

1 Elasticity generates indissoluble biomolecular condensates

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6 While biomolecular condensates are often liquid-like, many experiments found that condensates
7 also exhibit solid-like behaviors, making them indissoluble in conditions liquid condensates dissolve.
8 Despite the biological significance of indissoluble condensates to cellular fitness, the mechanisms
9 underlying the indissolubility of solid-like condensates are still unclear. In this work, we study
10 the effects of elasticity on the dissolution of biomolecular condensates. We demonstrate that the
11 bulk stress inside condensates may prevent the condensates from dissolution and obtain a new
12 mechanical equilibrium condition of elastic condensates. Moreover, we theoretically predict a phase
13 diagram of indissolubility for biomolecular condensates and identify a minimum bulk modulus for the
14 condensates to be indissoluble. To verify our theories, we simulate the two-fluid model in which the
15 slow component corresponding to biomolecules generates elastic stress. Our theoretical predictions
16 are nicely confirmed and independent of microscopic details. Our works show that elasticity makes
17 biomolecular condensates less prone to dissolution.

18 Biomolecular condensates are ubiquitous in various organisms, usually composed of proteins and RNAs [1–9].
19 They often have crucial biological functions [8, 10–16], such as adaptive responses to stresses, accelerating bio-
20 chemical reactions, and sequestering molecules from re-
21 actions. Therefore, the accurate regulation of biomole-
22 cular condensates’ formation and dissolution is critical.
23 Meanwhile, experiments have also found that biomole-
24 cular condensates are viscoelastic [17]: they are solid-like
25 on a short time scale and liquid-like on a long time scale.
26 Interestingly, they often exhibit aging behaviors, and the
27 viscoelastic relaxation time, which separates solid and
28 liquid behaviors, increases over time [18]. Indeed, aged
29 condensates may become indissoluble or infusible in con-
30 ditions where newly formed condensates can easily dis-
31 solve or fuse [3, 4, 9, 10, 14, 19–21]. The resistance to
32 dissolution of solid-like condensates is particularly signif-
33 icant when the condensates are induced by deep super-
34 saturation [7]. Dissolution of solid-like condensates may
35 need assistance by energy-consuming enzymes [20, 22–
36 24], therefore, reducing cellular fitness. Moreover, failure
37 to dissolve condensates during mitosis leads to aberrant
38 condensates that cause cell-cycle arrest and ultimately
39 cell death [25].

40 Despite the importance of solid-like nature on the dis-
41 solution of condensates [15, 18], theoretical studies on the
42 formation and dissolution of biomolecular condensates
43 have so far been limited to fluid models, in which the
44 elastic nature of condensates are neglected [26]. Among
45 the most common theoretical models, the hydrodynamic
46 model involving diffusion and advection, often called
47 Model H [26, 27], can successfully incorporate the physics
48 needed to describe the dynamics of phase separation and
49 droplet growth. However, it does not include the elas-
50 tic nature of condensates. Therefore, it cannot explain
51 the indissolubility of solid-like condensates in conditions
52 where liquid condensates dissolve.

53 In this work, we investigate the effects of elasticity on
54 the dissolubility of biomolecular condensates, combining
55 both theories and numerical simulations. In the follow-
56 ing, we first introduce our theoretical frameworks, fo-
57 cusing on elastic condensates subject to an abrupt con-
58 trol parameter change, e.g., some post-translational mod-
59 ifications that reduce the attractive strength between
60 biomolecules. Without elasticity, they are supposed to
61 dissolve. We derive the mechanical equilibrium condi-
62 tions for elastic condensates and find that the bulk stress
63 may prevent the dissolution and render the condensate
64 indissoluble. To test our theoretical predictions, we sim-
65 ulate the two-fluid model [28, 29] beyond the traditional
66 Model H, in which the biomolecule velocity field and the
67 elastic stress are dynamically coupled. Numerical simu-
68 lations of the two-fluid model nicely confirm our theories.
69 Our theories’ validity is independent of the microscopic
70 details, such as the free energy forms of the biomolecule
71 density field, as we confirm numerically.

