

1 Chiral and nematic phases of flexible active filaments

2 Zuzana Dunajova^{1*}, Batirtze Prats Mateu^{1*}, Philipp Radler^{1\$}, Keesiang Lim^{2\$}, Philipp Velicky¹,
3 Johann Georg Danzl¹, Richard W. Wong², Jens Elgeti³, Edouard Hannezo^{1#}, Martin Loose^{1#}

4

5 ¹Institute for Science and Technology Austria (ISTA), Am Campus 1, 3400 Klosterneuburg,
6 Austria.

7 ²WPI-Nano Life Science Institute, Kanazawa University, Kakuma-machi, Kanazawa, Ishikawa 920-
8 1192, Japan

9 ³ Theoretical Physics of Living Matter, Institute of Biological Information Processing and Institute
10 for Advanced Simulation, Forschungszentrum Jülich, 52425 Jülich, Germany.

11

12 *Contributed equally, \$contributed equally

13 #Correspondence: edouard.hannezo@ist.ac.at, martin.loose@ist.ac.at

15 Abstract

16 The emergence of large-scale order in self-organized systems relies on local interactions
17 between individual components. During bacterial cell division, the tubulin-homolog FtsZ
18 polymerizes into treadmilling filaments that further assemble into a cytoskeletal ring.
19 Although minimal *in vitro* assays have shown the striking self-organization capacity of
20 FtsZ filaments, such as dynamic chiral assemblies, how these large-scale structures
21 emerge and relate to individual filament properties remains poorly understood. To
22 understand this quantitatively, we combined minimal chiral active matter simulations
23 with biochemical reconstitution experiments. Using STED and TIRF microscopy as well
24 as high-speed AFM, we imaged the behavior of FtsZ filaments on different spatial scales.
25 Simulations and experiments revealed that filament density and flexibility define the
26 local and global order of the system: At intermediate densities, flexible filaments organize
27 into chiral rings and polar bands, while an effectively nematic organization dominates for
28 high filament densities and for mutant filaments with increased rigidity. Our predicted
29 phase diagram captured these features quantitatively, demonstrating how filament
30 flexibility, density and chirality cooperate with activity to give rise to a large repertoire
31 of collective behaviors. These properties are likely important for the dynamic
32 organization of soft chiral matter, including that of treadmilling FtsZ filaments during
33 bacterial cell division.

34

35 **Introduction**

36 In active systems, the emergence of large-scale order relies on a combination of local
37 interactions between components and microscopic energy consumption. One typical
38 property of such self-organizing systems is spontaneous motility of their constituents.
39 For example, cytoskeletal filaments can be transported due to the activity of motor
40 proteins or move due to treadmilling polymerization dynamics. Dynamic interactions
41 between active constituents can lead to complex collective behavior and phases not found
42 at equilibrium, which have been under intense experimental and theoretical
43 investigation in the past decade¹⁻⁷. Reconstituted mixtures of actin or microtubule
44 filaments with motor proteins self-organize into moving swarms, vortices, and travelling
45 waves^{1,5,8}. In living systems, such emergent behaviors can underlie a wealth of key
46 biological phenomena such as single and collective cell motility^{9,10}, cell division¹¹ and
47 organism morphogenesis^{12,13}.

48 Active matter systems can be classified according to the symmetry of its
49 constituents (e.g. polar or nematic)¹⁴⁻¹⁶. In particular, chiral active matter has recently
50 attracted attention, where the constituents are either asymmetric in shape or perform a
51 circular self-propelled motion. This includes curved cytoskeletal filaments, asymmetric
52 synthetic swimmers or cell types displaying chiral motions on 2D substrates¹⁷⁻¹⁹.
53 However, how to relate the microscopic properties of active constituents to the large-
54 scale outputs relevant for biology remains an outstanding challenge in the field¹⁶. For
55 instance, biological settings often involve highly dense systems with attraction, as well as
56 deformable constituents which can change their shape as a function of external forces or
57 crowding⁶. How flexible constituents self-organize and how their local deformability
58 contributes to large-scale collective features remain poorly understood, both
59 theoretically and experimentally.

60 One example of treadmilling filaments are polymers formed by the protein FtsZ, a
61 tubulin homolog that organizes cell division in almost all bacterial, and some archaeal,
62 species^{20,21}. FtsZ forms single stranded filaments that grow from one end and shrink from
63 the opposite end²², driven by the hydrolysis of guanosine triphosphate (GTP). FtsZ
64 filaments can also interact laterally, which facilitates the condensation of filaments from
65 a diffuse organization into a tight ring-like structure called the Z-ring²³⁻²⁶. The density of

66 this ring further increases during constriction until the cell is split into two^{23,27}. FtsZ also
67 forms treadmilling filaments *in vitro*, which organize into cytoskeletal patterns of moving
68 bands and chirally rotating rings²⁸. While lateral interactions between treadmilling
69 filaments play an important role for Z-ring assembly in living cells and the emergence of
70 cytoskeletal structures *in vitro*, the physical properties and interaction rules of
71 treadmilling filaments on a membrane surface are currently not well known. Here, by
72 studying the self-organization of FtsZ filaments at different spatial and temporal scales,
73 we elucidate the quantitative principles that govern the emergence of different collective
74 cytoskeletal organizations from the local interactions of active constituents.

75

76 **Modes of FtsZ filaments self-organization at different densities**

77 To understand the properties of FtsZ filament bundling, we first explored experimentally
78 the phase space of possible organizations of FtsZ filaments as a function of their density.
79 We used a previously established *in vitro* reconstitution assay, where treadmilling FtsZ
80 filaments are recruited to the surface of a supported bilayer by the membrane anchor
81 FtsA²⁸. Using total internal reflection fluorescence (TIRF) microscopy, we could record
82 the emergent behavior of the membrane-bound filaments. We found that we could
83 control the density of membrane-bound FtsZ filaments by changing the FtsZ
84 concentration in the buffer solution. When we increased the FtsZ bulk concentration from
85 around 0.6 to 5 μ M while keeping the FtsA concentration constant, the fluorescence
86 intensity of FtsZ on the membrane increased linearly until it saturated at concentrations
87 higher than 3 μ M (**Supplementary Fig. 1a**). At the same time, we found that the large-
88 scale organization of the filaments changed with their density (**Fig. 1a, Supplementary**
89 **Movie 1**).

90 At FtsZ concentrations lower than 0.6 μ M, individual filaments traveled across the
91 membrane surface²⁸. These filaments were too short to measure their intrinsic curvature
92 by fluorescence microscopy, but maximum intensity projections of time lapse movies
93 revealed that their trajectories followed a curved path corresponding to a circle with a
94 diameter of $1.16 \mu\text{m} \pm 0.35 \mu\text{m}$ and heavily biased in the clockwise direction (**Fig. 1b**,
95 **Supplementary Fig. 1b**).

96 When we increased the FtsZ bulk concentrations to 1.25 and 1.5 μ M, FtsZ
97 filaments organized into chiral rotating rings that persisted for around 5-6 min and
98 coexisted with comet-like structures and moving bundles of treadmilling filaments as

99 observed previously²⁸. When we calculated the directional autocorrelation of
100 cytoskeletal flows, we found a long decay time of 16 and 68 seconds at 1.25 and 1.5 μM
101 FtsZ respectively, consistent with persistent, long-range polar motion of filament
102 bundles. In fluorescence recovery after photobleaching experiments we found that the
103 mean lifetime of FtsZ monomers in filament bundles was only around 7 s, suggesting that
104 their directional motion is a collective property of the system (**Fig. 1c, Supplementary**
105 **Fig. 1c**).

