 [2021]. As we are focused on the formation process of differentiation patterns, we consider  
136 two intermediate cell types rather than specific cell types in an organism: germ-like and  
137 soma-like, which is inspired by the partial differentiation cell types in genus *Pandorina*, a  
138 closed genus of *Volvox* **Matt and Umen** [2016]. Here, the two cell types are allowed to differ-  
139 entiate into each other, and we investigate the possible differentiation process along with cell  
140 divisions, which we refer to as differentiation strategies. Different strategies lead organisms  
141 to different developmental trajectories and fitness. In the model, an organism's growth rate is  
142 a fitness proxy as it is the simplest direct way to measure organisms' fitness **Parker and Smith**  
143 [1990]. Next, we introduce the definition of differentiation strategies. We assume that each  
144 organism starts with a single germ-like cell, see Fig 1A. Cells divide synchronously, each  
145 cell producing two daughter cells at a time. After the  $i$ th cell division, organisms have  $2^i$   
146 cells in total. Organisms grow and mature until they reach a maturity size  $2^n$ , where  $n$  is the  
147 maximal cell division of organisms. Each germ-like cell is released from a mature organism  
148 as offspring to start a new life cycle. All soma-like cells in a mature organism die. For each  
149 division, cells have a set of probabilities to produce daughter cells of a certain type. Here,  
150  $g_{gg}^{(i)}$  is the probability of a germ-like cell producing two germ-like cells at the  $i$ th cell division.  
151 The probabilities  $g_{gg}^{(i)}$ ,  $g_{gs}^{(i)}$ ,  $g_{ss}^{(i)}$ ,  $s_{gg}^{(i)}$ ,  $s_{gs}^{(i)}$  and  $s_{ss}^{(i)}$  are defined in a similar manner, where we have  
152  $g_{gg}^{(i)} + g_{gs}^{(i)} + g_{ss}^{(i)} = 1$  and  $s_{gg}^{(i)} + s_{gs}^{(i)} + s_{ss}^{(i)} = 1$  for each growth stage  $i$ ,  $i = 1, 2, \dots, n$ . We  
153 denote  $d_i = [g_{gg}^{(i)}, g_{gs}^{(i)}, g_{ss}^{(i)}, s_{gg}^{(i)}, s_{gs}^{(i)}, s_{ss}^{(i)}]$  as the cell differentiation probabilities in the  $i$ th cell  
154 division. In addition,  $g_{g \rightarrow s}^{(i)} = \left( g_{ss}^{(i)} + \frac{g_{gs}^{(i)}}{2} \right)$  and  $s_{s \rightarrow g}^{(i)} = \left( s_{gg}^{(i)} + \frac{s_{gs}^{(i)}}{2} \right)$  are referred to as transi-

155 tion probabilities,  $i = 1, 2, \dots, n$ . The cell differentiation probabilities across the successive  
 156  $n$  rounds of cell divisions of an organism can be expressed in matrix form as

$$157 \quad \mathbf{D} = \begin{pmatrix} d^{(1)} \\ \vdots \\ d^{(i)} \\ \vdots \\ d^{(n)} \end{pmatrix} = \begin{pmatrix} g_{gg}^{(1)} & g_{gs}^{(1)} & g_{ss}^{(1)} & s_{gg}^{(1)} & s_{gs}^{(1)} & s_{ss}^{(1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{gg}^{(i)} & g_{gs}^{(i)} & g_{ss}^{(i)} & s_{gg}^{(i)} & s_{gs}^{(i)} & s_{ss}^{(i)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ g_{gg}^{(n)} & g_{gs}^{(n)} & g_{ss}^{(n)} & s_{gg}^{(n)} & s_{gs}^{(n)} & s_{ss}^{(n)} \end{pmatrix}, \quad (1)$$

158 where the  $i$ th row of the matrix contains the cell differentiation probabilities in the  $i$ th cell  
 159 division. We call  $\mathbf{D}$  stage-dependent cell differentiation, as the probabilities can change  
 160 between different division stages. We assume that this change is not larger than  $\delta$ , e.g.  
 161  $g_{g \rightarrow s}^{(i)} = g_{ss}^{(i)} + \frac{g_{gs}^{(i)}}{2} = g_{g \rightarrow s}^{(i-1)} \pm \delta_i$  with  $\delta_i \leq \delta$  sufficiently small such that all probabilities  
 162 remain well defined. If  $\delta = 0$  for  $i = 1, 2, \dots, n$ , then  $\mathbf{D}$  is a stage-independent cell differ-  
 163 entiation strategy, where the same type of cells follow a fixed set of probabilities to produce  
 164 daughter cells at each division. We should note that the stage-independent cell differentiation  
 165 is also defined by the values of  $g_{g \rightarrow s}^{(i)}$  and  $s_{s \rightarrow g}^{(i)}$  but which don't change across  $i$ . Different cell  
 166 differentiation strategies lead to different differentiation degrees. For example, if a strategy  
 167 has  $g_{g \rightarrow s}^{(i)} \equiv 0$  for  $i = 1, 2, \dots, n$ , then organisms have no cell differentiation. If a strategy  
 168 has  $g_{g \rightarrow s}^{(i)} = 1$  and  $s_{s \rightarrow g}^{(i)} = 1$ , then organisms have maximal degree of cell differentiation.  
 169 Stage-dependent differentiation allows many different trajectories. To distinguish them and  
 170 focus on differentiation patterns, we consider the probabilities in the last division (Fig 1B).  
 171 If  $g_{g \rightarrow s}^{(i)} \equiv 0$  for  $i = 1, 2, \dots, n$ , then we call the differentiation non-differentiation *ND*.  
 172 Otherwise, if germ-like cells differentiate soma-like cells at least one time before final cell  
 173 division, i.e.  $g_{g \rightarrow s}^{(i)} \neq 0$ ,  $i = 1, 2, \dots, n-1$ , but with NO differentiation for either cell type  
 174 at last division i.e.  $g_{g \rightarrow s}^{(n)} = 0$  or  $s_{s \rightarrow g}^{(n)} = 0$ , then we call it irreversible differentiation *ID*.  
 175 Strategy *ID* captures the process by which cells gradually lose their differentiation capabil-  
 176 ities. The rest differentiation is called reversible differentiation *RD*. We should stress the  
 177 limitation of this classification, in which different strategies could lead to a similar develop-  
 178 ment trajectory, especially in large organisms. Nevertheless, the classification is a simple way

179 that distinguish different differentiation patterns. For convenience, we use the upper script  
180  $i$  to show the stage-independent strategies where cells have fixed cell differentiation proba-  
181 bilities. From the definition of  $ND$ , we know it is a stage-independent cell differentiation,  
182 thus we use  $ND^i$  to denote it afterward. For strategy  $ID^i$ , only soma-like cells can possess  
183 irreversibility i.e.  $s_{s \rightarrow g} = 0$  [Gao et al. \[2021\]](#). In this work, the acronyms of differentiation  
184 strategies are stage-dependent unless otherwise stated.

185 We assume that cell differentiation impacts an organism's growth [Gao et al. \[2021\]](#). The  
186 effects of cell differentiation are further decomposed into cell differentiation benefits and  
187 costs on growth. We assume differentiated soma-like cells are beneficial and increase an  
188 organism's growth. The assumption is based on the division of labour in *Volvox*, where  
189 somatic cells are responsible for viability and germ cells are responsible for reproduction  
190 [Matt and Umen \[2016\]](#). Cell differentiation between germ-like cells and soma-like cells is  
191 costly and decreases growth. A direct impact of differentiation is the decreased number of  
192 offspring as the resource that could be used for reproduction to convert to newly typed somatic  
193 cells. Organisms grow faster with higher cell division rates and vice versa. Specifically,  $r^{(i)}$   
194 represents the growth rate in the  $i$ th cell division and is determined by two components

$$r^{(i)} = \frac{1 + F_b^{(i)}}{1 + F_c^{(i)}}, \quad (2)$$

195 where  $F_b^{(i)}$  and  $F_c^{(i)}$  are the effects of cell differentiation benefit and cell differentiation cost  
196 in the  $i$ th cell division,  $i = 1, 2, \dots, n$ .  $F_b$  is a function of the fraction of soma-like cells  $f_s$ ,

$$F_b = b f_s^\alpha, \quad (3)$$

197 where the  $b$  is the benefit scale,  $b \geq 0$ .  $\alpha$  controls the shape of the function, see Fig 1C.  $F_c$   
198 is a function of the fraction of cell differentiation between germ-like cell and soma-like cell  
199  $f_{g \rightarrow s}$  and  $f_{s \rightarrow g}$ ,

$$F_c = c(f_{g \rightarrow s} + \beta f_{s \rightarrow g}), \quad (4)$$

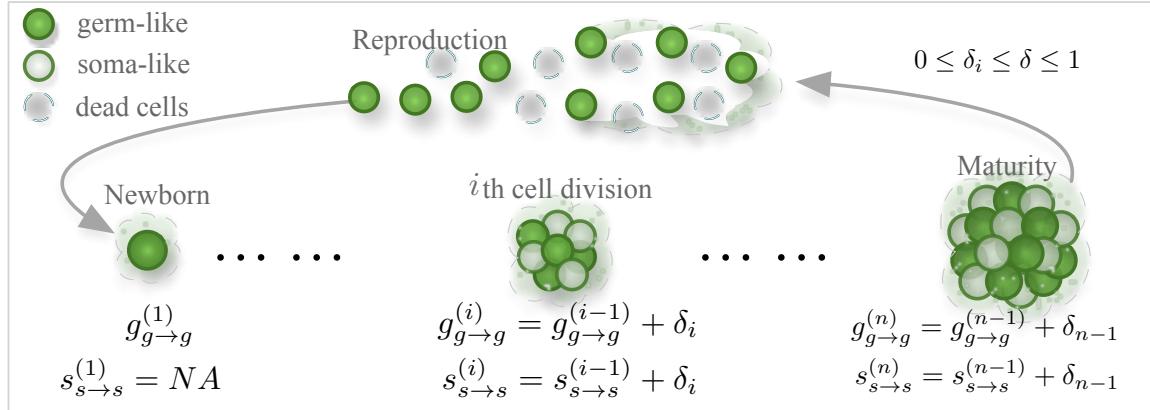
200 where  $c$  is the cost scale,  $c \geq 0$ .  $\beta$  measures the relative cost of differentiation from soma-like  
201 cell to germ-like cell, see Fig 1C. The fractions of cell differentiation in the  $i$ th cell division

202 are

$$\begin{aligned} f_{g \rightarrow s}^{(i)} &= f_g^{(i-1)} g_{g \rightarrow s}^{(i)} \\ f_{s \rightarrow g}^{(i)} &= f_s^{(i-1)} s_{s \rightarrow g}^{(i)}, \end{aligned} \tag{5}$$

205 where  $f_g^{(i-1)}$  and  $f_s^{(i-1)}$  are the fraction of germ-like cell and soma-like cell after the  $(i-1)$ th

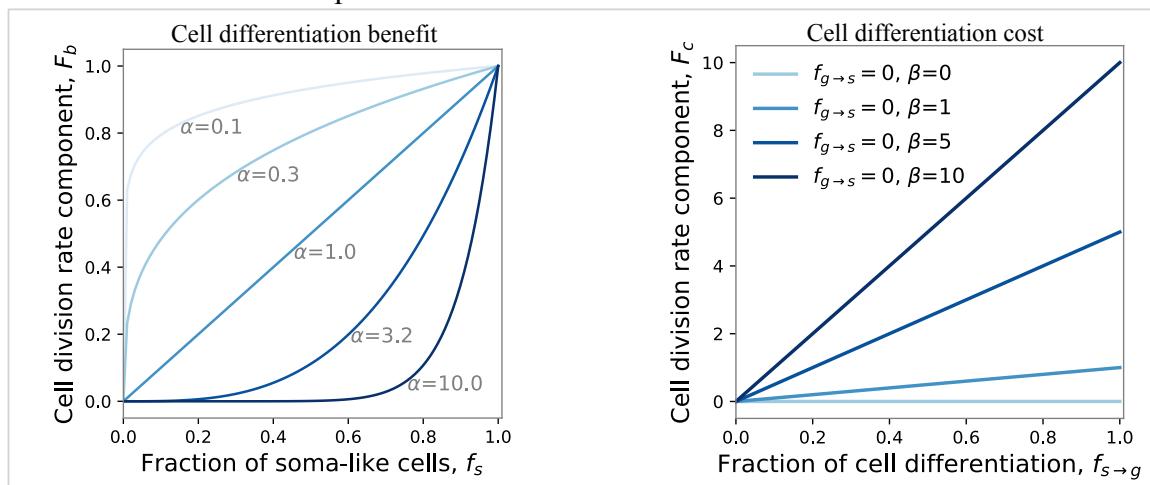
#### A. Dynamic developmental trajectories and cell differentiation categories



## B. Cell differentiation strategies

Differentiation probability	Differentiation category	
$g_{g \rightarrow s}^{(i)} = g_{g \rightarrow s}^{(i-1)} + \delta_i, s_{s \rightarrow g}^{(i)} = s_{s \rightarrow g}^{(i-1)} + \delta_i$	Stage-independent $\delta = 0$	Stage-dependent $\delta \neq 0$
$g_{g \rightarrow s}^{(i)} \equiv 0, i = 1, 2, \dots, n$	$ND^i$	$ND^i$
$g_{g \rightarrow s}^{(i)} \not\equiv 0, i = 1, 2, \dots, n-1.$ $g_{g \rightarrow s}^{(n)} = 0 \text{ or } s_{s \rightarrow g}^{(n)} = 0$	$ID^i$	$ID$
others	$RD^i$	$RD$

### C. Cell division rate components



**Figure 1: Illustration of the stage-dependent cell differentiation, differentiation strategies, and cell division rate components.** **A.** Schematic of an organism's life cycles. Organisms start from single germ-like cells and undergo  $n$  synchronous cell divisions before reproduction. For newborn organisms, the cell differentiation probability for soma-like cells is irrelevant as there are no soma-like cells. Cell differentiation probabilities can change from the  $(i - 1)$ th cell division to the  $i$ th cell division by a small quantity  $\delta_i$  ( $0 \leq \delta_i \leq 1$  and  $i = 1, 2, \dots, n$ ).  $\delta$  is the maximum change between successive cell differentiation probabilities i.e.  $0 \leq \delta_i \leq \delta \leq 1$ ,  $i = 1, \dots, n$ . **B.** Cell differentiation strategy classification. Based on the cell differentiation probabilities at the last cell division, we classify cell differentiation into three categories: non-differentiation  $ND$ , reversible differentiation  $RD$ , and irreversible differentiation  $ID$ . The upper script  $i$  means the strategy is stage-independent i.e.  $\delta = 0$ . For  $ND$ , since  $g_{g \rightarrow s}^{(i)} = 0$ ,  $i = 1, \dots, n$ , thus  $ND$  equals  $ND^i$ . **C.** Cell division rate components. The left panel shows the benefits of cell differentiation. We assume that the cell division rate increases with the fraction of soma-like cells  $f_s$ . For the associated benefit, we assume  $F_b = b(f_s)^\alpha$ , where the shape of the function is controlled by  $\alpha$ . The right panel shows the costs of cell differentiation. We assume that the cell division rate decreases with the fraction of cell divisions that turn a soma-like cell into a germ-like cell and vice versa. For the associated cost, we assume  $F_c = c(f_{g \rightarrow s} + \beta f_{s \rightarrow g})$ . Here, we show the values of  $F_c$  with varying  $f_{s \rightarrow g}$  and  $\beta$  by setting  $f_{g \rightarrow s} = 0$  (Parameters:  $b = 1$  in the left panel and  $c = 1$  in the right panel).