72 Furthermore, we theoretically predict a phase diagram
73 of indissolubility as a function of effective temperature
74 and the condensate’s bulk modulus. We demonstrate a
75 minimum bulk modulus for the condensate to be indis-
76 soluble. Numerical simulations nicely confirm our pre-
77 dictions regarding the boundary between the dissoluble
78 and indissoluble phases. Our results suggest that the
79 dissolution of elastic condensates can be facilitated by
80 decreasing biomolecules’ attractive strength and lower-
81 ing the condensates’ bulk moduli. Finally, we discuss the
82 biological implications of our work and propose potential
83 experiments to test our predictions.

84 *Equilibrium conditions of elastic condensates.*—
85 Biomolecular condensates usually have well-defined vis-
86 coelastic relaxation times, below which the condensates
87 behave as elastic materials [17, 18]. In this work, we
88 simplify the problem by considering aged condensates
89 with their viscoelastic relaxation times much longer than

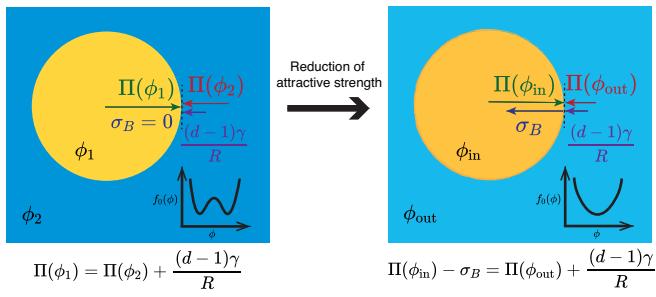


FIG. 1. Mechanical equilibrium conditions of elastic condensates. Liquid-like condensates are initially formed, which then become aged and solid-like. An abrupt change in the attractive strength between biomolecules is introduced, corresponding to a change in the shape of the biomolecule free energy $f_0(\phi)$ from two minima to a single minimum. Here ϕ is the biomolecule density. The condensate's bulk stress σ_B is involved in the new equilibrium condition.

92 the time scales of biological interests, e.g., the duration
 93 of cell-cycle phases. Therefore, the viscoelastic relax-
 94 ation time can be taken as infinite, which is the main
 95 focus of this work. We introduce an abrupt reduc-
 96 tion to the attractive strength between the biomolecules.
 97 Biologically, one of the most relevant perturbations is
 98 the post-translational modification such as phosphory-
 99 lation, which introduces Coulomb repulsive interaction
 100 between biomolecular monomers and reduces the attrac-
 101 tive interaction [5, 13, 30]. Other perturbations that
 102 can lead to similar effects include changing temperature,
 103 pH, and salt concentrations. In the absence of elasticity,
 104 the condensates will dissolve because the free energy of
 105 biomolecule density changes from a form with two min-
 106 imums to a form with only one minimum (Figure 1).
 107 However, as shown in the following, the elastic force may
 108 prevent the dissolution.

109 In liquid-liquid phase separation, a stable conden-
 110 sate requires mechanical equilibrium [31, 32], such that
 111 $\Pi_{in} = \Pi_{out} + (d-1)\gamma/R$. Here, $\Pi_{in(out)}$ is the osmotic
 112 pressure inside (outside) the condensate, d is the spatial
 113 dimension, γ is the surface tension constant and R is
 114 the condensate radius. In the presence of elastic stress,
 115 we need to take account of the elastic energy F_{el} of the
 116 condensate so that the equilibrium condition becomes

$$\Pi_{in} - \sigma_B = \Pi_{out} + \frac{(d-1)\gamma}{R}, \quad (1)$$

117 Here $\sigma_B = \partial F_{el}(V_{in})/\partial V_{in}$ is the bulk stress inside the
 118 condensate due to elasticity where V_{in} is the volume of
 119 the condensate (Figure 1). As we show later, the inclu-
 120 sion of bulk stress compensates the imbalance of osmotic
 121 pressures. To find the expression of σ_B , we use the con-
 122 stitutive equation of the bulk stress and the continuity

123 equation of biomolecule density (ϕ)

$$\frac{\partial \sigma_B}{\partial t} = G_B \nabla \cdot \mathbf{v}_p, \quad (2)$$

$$\frac{\partial \phi}{\partial t} = -\phi \nabla \cdot \mathbf{v}_p. \quad (3)$$