106 At bulk concentrations of 3 μM and higher, filaments densely covered the
107 membrane surface without apparent large-scale organization or directional movements.
108 FtsZ filaments still continuously exchanged monomers at these high densities, with a
109 recovery half-time of around 15s (**Supplementary Fig. 1d**).

110 These features were also recapitulated using time-lapse stimulated emission depletion
111 (STED) microscopy. Compared to TIRF microscopy, STED provided higher resolution but
112 did not allow for long-term imaging of the filament pattern. It still showed that rings form
113 preferentially at intermediate densities, and are comprised of many transiently
114 interacting FtsZ filaments (**Fig. 1d, Supplementary Movie 2, Supplementary Fig. 1e**).
115 The apparent weak interactions between filaments suggest that their local polar
116 orientation is an emergent property of the ensemble, rather than the result of highly
117 specific static interactions, such as residue contacts between individual filaments
118 (**Supplementary Fig. 1f, g**).

119 Together, these observations and quantifications show that there are density-
120 dependent transitions of the large-scale organization and motion of treadmilling
121 filaments.

122

123 **Collective filament organization as a function of bending rigidity and attraction.**

124 Given our experimental findings that single FtsZ filaments displace along curved, chiral
125 paths, and previous theoretical work showing that non-adhesive chiral self-propelled
126 filaments could organize into ring-like phases at intermediate densities², we explored
127 whether a minimal coarse-grained model could quantitatively reproduce the observed
128 phenomenology. To explore the limits of strong adhesion, high densities and low bending
129 rigidity, we modeled this system on a mesoscopic level by a collection of overdamped
130 self-propelled semi-flexible filament bundles in two dimensions^{29,30} (see section Material
131 & Methods for more details on the simulation framework). Each semi-flexible filament

132 bundle is simulated as a worm-like chain, with bending rigidity described by the potential
133 V_{bend} , and is self-propelled with tangential force F_p^i to describe an effective treadmilling
134 velocity $v_0 = F_p^i / \gamma$, where γ is the friction coefficient with the membrane. In the following,
135 we use the flexure number²⁹ \mathcal{F} , defined as the ratio of self-propulsion forces to bending
136 rigidity $\mathcal{F} = \frac{F_p L_f^3}{k_{bend}}$, as a measure of filament flexibility (inverse of rigidity). To account for
137 the observed chirality, filaments are considered to have spontaneous signed curvature
138 with rest angle θ_0 . An effective thermal noise force $F_{k_B T}^i$ accounts for the many sources of
139 noise in the system (**Fig. 2a**). Finally, based on previous studies of FtsZ filament-filament
140 interactions^{26,32,33} and our own observations (**Fig. 1d**), we considered mid-range
141 attractive interactions between filaments (V_{pair}).

142 We observed that the filament self-organization was markedly different with
143 varying filament flexibility (**Fig. 2b, Supplementary Fig. 2a, Supplementary Movie 3**).
144 In the case of very rigid, curved filaments, ring-like patterns and vortices dominated the
145 system throughout the experimentally explored density range. However, this was not the
146 case for semiflexible filaments, which displayed two density-driven transitions as seen in
147 experiments. Moreover, in this regime we could observe a co-existence, both temporal
148 and spatial, of chiral rings and nematic-like, straighter traveling bands characterized by
149 low spontaneous curvature. Interestingly, this was in strong qualitative agreement with
150 our experimental observations, where filaments can adopt a wide range of curvatures
151 even at intermediate densities³⁴ and where rings are interspersed with less ordered
152 filament assemblies (**Supplementary Fig. 1e**). In addition, as in our experimental data,
153 we observed dynamical interconversion between ring and band patterns (**Fig. 2c**,
154 **Supplementary Fig. 2b, Supplementary Movie 4**). At the other extreme, filaments with
155 very low bending rigidity deformed too easily and formed only rare and unstable chiral
156 rings at all density tested. Together, this phase diagram argues that filament flexibility
157 could be a key parameter for FtsZ self-organization.

158 More quantitatively, by analyzing the average filament curvature in our
159 simulations, we found that the nature of self-organization in high densities was markedly
160 different with varying filament flexibility (**Fig. 2b, d**). In the case of very rigid filaments,
161 even above the critical density where well-defined rings disappeared in simulations, we
162 found that individual filaments were still highly curved and could self-organize into spiral
163 patterns. These were characterized by the presence of +1 topological defects and

164 displayed chiral rotation dynamics (**Fig. 2e**). However, semiflexible filaments displayed
165 pronounced decrease in their average curvature with increasing density, so that they
166 formed effectively an active nematic phase, characterized by spontaneous appearance
167 and movement of +1/2 and -1/2 topological defects (**Fig. 2f, Supplementary Movie 5**).

168 Before turning to a more quantitative assessment of the match between data and
169 modelling, we also investigated the effect of attraction and noise (Peclet number) for the
170 phase diagram. For low attraction, we systematically observed a transition from
171 disordered patterns to rings above a critical density, as previously reported².
172 Interestingly however, for strong FtsZ attraction, this transition was largely lost, with
173 rings able to form even at the lowest experimental densities (**Supplementary Fig. 2c,**
174 **3a**), similarly to the case of very rigid filaments. From a physical perspective, this is due
175 to ring formation being energetically favored, instead of being an active kinetic state in
176 the case of purely repulsive self-propelled filaments. The Peclet number also strongly
177 affected the first transition, as well as the overall density of rings in the system
178 (**Supplementary Fig. 2d, 3b**). Above a second threshold of density, we could observe in
179 all cases a loss of ring patterns.

180

181 Quantitative comparison between model and experiments.

182 To more systematically and quantitatively compare simulations and experiments,
183 we first sought to constrain model parameters. From our observations of single-filament
184 trajectories (**Fig. 1b**) and previously published values^{28,35} we considered a treadmilling
185 speed of $v_0 = 0.04 \mu\text{m/s}$, and estimated the packing fraction of filaments ϕ based on our
186 calibration experiments (**Supplementary Fig. 1a**). The aspect ratio of filaments in our
187 mesoscopic simulations corresponded to bundles with thickness of 5 FtsZ filaments of
188 length 400 nm (see section Material & Methods for more details on parameter
189 estimation). Estimation of filament chirality based on curvature of single-filament
190 trajectories was noisy and limited by the resolution of fluorescence microscopy, but still
191 allowed us to guide parameter space by estimating θ_0 to yield the single-filament rotation
192 diameter of 500-1000 nm, as well as a lower bound for the Peclet number of ~ 100
193 (**Supplementary Fig. 3k**). Therefore, after this parameter estimation, we were left with
194 only three fitting parameters, which we systematically explored in our simulations: the
195 adhesion strength ε , the filament flexibility measured by flexure number \mathcal{F} and noise
196 strength measured by Peclet number Pe .