206 cell division, respectively. Note that  $f_g^{(i-1)} + f_s^{(i-1)} = 1$ ,  $g_{g \rightarrow g}^{(i)} + g_{g \rightarrow s}^{(i)} = 1$ , and  $s_{s \rightarrow s}^{(i)} + s_{s \rightarrow g}^{(i)} =$   
 207 1,  $i = 1, 2, \dots, n$ . Specifically,  $f_g^{(i)}$  and  $f_s^{(i)}$  are calculated by using transition probabilities,  
 208 see Eq (11) in [S1 Appendix](#). Taking Eq (2), Eq (3), Eq (4) and Eq (5) together, we have

$$r^{(i)} = \frac{1 + b(f_s^{(i-1)})^\alpha}{1 + c(f_{g \rightarrow s}^{(i)} + \beta f_{s \rightarrow g}^{(i)})}. \quad (6)$$

209 After the  $(i - 1)$ th cell division, the waiting time before the  $i$ th cell division occurring  $t^{(i)}$   
 210 follows the exponential distribution  $f(t^{(i)}) = r^{(i)} e^{-r^{(i)} t^{(i)}}$ ,  $i = 1, 2, \dots, n$ . Thus the expected  
 211 waiting time from the  $(i - 1)$ th to the  $i$ th cell division is  $t^{(i)} = \frac{1}{r^{(i)}}$  [Allen \[2010\]](#). The expected  
 212 growth time of organisms with  $n$  rounds of cell divisions is

$$213 t = \sum_{i=1}^n t^{(i)} = \sum_{i=1}^n \frac{1}{r^{(i)}} = \sum_{i=1}^n \frac{1 + c \left( f_g^{(i-1)} g_{g \rightarrow s}^{(i)} + \beta f_s^{(i-1)} s_{s \rightarrow g}^{(i)} \right)}{1 + b \left( f_s^{(i-1)} \right)^\alpha}. \quad (7)$$

215 We consider a density-independent population, see [Ress et al. \[2022\]](#) for a discussion when  
216 density dependence is relevant in a related model. The growth rate of an organism only de-  
217 pends on the number of offspring and their growth time. As organisms divide synchronously,  
218 the number of offspring that an organism produces during its life is  $2^n f_g^{(n)}$ . The expected  
219 number of offspring per unit time of an organism captures the effect of a given strategy on  
220 organisms. Therefore, organisms grow exponentially and an organism's growth rate can be  
221 approximated by

$$222 \quad \lambda = \frac{\ln N}{t} = \frac{\ln(2^n f_g^{(n)})}{\sum_{i=1}^n \frac{1}{r^{(i)}}} = \frac{\ln(2^n f_g^{(n)})}{\sum_{i=1}^n \frac{1 + c \left( f_g^{(i-1)} g_{g \rightarrow s}^{(i)} + \beta f_s^{(i-1)} s_{s \rightarrow g}^{(i)} \right)}{1 + b \left( f_s^{(i-1)} \right)^\alpha}}, \quad (8)$$

223

224 where  $n$  is the number of cell divisions an organism undergoes before maturity.  $f_g^{(i)}$  and  $f_s^{(i)}$   
225 are fractions of germ-like cell and soma-like cell after the  $i$ th cell division. Here,  $g_{g \rightarrow s}^{(i)}$  and  
226  $s_{s \rightarrow g}^{(i)}$  are the transition probabilities between germ-like cell and soma-like cell in the  $i$ th cell  
227 division ( $1 \leq i \leq n$ ), see the [S1 Appendix](#). We provide the calculation details of the growth  
228 rate in [S1 Appendix](#) and [S2 Appendix](#).

229 It should be noted that the growth rate calculated here is not the exact growth rate for each  
230 realization. As each strategy in the model is stochastic, each strategy has different potential  
231 developmental trajectories. Therefore the growth rate of an organism under a strategy is a ran-  
232 dom variable depending on the probability of each trajectory that an organism can develop.  
233 Thus, the growth rate calculated via Eq (8) is an approximation of the mean growth rate.  
234 We test the robustness of the approximation in [S3 Appendix](#). Our results show that the ap-  
235 proximation is consistent with the mean growth rate of an organism. This model generalizes  
236 our previous study of stage-independent developmental trajectories using individual-based  
237 simulations [Gao et al. \[2021\]](#). Here, however, we investigate the mean developmental trajec-  
238 tory numerically which is more efficient than individual-based simulations, especially for the  
239 complex developmental trajectories under stage-dependent scenario. In addition, as each or-  
240 ganism starts with a single germ-like cell,  $f_g^{(0)} = 1$  and  $f_s^{(0)} = 0$ . In  $ND^i$ , cells only produce  
241 germ-like cells, thus  $g_{g \rightarrow g}^{(i)} = 1$  for all  $i$ , and all other probabilities are irrelevant. Therefore,

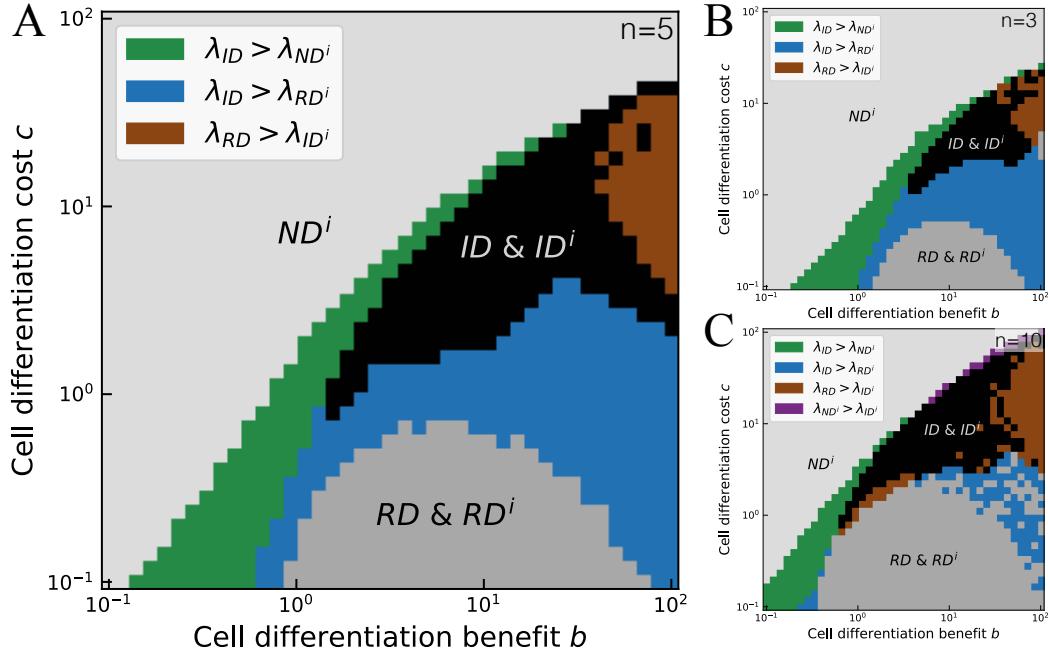
<sup>242</sup>  $f_g^{(i)} \equiv 1$ ,  $f_s^{(i)} \equiv 0$ . From Eq (8), the growth rate of  $ND^i$  is  $\lambda_{ND^i} = \ln 2$ . Biologically, it  
<sup>243</sup> means that organisms double their size per unit of time when they take the  $ND^i$  strategy.

## <sup>244</sup> Results

### <sup>245</sup> Stage-dependent cell differentiation promotes irreversible cell differenti- <sup>246</sup> ation in small organisms

<sup>247</sup> Theoretically, differentiation probabilities at different cell divisions could be arbitrary values.  
<sup>248</sup> Therefore, differentiation probabilities can be arbitrary values between 0 and 1. However, we  
<sup>249</sup> rarely observe drastic changes in cell differentiation probabilities during an organism's devel-  
<sup>250</sup> opment – instead, these probabilities change slowly during development. For instance, cells  
<sup>251</sup> in a series of closely relative species in *Volvox* family show gradual degrees of germ-soma  
<sup>252</sup> differentiation [Matt and Umen \[2016\]](#). Thus, it is natural to assume that the maximum value  
<sup>253</sup> of the change of two successive differentiation probabilities is small. Therefore, we restrict  
<sup>254</sup> attention to a small range of  $\delta$  and set  $\delta = 0.1$  (i.e.  $0 \leq \delta_i \leq \delta = 0.1$ ,  $i = 1, \dots, n$ )  
<sup>255</sup> in this section and the effects of large  $\delta$  will be investigated in the third section. We found  
<sup>256</sup> that stage-dependent differentiation promotes the evolution of irreversible strategies  $ID$  com-  
<sup>257</sup> pared with stage-independent differentiation  $ID^i$  in small organisms, see Fig 2. Specifically,  
<sup>258</sup> stage-dependent differentiation  $ID$  evolves at more parameter space of differentiation ben-  
<sup>259</sup>efits and costs than stage-independent  $ID^i$  in small organisms. However, stage-dependent  
<sup>260</sup>  $ID$  gradually loses its advantages when organismal size increases. It has been shown that  
<sup>261</sup> stage-independent differentiation  $ID^i$  is more likely to evolve in large organisms, see  $ID^i$  in  
<sup>262</sup> Fig 2. The conclusion about  $ID^i$  is consistent with the previous findings [Gao et al. \[2021\]](#).

<sup>263</sup> Next, with the constraint of  $\delta$ , we investigate the effects of stage-dependent differentiation  
<sup>264</sup> on an organism's growth rate under varying differentiation benefits and costs, comparing it  
<sup>265</sup> with the results of stage-independent differentiation strategies. We first focus on the parame-  
<sup>266</sup>ter space where both stage-independent and stage-dependent differentiation evolve the same



**Figure 2: Comparison of optimal strategies between stage-independent and stage-dependent differentiation.** Comparison of the parameter space of the optimal strategy between stage-independent and stage-dependent cell differentiation under maximal cell number of division rounds of  $n = 5$  (panel **A**),  $n = 3$  (panel **B**) and  $n = 10$  (panel **C**). The grey, dark grey, and black areas represent the parameter space where the optimal strategies are the same under both stage-independent and stage-dependent cell differentiation. The green strip represents stage-dependent *ID* leading to a larger growth rate than stage-independent  $ND^i$ . Similarly, the blue area and the brown area represent *ID* and *RD* leading to higher growth rates than stage-independent  $RD^i$  and  $ID^i$ , respectively. Purple color represents  $ND^i$  leads to a higher growth rate than  $ID^i$  in panel **C**. Parameters of all panels:  $0 \leq \delta_i \leq 0.1$ , and  $\alpha = \beta = 1$ . Parameters of calculating optimal strategy: the number of initial sampling  $d^{(1)}$ ,  $M = 1000$ , the number of stage-dependent strategies starting with a given  $d^{(1)}$ ,  $R = 100$ , for more detail, see [S2 Appendix](#). At each pixel, the frequency of each optimal strategy was calculated across 100 replicates in panel **A** and 20 replicates in the rest panels.