124 Here \mathbf{v}_p is the velocity field of the biomolecules, which is
 125 responsible for the bulk stress. G_B is the bulk modulus.
 126 In writing the above two equations, we assume that ϕ and
 127 σ_B are uniform inside the condensate, which we confirm
 128 numerically later. Combining Eqs. (2, 3), we obtain

$$\sigma_B = G_B \ln \left(\frac{\phi_1}{\phi_{in}} \right) \quad (4)$$

129 where ϕ_1 and ϕ_{in} are the biomolecule densities inside
 130 condensates before and after the reduction of attrac-
 131 tive strength between the biomolecules. The equilibrium
 132 condition must be satisfied for all condensates in a sys-
 133 tem of multiple condensates with the bulk stress deter-
 134 mined by Eq. (4), and Π_{in} and Π_{out} determined by ϕ_{in}
 135 and ϕ_{out} , respectively. Each condensate can have dif-
 136 ferent ϕ_{in} 's although they share the same biomolecule
 137 density ϕ_{out} outside them. Given the radii R 's and the
 138 densities ϕ_{in} 's of each condensate, ϕ_{out} can be calcu-
 139 lated using the conservation of total molecular number:
 140 $V\phi_0 = \sum_i V_i \phi_{in,i} + (V - \sum_i V_i)\phi_{out}$. Here, V is the to-
 141 tal volume, V_i is the volume of condensate i assuming a
 142 spherical shape, and the summation is over all conden-
 143 sates. ϕ_0 is the average density over the total volume.

144 To uniquely determine the densities inside conden-
 145 sates, we still need one more equation. For liquid conden-
 146 sates, it is a uniform chemical potential. However, in our
 147 case, the condensate is solid, so the exchange of molecules
 148 is suppressed as soon as the mechanical equilibrium is
 149 established [3, 8, 15]. Instead, we propose that conden-
 150 sate size does not change upon the weakening of attrac-
 151 tive interaction between biomolecules, namely, $R = R_0$,
 152 where R and R_0 are the radii of an elastic condensate
 153 before and after the condition changes. We confirm this
 154 assumption numerically later. We note that the conden-
 155 sate's constant radius and decreased density do not con-
 156 flict with the suppressed exchange of molecules since the
 157 density change happens when the system is initially out-
 158 of-equilibrium after the attractive strength suddenly de-
 159 creases. As a result, the density decreases initially until
 160 the condensate reaches mechanical equilibrium. Finally,
 161 we remark that in our case, the bulk stress inside the
 162 condensate stabilizes the condensate, in contrast to the
 163 bulk stress outside a condensate, e.g., due to the sur-
 164 rounding polymer network that suppresses the formation
 165 of condensates [33–40].

166 *Simulations of the two-fluid model.*— We numerically
 167 simulate the two-fluid model in two dimensions [28, 29] to
 168 test our theories with two components: the slow compo-
 169 nent corresponding to the biomolecules and the fast compo-
 170 nent corresponding to the solvent. It is the biomolecule

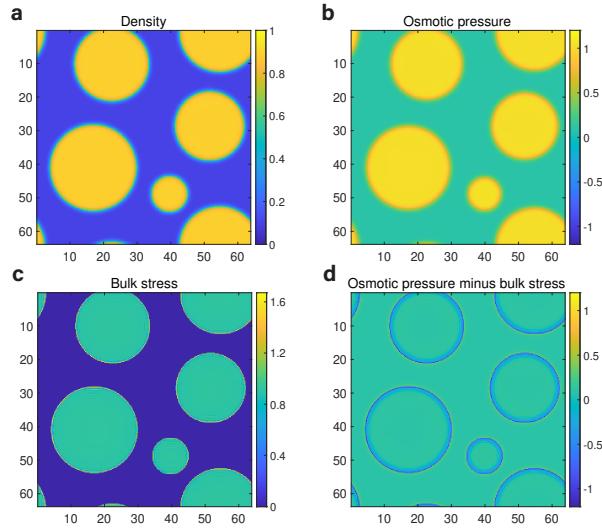


FIG. 2. Simulations of multiple coexisting elastic condensates. (a) The density field ϕ after decreasing the χ parameter in the Flory-Huggins free energy from 3.0 to 1.5. For liquid condensates, they will simply dissolve. In contrast, the elastic condensates can be indissoluble. (b) The osmotic pressure Π from the same simulation of (a). (c) The bulk stress σ_B from the same simulation of (a). (d) $\Pi - \sigma_B$ from the same simulation of (a). In this figure, we take $\phi_0 = 0.45$, $G_B = 20$, $G_S = 20$, and $\phi_c = 0.5$.