197 Interestingly, intermediate values of filament flexibility, as well as low to
198 intermediate attraction, provided a good match for a number of qualitative and
199 quantitative features of our dataset. Firstly, we could quantitatively reproduce not only
200 the two thresholds of appearance and disappearance of rings as a function of density, but
201 also the absolute probability of ring formation in the intermediary density region (**Fig.**
202 **2g**). Strikingly, when measuring ring life-time (normalized by period of filament rotation)
203 in both experiments and simulations, we found excellent agreement with our predicted
204 parameter regime (**Fig. 2h, Supplementary Fig. 3c, d**). Finally, our simulations
205 predicted that the diameter and thickness of rings should monotonously increase with
206 density, a feature in agreement with the data (**Fig. 2i, Supplementary Fig. 3e, f**). Overall,
207 this demonstrates that the transitions seen in the data as a function of density can be
208 quantitatively explained by a simple theoretical framework of flexible chiral active
209 filaments. Interestingly, this analysis suggests FtsZ filaments being much more flexible
210 than previously anticipated^{2,30}. This prompted us to test more directly how FtsZ filament
211 conformation changes at the microscopic level as a function of increasing density.

212

213 **High-speed-AFM imaging of dynamic FtsZ filaments**

214 As fluorescence microscopy was not sufficient to visualize individual filaments inside of
215 dense filament bundles, we applied high speed atomic force microscopy (HS-AFM) on
216 treadmilling, membrane-bound filaments, which allowed temporal imaging of biological
217 samples at high lateral resolution down to 2-3 nm and at up to 20 frames per second^{36,37}.
218 We were able to image individual filaments that dynamically moved across the
219 membrane surface and increased in density during the course of the experiments. These
220 filaments rapidly moved in and out of the field of view, which made it difficult to track
221 individual filaments over time. However, these time-lapse movies confirmed that
222 neighboring filaments did not form stable lateral contacts; instead they displayed fast
223 lateral fluctuations on the membrane surface (**Fig. 3 a, Supplementary Movie 6**). This
224 suggests that the attraction energy was comparable or smaller to fluctuations, consistent
225 with our numerical simulations. We could also reconstruct filament shape as a function
226 of density. Interestingly, we found a gradual decrease in filament curvature as a function
227 of overall surface density, which fitted well our theoretical prediction for semiflexible
228 filaments (**Fig. 3b**). At the highest densities, filaments had no discernable intrinsic
229 curvature and showed hallmarks of active nematics, such as spontaneous formation of

230 +1/2 and -1/2 topological defects (**Fig. 3c**), another feature generically observed in our
231 simulations (**Fig. 2f**). This confirms that their organization at high densities is governed
232 by nematic rather than polar filament-filament interactions.

233

234 **Testing the model by changing filament conformation**

235 Since our theory suggests that semiflexible filaments and intrinsic chirality are crucial to
236 explain the density-dependent transitions that we observed experimentally, we sought
237 to challenge it further by using constituents with altered properties, i.e. straight filaments
238 with enhanced bending rigidity. We reviewed previously described FtsZ mutants and
239 identified FtsZ L169R as an interesting candidate protein. This mutant was previously
240 described to have increased lateral interactions, as it shows enhanced bundling *in vitro*
241 and *in vivo*^{38,39}. We used AlphaFold^{40,41} predictions of a FtsZ-FtsZ dimer using the crystal
242 structure of FtsZ filaments from *Staphylococcus aureus* as a template⁴² and found that the
243 positively charged Arginine residue at position 169 is in fact located at the longitudinal
244 interface, facing towards a negatively charged Glutamic acid residue at position 276 of
245 the neighboring monomer (**Fig. 4a**). Accordingly, the L169R mutation could result in an
246 additional salt-bridge between two monomers in the filament, which could straighten
247 and stiffen the resulting filaments. Indeed, our HS-AFM experiments with this FtsZ
248 mutant confirmed our hypothesis and also revealed a number of key differences from the
249 wildtype protein. First, while wildtype filaments moved in and out of the field of view,
250 filaments of FtsZ L169R mutant protein appeared to be more static. This loss of mobility
251 suggests that their kinetic polarity is perturbed (**Supplementary Movie 7**). We indeed
252 found that many filaments displayed bidirectional growth and sudden shrinkage events
253 rather than treadmilling behavior. Second, our HS-AFM experiments showed that single
254 filaments appeared to have lower intrinsic curvature and less fluctuations than filaments
255 of the wildtype protein even at very low densities (**Fig. 4b**). When we analyzed the shape
256 of individual filaments at different densities, we found that the curvature of wildtype
257 filaments decreased two-fold with increasing density, while mutant filaments were
258 almost straight at all densities studied (**Fig. 4c**). At the same time the persistence and
259 contour lengths were about two times higher for FtsZ L169R filaments and further
260 increased with their densities (**Fig. 4d, e**). We also found that mutant filaments showed
261 transient filament interactions and height profile similar to the wildtype filaments, but
262 with a two times smaller mean filament distance (**Supplementary Fig. 4a-c**)³⁸.

263 Given that we previously had assumed a local polar order between filaments in
264 our mesoscopic bundle simulations, we now wanted to test the effect of the filament
265 properties of FtsZ L169R. We thus quantified local filament-filament alignment in toy-
266 simulations with different filament parameters. Interestingly, we found that all changes
267 observed in the FtsZ L169R mutant (longer, less chiral, or more persistent filaments with
268 slower treadmilling) went in the same direction of decreasing polar filament alignment
269 (**Supplementary Fig. 4e-h**). Combining these three experimentally observed static
270 properties thus predicted a strong decrease of local polarity sorting (**Fig. 5a, b**,
271 **Supplementary Fig. 4d, Supplementary Movie 8**). We then examined the experimental
272 large-scale patterns of mutant FtsZ filaments via TIRF microscopy. FtsZ L169R
273 polymerized into a cytoskeletal network of dynamic filament bundles at concentrations
274 between 0.6 μ M and 3 μ M (**Fig. 5c, Supplementary Movie 9**). We found that while the
275 mutant reached saturation on membranes with FtsA similarly to the wildtype, its
276 turnover was 2-3x slower and that it hydrolyzed GTP 3x slower compared to the wildtype
277 (**Supplementary Fig. 4i-k**). Furthermore, we did not observe any directional flows or
278 chirally rotating rings on the membrane surface. In addition, differential imaging did not
279 generate directional moving speckles, confirming that local polar arrangement of
280 filaments is strongly impaired (**Supplementary Fig. 4l**)³⁵. These features were well-
281 recapitulated by the same large-scale bundle simulations as before, but with the HS-AFM-
282 derived properties for the mutant filaments: longer, less chiral and more persistent
283 filaments with slower kinetics F_p (**Fig. 5d, Supplementary Movie 10**). Altogether, this
284 shows that our simplified model can provide a powerful framework to link the local
285 microscopic structure of active filaments to the large-scale collective phases that they
286 form as a function of density.