267 strategies.  $ND^i$  dominates under both stage-independent and stage-dependent differentiation  
268 at high differentiation costs which largely decreases an organism's growth rates under dif-  
269 ferentiation strategies ( $RD^i$ ,  $ID^i$ ,  $RD$  and  $ID$ ). In the absence of differentiation benefits,  
270 i.e.  $b = 0$  and  $c > 0$ , we show that  $ND^i$  is optimal analytically ([S4 Appendix](#)). Addition-  
271 ally, if there is only a single cell division ( $n = 1$ ),  $ND^i$  is still optimal in the absence of  
272 differentiation costs, i.e.  $c = 0$  and  $b > 0$  ([S4 Appendix](#)). This is because differentiation  
273 benefits  $F_b^{(i)}$  at the  $i$ th division are based on the fraction of soma-like cells after the  $(i - 1)$ th  
274 division. Similarly, under the scenario of high costs and low benefits, stage-dependent dif-  
275 ferentiation bears huge costs, thus only  $ND^i$  is chosen. When benefits are much higher than  
276 costs, then reversible differentiation ( $RD^i$ ,  $RD$ ) is chosen under both stage-independent and  
277 stage-dependent differentiation. This is because differentiation benefits will cover the dif-  
278 ferentiation costs caused by cell differentiation among divisions, see [S5 Appendix](#) for the  
279 optimal strategy under larger scales of benefits and costs.

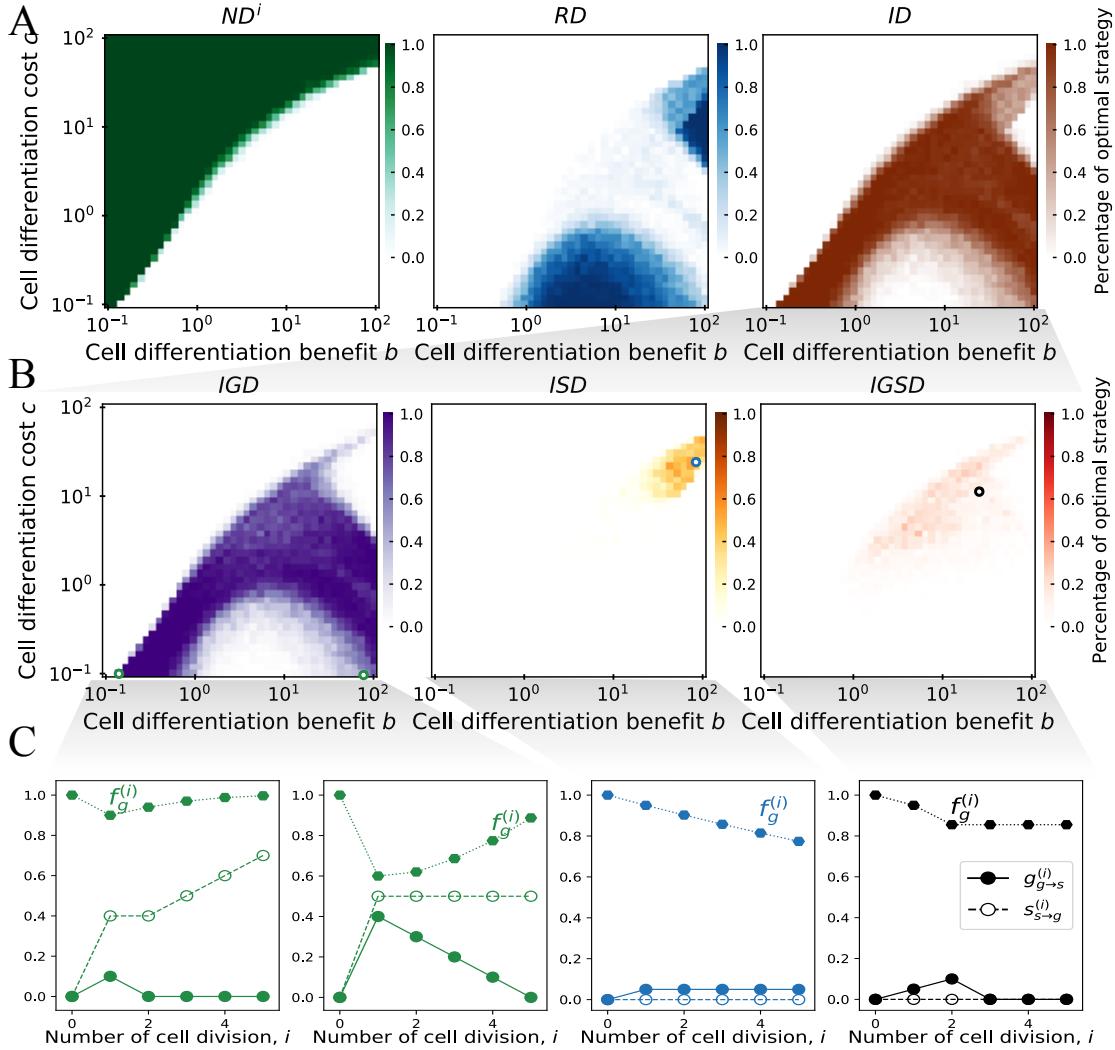
280 Then, we analyze the effects of organism size on the effects of the occurrence of stage-  
281 dependent  $ID$ . In the model, cell differentiation plays a dual role in growth rate. It pro-  
282 vides benefits, but also incurs costs on the growth rate. The best strategy is the one that can  
283 maximally use cell differentiation benefits and at the same time reduce costs. So under the  
284 conditions of high benefits or high costs, only  $ND$  is selected. Due to the randomness of  
285 cell differentiation probabilities, stage-dependent strategies contain the one that can adjust  
286 the fraction of germ-like cells to gain differentiation benefits and avoid differentiation costs  
287 during growth, especially in large organisms that contain more cell divisions, see Fig 2. In  
288 small organisms, due to the constraints on the fluctuations of two successive cell differen-  
289 tiation probabilities, stage-dependent  $ID$  strategies only accumulate limited differentiation  
290 benefits. Since stage-dependent  $ID$  needs to undergo the differentiation from germ-like to  
291 soma-like first and then at least cell type turns irreversible, thus higher cell differentiation  
292 costs, especially high differentiation from germ-like to soma-like, will prohibit it from being  
293 the optimal strategy. But for  $ID$  in large organisms that need more cell divisions to mature,  
294 the differentiation from germ-like to soma-like can occur only in the first several cell divi-

295 sions to gain benefits, then cells can remain irreversible to avoid costs in the following cell  
296 divisions cells. Thus, stage-dependent differentiation strategies (either *RD* or *ID*) can lead  
297 to higher growth rates than stage-independent ones (either  $RD^i$  or  $ID^i$ ) in small organisms  
298 for the account of their flexible adjustment of the differentiation probability patterns.

299 **Irreversible germ differentiation dominates among optimal stage-dependent  
300 irreversible cell differentiation**

301 To further analyze why stage-dependent irreversible differentiation is favored over stage-  
302 independent irreversible differentiation in small organisms, we should further study the pos-  
303 sible irreversible differentiation forms in *ID*. In the model, an organism can contain two  
304 cell types, thus irreversibility can occur on either cell type. Therefore, the stage-dependent  
305 irreversible differentiation (*ID*) can further be classified into three subcategories: irreversible  
306 germ differentiation *IGD* ( $g_{g \rightarrow s}^{(n)} = 0$  and  $s_{s \rightarrow g}^{(n)} \neq 0$ ), irreversible soma differentiation *ISD*  
307 ( $s_{s \rightarrow g}^{(n)} = 0$  and  $g_{g \rightarrow s}^{(n)} \neq 0$ ), and irreversible germ and soma differentiation *IGSD* ( $g_{g \rightarrow s}^{(n)} =$   
308  $s_{s \rightarrow g}^{(n)} = 0$ ). Next, we investigate the occurrence conditions of each sub-strategy.

309 The results show that among the optimal *ID* in small organisms, *IGD* evolves at most  
310 parameter space of benefits and costs, see Fig 3 A and B. *IGD* leads stage-dependent *ID*  
311 replaces  $ND^i$  as the optimal strategy in small organisms at small  $c$ , see Fig 2A and Fig 3B.  
312 Specifically, we first found that the *IGD* strategy replaces  $ND^i$  when  $b$  is slightly larger  
313 than  $c$ . Under this scenario, the best strategy would be to produce a few soma-like cells to  
314 use differentiation benefits, but decrease the differentiation probabilities between cell types  
315 to avoid differentiation costs as the growth rate is a tradeoff between differentiation benefits  
316 and differentiation costs based on Eq (8). Thus, the *IGD* that produces few soma-like cells  
317 in the first few cell divisions and then turns into irreversible becomes optimal (the first panel  
318 in Fig 3C). The *IGD* strategy can keep a high fraction of germ-like cells which increases  
319 the growth rate by increasing the number of offspring i.e.  $2^n f_g^{(n)}$ . Under this *IGD* strategy,  
320 although the differentiation probabilities of soma-like cells  $s_{s \rightarrow g}^{(n)}$  is not small, we should note  
321 that the differentiation costs are still low as the number of soma-like cells is small, which is



**Figure 3: Irreversible germ differentiation evolved mostly among irreversible differentiation under stage-dependent differentiation.** **A.** Fractions of three stage-dependent cell differentiation strategies being optimal under differentiation benefits and costs. **B.** Fractions of three sub-irreversible stage-dependent strategies being optimal under differentiation benefits and costs. **C.** Cell differentiation probabilities ( $g_{g \rightarrow s}^{(i)}$ ,  $s_{s \rightarrow g}^{(i)}$ ) and the frequencies of germ-like cell ( $f_g^{(i)}$ ) of the optimal irreversible strategy via cell divisions at the parameter space indicated by circles in panel **B**. The circle color follows that in Fig 3 Parameters of all panels: maximal cell number of division rounds  $n = 5$ ,  $0 \leq \delta_i \leq 0.1$ , and  $\alpha = \beta = 1$ . At each pixel, the frequency of each optimal strategy was calculated across 100 replicates. Parameters of calculating optimal strategy: the number of initial sampling  $d^{(1)}$ ,  $M = 1000$ , the number of stage-dependent strategies starting with a given  $d^{(1)}$ ,  $R = 100$ , replicates for each pixel is 100, for more detail, see [S2 Appendix](#).

322  $2^i(1 - f_g^{(i)})$  after the  $i$ th cell division. Then, we found that  $IGD$  is optimal when  $b$  is much  
323 larger than small  $c$  in small organisms (the second panel in Fig 3C). Under this scenario,  
324 due to the tradeoff between differentiation benefits and costs, the  $IGD$  strategy with higher  
325 germ-like differentiation probabilities  $g_{g \rightarrow s}$  at first several cell divisions becomes optimal.  
326 Taken together, we found that  $ID$ 's sub-strategy  $IGD$  evolves at low  $c$ . Furthermore, we  
327 show an analytical proof that except for  $n = 1$  (S4 Appendix), either  $RD$  or  $ID$  is optimal  
328 in the absence of cell differentiation costs, i.e.  $c = 0$  and  $b > 0$  (S6 Appendix). The finding  
329 indicates that without the punishment of differentiation costs  $ND$  cannot be selected.

330 Meanwhile, We found that  $ISD$  and  $IGSD$ , the other subcategories of  $ID$ , evolve at both  
331 intermediate values of differentiation benefits and costs, see the last two panels in Fig 3B. We  
332 analytically proved that both cell differentiation benefits and costs are indispensable factors  
333 for the evolution of  $ISD$  and  $IGSD$ , see the proof in S7 Appendix. The subcategory strategy  
334  $IGSD$  and  $ISD$  of  $ID$  evolves at both high  $b$  and high  $c$ . Specifically,  $IGSD$  evolves at both  
335 higher  $b$  and  $c$  than the strategy of  $ISD$ . This is an account of the differences in irreversibility  
336 features of cell types between  $IGSD$  and  $ISD$ . Compared with  $ISD$  strategies,  $IGSD$  with  
337 both irreversible cell types at last cell division bears lower cell differentiation costs, thus it  
338 can evolve either at higher  $c$  or at low conditions of  $b$  and  $c$  than  $ISD$  (the last two panels  
339 of Fig 3C). Meanwhile,  $IGSD$  has relatively higher fractions of germ-like cells than  $ISD$ ,  
340 which leading a larger number of offspring i.e.  $2^n f_g^{(n)}$  and then leads to a higher growth rate  
341 based on Eq (8). Additionally, it is noteworthy that for the evolution conditions of stage-  
342 independent irreversible soma differentiation  $ISD^i$ , the only  $ID^i$  under stage-independent  
343 cell differentiation, the result is consistent with our previous study Gao et al. [2021].

344 In addition, compared with the previous work which investigated cell differentiation un-  
345 der stage-independent differentiation Gao et al. [2021], stage-dependent differentiation also  
346 promotes the evolution of irreversible differentiation under the effects of  $\alpha$  and  $\beta$ , see S8  
347 Appendix. Meanwhile, we found that under stage-dependent differentiation  $\alpha$  plays a similar  
348 role as that of stage-independent differentiation. That is, irreversible differentiation strategies  
349 are optimal when  $\alpha < 1$ , i.e. the cell division rate component  $F_b$  accelerates with  $\alpha$ , see Fig 8

350 in S8 Appendix. However, the effects of  $\beta$ , measuring the relative weight of cell transition  
 351 between germ-like and soma-like cells, leads to different results between stage-dependent  
 352 and stage-independent differentiation. Specifically, we found that irreversible differentiation  
 353 evolves across all values of  $\beta$ . As the subcategory *IGD* of *ID* evolves when  $\beta$  is small, and  
 354 *ISD* and *IGSD* of *ID* evolve when  $\beta$  is large.