171 component that generates the elastic stress. The average
 172 velocity field $\mathbf{v} = \phi \mathbf{v}_p + (1 - \phi) \mathbf{v}_s$ where \mathbf{v}_p are \mathbf{v}_s are
 173 respectively the biomolecule and solvent velocity field.
 174 In the two-fluid model, the biomolecule density field ϕ is
 175 spatially dependent with its value between 0 and 1. The
 176 dynamics of the biomolecule density and velocity field
 177 follows (see details in Supplementary Information)

$$\frac{\partial \phi}{\partial t} = -\nabla \cdot (\phi \mathbf{v}) + \nabla \left(\frac{\phi(1-\phi)^2}{\zeta} (\nabla \cdot \mathbf{\Pi} - \nabla \cdot \boldsymbol{\sigma}) \right), \quad (5)$$

$$-\nabla \cdot \mathbf{\Pi} + \nabla \cdot \boldsymbol{\sigma} - \nabla p + \eta \nabla^2 \mathbf{v} = 0. \quad (6)$$

178 Here, ζ is the friction constant between biomolecules and
 179 solvent, and η is the viscosity. The pressure p is deter-
 180 mined by the incompressible condition: $\nabla \cdot \mathbf{v} = 0$. The
 181 stress tensor $\boldsymbol{\sigma} = \boldsymbol{\sigma}_S + \sigma_B \mathbf{I}$ where σ_B is the bulk stress and
 182 $\boldsymbol{\sigma}_S$ is the shear stress tensor. They follow the Maxwellian
 183 dynamics such that (Supplementary Information),

$$\frac{\partial \sigma_B}{\partial t} = -(\mathbf{v}_p \cdot \nabla) \sigma_B - \frac{1}{\tau_B(\phi)} \sigma_B + G_B(\phi) \nabla \cdot \mathbf{v}_p. \quad (7)$$

184 where G_B is the bulk modulus. To simulate elastic con-
 185 densates, we take the relaxation time of bulk stress to
 186 be diverging at a critical density ϕ_c such that $\tau_B^{-1}(\phi) =$
 187 $(\phi_c - \phi) \Theta(\phi_c - \phi)$, where $\Theta(x)$ is the Heaviside function.
 188 We remark that if we take the stress tensor to be zero
 189 in Eqs. (5, 6), they are reduced to the classical Model
 190 H [27]. The osmotic stress tensor is determined by the

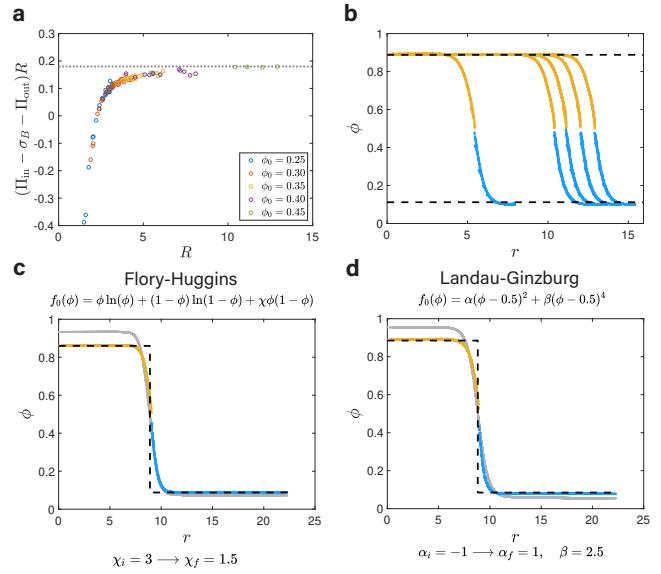


FIG. 3. Computations of the surface tension constant γ and predictions of the density field using different types of free energy. (a) The inferred surface tension constant $\gamma = (\Pi_{in} - \sigma_B - \Pi_{out})R$ approaches an asymptotic value in the large radius limit. (b) A comparison of the theoretical predictions of ϕ_{in} and ϕ_{out} (black dashed lines) and the simulations (yellow dots above ϕ_c and blue dots below ϕ_c) [same as Figure 2(a)]. Each yellow and blue curve represents one condensate and r is the distance from the condensate center. (c) For the Flory-Huggins free energy, the initial density field (gray dots) cannot be maintained after χ decreases and the final density field is established. (d) For the Landau-Ginzburg free energy, the equilibrium density field can also be predicted by our theories after the control parameter α increases from -1 to 1. In (a) and (b), $G_B = 20$, $\chi_i = 3.0$, $\chi_f = 1.5$, $\phi_0 = 0.45$ in (b). In (c) and (d), we simulate a single condensate (see details in Supplementary Information) with $R_0 = 9$. $G_B = 10$ in (c) and $G_B = 20$ in (d). In all figures, $G_S = 20$ and $\phi_c = 0.5$.