287

288 **Discussion**

289 In this study, we have identified density-driven transitions in the self-organization of FtsZ
290 filaments between three phases: a low density disordered phase where individual
291 filaments treadmill in a chiral manner with little interactions, an intermediate density
292 phase where coherent chiral vortices form - as previously shown theoretically and
293 experimentally^{2,28,30}, and a high density phase where individual filaments straighten and
294 where the system displays a uniform, nematic-like organization. Using a minimal model

295 of active, chiral and flexible filaments we computationally analyzed how the physical
296 properties of self-propelled filaments determine their large-scale self-organization.
297 Although the presence of rings in intermediate densities can arise in the context of purely
298 repulsive rigid filaments, a key finding from our computational work is that varying
299 filament bending rigidity and attractive interactions changes qualitatively and
300 quantitatively the phase diagram of possible morphologies. We find that the
301 experimental system is consistent with intermediate values of both parameters, as
302 filament flexibility in particular significantly modifies ring statistics and lifetimes. With
303 semiflexible filaments, we observe indeed a competition between chiral shape, active
304 treadmilling and inter-filament interactions, which at intermediate densities results in
305 the co-existence and interconversion of chiral vortices and nematic-like traveling bands.
306 This recapitulates well our experimental observations made on different spatial scales:
307 while TIRF and STED microscopy allowed us to visualize the large-scale spatiotemporal
308 dynamics of FtsZ filaments - and thus the transitions between different phases - HS-AFM
309 revealed the behavior of individual filaments at different filament densities on a
310 membrane surface. This data confirmed that the transition from chiral vortices to a
311 nematic-like organization with increasing density goes along with straightening of the
312 filaments, arguing for a relatively low FtsZ bending rigidity.

313 Furthermore, we demonstrated that a local perturbation, in this case a specific
314 residue in the primary sequence of a protein, can dramatically change both individual
315 filament properties and collective self-organization. Specifically, we found that the
316 presence of an additional salt bridge between two monomers in filaments of FtsZ L169R
317 increases their rigidity and lowers their intrinsic curvature. As a consequence, this
318 mutant does not show a density dependent transition between different phases and
319 instead always displays achiral, non-directional large-scale cytoskeletal networks. FtsZ
320 L169R was originally described as bundling mutant that shows enhanced lateral
321 interactions^{38,39}. Our data suggests that enhanced lateral interactions are a consequence
322 of mutant filaments being longer, more rigid and straighter and that the closer contact
323 between them is only a secondary effect. The lack of a preference for a polar orientation
324 of straight filaments are also in agreement with the observation of aberrant rings and
325 spiral structures of FtsZ L169R found *in vivo*³⁸. Although the intrinsic curvature of
326 wildtype FtsZ filaments is much lower than that of MreB⁴³ and FtsA filaments⁴⁴, it could

327 contribute to the correct alignment of the Z-ring perpendicular to the long cell axis in
328 particular at early stages of cell division⁴⁵.

329 Several experimental examples of chiral active matter have emerged in the past
330 few years, across many different length scales. At the organismal scale, malaria parasites
331 have been shown to have flexible rod-like shape and migrate actively in a chiral manner
332 with different parasites having opposite chirality. This leads to sorting based on chirality,
333 which is favored by mechanical flexibility⁴⁶. Starfish embryos, although spherical in
334 shapes, have also recently been shown to swim in a chiral manner, and collectively form
335 crystal-like structures with odd elastic behavior⁴⁷. At the cellular scale, chirality can bias
336 the active nematic instabilities observed in confluent monolayers ⁴⁸ and biofilms⁴⁹. This
337 has further been proposed to arise from cytoskeletal organization, due to the polar
338 helicoidal structure of active filaments⁵⁰, highlighting the need to a better understanding
339 of the collective dynamics bridging different scales. In particular, it would be interesting
340 in the future to investigate more complex models of filament treadmilling, for instance
341 incorporating finite lifetimes or stress-dependent polymerization. Furthermore, our
342 experimental and theoretical data suggests that weak and transient lateral interactions
343 lacking any apparent biochemical specificity are sufficient for the alignment of
344 treadmilling filaments on a membrane surface. Indeed, *in vivo* filaments in the Z-ring can
345 move either in the same or opposite direction^{51,52} and recent experiments in *Bacillus*
346 *subtilis* suggest that filament treadmilling facilitates their encounters promoting
347 condensation of filaments into the Z-ring²³. Relying on weak, non-specific interactions
348 instead of specific residue contacts is therefore likely advantageous for the cell as it
349 allows for the condensation of the Z-ring, while still permitting its dynamic
350 reorganization. The relatively low bending rigidity that we observe for FtsZ could also be
351 key to allow the Z-ring to adapt to the decreasing diameter of the constricting cell septum.
352 Overall, our study highlights how minimal models of active matter based on symmetries
353 can provide quantitative insights into fundamental biological functions.

354

355

356 **Figure captions**

357 **Figure 1: The orientation of FtsZ filament bundles changes with increasing density**
358 **on SLBs**

359 **a**, Representative TIRF micrographs of Alexa488-FtsZ at increasing FtsZ and constant
360 FtsA concentrations. Below 0.625 μM FtsZ, filaments do not form higher order structures.
361 At 1.25 and 1.5 μM , FtsZ forms rotating rings and directionally moving filament bundles.
362 This organization is lost at 3 μM and 5 μM FtsZ, at which filaments densely cover the
363 membrane surface. Scale bars are 2 μm . **b**, Representative images of a trajectory of a single
364 FtsZ filament at 0.5 μM (left) and distributions of measured curvatures. The filament
365 moves along a curved path, corresponding to an apparent diameter of $1.15 \pm 0.346 \mu\text{m}$
366 (std, $n = 105$). Scale bars are 5 and 1 μm . **c**, Decay constants from fitting a bi-exponential
367 function to directional autocorrelation curves from treadmilling trajectories. For
368 intermediate FtsZ concentrations, the curves are best fitted assuming a fast and slow
369 decay constant, consistent with persistent directional flows.
370 **d**, Representative STED micrographs of 1.5 μM Atto633-FtsZ tethered to SLBs by 0.2 μM
371 FtsA at low (5 minutes after starting the experiment, top) and high (50 minutes, bottom)
372 densities. First, rotating rings and moving bundles first coexisted on the membrane
373 surface. With increasing filament density ring-like structures disappeared. Scale bars: 20
374 or 2 μm .

375

376 **Figure 2: Numerical simulation of FtsZ WT self-organization**

377 **a**, Scheme of the simulation model. **b**, Phase diagram of the large-scale patterns
378 ($L = 212 d$) with varying filament flexibility (measured by flexure number \mathcal{F} , vertical
379 axis) and density (horizontal axis). Filaments are color-coded according to the
380 orientation of the bond vectors between beads. We observe ring-like self-organization of
381 rigid filaments ($\mathcal{F} = 5$), spatial coexistence of chiral rings and polar bands in regime of
382 semiflexible filaments ($\mathcal{F} = 40$) and disordered patterns with flexible filaments ($\mathcal{F} = 200$).
383 **c**, Temporal coexistence of chiral rings and polar bands in a small simulated system
384 ($L = 42 d$) of intermediate density ($\Phi = 0.5$) and filament flexibility ($\mathcal{F} = 40$). Filaments
385 are color-coded according to the orientation of the bond vectors between beads. **d**,
386 Average filament curvature with varying density, showing density-driven straightening.
387 Three values of filament flexibility are shown as in the phase diagram in b. **e**, Spiral defect