355 **Large changes in two successive probabilities of cell differentiation prevent irreversible differentiation from becoming optimal**  
 356

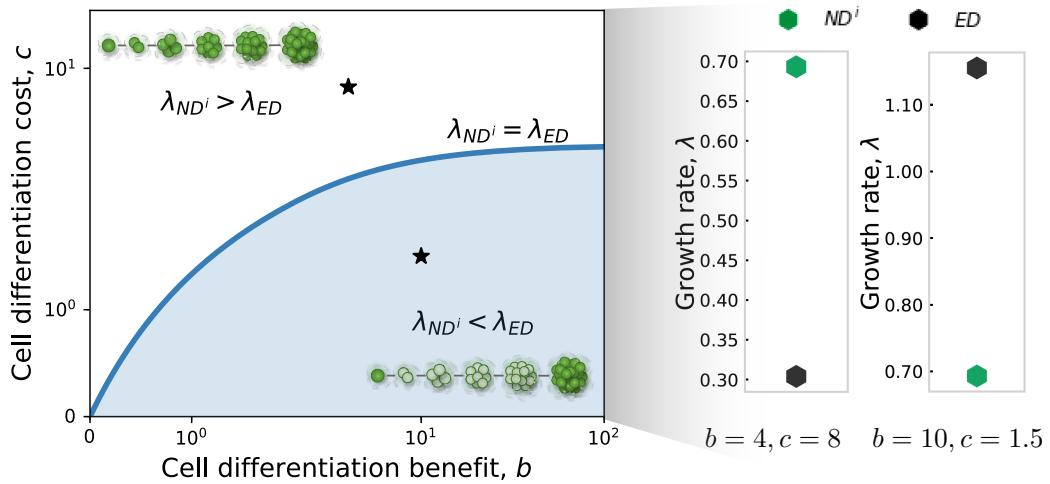
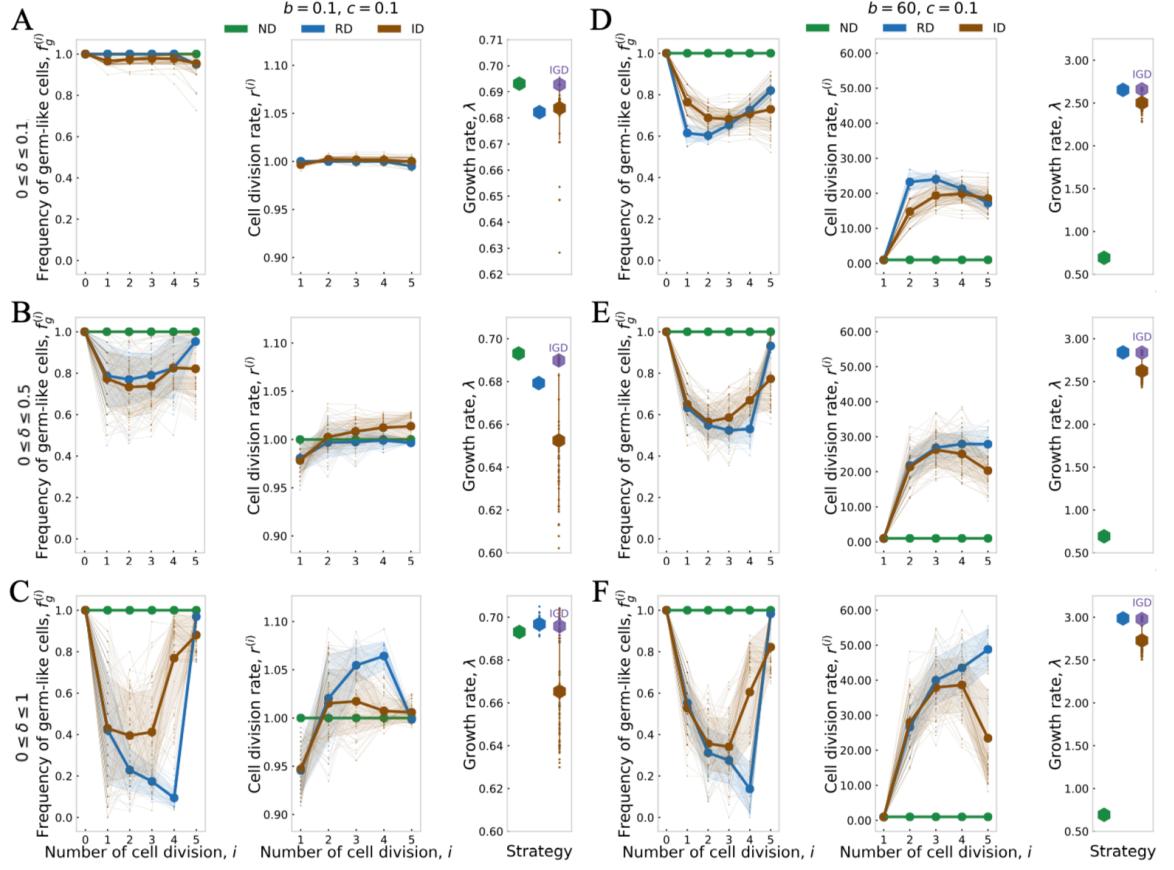


Figure 4: **The evolutionary conditions for non-differentiation  $ND^i$  and extreme differentiation  $ED$ , and their corresponding growth rates.** The blue line represents the condition for  $\lambda_{ND^i} = \lambda_{ED}$ . The shaded area represents where  $\lambda_{ND^i} < \lambda_{ED}$ . We found that  $ND^i$  is optimal under high  $c$  and  $ED$  is optimal under high  $b$ . The black stars correspond to the parameter combinations where growth rates have been calculated in the right panel. Parameters:  $n = 5$  and  $\alpha = \beta = 1$ .

357 Without constraints, the differences of cell differentiation probabilities in successive cell  
 358 divisions can take any value, i.e.  $0 \leq \delta \leq 1$ , where  $0 \leq \delta_i \leq \delta \leq 1$ . Then, cells' differ-  
 359 entiation probabilities and related organism's growth are completely arbitrary. An extreme  
 360 example that optimally exploits the potential of somatic cells would be that both types of cells  
 361 produce soma-like cells in the first  $(n - 1)$  divisions, and then all produce germ-like cells in

362 the last division. We refer to this cell differentiation as “extreme differentiation” (*ED*). We  
363 will first take *ED* as a typical example to investigate the effects of stage-dependent differen-  
364 tiation without any constraints. *ED* is a strategy that fully uses cell differentiation benefits.  
365 In contrast, *ND<sup>i</sup>* is a strategy that does not receive cell differentiation benefit or cell differ-  
366 entiation cost. Next, we compare the evolving conditions for *ED* and *ND<sup>i</sup>*. Naturally, we  
367 expect that when  $b \gg 1$  and  $c \ll 1$ , *ED* is optimal, and when  $c \gg 1$  and  $b \ll 1$ , *ND<sup>i</sup>* is  
368 optimal. Based on Eq (8), the growth rate of *ED* is  $\lambda_{ED} = \frac{\ln 2}{n+b+(1+b+\beta)c}$  and the growth rate  
369 of *ND<sup>i</sup>* is  $\lambda_{ND^i} = \ln 2$ . Thus, when  $c < \frac{(n-1)b}{1+b+\beta}$ , we have  $\lambda_{ED} > \lambda_{ND^i}$  and when  $c > \frac{(n-1)b}{1+b+\beta}$ ,  
370 we have  $\lambda_{ED} < \lambda_{ND^i}$ , see Fig 4. The outcome indicates that if the costs of differentiation are  
371 high, the strategy of no differentiation will be chosen, and if the benefits of differentiation are  
372 high, differentiation strategies will be selected.

373 We next investigate the effects of the maximum change of two successive differentiation  
374 probabilities i.e. parameter  $\delta$  on the growth rates of the general stage-dependent strategies  
375 *ND<sup>i</sup>*, *RD*, and *ID*. We found that irreversible differentiation cannot be optimal under large  
376  $\delta$ , see Fig 5. Large  $\delta$  means more randomness of cell differentiation during an organism’s  
377 growth. A higher value of  $\delta$  intensifies the spectrum of an organism’s growth rate except for  
378 *ND<sup>i</sup>* whose differentiation probabilities don’t change with  $\delta$ . For instance, we found that the  
379 growth rate of the optimal differentiation strategies including *RD* and *ID* all increase with  
380 increasing  $\delta$ . However, *RD* has a relatively greater increase than *ID* (Fig 5). Furthermore, we  
381 found that *RD* outcompetes *ID* and turns into the optimal strategy when  $\delta$  is 1. Specifically,  
382 when  $\delta = 0.1$ , we found that the sub-strategy *IGD* of *ID* leads to a larger growth rate  
383 than *RD*, whereas when  $\delta = 1$ , *RD* outcompetes *IGD* and leads to the largest growth  
384 rate, see Fig 5 A-C and D-F. Based on Eq (8), we know that the growth rate depends both  
385 on cell division rates and the number of offspring (the fraction of germ-like cells after the  
386  $n$ th division). Furthermore, the cell division rate is proportional to the fraction of soma-like  
387 cells but inversely proportional to the differentiation probabilities which cause differentiation  
388 costs. Taken together, the largest growth rate favors the strategy with a higher fraction of  
389 soma-like cells all the time, a higher fraction of germ-like cells after the last cell division,



**Figure 5: The effect of the maximum change of two successive differentiation probabilities  $\delta$  on the growth rate of optimal strategies.** Frequencies of germ-like cells, cell division rates, and growth rate of the optimal stage-dependent strategy of each category under  $\delta = 0.1$ ,  $\delta = 0.5$  and  $\delta = 1$  respectively,  $i = 1, 2, \dots, n$ . Small dots are the values of each interesting feature at each cell division. Thick lines are the averaged values at each cell division. The shaded areas indicate the standard deviation.  $\alpha = \beta = 1$  and colors correspond with those in Fig 3. Parameters:  $n = 5$ , and  $\alpha = \beta = 1$ , we chose 20 duplicates for generating the optimal strategies in each category which includes subcategories.

390 and lower differentiation probabilities. *RD* contains the strategy to increase the fraction of  
 391 soma-like cells in the middle stages of cell divisions and the number of offspring which is the  
 392 number of germ-like cells after the  $n$ th cell division. Thus, higher  $\delta$  prohibits the emergence  
 393 of strategy *ID* being optimal.

## 394 Conclusion and discussion

395 We investigated the effect of stage-dependent differentiation on an organism's growth and  
396 compared it with stage-independent cell differentiation. Stage-independent cell differenti-  
397 ation only allows a fixed cell differentiation probability for a cell type. Stage-dependent  
398 differentiation, by contrast, refers to being capable of changing differentiation probabilities  
399 in consecutive cell divisions. The most extreme case would be an organism that is entirely  
400 consisting of soma-like cells until the last cell division, where all cells turn into germ-like  
401 cells to produce as many offspring as possible. Stage-dependent differentiation intensifies  
402 the fluctuation of the germ-soma ratio during an organism's growth, which further increases  
403 the complexity of competition between different strategies. We used the growth rate of an or-  
404 ganism as a proxy to investigate the growth competition of different strategies under different  
405 benefits and costs. Based on the differentiation probabilities in the last division, we classify  
406 stage-dependent differentiation into three categories: non-differentiation  $ND^i$ , reversible dif-  
407 ferentiation  $RD$ , and irreversible differentiation  $ID$ . The evolution of irreversible differenti-  
408 ation under stage-independent differentiation has been demonstrated by previous work to be  
409 challenging [Gao et al. \[2021\]](#). Contrary to our expectations, we found that stage-dependent  
410 differentiation favors  $ID$  (in the last division step) more than stage-independent irreversible  
411 differentiation  $ID^i$  in smaller organisms. Specifically,  $IGD$ , a sub-strategy of  $ID$ , leads to  
412 a higher growth rate than other strategies in small organisms. Additionally,  $ISD$  and  $IGSD$   
413 evolved in the parameter space with intermediate benefits and costs, consistent with previous  
414 findings [Gao et al. \[2021\]](#). Finally, we found that large differentiation probability variation  
415 prohibits irreversible differentiation  $ID$  from becoming the optimal strategy. The findings in-  
416 dicate that stage-dependent differentiation favors the evolution of irreversible differentiation  
417 in small organisms and with limited variations between successive cell divisions.

418 That irreversible differentiation is favored in small organisms is contrary to the intuition  
419 provided by stage-independent differentiation, where irreversible differentiation is favored  
420 in large organisms [Gao et al. \[2021\]](#). Our previous work has shown that the minimum size  
421 for irreversible differentiation occurring is  $n = 6$  [Gao et al. \[2021\]](#). This discrepancy arises

422 because of the flexibility of the developmental trajectories under stage-dependent differenti-  
423 ation. These complex developmental trajectories in different categories increase the growth  
424 differences between different strategies. Thus, we found that the optimal strategies of dif-  
425 ferentiation categories can lead to divergent growth rates. In addition, stage-dependent ir-  
426 reversible differentiation evolves two more subcategories than stage-independent one: irre-  
427 versible germ differentiation *IGD* and irreversible germ and soma differentiation *IGSD*.  
428 The broad form of stage-dependent differentiation strategies can capture more cell differenti-  
429 ation patterns in reality. For example, the evolution of *IGSD* can help us to understand cell  
430 lineage segregation in nature [Matt and Umen \[2016\]](#). Our model can screen the stage where  
431 irreversible differentiation emerges, in line with the question of early segregation of germ and  
432 soma in animals [Buss \[1983\]](#), [Knaut et al. \[2000\]](#), [Buehr \[1997\]](#), [McLaren \[2003\]](#), [Extavour](#)  
433 and [Akam \[2003\]](#), but late in most plants [Lanfear \[2018\]](#). To identify the segregation, we  
434 need to investigate the irreversible developmental states of germ-like and soma-like cells in  
435 our model. Future work is necessary for seeking and analyzing the conditions where different  
436 segregation occurs.