191 biomolecule free energy $f(\phi)$, $\nabla \cdot \mathbf{\Pi} = \phi \nabla f'(\phi)$ where
 192 $f(\phi) = f_0(\phi) + \frac{C}{2} (\nabla \phi)^2$ and C is a constant. If not men-
 193 tioned explicitly, we use the Flory-Huggins free energy
 194 density: $f_0(\phi) = \epsilon_0(\phi \ln(\phi) + (1-\phi) \ln(1-\phi) + \chi \phi(1-\phi))$.
 195 Here $\epsilon_0 = k_B T / V_0$ where k_B is the Boltzmann con-
 196 stant, T is the temperature, and V_0 is the biomolecular
 197 monomer volume. We note that the condition of stable
 198 liquid condensate for the Flory-Huggins model is that
 199 the parameter $\chi > 2$. We use the osmotic pressure Π to
 200 represent the scalar osmotic stress computed from $f_0(\phi)$,
 201 $\Pi = \phi f'_0(\phi) - f_0(\phi)$. By non-dimensionalizing Eqs. (5,
 202 6), we choose the unit of elastic modulus as ϵ_0 , the time
 203 unit $t_0 = \eta / \epsilon_0$, and the length unit $l_0 = \sqrt{\eta / \zeta}$ (see esti-
 204 mations in Supplementary Information).

205 We note that the osmotic stress and the elastic stress
 206 are fundamentally different as the osmotic stress is a func-
 207 tion of the instantaneous density field while the elastic
 208 stress depends on the accumulated change of the den-
 209 sity field. One may attempt to compute an effective

210 elastic free energy that is a function of ϕ so that the
 211 elastic bulk stress is included in the osmotic pressure as
 212 $\phi f'_{\text{el}}(\phi) - f_{\text{el}}(\phi) = -\sigma_B$. However, the resulting effective
 213 chemical potential $\mu_{\text{eff}} = f'(\phi) + f'_{\text{el}}(\phi)$ is not uniform
 214 across the system, which suggests that the elastic stress
 215 cannot be included in the free energy as a function of the
 216 density field (Figure S1).

217 *Tests of theoretical predictions.*— We simulate multiple
 218 coexisting condensates, first generated by the two-fluid
 219 model without elasticity from a uniform density ϕ_0 as the
 220 initial condition (see details in Supplementary Informa-
 221 tion). The average density is therefore set by the initial
 222 condition. The initial Flory-Huggins parameter $\chi_i = 3$.
 223 We add elasticity to the condensates after the formation
 224 of multiple spherical condensates to mimic the aging pro-
 225 cess. After a short time, we change χ to $\chi_f = 1.5$ so that
 226 the free energy $f_0(\phi)$ changes from a form with two min-
 227 imums to a form with only one minimum (Figure 1).

228 After the reduction of χ , we find that these conden-
 229 sates are indissoluble under the parameters we take. The
 230 density field inside condensates is indeed uniform as as-
 231 sumed (Figure 2a). We also confirm our assumptions of
 232 uniform bulk stress (Figure 2c) and constant radii (Fig-
 233 ure S2). An example of simulations is shown in Movie
 234 S1. The osmotic pressure is significantly different across
 235 the boundaries of condensates (Figure 2b). For liquid
 236 condensates, they will quickly dissolve due to the large
 237 pressure difference. In contrast, the bulk stress balances
 238 the osmotic pressure difference for elastic condensates.
 239 Indeed, we find that the difference between the osmotic
 240 pressure and bulk stress ($\Pi - \sigma_B$) is uniform across the
 241 boundaries (Figure 2d). We note that the uniform $\Pi - \sigma_B$
 242 is not valid near the condensate boundaries due to the
 243 surface tension. Using the variable sizes of condensates,
 244 we compute the radius dependence of surface tension con-
 245 stant γ , and find that γ converges to a constant value in
 246 the large radius limit (Figure 3a), suggesting that it is
 247 well defined in the thermodynamic limit.