388 for rigid filaments ($\mathcal{F} = 5$) and high density ($\phi = 0.9$). Only bonds of the filaments (without
389 the full diameter of beads) are presented for clarity. Filaments are color-coded according
390 to the orientation of the bond vectors between beads. **f**, Nematic defects in high density
391 ($\phi = 0.9$) of semiflexible filaments ($\mathcal{F} = 40$). Only bonds of the filaments (without the full
392 diameter of beads) are presented for clarity. Filaments are color-coded according to the
393 orientation of the bond vectors between beads. **g-i**, Quantitative comparison of ring
394 density, lifetime and diameter between simulations and experiments (see Methods for
395 details on the quantification and comparisons). Red solid line corresponds to the best fit
396 of filament flexibility ($\mathcal{F} = 40$).
397

398 **Figure 3: High-speed AFM imaging shows nematic organization at high densities of**
399 **FtsZ filaments**

400 **a**, Representative HS-AFM time-lapse experiment, showing an increase in density of FtsZ
401 WT filaments with passing time. The filaments move less and become more stable with
402 increasing time. Scale bars are 500 nm. **b**, Curvature of FtsZ WT filaments at increasing
403 densities from experiments (orange) and simulations (gray). With increasing densities,
404 the curvature of individual filaments decreases. **c**, At high densities FtsZ WT filaments
405 show nematic order with topological defects ($[FtsZ] = 1.5 \mu M$, left and $4.0 \mu M$, right). Scale
406 bars: 500 nm.
407

408 **Figure 4: A point mutation in FtsZ L169R changes the properties of FtsZ filaments**

409 **a**, Ribbon model of the *Staphylococcus aureus* FtsZ filament (PDB: 3VOB), left, and the
410 longitudinal interface predicted for *E. coli* FtsZ WT (right, top) and L169R (right, bottom).
411 The Leucine to Arginine mutation likely enables a novel salt bridge, which stabilizing the
412 FtsZ filament. **b**, Representative HS-AFM time-lapse experiment, showing an increase in
413 density of FtsZ L169R filaments. Already at low densities the mutant filaments are less
414 dynamic and more rigid. Scale bars are 500nm. **c**, Curvature of FtsZ L169R filaments as a
415 function of density, showing that it is lower than of FtsZ WT filaments at all tested
416 densities. **d**, Persistence length of FtsZ L169R filaments as a function of density, showing
417 a two times higher value compared to FtsZ WT. **e**, Contour length of FtsZ L169R as a
418 function of density, increasing faster than for FtsZ L169R. Data shown in **d, e** is taken
419 from experiments at low densities (<40%), as FtsZ L169R filaments at higher densities
420 can be longer than the field of view and limiting quantification.

421

422 **Figure 5: Microscopic sorting and large-scale organization of FtsZ L169R filaments**

423 **a**, Toy simulations studying the polarity sorting kinetics of filaments with varying
424 intrinsic curvature, length and bending rigidity - snapshot of the initial (left) and final
425 configuration (right) of FtsZ L169R filaments in high density $\phi = 0.88$. Only bonds of the
426 filaments (without the full diameter of beads) are presented for clarity. Filaments are
427 color-coded according to the orientation of the bond vectors between beads. **b**, Fraction
428 of parallel (polar) alignment of filaments as the function of simulation time. The FtsZ
429 L169R was simulated with two times higher persistence and length than WT, no intrinsic
430 curvature and 8x lower F_p (parameters $Pe = 300$, $\mathcal{F} = 20$, $k_{bend} = 107 \text{ } k_B T/\text{rad}^2$, $L_{fL169R} =$
431 $16d$, $\theta_0 = \pi$, $\phi = 0.22 - 0.88$). While FtsZ WT prefers to align in a parallel orientation, FtsZ
432 L169R stays aligned in nematic fashion until the end of simulations. **c**, Representative
433 TIRF micrographs of Alexa488-FtsZ L169R at increasing FtsZ L169R and constant FtsA
434 concentration. We observe very static thread-like self-organization of filaments with no
435 concentration-dependent transitions. **d**, Snapshots of large-scale FtsZ L169R simulations
436 with increasing density. The FtsZ L169R filaments were modeled with altered properties
437 according to the HS-AFM analysis: filaments were 2x longer and more rigid, had no
438 intrinsic curvature and 8x lower F_p resulting in 3x lower Peclet number and 2x lower
439 flexure number than FtsZ WT. (parameters $Pe = 300$, $\mathcal{F} = 20$, $k_{bend} = 107 \text{ } k_B T/\text{rad}^2$ $\theta_0 = \pi$,
440 $L_{fL169R} = 16 \text{ } d$). FtsZ L169R exhibits static thread-like pattern in all densities, as in the
441 experiment. Filaments are color-coded according to the orientation of the bond vectors
442 between beads.

443

444 **Supplementary Figure captions**

445 **Supplementary Figure 1: Characterization of FtsZ WT organization at increasing**
446 **densities with TIRF and STED**

447 **a**, Quantification of the intensity of Alexa488-FtsZ WT during a TIRF titration experiment.
448 The density of FtsZ filaments on the supported membrane is saturated at $\sim 3 \mu\text{M}$. **b**,
449 Snapshots of time lapse movie of a single FtsZ filament treadmilling on a membrane
450 surface. The maximum intensity projection reveals the curved trajectory of the filament.
451 **c**, Representative fits of mono- and bi-exponential functions to the directional
452 autocorrelation of treadmilling trajectories. **d**, Quantification of the membrane residence

453 time of FtsZ WT by FRAP experiments with increasing bulk concentrations. **e**,
454 Representative micrographs of STED time-lapse experiments of Atto633-FtsZ WT. The
455 insets show either rings which dynamically re-arrange into bundles (red) or bundles
456 which re-arrange into rings (orange). The scale bars are 1 μ m. **f**, Model drawings showing
457 the different types of collisions FtsZ filaments can undergo. In the aftermath of collision
458 events FtsZ filaments align either parallel or antiparallel. **g**, Probability of parallel and
459 antiparallel alignment as a function of impact angle.

460

461 **Supplementary Figure 2: Numerical model of FtsZ WT - self-organization with**
462 **varying rigidity, attraction and noise**

463 **a**, Extended visual phase diagram of the large-scale patterns ($L = 212 d$) with varying
464 filament flexibility (measured by flexure number \mathcal{F} , vertical axis) and density (horizontal
465 axis). Filaments are colored according to the orientation of the bond vectors between
466 beads. **b**, Evolution of average curvature of the system showing the temporal coexistence
467 of chiral rings and polar bands in a small system size ($L = 42 d$) of intermediate density
468 ($\Phi = 0.5$). The vertical lines represent the timepoints of the snapshots in Figure 2c. **c**,
469 Dependence of density of rings in the large-scale system ($L = 212 d$) on the packing
470 fraction, for simulations with varying filament attraction. **d**, Dependence of density of
471 rings in the large-scale system on the packing fraction, for simulations with varying Peclet
472 number.