437 Previous investigations of cell differentiation mostly focused on the state with a group  
438 of undifferentiated clonal cells [Michod \[2007\]](#), [Gavrilets \[2010\]](#), [Rodrigues et al. \[2012\]](#),  
439 [Goldsby et al. \[2014\]](#), [Cooper and West \[2018\]](#), [Yanni et al. \[2020\]](#), [Liu et al. \[2021\]](#), [Cooper](#)  
440 [et al. \[2021, 2022\]](#) or cells with randomly chosen initial cell types (similar to aggregated  
441 organisms) [Rodrigues et al. \[2012\]](#). The focus of these studies was on the final static condi-  
442 tions that lead to the division of labor rather than the dynamic process during an organism's  
443 growth. These models ignored the dynamic developmental trajectories of organisms from  
444 newborn to maturity. In our model, the developmental trajectories of each organism are  
445 recorded by stage-dependent differentiation probabilities, allowing us to know the dynamic  
446 fractions of each cell type during an organism's growth, which further allow us to investigate  
447 cell differentiation patterns. In addition, Rodrigues et al. have considered cell differentiation  
448 probability as an evolving trait to understand the evolution of differentiation [Rodrigues et al.](#)  
449 [\[2012\]](#). They concluded that differentiation costs, compared with the difference in division

450 rates between cell types, have less impact on the evolution of terminal and reversible differ-  
451 entiation. They also found that differentiation costs played a crucial role in the evolution of  
452 diversity differentiation strategies. Moreover, Rodrigues et al. investigated developmental  
453 strategies in filament multicellular organisms with two essential tasks, and they found that  
454 high differentiation costs can promote the evolution of symbioses. In the model, we employ  
455 functions to demonstrate differentiation benefits and costs (Eq (3), Eq (4) as they can capture  
456 more general forms of benefits and costs by varying relevant parameters.

457 In our model, since we focus on the evolution process that cells reach the final specialized  
458 types, thus we assumed that differentiation occurs randomly and both cell types are capable  
459 of cell differentiation Gao et al. [2021]. The assumption is based on the cell differentiation  
460 situation of species observed in genus *Volvox*, which reveals that cell types undergo an in-  
461 termediate and partial differentiation stage in some closed related species before eventually  
462 becoming specialized cell types Matt and Umen [2016]. We classify the stage-dependent dif-  
463 ferentiation strategy based on its differentiation probability at the last round of cell division.  
464 The classification is based on the idea that the differentiation strategy (reversible and irre-  
465 versible) describes the changes in differentiation capability along the cell division process.  
466 Nevertheless, we stress that this classification is imperfect, especially for large organisms  
467 with more cell divisions, where a more refined classification criterion is needed. However,  
468 owing to the simple classification, the current classification can still largely reflect the evolv-  
469 ing situation of the specific strategies interested. For instance, the strategy that cells all turn  
470 into specialized types after half a round of cell divisions is a subset strategy of *ID*, thus it can  
471 only evolve in the parameter space that *ID* emerged. Meanwhile, we assumed that organ-  
472 isms are clonal, growing from a single founding cell. The reasons for our clonal assumption  
473 are that multicellularity is formed commonly by clonal division rather than cell aggregation  
474 Fisher et al. [2013], Grosberg and Strathmann [1998], Tarnita et al. [2013], Brunet and King  
475 [2017], Pentz et al. [2020], Márquez-Zacarías et al. [2021a]. and clonal organisms with iden-  
476 tical genes have advantages at purging deleterious mutations and reducing conflicts among  
477 cells Grosberg and Strathmann [1998, 2007]. Therefore, clonal multicellularity is predicted

478 to be evolutionarily stable [Mikhailov et al. \[2009\]](#). In the cell differentiation models of aggregated multicellularity, a relatedness parameter can be used to evaluate the level of cooperation  
479 between cell types [Ispolatov et al. \[2012\]](#), [Cooper and West \[2018\]](#), [Madgwick et al. \[2018\]](#),  
480 [Liu et al. \[2021\]](#). Additionally, the maturity size is fixed in the model as previous work has  
481 shown that selection favors life cycles where all organisms grow to the same size and frag-  
482 ment into pieces with the same pattern [Pichugin et al. \[2017\]](#). The assumption is generally in  
483 line with the size observation in some species such as *Volvox* [Matt and Umen \[2016\]](#).

485 We assumed that cell differentiation costs influence an organism's growth. In nature,  
486 cell differentiation and cell plasticity usually originally occur under severe environmental  
487 conditions, indicating a differentiation cost involved [Gallon \[1992\]](#), [Claessen et al. \[2014\]](#),  
488 [Aguirre et al. \[2005\]](#), [Loenarz et al. \[2011\]](#). Differentiation cost has been considered in pre-  
489 vious theoretical research via varying forms [DeWitt et al. \[1998\]](#), [Gavrillets \[2010\]](#), [Ispolatov](#)  
490 [et al. \[2012\]](#), [Rodrigues et al. \[2012\]](#), [Goldsby et al. \[2012\]](#), [Staps and Tarnita \[2022\]](#). But  
491 the modeling purpose of cell differentiation costs is the same, i.e. reducing an organism's  
492 fitness. In our model, we are interested in the relative growth advantage between different  
493 differentiation strategies. Therefore, we assume that differentiation costs affect the growth  
494 rate, reducing cell division rates. Finally, we suppose that cells undergo synchronous cell  
495 divisions. This is not true for large multicellularity with many more cell divisions [New-](#)  
496 [port and Kirschner \[1982\]](#), [Matt and Umen \[2016\]](#). Asynchronous cell division has been  
497 explored under stage-independent differentiation in previous studies, leading to the same  
498 predictions as the synchronous one [Gao et al. \[2021\]](#). Yet, it still needs to be investigated  
499 whether asynchronous cell division leads to the same conclusion as synchronous ones under  
500 stage-dependent differentiation in the future. Our model could be further extended by includ-  
501 ing cell death or differentiation costs related to the risk of organism death. Yet, our model  
502 gives first insights into understanding the effects of dynamic differentiation on the evolution  
503 of cell differentiation in multicellularity.

## 504 Supporting information

505 **S1 Appendix. Growth rate.** In our model, we treat both the number of cells and growth  
 506 time as continuous, distilling the stochastic process down to two quantities for calculating the  
 507 growth rate: the expected offspring number of germ-like cells  $N$  and the amount of growth  
 508 time for an organism to grow  $t$ . The expected growth rate  $\lambda$  can be calculated by the following  
 509 equation approximately

$$\lambda = \frac{\ln N}{t}. \quad (9)$$

510 The robustness of the approximation is tested in [S3 Appendix](#). Here, we use  $f_g(i)$  and  $f_s(i)$   
 511 to denote the fractions of germ-like cells and soma-like cells after the  $i$ th cell division. Since  
 512 each organism starts with a single germ-like cell,  $f_g(0) = 1$  and  $f_s(0) = 0$ . We use  $p_{x \rightarrow y}^{(i)}$  to  
 513 denote the transition probability from cell type  $x$  to  $y$  in the  $i$ th cell division, where  $x$  and  $y$   
 514 are either germ-like cells or soma-like cells. Based on Eq (1), we have

$$\begin{aligned} g_{g \rightarrow g}^{(i)} &= g_{gg}^{(i)} + \frac{g_{gs}^{(i)}}{2} \\ g_{g \rightarrow s}^{(i)} &= g_{ss}^{(i)} + \frac{g_{gs}^{(i)}}{2} \\ s_{s \rightarrow g}^{(i)} &= s_{gg}^{(i)} + \frac{s_{gs}^{(i)}}{2} \\ s_{s \rightarrow s}^{(i)} &= s_{ss}^{(i)} + \frac{s_{gs}^{(i)}}{2}. \end{aligned} \quad (10)$$

515 After the  $i$ th cell division, the expected fraction of germ-like cells is  $f_g^{(i)} = g_{g \rightarrow g}^{(i)} f_g^{(i-1)} +$   
 516  $s_{s \rightarrow g}^{(i)} f_s^{(i-1)}$  and the expected fraction for soma-like cells is  $f_s^{(i)} = g_{g \rightarrow s}^{(i)} f_g^{(i-1)} + s_{s \rightarrow s}^{(i)} f_s^{(i-1)}$ ,  
 which can be expressed in

$$\begin{pmatrix} f_g^{(i)} \\ f_s^{(i)} \end{pmatrix} = \begin{pmatrix} g_{g \rightarrow g}^{(i)} & s_{s \rightarrow g}^{(i)} \\ g_{g \rightarrow s}^{(i)} & s_{s \rightarrow s}^{(i)} \end{pmatrix} \begin{pmatrix} f_g^{(i-1)} \\ f_s^{(i-1)} \end{pmatrix}. \quad (11)$$

The expected  $f_g^{(n)}$  and  $f_s^{(n)}$  can be calculated recursively by Eq (11)

$$\begin{pmatrix} f_g^{(n)} \\ f_s^{(n)} \end{pmatrix} = \begin{pmatrix} g_{g \rightarrow g}^{(n)} & s_{s \rightarrow g}^{(n)} \\ g_{g \rightarrow s}^{(n)} & s_{s \rightarrow s}^{(n)} \end{pmatrix} \cdots \begin{pmatrix} g_{g \rightarrow g}^{(1)} & s_{s \rightarrow g}^{(1)} \\ g_{g \rightarrow s}^{(1)} & s_{s \rightarrow s}^{(1)} \end{pmatrix} \begin{pmatrix} f_g^{(0)} \\ f_s^{(0)} \end{pmatrix}. \quad (12)$$

517 Since cells divide synchronously and no cell dies during growth, the expected number of  
 518 germ-like cells  $N_g^{(n)}$  and soma-like cells  $N_s^{(n)}$  after the  $n$ th cell division are

$$519 \quad \begin{pmatrix} N_g^{(n)} \\ N_s^{(n)} \end{pmatrix} = 2^n \begin{pmatrix} f_g^{(n)} \\ f_s^{(n)} \end{pmatrix}, \quad (13)$$

520  
 521 where  $0 \leq f_g^{(n)}, f_s^{(n)} \leq 1$ .

522 The cell division rate determines the growth duration of organisms. Since cells divide with  
 523 a rate  $r^{(i)} = \frac{1+b[f_s^{(i-1)}]^\alpha}{1+c[f_g^{(i)}_{\rightarrow s} + \beta f_s^{(i)}_{\rightarrow g}]}$  during the  $i$ th cell division, the waiting time for a cell division  
 524  $t^{(i)}$  follows the exponential distribution  $f(t^{(i)}) = r^{(i)}e^{-r^{(i)}t^{(i)}}$ , where  $f_g^{(i)}_{\rightarrow s} = f_g^{(i-1)}s_{s \rightarrow g}^{(i)}$  and  
 525  $f_s^{(i)}_{\rightarrow g} = f_s^{(i-1)}s_{s \rightarrow g}^{(i)}$ , see Eq (6). Thus the expected waiting time from the  $i$ th cell division to  
 526 the  $(i+1)$ th cell division is  $t^{(i)} = \frac{1}{r^{(i)}}$ . The expected growth time for organisms with total  $n$   
 527 cell divisions is

$$528 \quad t = \sum_{i=1}^n t^{(i)} = \sum_{i=1}^n \frac{1}{r^{(i)}} = \sum_{i=1}^n \frac{1 + c[f_g^{(i-1)}g_{g \rightarrow s}^{(i)} + \beta f_s^{(i-1)}s_{s \rightarrow g}^{(i)}]}{1 + b[f_s^{(i-1)}]^\alpha}. \quad (14)$$

529  
 530 Substituting Eq (13) and Eq (14) into Eq (9), we have

$$531 \quad \lambda = \frac{\ln N}{t} = \frac{n \ln 2 + \ln f_g^{(n)}}{\sum_{i=1}^n \frac{1 + c[f_g^{(i-1)}g_{g \rightarrow s}^{(i)} + \beta f_s^{(i-1)}s_{s \rightarrow g}^{(i)}]}{1 + b[f_s^{(i-1)}]^\alpha}}, \quad (15)$$

532  
 533 where  $n$  is the number of total cell divisions of organisms,  $f_g^{(i)}$  and  $f_s^{(i)}$  are fractions of  
 534 germ-like cell and soma-like cell after the  $i$ th cell division,  $g_{g \rightarrow s}^{(i)}$  and  $s_{s \rightarrow g}^{(i)}$  are the transition  
 535 probabilities between germ-like cell and soma-like cell at the  $i$ th cell division ( $1 \leq i \leq n$ ).

536 We have  $f_g^{(0)} = 1$  and  $f_s^{(0)} = 0$ . For the non-differentiation strategy  $ND^i$ , no soma-like  
 537 cells are produced during growth, i.e.  $g_{g \rightarrow g} = 1$  and  $g_{g \rightarrow s} = s_{s \rightarrow g} = s_{s \rightarrow s} = 0$ . Therefore,  
 538  $f_g^{(i)} = 1$ ,  $f_s^{(i)} = 0$ . Thus from Eq (15) the growth rate of  $ND^i$  which is denoted by  $\lambda_{ND^i}$   
 539 is  $\ln 2$ . Biologically, the growth rate of  $ND^i$  describes the number of cells doubling per unit  
 540 of time. As we defined strategies based on the series of cell differentiation probabilities, the  
 541 growth rate of a strategy should be a distribution rather than a fixed value. But for calculation  
 542 convenience, we took Eq (15) as an approximation of a stochastic differentiation strategy. In  
 543 appendix S3, we show the approximation is reliable in finding the optimal differentiation  
 544 strategy.