248 To compute the predicted densities inside and outside
 249 condensates, in principle, we need to solve n equations
 250 of Eq. (1) with a shared $\Pi_{\text{out}}(\phi_{\text{out}})$ because each con-
 251 densate has a different radius. Here, n is the number
 252 of condensates. However, if we neglect the contribution
 253 of surface tension in Eq. (1), we can combine all con-
 254 densates to find the common ϕ_{in} that works for all con-
 255 densates. We find that the surface tension constant is
 256 relatively small in our simulations, so our predictions of
 257 the densities ϕ_{in} and ϕ_{out} with $\gamma = 0$ (Figure 3b) are
 258 very close to the predictions with a finite γ (Figure S3).
 259 Therefore, we neglect the surface tension in the following
 260 theoretical calculations.

261 To test the generality of our theories, we also use the
 262 Landau-Ginzburg free energy and find that our theories
 263 are equally applicable (Figure 3c, d). In both cases, the
 264 condensates can be indissoluble after changing the forms
 265 of free energies from two minima to a shape with only

266 one minimum. We find that the numerical densities ϕ
 267 as a function of the distance from the condensate center
 268 precisely match the theoretical predictions. We also test
 269 our theories using asymmetric Flory-Huggins model and
 270 again obtain satisfying agreements between simulations
 271 and theoretical predictions (Figure S4a).

272 To test the effects of the shear modulus, we repeat the
 273 above simulations with the Flory-Huggins free energy and
 274 change the shear modulus with the same bulk modulus.
 275 We find that the density distributions are insensitive to
 276 the values of shear moduli (Figure S5), corroborating the
 277 major role of bulk stress in the mechanical equilibrium
 278 conditions. Finally, we also test the effects of the critical
 279 density and find that our results are valid independent of
 280 ϕ_c (Figure S4b, S6-S8).

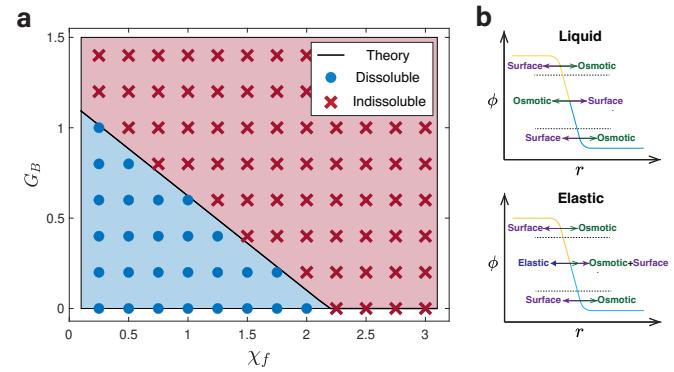


FIG. 4. A critical bulk modulus $G_{B,c}$ above which condensates are indissoluble. (a) Phase diagram of condensate indissolubility with control parameters χ_f and G_B . The theoretical predicted $G_{B,c}$ is the black line and the simulation results are the blue dots and red crosses. (b) Schematics for the force balance in the crossover regime of condensates.

281 *Phase diagram of indissolubility.*— In the following, we
 282 systematically investigate the indissoluble conditions of
 283 elastic condensates. We compute the theoretically pre-
 284 dicted ϕ_{in} after χ is lowered from $\chi_i = 3$ to χ_f as a
 285 function of G_B using Eq. (1). We find that ϕ_{in} decreases
 286 as G_B decreases (Figure S9a). From this calculation, we
 287 predict the critical bulk modulus $G_{B,c}$ for the condens-
 288 ate to be indissoluble given χ_f , based on the condition
 289 $\phi_{\text{in}} > \phi_c$ where ϕ_c is the critical density above which the
 290 condensates become elastic. Because both χ and G_B ap-
 291 pear linearly in Eq. (1), the phase boundary separating
 292 the dissoluble and indissoluble phase must be linear in
 293 the χ_f - G_B parameter space.