473

474 **Supplementary Figure 3: Numerical model of FtsZ WT - self-organization with**
475 **varying attraction and noise, quantitative analysis**

476 **a**, Visual phase diagram of the large-scale patterns ($L = 212 d$) with varying attraction
477 (vertical axis) and density (horizontal axis). The rings are less stable without lateral
478 attraction, while with the attraction being too strong, rings are favored in small density
479 ($\Phi < 0.25$), while in the intermediate density the cluster formation is very rapid resulting
480 in lower occurrence of rings. Filaments are colored according to the orientation of the
481 bond vectors between beads. **b**, Visual phase diagram of the large-scale patterns
482 ($L = 212 d$) with varying noise (measured by Peclet number Pe , vertical axis) and density
483 (horizontal axis). The rings are more abundant and stable with higher Peclet numbers
484 with strongest differences appearing in lower density ($0.05 < \Phi < 0.25$). Filaments are
485 colored according to the orientation of the bond vectors between beads. **c**, Quantitative

486 comparison of ring lifetime between large-scale simulations with varying filament
487 attraction and experiments. The remaining simulation parameters were kept constant. **D**,
488 Quantitative comparison of ring lifetime between large-scale simulations with varying
489 Peclet number (noise) and experiments. The remaining simulation parameters were kept
490 constant. **E**, Quantitative comparison of ring diameter between large-scale simulations
491 with varying filament attraction and experiments. The remaining simulation parameters
492 were kept constant. **F** Quantitative comparison of ring diameter between large-scale
493 simulations with varying Peclet number (noise) and experiments. The remaining
494 simulation parameters were kept constant. **H**, Example illustrating the automatic
495 detection of rings. Left box depicts a simulation snapshot with filaments forming rings
496 and bands (with filaments are colored according to the orientation of the bond vectors
497 between beads), while the right box shows the result of clustering of filament rotation
498 centers in space after applying all cutoffs (cluster size, polarity and effective radius). **i**,
499 Result of the ring detection algorithm for the whole simulation after applying the time
500 clustering algorithm to detect the rings in all calculated simulation snapshots. **j**, Analysis
501 of filament persistence in the numerical simulations based on the distribution of local
502 curvatures. Persistence length was extracted from variance of Gaussian function fitted to
503 the data. **k**, Single filament trajectories (10 independent simulations) with varying Peclet
504 number. The comparison to single-filament trajectories analyzed in **Fig. 1b** allowed us to
505 set a lower bound to Peclet number in simulations $Pe = 100$ (see Methods), as below this
506 value the intrinsic curvature of filament trajectories wouldn't be detectable due to
507 stochasticity of movement.

508

509 **Supplementary Figure 4: Characterization of FtsZ L169R and effect of filament**
510 **properties on polarity sorting**

511 **a**, The interfilament distance of FtsZ WT measured by HS-AFM. The distance decreases
512 with increasing bulk concentrations and membrane densities of FtsZ WT. **b**, The
513 interfilament distance of FtsZ L169R measured by HS-AFM. Filament distances are
514 smaller than for the wildtype filaments even at lower FtsZ L169R concentrations. **c**, FtsZ
515 WT (red) and L169R (blue) filaments shows no significant difference in their heights. **d**,
516 Polarity alignment of simulations with FtsZ WT and FtsZ L169R in individual densities.
517 The properties of mutant significantly slow down the polar alignment of filaments in all
518 the studied packing fractions. The analyzed simulations correspond to the **Fig. 5b. e-h**,

519 effect of individual filament properties on the polarity sorting. In each plot, simulations
520 with FtsZ WT are presented to be compared with simulations of FtsZ WT with one
521 changed parameter (intrinsic curvature, length, treadmilling speed, persistence length,
522 respectively). All studied properties decrease the polar sorting with persistence length
523 having the smallest effect. Each plot represents averaged results over four packing
524 fractions in range to $\phi = 0.29$ to $\phi = 0.88$. **i**, Quantification of the membrane residence
525 time of FtsZ L169R by FRAP experiments. Monomer turnover slows down with increasing
526 FtsZ concentrations. **j**, Quantification of the TIRF intensities of Alexa488-FtsZ L169R
527 (blue) at different protein concentrations. The intensity as well as the filament density
528 saturates at $\sim 3\mu\text{M}$, similar to FtsZ WT (red). **k**, Quantification of the GTPase hydrolysis
529 rate of FtsZ WT (red) and FtsZ L169R (blue). The production of free phosphate is
530 measured at a protein concentration of $5\mu\text{M}$. The rates are 2.2 ± 0.03 and 0.78 ± 0.03
531 GTP/FtsZ/min for WT and L169R respectively. **l**, TIRF micrographs and representative
532 kymographs after differential imaging for FtsZ WT (left, red) and L169R (right, blue). The
533 kymographs were obtained along the red or blue dashed lines. Diagonal lines in the
534 kymograph of FtsZ WT correspond to directional polymerization dynamics of filament
535 bundles, which are missing for L169R. Scale bars are $2\mu\text{m}$.

536

537 **Supplementary movies captions**

538 **Supplementary movie 1: TIRF time-lapse movies of FtsZ WT**

539 TIRF time-lapse movies of Alexa488-FtsZ WT at different concentrations ([FtsA] =
540 $0.2\mu\text{M}$). With increasing FtsZ concentration, the pattern changes from rotating rings and
541 directional moving filament bundles to a more nematic pattern. Movies were acquired at
542 0.5 frame per second and correspond to **Fig. 1a**.

543

544 **Supplementary movie 2: STED time-lapse experiments of FtsZ WT**

545 STED time-lapse movies of Atto633-FtsZ WT. The FtsZ concentration was $1.25\mu\text{M}$ and
546 FtsA $0.2\mu\text{M}$. The experiments were recorded at 1 frame every 4-6s, where the acquisition
547 rate depends on the field of view. The movies display the co-existence of rings and
548 bundles of FtsZ filaments. The movies correspond to **Supplementary Fig. 1c**.

549

550

551 **Supplementary movie 3: FtsZ phase diagram**

552 Large scale FtsZ patterns ($L = 212 d$) with varying filament flexibility (measured by
553 flexure number \mathcal{F} , vertical axis) and packing fractions (horizontal axis). Filaments are
554 colored according to the orientation of the bond vectors between beads. We observe ring-
555 like self-organization of rigid filaments ($\mathcal{F} = 5$), spatial coexistence of chiral rings and
556 polar bands in regime of semiflexible filaments ($\mathcal{F} = 40$) and disordered patterns with
557 flexible filaments ($\mathcal{F} = 200$). Movies correspond to **Fig. 2b**.

558

559 **Supplementary movie 4: Temporal coexistence of rings and bands**

560 Temporal coexistence of chiral rings and polar bands in a small system ($L = 42 d$) of
561 intermediate density ($\phi = 0.5$) and filament flexibility ($\mathcal{F} = 40$). With increasing density,
562 the ring state becomes unstable and filaments organize only in bands. Filaments are
563 colored according to the orientation of the bond vectors between beads. Movie
564 corresponds to **Fig. 2c** and **Supplementary Fig. 2b**.

565

566 **Supplementary movie 5: FtsZ topological defects**

567 Topological defects in high filament density ($\phi = 0.9$) and small system size ($L = 42 d$). The
568 rigid filaments ($\mathcal{F} = 5$) form spiral (+1) topological defects, whereas semiflexible
569 filaments ($\mathcal{F} = 40$) form only nematic defects (+1/2 and -1/2) due to the filament
570 straightening. Only bonds of the filaments (without the full diameter of beads) are
571 presented for clarity. Filaments are colored according to the orientation of the bond
572 vectors between beads. Movies correspond to **Fig. 2e,f**.