**S2 Appendix. Numerical calculation of the growth rate of the stage-dependent differentiation strategy.** We introduce the method of calculating growth rate numerically in stage-dependent cell differentiation. To find the optimal strategy, at a fixed benefit and cost condition, we use the Monte-Carlo methods randomly to sample differentiation strategies and then calculate and compare the growth rates of organisms under these strategies. Grid search is used to find the optimal strategy in different conditions of benefits and costs. We first look at the cell differentiation probabilities in the first division step  $d^{(1)} = [g_{gg}^{(1)}, g_{gs}^{(1)}, g_{ss}^{(1)}, s_{gg}^{(1)}, s_{gs}^{(1)}, s_{ss}^{(1)}]$ . Each probability can take the value 0 or other values by increasing 0.1 from 0 at each time until reaching the highest value 1, thus there are 11 possible values for each probability, i.e  $0, 0.1, 0.2, \dots, 1$ . We first define the number of probability combinations. Since  $g_{gg}^{(1)} + g_{gs}^{(1)} + g_{ss}^{(1)} = 1$  and  $g_{ss}^{(1)} = 1 - g_{gg}^{(1)} + g_{gs}^{(1)}$ , as long as we know the values of  $g_{gg}^{(1)}$  and  $g_{gs}^{(1)}$ , we know  $g_{ss}^{(1)}$ . When  $g_{gg} = 0$ ,  $g_{gs}$  can take the 11 values from 0 to 1, Thus, there are totally  $\sum_{i=1}^{11} i = 66$  combinations for  $g_{gg}^{(1)}, g_{gs}^{(1)}, g_{ss}^{(1)}$ . The same number of combinations exist for soma-like cells. Thus, there are a total of  $66 \times 66 = 4356$  combinations for  $d^{(1)}$ . As long as  $d^{(1)}$  is chosen, we need to identify  $d^{(2)}, d^{(3)}, \dots, d^{(n)}$ .  $d^{(2)}$  deviates from  $d^{(1)}$  by either a 0 or  $\delta_2$ . That is  $0 \leq |g_{gg}^{(1)} - g_{gg}^{(2)}| \leq \delta_2$ . The same for the other probabilities  $g_{gs}, g_{ss}, s_{gg}, s_{gs}, s_{ss}$ . The choice of  $d^{(i+1)}$  depends on number of neighbours of  $d^{(i)}$ , which further depends on the elements  $d^{(i)}$ . For  $\delta^{(i+1)} = 0.1$ , if  $g_{gg} = s_{ss} = 1$  in  $d^{(i)}$ , then  $g_{gg}$  and  $s_{ss}$  can only be decreased or be constant. Thus, there are 5 choices for  $d^{(i+1)}$ . However, if the elements in  $d^{(i)}$  are either 0.3 or 0.4, then each element can be increased, decreased, or unchanged. Therefore, it has 13 choices for choosing  $d^{(i+1)}$ . Let's take  $d_1 = [0.3, 0.3, 0.4, 0.3, 0.3, 0.4]$  as an example, then

it's neighbours are

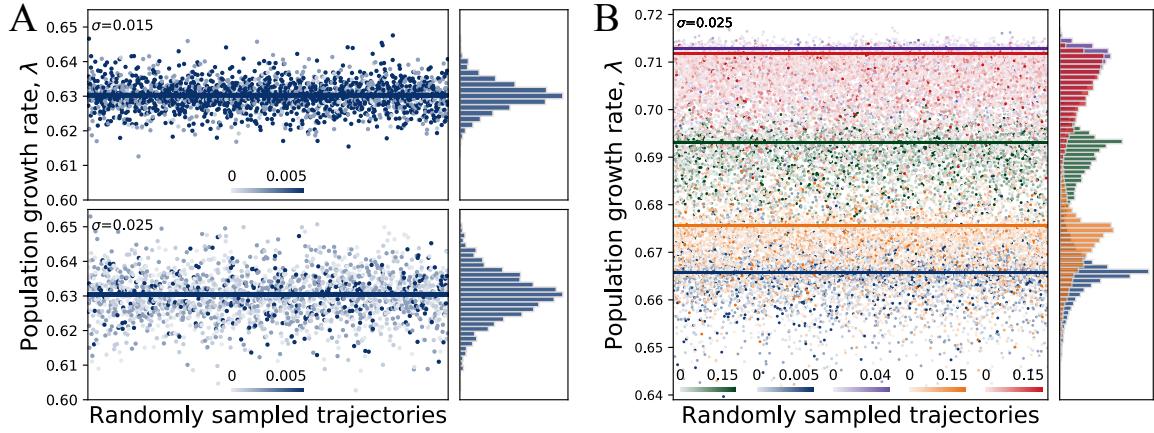
$$\begin{aligned} & [0.3, 0.3, 0.4, 0.3, 0.3, 0.4], \\ & [0.4, 0.2, 0.4, 0.3, 0.3, 0.4], \\ & [0.4, 0.3, 0.3, 0.3, 0.3, 0.4], \\ & [0.2, 0.4, 0.4, 0.3, 0.3, 0.4], \\ & [0.2, 0.3, 0.5, 0.3, 0.3, 0.4], \\ & [0.3, 0.4, 0.3, 0.3, 0.3, 0.4], \\ & [0.3, 0.2, 0.5, 0.3, 0.3, 0.4], \\ & [0.3, 0.3, 0.4, 0.4, 0.2, 0.4], \\ & [0.3, 0.3, 0.4, 0.4, 0.3, 0.3], \\ & [0.3, 0.3, 0.4, 0.2, 0.4, 0.4], \\ & [0.3, 0.3, 0.4, 0.2, 0.3, 0.5], \\ & [0.3, 0.4, 0.3, 0.3, 0.3, 0.4], \\ & [0.3, 0.3, 0.4, 0.3, 0.2, 0.5]. \end{aligned}$$

545 Note that  $d^{(1)}$  is considered as one of its neighbours. To generate a stage-dependent differen-  
546 tiation strategy, we first chose  $d^{(1)}$  from the combination pool and then chose  $d^{(2)}$  from  $d^{(1)}$ 's  
547 neighbors and repeat the process until obtaining  $d^{(n)}$ . We choose each strategy randomly fol-  
548 lowing a uniform distribution. As long as we have classified strategies, we will have a pool  
549 of each strategy and then we choose strategies from the pools. Specificity, we first choose  
550 the last probabilities and then randomly choose other probabilities backward in rounds of cell  
551 division. For example, for choosing a *RD* strategy, we first randomly pick the probabilities at  
552 the  $n$ th round of cell division, which should satisfy  $0 < g_{g \rightarrow s}^{(n)}, s_{s \rightarrow g}^{(n)} < 1$ . Then we randomly  
553 choose the probabilities at the  $(n - 1)$ th round of cell division and so on until the first one.

554 In stage-independent cell differentiation, we calculate the growth rates of each strategy  
555 in the cell differentiation probabilities pool. We seek the optimal strategy which leads to the  
556 fastest growing among these 4356 strategies. To find the optimal strategy at a given parameter

557 point, we first chose  $M = 1000$  values for  $d^{(1)}$  from the cell differentiation pool. Then for  
558 each chosen  $d^{(1)}$ , we randomly chose  $R = 100$  stage-dependent strategies, all generated  
559 from this  $d^{(1)}$ .  $R = 100$  is the sampling size of the stage-dependent strategies from the same  
560 initial differentiation probabilities  $d^{(1)}$ . Then we compute the growth rate of the  $10^5$  strategies  
561 and choose the strategy leading to the largest growth rate. Next, we optimize that strategy  
562 further. For the optimal strategy with the largest growth rate, we compare its growth rate  
563 with a slightly modified strategy. The modified strategies include the one removing  $d^{(1)}$  but  
564 compensating with an  $d^{(n+1)}$  or removing  $d^{(n)}$  by compensating with an  $d^{(0)}$ . Specifically,  
565 for the focused  $D = [d^{(1)}, d^{(2)}, \dots, d^{(n)}]$ , we check whether  $D' = [d^{(0)}, d^{(1)}, d^{(2)}, \dots, d^{(n-1)}]$   
566 or  $D' = [d^{(2)}, \dots, d^{(n)}, d^{(n+1)}]$  leads to a higher growth rate over  $D$ . Here the  $d^{(0)}$  is one  
567 neighbour of  $d^{(1)}$ , and  $d^{(n+1)}$  is one neighbour of  $d^{(n)}$ . If the  $D'$  leads to a higher growth rate,  
568 we keep the process until we find the  $D'$  which makes the growth rate stay at the maxima.  
569 We aim to find a local optimum close to the strategy that was identified in our grid search.  
570 Local optimization stops when the largest steady growth rate in the local neighbourhood  
571 is identified. Overall, we first search the optimal  $D$  globally by randomly choosing  $d^{(1)}$ ,  
572 represented by  $M$  and  $R$ . The values of  $M$  and  $R$  and the number of duplications used in  
573 the main text were chosen to ensure the optimal strategy converging to a unique strategy.  
574 Then, we used a local grid search by modifying  $d^{(1)}$  or  $d^{(n)}$  of a strategy until finding the  
575 optimal  $D$ . Besides, we have constructed initial sampling strategies from the middle of  $d^{(i)}$   
576 sequences. We first identified  $d^{(\frac{n+1}{2})}$  if  $n$  is odd and  $d^{(\frac{n}{2})}$  if  $n$  is even, and then constructed  
577 the rest  $d^{(i)}$ s. The results show that there is almost no differences in terms of searching for  
578 optimal strategies between the two methods.

579 **S3 Appendix. Robustness of the growth rates of stochastic differentiation strategies.**  
580 In our model, we calculated the growth rate of a stochastic strategy based on its expected  
581 growth time and expected number of germ-like cells. That is, we treat a stochastic differenti-  
582 ation strategy that may contain many potential developmental trajectories as a deterministic  
583 one. Theoretically, the growth rate of a stochastic strategy should be a random variable.



**Figure 6: Comparison of growth rates by approximation and random sampling.** **A.** Growth rate comparison of potential random trajectories (strategies) of a randomly chosen  $RD$  strategy under Gaussian distribution with variance 0.015 and 0.025 respectively. The small blue dots represent the potential trajectories. The lines represent the expected growth rates calculated based on Eq (15). The color of the dots represents the probability of the randomly chosen  $RD$  choosing the dots. The histograms represent the distribution frequency of  $\lambda$ . **B.** Growth rate of randomly sampled optimal strategies of each category ( $ND^i$ ,  $RD$ ,  $IGD$ ,  $ISD$  and  $IGSD$ ). The optimal strategy is obtained based on the calculation in [S1 Appendix](#) and the grid search method in [S2 Appendix](#). The color of the dots represents the probability of the optimal given strategy choosing the strategy. The histograms represent the distribution frequency of  $\lambda$ . The colors represent the same strategy as that in Fig 3. Parameters for all panels  $\delta = 0.05$ ,  $n = 5$  and  $b = c = 1$ . For calculating the growth rate of each strategy, see the appendix [S2 Appendix](#).

584 Next, we show that the method used in the model is a good approximation for seeking the  
 585 optimal strategy in an average sense. To simulate the consecutive stochastic differentiation  
 586 probabilities, at a given stage we need to know the differentiation probability distribution  
 587 that the next consecutive probabilities follow. Without loss of generality, we assume that the  
 588 differentiation probabilities follow Gaussian distribution. Then, the coming cell differentia-  
 589 tion probability of a cell type is a variable with the last past differentiation probability as the  
 590 mean. For an arbitrary strategy  $D = [d^{(1)}, d^{(2)}, \dots, d^{(n)}]$ , we can get  $g_{g \rightarrow s}^{(i)}$  and  $s_{s \rightarrow g}^{(i)}$  for each  $i$ ,  
 591  $i = 0, 1, \dots, n$ . Then the variable  $g_{g \rightarrow s}^{(i+1)}$  follows the Gaussian distribution  $g_{g \rightarrow s}^{(i+1)} \sim \mathcal{N}(\mu, \sigma^2)$ ,

592 where  $\mu = g_{g \rightarrow s}^{(i)}$ . To capture the growth rate of the stochastic differentiation strategy  $D$ , we  
593 randomly choose the new  $g_{g \rightarrow s}^{(i)}$  and the new  $s_{s \rightarrow g}^{(i)}$  from the Gaussian distribution with mean  
594  $g_{g \rightarrow s}^{(i)}$  and  $s_{s \rightarrow g}^{(i)}$  respectively,  $i = 1, 2, 3, \dots, n$ . Each sampling will generate a new strategy  
595  $D^*$ , which is a potential developmental strategy based on  $D$ . For each  $D^*$ , we can calculate  
596 its growth rate based on Eq (15). Then we adopt the Monte Carlo method to capture the po-  
597 tential growth rate distribution by randomly choosing a large number of  $D^*$  and calculating  
598 their growth rate. Based on our numerical calculation, we found that our approximation is  
599 along well with the expected growth rate of a randomly chosen strategy (Fig 6A). The value  
600 of variance  $\sigma$  is undefined. As here we focus on the mean behavior of a strategy, thus vari-  
601 ance only impacts the range of growth rate. Furthermore, we testified whether the conclusion  
602 under the approximation is consistent with the statistical results introduced above. We found  
603 that the optimal strategies are the same (Fig 3 and Fig 6B), indicating the robustness of the  
604 approximation method. However, we should note the expected growth rate of a stochastic  
605 differentiation strategy may not be equal to our approximation. The former is  $\sum_{\lambda} \lambda_k p_k$ , where  
606  $k$  is the all possible trajectories of  $D^*$ ,  $p_k$  is the corresponding probability of choosing trajec-  
607 tory  $k$ , and  $\lambda_k$  is the growth rate under trajectory  $k$ .  $p_k$  is multiplication of  $p_k^i$  which is the  
608 probability of choosing a differentiation probability for either germ-like cell  $g_{g \rightarrow s}^{(i)}$  or soma-  
609 like cell  $s_{s \rightarrow g}^{(i)}$  in  $D^*$ , where  $i = 1, 2, \dots, n$ . In the numerical calculation (Fig 6), we roughly  
610 classify 8 intervals i.e. 8 different probabilities for generating  $g_{g \rightarrow s}^{(i)}$  or  $s_{s \rightarrow g}^{(i)}$  for a given  $i$ . The  
611 8 intervals are classified based on boundaries of  $\mu + j * \sigma$ , where  $j = -3, -2, -1, 1, 2, 3$ .  
612 As we seek the optimal strategy, which depends on the relative difference between different  
613 strategies i.e. the rank of the growth rate of different strategies, we employ the approximation  
614 to seek the optimal strategy in the model.