294 To test our predictions, we simulate a single conden-
 295 sate with two control parameters χ_f and G_B and monitor
 296 its dissolution dynamics after χ is reduced from $\chi_i = 3$
 297 to χ_f . We label the condensate as dissoluble or indissol-
 298 uble depending on if the system becomes uniform or not
 299 after a long waiting time $t = 10^4$ (see examples in Figure
 300 S9b, c and Movie S2). As expected, when $G_B = 0$, the
 301 condensate is stable only if $\chi_f > 2$. For $\chi_f < 2$, the

302 condensate becomes indissoluble if the bulk modulus is
303 larger than a critical value. The numerically simulated
304 phase diagram nicely matches the predicted phase dia-
305 gram (Figure 4a). Our results are not sensitive to the
306 values of ϕ_c as we get similar results using different ϕ_c
307 (Figure S10).

308 We find that to match the predicted phase boundary to
309 the simulations accurately, we need to solve the equation
310 $\phi_{in} = \theta\phi_c$ with $\theta = 1.1$ to find the critical bulk modulus
311 (Figure 4a and Figure S9a). To understand why θ is close
312 but slightly larger than 1, we remark that for the elastic
313 condensate to be stable, ϕ_{in} must be larger than ϕ_c to en-
314 sure force balance across the condensate boundary. The
315 biomolecules are subject to two types of force: the force
316 from the gradient of the osmotic tensor ($-\nabla \cdot \Pi$) and the
317 force from the gradient of the elastic stress ($\nabla \cdot \sigma$). We
318 can further decompose the former force into two parts,
319 one is from the free energy $f_0(\phi)$, which we call the os-
320 motic force, and the other is from the $\frac{C}{2}(\nabla\phi)^2$ term in the
321 free energy, which we call the surface tension force. For
322 a liquid condensate, the osmotic force always balances
323 the surface tension force across the condensate bound-
324 ary. Because the osmotic pressure is a non-monotonic
325 function of density, in this case, the crossover regime can
326 be separated into three parts in which both the surface
327 tension force and the osmotic force change their signs (see
328 the schematic in Figure 4b and numerical simulations
329 in Figure S11a). For an elastic condensate, the osmotic
330 force always points outwards from the condensates since
331 the osmotic pressure is now a monotonically increasing
332 function of ϕ , while the surface tension force still changes
333 its sign. Therefore, in this case, an inward elastic force
334 must exist to balance the sum of surface tension force
335 and osmotic force in the crossover regime (Figure 4b and
336 Figure S11b). In conclusion, ϕ_{in} should be larger than
337 ϕ_c to ensure a finite elastic force in the crossover regime;
338 therefore, $\theta \gtrsim 1$.

339 *Discussion.*—Our work provides the first mechanis-
340 tic understanding of the indissolubility of solid-like
341 biomolecular condensates. We show that the bulk stress
342 can balance the osmotic pressure difference inside and
343 outside the condensates, therefore preventing the dis-
344 solution. Numerical simulations of the two-fluid model
345 nicely confirm our theoretical predictions of the mechan-
346 ical equilibrium condition. Moreover, we theoretically
347 and numerically obtain a phase diagram of indissolubil-
348 ity for elastic condensates and obtain a minimum bulk
349 modulus for condensates to be indissoluble. There re-
350 main some open questions, including the effects of shear
351 modulus. While we numerically find that our simulations
352 are mostly independent of the values of shear moduli, a
353 small but finite shear modulus is nevertheless necessary
354 to maintain the spherical shapes of condensates.

355 Finally, our results demonstrate that the bulk mod-
356 ulus is the primary material property determining con-
357 densates' indissolubility. The phase diagram of indissol-

358 ubility (Figure 4) can be experimentally tested, e.g., by
359 measuring the critical pH or salt concentrations (corre-
360 sponding to the χ parameter in the Flory-Huggins free
361 energy) to dissolve the condensates and in the meantime,
362 measuring the bulk modulus separately. We note that the
363 shear modulus of gel is usually the order of $nk_B T$ where n
364 is the cross-linker density, and meanwhile, the bulk mod-
365 ulus is typically hundreds of shear modulus [31]. Note
366 that the unit of the bulk modulus is $k_B T/V_0$ in the two-
367 fluid model, which suggests that the critical bulk moduli
368 that are of order 1 in the phase diagram (Figure 4d) may
369 be biologically relevant. Our results may have implica-
370 tions for developing condensate-targeting drugs to change
371 condensate properties inside cells, e.g., lowering the bulk
372 moduli to dissolve irreversible condensates.

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