573

574 **Supplementary movie 6: HS-AFM time-lapse movies of FtsZ WT**

575 HS-AFM movies of FtsZ WT filaments. As the density of FtsZ WT filaments on the
576 supported lipid bilayer increases, they become less dynamic and straighter. At high
577 densities, the filaments show a nematic order with topological defects. Time-lapse movies
578 were acquired with 3 and 2 frames per second. The movies correspond to **Fig. 3a, 3d**.

579

580

581 **Supplementary movie 7: HS-AFM time-lapse experiments of FtsZ L169R**

582 HS-AFM videos of FtsZ L169R filaments. As the density of FtsZ L169R increases, but the
583 filaments remain straight and static. At high densities, the filaments pack extremely tight
584 together. Images were acquired with 3 and 2 frames per second. The movies correspond
585 to **Fig. 4b**.

586

587 **Supplementary movie 8: Polar sorting of filaments with different properties**

588 Polar sorting of filaments with properties of FtsZ WT and FtsZ L169R in two different
589 densities. FtsZ L169R is 2x longer and more rigid, non-chiral and has lower self-
590 propulsion than FtsZ WT, resulting in 3x lower Peclet number and 2x lower flexure
591 number. Only bonds of the filaments (without the full diameter of beads) are presented
592 for clarity. Filaments are colored according to the orientation of the bond vectors
593 between beads.

594

595 **Supplementary movie 9: TIRF time-lapse movies of FtsZ L169R**

596 TIRF time-lapse movies of Alexa488-FtsZ L169R at increasing concentrations ([FtsA] =
597 0.2 μ M). FtsZ L169R does not form rings as seen for FtsZ WT and the pattern appears less
598 dynamic. Movies were acquired at 0.5 frame per second and correspond to **Fig. 5c**.

599

600 **Supplementary movie 10: Large-scale simulations of FtsZ L169R**

601 Large-scale simulations with increasing concentration of FtsZ L169R. The FtsZ L169R
602 filaments are non-chiral, 2x longer and more rigid than FtsZ WT and are self-propelled
603 with 8x lower speed, resulting in $Pe = 200$ and $\mathcal{F} = 20$ and filament persistence length 2x
604 longer than FtsZ WT. FtsZ L169R does not form rings and self-organizes into less dynamic
605 pattern. Filaments are colored according to the orientation of the bond vectors between
606 beads. Movies correspond to **Fig. 5d**.

607

608 **Material & Methods**

609 **Experiments**

610 **Protein biochemistry**

611 Proteins used in this study, wildtype FtsZ, FtsZ L169R and FtsA, were purified as
612 previously described⁵³. FtsZ, L169R was obtained by site-directed mutagenesis (SDM).

613 Leucine 169 was replaced with Arginine, by exchanging two nucleotides (CTG → CGC).
614 FtsZ L169R was purified as the wild-type protein as described before for the wild-type
615 protein.

616

617 **Preparation of coverslips**

618 We used piranha solution (30 % H₂O₂ mixed with concentrated H₂SO₄ at a 1:3 ratio to
619 clean the glass coverslips for 60 min. This was followed by extensive washes with double-
620 distilled H₂O, 10 min sonication in ddH₂O and again washing in ddH₂O. The coverslips
621 were used within one week and were stored in ddH₂O water. Furthermore, before
622 coverslips were used to form supported lipid bilayers, they were dried with compressed
623 air and treated for 10 min with a Zepto plasma cleaner (Diener electronics) at maximum
624 power. As reaction chambers we used 0.5 ml Eppendorf tubes missing the conical end,
625 which were glued on the coverslips with UV glue (Norland Optical Adhesive 63) and
626 exposed to ultraviolet light for 10 min.

627

628 **Preparation of small unilamellar vesicles (SUVs)**

629 DOPC (1,2-dioleoyl-sn-glycero-3-phosphocholine) and DOPG (1,2-dioleoyl-sn-glycero-3-
630 phospho-(1'-rac-glycerol)), which were purchased from Avanti Polar Lipids, at a ratio of
631 67:33 mol% were used. The lipids in chloroform solution were mixed inside a glass vial
632 in the appropriate volumes and dried with filtered N₂ for a thin lipid film. Remaining
633 solvent was removed by putting the lipids in a vacuum desiccator for 2 h. Afterwards
634 swelling buffer (50 mM Tris-HCl [pH 7.4] and 300 mM KCl) was added to the lipid film to
635 obtain a lipid concentration of 5 mM. After incubating the suspension for 30 min at room
636 temperature, the multilamellar vesicles were vortexed rigorously and freeze-thawed
637 (8x) in dry ice or liquid N₂. The liposomes were tip-sonicated using a Q700 Sonicator
638 equipped with a ½ mm tip (amplitude = 1, 1 second on, 4 seconds off) for 25 min on ice
639 to obtain SUVs. Finally, the vesicles were centrifuged for 5 min at 10,000 g and the
640 supernatant was stored at 4 °C in an Argon atmosphere and used within one week.

641

642 **Preparation of supported lipid bilayers (SLB) for TIRF**

643 SLBs were prepared by diluting the SUV suspension to a concentration of 0.5 mM with
644 reaction buffer supplemented with 5 mM CaCl₂. SLBs were incubated for 30 min at 37 °C
645 and non-fused vesicles were washed away by 8x200 µL washes with reaction buffer
20

646 (50 mM Tris-HCl [pH 7.4], 150 mM KCl and 5 mM MgCl₂). The membranes were used
647 within 4 hours.

648

649 **Total internal reflection fluorescence (TIRF) microscopy**

650 Experiments were performed using a Visitron iLAS2 TIRF microscope, equipped with a
651 100xOlympus TIRF NA 1.46 oil objective. The fluorophore Alexa488 was excited with a
652 laser line at 488nm. The emitted fluorescence from the sample was filtered using a Laser
653 Quad Band Filter (405/488/561/640 nm). A Cairn TwinCam camera splitter equipped
654 with a spectral long pass of 565 nm and a band pass filter of 525/50 nm was used. Time
655 series were recorded using Photometrics Evolve 512 EMCCD (512 x 512 pixels, 16 x 16
656 μm^2) operating at a frequency of 5 Hz.

657

658 **Stimulated emission depletion (STED) microscopy**

659 STED microscopy was performed at room temperature on an inverted Expert Line STED
660 microscope (Abberior Instruments) with pulsed excitation and STED lasers. A 640 nm
661 laser was used for excitation and a 775 nm laser for stimulated emission. A oil immersion
662 objective with 1.4 NA (Olympus, UPLSAPO 100XO) was used for image acquisition. The
663 fluorescence signal was collected in a confocal arrangement with a pinhole size of 0.8 airy
664 units. For detection a 685/70 nm bandpass filter (Chroma, #F49-686) and a photon
665 counting avalanche photodiode (Laser Components, Count-T100) were used. The pulse
666 repetition rate was 40 MHz and fluorescence detection was time-gated. Data were
667 acquired with 10 μs pixel dwell time and 30 nm pixel size for time lapse imaging and 20
668 μs with 20 nm pixel size for overview images, 5 – 6.5 μW excitation laser power and 30 –
669 40 mW STED laser power. The power values refer to the power at the sample, measured
670 with a slide powermeter head (Thorlabs, S170C). A spatial light modulator (SLM)
671 imprinted the STED phase pattern for lateral resolution increase. Image acquisition and
672 microscope control were performed with Imspector software version 14.0.3052.