615 **S4 Appendix. Optimality of non-differentiation strategy  $ND^i$ .**

616  **$ND^i$  is optimal in the absence of cell differentiation benefits for any maximal cell divi-  
617 sion number  $n$**

618 When the cell differentiation benefit is absent, i.e.  $b = 0$  and  $c > 0$ , we find that  $ND^i$  is the  
619 optimal strategy based on Eq (15).

620  **$ND^i$  is optimal in the absence of costs for  $n = 1$**

621 Next, when there is only one cell division ( $n = 1$ ), we prove that  $ND^i$  leads to the largest  
622 growth rate under  $b > 0$  and  $c = 0$ . Since  $f_g^{(0)} = 1$  and  $f_s^{(0)} = 0$ , based on Eq (14), the  
623 growth time is

$$624 \quad t = \sum_{i=1}^{n=1} t^{(i)} = \frac{1}{r^{(1)}} = \frac{1}{1 + b[f_s^{(0)}]^\alpha} = 1. \quad (16)$$

626 Substituting Eq (16) into Eq (15) and using Eq (12), we have

$$627 \quad \lambda = \frac{\ln N}{t} = \ln 2 + \ln f_g^{(1)} = \ln 2 + \ln g_{g \rightarrow g}^{(1)}. \quad (17)$$

629 Since  $0 \leq g_{g \rightarrow g}^{(1)} \leq 1$ , the optimal strategy is  $ND^i$  which has  $g_{g \rightarrow g}^{(1)} = 1$ . Thus,  $ND^i$  is the  
630 optimal strategy under  $b > 0$ ,  $c = 0$  and  $n = 1$ .

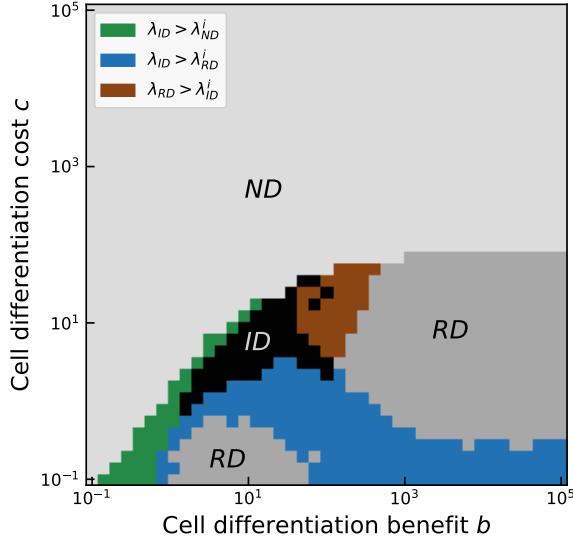
631 **S5 Appendix. Optimal strategies of  $n = 5$  under a larger range of parameter space.**

632 Here, we show that  $RD$  is optimal when benefits are far larger than costs, see Fig 7.  $ND^i$  is  
633 optimal when differentiation costs are far larger than benefits.

634 **S6 Appendix. Either  $IGD$  or  $RD$  is optimal in the absence of cell differentiation costs  
635 when maximal cell division  $n > 1$ .** To show the optimal strategy is either  $IGD$  or  $RD$  under  
636  $b > 0$  and  $c = 0$ . We first prove  $\lambda_{IGD} > \lambda_{IGSD} > \lambda_{ND^i}$ , and then prove  $\lambda_{RD} > \lambda_{ISD} >$   
637  $\lambda_{ND^i}$ .

638  **$IGD$  is optimal among  $IGD$ ,  $IGSD$  and  $ND^i$**

639 To prove  $\lambda_{IGD} > \lambda_{IGSD} > \lambda_{ND^i}$ , we begin with the proof of  $\lambda_{IGSD} > \lambda_{ND^i}$ . Unlike the  
640  $ND^i$  strategy,  $ISD$ ,  $RD$ ,  $IGD$  and  $IGSD$  are categories which include many strategies. As



**Figure 7: Comparison of the optimal strategies between stage-independent differentiation and stage-dependent differentiation under the large scale of benefits and costs.** The colors show the same meaning as that in Fig 2. Parameters:  $0 \leq \delta_i \leq 0.1$ ,  $\alpha = \beta = 1$ , and  $n = 5$ . Parameters of calculating optimal strategy: the number of initial sampling  $d^{(1)}$ ,  $M = 1000$ , the number of stage-dependent strategies starting with a given  $d^{(1)}$ ,  $R = 100$ , for more detail, see [S2 Appendix](#). At each pixel, the frequency of each optimal strategy was calculated across 20 replicates..

641 long as one strategy in *IGSD* has a greater growth rate than  $ND^i$ , we say *IGSD* is more  
 642 optimal than  $ND^i$ . Think of an *IGSD* strategy with only a non-zero cell differentiation  
 643 probability from germ-like cells to soma-like cells and zero differentiation probabilities the  
 644 other way around. Let's assume it is the  $i$ th cell division that makes  $g_{g \rightarrow s}^{(i)} > 0$ , thus we have  
 645  $g_{g \rightarrow s}^{(j)} = 0$  for  $j \neq i$  and  $s_{s \rightarrow g}^{(i)} = 0$  for any  $i$ . Based on Eq (12), the cell frequencies after the  
 646  $n$ th division are

$$\begin{aligned}
 \begin{pmatrix} f_g^{(n)} \\ f_s^{(n)} \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdots \begin{pmatrix} g_{g \rightarrow g}^{(i)} & 0 \\ g_{g \rightarrow s}^{(i)} & 1 \end{pmatrix} \cdots \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} f_g^{(0)} \\ f_s^{(0)} \end{pmatrix} \\
 &= \begin{pmatrix} g_{g \rightarrow g}^{(i)} \\ g_{g \rightarrow s}^{(i)} \end{pmatrix},
 \end{aligned} \tag{18}$$

647

648

649 where  $f_g^{(0)} = 1$  and  $f_s^{(0)} = 0$ . For convenience, we denote  $g_{g \rightarrow g}^{(i)} = g^*$  and  $g_{g \rightarrow s}^{(i)} = 1 - g^*$ .

650 From Eq (15), we have

$$\begin{aligned}
 \lambda_{IGSD} &= \frac{n \ln 2 + \ln g^*}{\frac{n-1}{1+b} + \frac{1}{1+b(1-g^*)^\alpha}} \\
 &\geq \frac{n \ln 2 + \ln g^*}{\frac{n+b}{1+b}} \\
 &= \ln 2 + \frac{(n-1)b}{n+b} \ln 2 + \frac{1+b}{n+b} \ln g^* \\
 &= \lambda_{ND^i} + \frac{\ln 2^{(n-1)b} (g^*)^{1+b}}{n+b}, \\
 \end{aligned} \tag{19}$$

651  
652 where we use  $(1 - g^*) \geq 0$  to obtain the inequality and  $\lambda_{ND^i} = \ln 2$  (appendix ). Since  $b > 0$  and  $n > 1$ , as long as  $2^{(n-1)b} (g^*)^{1+b} \geq 1$ , we obtain  $\lambda_{IGSD} > \lambda_{ND^i}$ . That is  $g^* \geq \frac{1}{2^{\frac{(n-1)b}{1+b}}}$ . Therefore, when  $b > 0$ , we can always find an *IGSD* strategy with a  $g_{g \rightarrow s}^{(i)} \leq 1 - \frac{1}{2^{\frac{(n-1)b}{1+b}}}$  and all other  $g_{g \rightarrow s}^{(i)} = 0$  and  $s_{s \rightarrow g}^{(i)} = 0$ , which leads to higher growth rate than  $ND^i$ . Thus,  $\lambda_{IGSD} > \lambda_{ND^i}$ . The proof of  $\lambda_{IGD} > \lambda_{IGSD}$  is in the appendix . Taken these together, we have  $\lambda_{IGD} > \lambda_{IGSD} > \lambda_{ND^i}$ .

653  
654 **655 RD is optimal among RD, ISD and ND<sup>i</sup>**

656 Next, we prove  $\lambda_{RD} > \lambda_{ISD} > \lambda_{ND^i}$ . We first prove  $\lambda_{ISD} > \lambda_{ND^i}$ . We prove that there exists an *ISD* strategy leading to a higher  $\lambda$  than  $\lambda_{ND^i} = \ln 2$  (appendix ). Consider the *ISD* with  $s_{s \rightarrow s}^{(i)} = 1$ , but with at least one  $i$  which makes  $g_{g \rightarrow s}^{(i)} > 0$  i.e.  $g_{g \rightarrow g}^{(i)} = 1 - g_{g \rightarrow s}^{(i)} < 1$  for  $1 \leq i \leq n$ . The above constraint corresponds with the definition of the *ISD* strategy.

657 Based on Eq (12), the cell frequencies after the  $n$ th division are

$$\begin{aligned}
 \begin{pmatrix} f_g^{(n)} \\ f_s^{(n)} \end{pmatrix} &= \begin{pmatrix} g_{g \rightarrow g}^{(n)} & 0 \\ g_{g \rightarrow s}^{(n)} & 1 \end{pmatrix} \cdots \begin{pmatrix} g_{g \rightarrow g}^{(i)} & 0 \\ g_{g \rightarrow s}^{(i)} & 1 \end{pmatrix} \cdots \begin{pmatrix} g_{g \rightarrow g}^{(1)} & 0 \\ g_{g \rightarrow s}^{(1)} & 1 \end{pmatrix} \begin{pmatrix} f_g^{(0)} \\ f_s^{(0)} \end{pmatrix} \\
 &= \begin{pmatrix} \prod_{i=1}^n g_{g \rightarrow g}^{(i)} \\ 1 - \prod_{i=1}^n g_{g \rightarrow g}^{(i)} \end{pmatrix}, \\
 \end{aligned} \tag{20}$$

667 where  $n \geq 1$ . Substituting Eq (20) into Eq (15) and together with  $c = 0$ , we find the growth  
 668 rate of the ISD strategy

$$669 \quad \lambda_{ISD} = \frac{n \ln 2 + \ln \prod_{i=1}^n g_{g \rightarrow g}^{(i)}}{1 + \sum_{i=2}^n \frac{1}{1 + b[1 - \prod_{k=1}^{i-1} g_{g \rightarrow g}^{(k)}]^\alpha}}, \quad (21)$$

670

671 where the first item in the denominator represents the time for the first cell division  $t_1 = 1$   
 672 because of  $f_s^{(0)} = 0$ . We define the second item of the denominator of Eq (21) as  $F(n) =$   
 673  $\sum_{i=2}^n f^{(i)}$ , where  $f^{(i)} = \frac{1}{1 + b[1 - \prod_{k=1}^{i-1} g_{g \rightarrow g}^{(k)}]^\alpha}$ . Next, we prove  $F(n)$  is a bounded function.  
 674 Since  $0 \leq g_{g \rightarrow g}^{(k)} \leq 1$ , thus sequence  $\{\prod_{k=1}^{i-1} g_{g \rightarrow g}^{(k)}\}_{i=2}^\infty$  decreases with increasing  $i$ . That is,  
 675  $\{f^{(i)}\}_{i=2}^\infty$  is a positive but decreasing sequence.  $f^{(2)}$  is the largest one in  $\{f^{(i)}\}_{i=2}^\infty$ . Therefore,  
 676 the sequence  $\{F(n) = \sum_{i=2}^n f^{(i)}\}$  is an accelerating discrete sequence with respect to  $n$ . We  
 677 have

$$678 \quad F(n) \leq (n-1)f^{(2)} = \frac{n-1}{1 + b(1 - g_{g \rightarrow g}^{(1)})^\alpha}. \quad (22)$$

679

680 Substituting the right-hand side of inequality (22) into Eq (21), we have

$$681 \quad \begin{aligned} \lambda_{ISD} &= \frac{n \ln 2 + \ln \prod_{i=1}^n g_{g \rightarrow g}^{(i)}}{\sum_{i=1}^n \frac{1}{1 + b(1 - \prod_{i=1}^n g_{g \rightarrow g}^{(i)})^\alpha}} \\ &\geq \frac{n \ln 2 + \ln \prod_{i=1}^n g_{g \rightarrow g}^{(i)}}{1 + \frac{n-1}{1 + b(1 - g_{g \rightarrow g}^{(1)})^\alpha}} \\ &= \frac{\left( n \ln 2 + \ln \prod_{i=1}^n g_{g \rightarrow g}^{(i)} \right) [1 + b(1 - g_{g \rightarrow g}^{(1)})^\alpha]}{n + b(1 - g_{g \rightarrow g}^{(1)})^\alpha} \\ &= \ln 2 + \frac{(n-1)b(1 - g_{g \rightarrow g}^{(1)})^\alpha \ln 2 + [1 + b(1 - g_{g \rightarrow g}^{(1)})^\alpha] \ln \prod_{i=1}^n g_{g \rightarrow g}^{(i)}}{n + b(1 - g_{g \rightarrow g}^{(1)})^\alpha}. \end{aligned} \quad (23)$$

682

683 As long as there exist a strategy which makes the right side of Eq (23) greater than  $\ln 2$ , we  
 684 have  $\lambda_{ND^i} < \lambda_{ISD}$ . Then we need to identify the conditions for

$$685 \quad (n-1)b\left(1-g_{g \rightarrow g}^{(1)}\right)^{\alpha} \ln 2 + (1+b\left(1-g_{g \rightarrow g}^{(1)}\right)^{\alpha}) \ln \prod_{i=1}^n g_{g \rightarrow g}^{(i)} > 0 \quad (24)$$

687 to hold. As  $b > 0$  and  $\alpha > 0$ ,  $(n-1)b\left(1-g_{g \rightarrow g}^{(1)}\right)^{\alpha} \ln 2 \geq 0$ . Since  $0 \leq g_{g \rightarrow g}^{(i)} \leq 1$ ,  
 688  $\prod_{i=1}^n g_{g \rightarrow g}^{(i)} \leq 1$  and  $\ln \prod_{i=1}^n g_{g \rightarrow g}^{(i)} \leq 0$ . The second item of Eq (24) is negative. There exists  
 689 a sequence  $\{g_{g \rightarrow g}^{(i)}\}$ , which makes  $\prod_{i=1}^n g_{g \rightarrow g}^{(i)} \rightarrow 1^-$  and

$$690 \quad \begin{aligned} \ln \prod_{i=1}^n g_{g \rightarrow g}^{(i)} &> -\frac{(n-1)b\left(1-g_{g \rightarrow g}^{(1)}\right)^{\alpha} \ln 2}{1+b\left(1-g_{g \rightarrow g}^{(1)}\right)^{\alpha}} \\ &= -\frac{(n-1) \ln 2}{1+\frac{1}{b\left(1-g_{g \rightarrow g}^{(1)}\right)^{\alpha}}}, \end{aligned} \quad (25)$$

691 which makes the Eq (24) hold. With the above proof, we conclude that  $\lambda_{ISD} > \lambda_{ND^i}$  with  
 692 only cell differentiation benefit. From Eq (25), we found that more *ISD* strategies are better  
 693 than *ND<sup>i</sup>* under high benefits  $b$ . However, when  $b$  is small, only *ISD* with  $g_{g \rightarrow g}^{(i)} \rightarrow 1$  leads  
 694 higher growth rate than *ND<sup>i</sup>*. The proof of  $\lambda_{RD} > \lambda_{ISD}$  can be found in the appendix . Thus,  
 695 we have  $\lambda_{RD} > \lambda_{ISD} > \lambda_{ND^i}$ . The results show that when there is a benefit and no costs,  
 696 differentiation strategies (*ISD*, *IGSD*, *IGD* and *RD*) are better over *ND<sup>i</sup>*. Either *IGD* or  
 697 *RD* is optimal under  $b > 0$  and  $c = 0$ .  
 698

699 **S7 Appendix. *IGSD* and *ISD* cannot be optimal in the absence of either cell differen-  
 700 tiation benefit or cost.** In the appendix , we have proved *ND<sup>i</sup>* is optimal in the absence of  
 701 differentiation benefits, i.e.  $b = 0$  and  $c > 0$ . Thus, we prove *IGSD* and *ISD* can be optimal  
 702 in the absence of differentiation costs, i.e.  $c = 0$  and  $b > 0$ . Since we also have proved that  
 703 *ND<sup>i</sup>* is optimal under  $n = 1$  when  $c = 0$  and  $b > 0$  in appendix . Therefore, we only need to  
 704 prove that the optimal strategy can neither be *IGSD* nor *ISD* when  $b > 0$ ,  $c = 0$  and  $n \geq 2$ .

705 We first prove  $\lambda_{IGD} > \lambda_{IGSD}$ . For a given *IGSD* strategy, we can always modify it  
 706 and obtain an *IGD* strategy, which leads to a higher  $\lambda$  than the given *IGSD* strategy. For a  
 707 given *IGSD* strategy, we know its transition probabilities  $s_{s \rightarrow g}^{(n)} = 0$ . We modify the *IGSD*

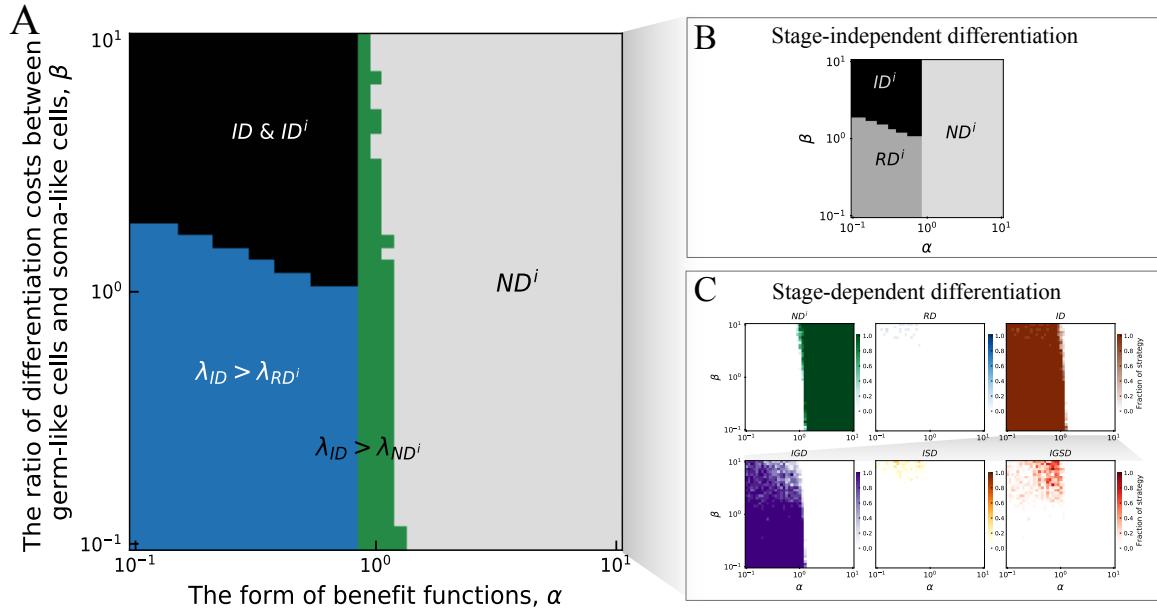
708 strategy by setting  $0 < s_{s \rightarrow g}^{(n)} = k \leq 1$  to get a *IGD* strategy. The constructed *IGD* strategy  
 709 produces more offspring than the given *IGSD* strategy as its final number of germ-like cells  
 710 is  $N = 2^n(f_g^{(n-1)}g_{g \rightarrow g}^{(n)} + f_s^{(n-1)}s_{s \rightarrow g}^{(n)})$ , which is greater than that of the *IGSD* as  $s_{s \rightarrow g}^{(n)} = k > 0$   
 711 in the *IGD* strategy. Since there is no cell differentiation cost ( $c = 0$ ), cell division rates are  
 712 the same among all strategies. Thus,  $\lambda_{IGD} > \lambda_{IGSD}$ .

713 Next, we prove  $\lambda_{RD} > \lambda_{ISD}$ . Given an *ISD* strategy, we have  $s_{s \rightarrow g}^{(n)} = 0$ . Construct a *RD*  
 714 strategy that has the same transition probability matrixes as the given *ISD* strategy for the  
 715 first  $(n-1)$  cell divisions. For the  $n$ th transition probability matrix, we keep the  $g_{g \rightarrow g}^{(n)}$  and  $g_{g \rightarrow s}^{(n)}$   
 716 the same as that in the given *ISD* strategy. However, we set  $s_{s \rightarrow g}^{(n)} > 0$  rather than  $s_{s \rightarrow g}^{(n)} = 0$   
 717 as that in *ISD* strategy.  $s_{s \rightarrow g}^{(n)} > 0$  implies  $s_{s \rightarrow s}^{(n)} = 1 - s_{s \rightarrow g}^{(n)} < 1$ . Then, *ISD* and *RD* have  
 718 the same germ-like cells during the first  $n-1$  cell divisions. The fraction of germ-like cells  
 719 for the *ISD* strategy after the  $n$ th cell divisions is  $f_g^{(n-1)}g_{g \rightarrow g}^{(n)} + f_s^{(n-1)}s_{s \rightarrow g}^{(n)} = f_g^{(n-1)}g_{g \rightarrow g}^{(n)}$   
 720 as  $s_{s \rightarrow g}^{(n)} = 0$ . Whereas, the fraction of germ-like cells for the *RD* strategy after the  $n$ th cell  
 721 divisions is  $f_g^{(n-1)}g_{g \rightarrow g}^{(n)} + f_s^{(n-1)}s_{s \rightarrow g}^{(n)}$ . Thus, the constructed *RD* has an extra  $2^n f_s^{(n-1)}s_{s \rightarrow g}^{(n)}$   
 722 germ-like cells compared with the *ISD* strategy. Since the cell division rate at the  $n$ th cell  
 723 division depends on the fraction of soma-like cells at the  $(n-1)$ th cell division, the cell  
 724 division rates  $r^{(i)}$  for the two strategies are the same,  $1 \leq i \leq n$ . Thus, from Eq (15), we  
 725 have

$$\begin{aligned}
 \lambda_{RD} &= \frac{\ln\{2^n[f_g^{(n-1)}g_{g \rightarrow g}^{(n)} + f_s^{(n-1)}s_{s \rightarrow g}^{(n)}]\}}{\sum_{i=1}^n \frac{1}{1 + b[f_s^{(i-1)}]^\alpha}} \\
 &> \frac{\ln\{2^n[f_g^{(n-1)}g_{g \rightarrow g}^{(n)}]\}}{\sum_{i=1}^n \frac{1}{1 + b[f_s^{(i-1)}]^\alpha}} \\
 &= \lambda_{ISD}.
 \end{aligned} \tag{26}$$

727  
 728 Therefore,  $\lambda_{RD} > \lambda_{ISD}$ .

729 **S8 Appendix. Stage-dependent differentiation promotes irreversible cell differentiation**  
 730 **under the effects of benefit function forms  $\alpha$  and the ratio of differentiation costs**  
 731 **between germ-like cells and soma-like cells  $\beta$ .**



**Figure 8: The effects of  $\alpha$  and  $\beta$  on the growth rates of cell differentiation strategies.** **A.** Comparison of the optimal strategy evolved in stage-independent and stage-dependent differentiation strategies depending on  $\alpha$  and  $\beta$ . The areas of grey and black represent the parameter space in which the same strategy are optimal both under stage-independent and stage-dependent cell differentiation. The green area represents stage-dependent  $ID$  leading to a larger growth rate than stage-independent  $ND^i$ . The blue strip represents stage-dependent  $ID$  leading to a larger growth rate than stage-independent  $RD^i$ . **B.** The parameter space of optimal stage-independent differentiation strategy at different values of  $\alpha$  and  $\beta$ . **C.** The frequencies of each stage-dependent strategy depending on  $\alpha$  and  $\beta$ . Parameters for all panels  $\delta = 0.1$ ,  $n = 5$  and  $b = c = 1$ . For calculating the growth rate of each strategy, see the appendix .

732 Under the effects of  $\alpha$  and  $\beta$ , we found that stage-dependent differentiation favors irre-  
 733 versible cell differentiation over stage-independent cell differentiation.  $IGD$  replaces stage-  
 734 independent  $RD$  when  $\alpha$  and  $\beta$  are both small, see Fig 8. Under this scenario, the cell  
 735 transition probability  $s_{s \rightarrow g}$  has a smaller effect in decreasing the growth rate than the transi-  
 736 tion probability  $g_{g \rightarrow s}$ . Thus,  $IGD$  produces a higher fraction of germ-like cells and bears less  
 737 cell differentiation costs, leading to a higher growth rate. When  $\alpha$  is around 1,  $IGD$  leads to  
 738 faster growth than  $ND^i$ . The reason is analogous to the one given in the main text.

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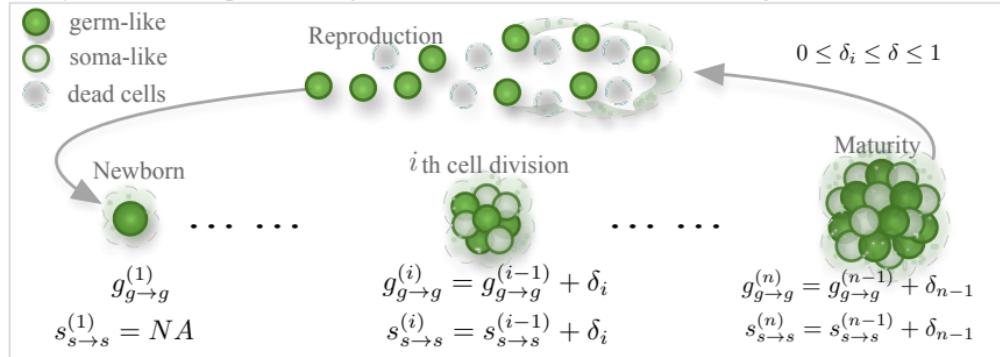
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## A. Dynamic developmental trajectories and cell differentiation categories



## B. Cell differentiation strategies

Differentiation probability	Differentiation category	
$g_{g \rightarrow s}^{(i)} = g_{g \rightarrow s}^{(i-1)} + \delta_i, s_{s \rightarrow g}^{(i)} = s_{s \rightarrow g}^{(i-1)} + \delta_i$	Stage-independent $\delta = 0$	Stage-dependent $\delta \neq 0$
$g_{g \rightarrow s}^{(i)} \equiv 0, i = 1, 2, \dots, n$	$ND^i$	$ND^i$
$g_{g \rightarrow s}^{(i)} \not\equiv 0, i = 1, 2, \dots, n-1$ $g_{g \rightarrow s}^{(n)} = 0$ or $s_{s \rightarrow g}^{(n)} = 0$	$ID^i$	$ID$
others	$RD^i$	$RD$

## C. Cell division rate components