673

674 **High speed atomic force microscopy (HS-AFM)**

675 A laboratory-built tapping mode (2 nm free amplitude, ~2.2 MHz) high-speed atomic
676 force microscope (HS-AFM) equipped with a wide-range scanner (6 μm x 6 μm) was used
677 to visualize the dynamics of the system. BL-AC10DS-A2 (Olympus) cantilevers were used
678 as HS-AFM scanning probes. The cantilever has a spring constant (k) of 0.1N/m and a

679 resonance frequency (f) of 0.6MHz in water or 1.5MHz in air. The dimensions of the
680 cantilever are: 9 μm (length), 2 μm (width), and 0.13 μm (thickness). To achieve high
681 imaging resolution, a sharpened and long carbon tip with low apical radius was made on
682 the existing tip of the cantilever using electron-beam deposition (EBD) as described
683 previously⁵⁴⁻⁵⁶. Scanning speed varied from 0.2s to 5s per frame. The number of pixels
684 acquired were adjusted for every measurement depending on the scan size (min: 2nm,
685 max: about 50nm). The in-house designed program “Kodec” was used to read the data
686 generated by HS-AFM. The ware stores all parameters, calibration and description given
687 during the measurement and allows to load a whole folder or several movies.

688

689 **FtsZ TIRF and STED experiments on SLBs**

690 To study the organization of increasing concentrations of treadmilling FtsZ filaments on
691 supported lipid bilayers, we used 0.2 μM FtsA and increasing concentrations of Alexa488-
692 FtsZ wt or L169R (0.625/1.25/1.5/3/5 μM) in 100 μl of reaction buffer. Additionally, the
693 reaction chamber contained 4 mM ATP/GTP and a scavenging system to minimize
694 photobleaching effects: 30 mM d-glucose, 0.050 mg ml^{-1} Glucose Oxidase, 0.016 mg ml^{-1}
695 Catalase, 1 mM DTT and 1 mM Trolox. Prior addition of all components a corresponding
696 buffer volume was removed from the chamber to obtain a total reaction volume of 100
697 μl . The FtsZ filaments was imaged by TIRF at one frame per two seconds and 50 ms
698 exposure time.

699

700 STED microscopy exposes fluorescent labelled proteins to a much higher laser intensity
701 compared to TIRF and imposes bleaching and photo-toxic effects on the FtsZ filaments.
702 We adjusted our imaging setup accordingly to avoid bleaching and photo-induced
703 bundling of FtsZ filaments. First we replaced Alexa488 with Atto633, a widely used STED
704 dye with higher photon yield and improved resistance to bleaching. Furthermore, we
705 optimized the scavenging solution used for imaging: 60 mM d-glucose, 0.10 mg ml^{-1}
706 Glucose Oxidase, 0.032 mg ml^{-1} Catalase, 20 mM DTT and 2 mM Trolox. Finally, we
707 changed the acquisition protocol to pixel-step based excitation/STED cycles. Here, we
708 introduce short breaks in between the excitation cycles as follows: 5 μs excitation/STED
709 - 10 μs break - 5 μs excitation/STED - 10 μs break⁵⁷. Together, these changes allowed us to
710 observe the same behavior as in TIRF microscopy experiments, but with increased
711 resolution for longer than 10 minutes. For the experiments shown, we used 0.2 μM FtsA

712 and 1.5 μ M Atto633-FtsZ. In the time-lapse montages shown in S1 the FtsZ pattern was
713 imaged at one frame per five seconds.

714

715 **Preparation of SLBs for HS-AFM**

716 SUVs were prepared as described above. An ultra-flat muscovite mica layers (1.5 mm Ø)
717 substrate was mounted on a glass stage using a standard 2-component glue. The glass
718 stage was then attached to the scanner with a thin film of nail polish. A drop of acetone
719 was deposited on the stage/scanner interface to ensure a flat nail polish layer. The
720 mounted stage was dried at RT for about 30 min. A fresh cleaved mica layer was used as
721 substrate to form a supported lipid bilayer (SLB) by depositing ~4 μ L of a mix of 1 mM
722 SUVs suspension in reaction buffer with additional 5mM CaCl₂. To avoid drop breakage,
723 the scanner was flipped upside down and inserted in a custom-made mini-chamber with
724 a thin water film at the bottom (a 500 μ L tube cut on the bottom and glued to a petri dish).
725 The drop was incubated on the stage for at least 30 min. After, the drop was exchanged
726 5-10 times with 5 μ L of fresh reaction buffer. The stage was immediately inserted in the
727 HS-AFM chamber containing about 80 μ L of the same reaction buffer. Prior to the addition
728 of the proteins, HS-AFM imaging and indentation were performed to assess the quality of
729 the SLB. When the force-distance curve showed the typical lipid bilayer indentation
730 profile (~2-4 nm) the SLB was used in the next steps.

731

732 **FtsZ HS-AFM experiments on SLBs**

733 The selected proteins (FtsZ WT or L169R, FtsA) were added to the chamber with
734 ATP/GTP (4mM each) and DTT (1mM) (final concentrations). Optimal protein
735 concentration and ratios were tested as well for a range of ~0.3-4.5 μ M for FtsZ and 0.05-
736 1.2 μ M FtsA.

737

738 **Image processing and analysis**

739 For data analysis of TIRF and STED experiments, the movies were imported to the ImageJ
740 Version 2.9.0/1.53t⁵⁸ software and raw, unprocessed time lapse videos were used. The
741 contrast of micrographs in the manuscript was adjusted to improve visibility of filaments.
742 HS-AFM data was exported to ImageJ, where the post-processing, such as noise reduction
743 and smoothing, was carried out. Noise reduction and smoothing were performed using a

744 bandpass filter of different size depending on the image features. Analysis of HS-AFM
745 movies was performed in MatLab, using the AFM analysis package FiberApp⁵⁹ (Version
746 2017b of MatLab). HS-AFM micrographs in the manuscript are raw data, where the
747 contrast was optimized for best quality.

748

749 **FtsZ intensity analysis**

750 To estimate the saturating coverage of FtsZ on SLBs we titrated bulk FtsZ concentrations
751 from 0.625 – 5 μ M in two independent experiments on the same day, to avoid any changes
752 in the power of the microscope. After reaching equilibrium (20 minutes), we measured
753 the intensity at three different field of views at each concentration. Finally, we normalized
754 these intensity values by a min-max normalization and fitted a Hill Fit $y = A + (B -$
755 $A) * (\frac{x^n}{k^n} + x^n)$, where A is the starting point, B the ending point and n the Hill coefficient

756 **(Supplementary Fig. 1a and Supplementary Fig. 3j).**

757

758 **FtsZ trajectory analysis**

759 To estimate the diameter of trajectories of FtsZ single filaments, we performed a
760 maximum intensity projection of selected regions of interest (ROI) showing isolated
761 single filaments. Next, a circle was manually drawn on top of the filament trajectory and
762 its diameter measured **(Fig. 1b)**.

763

764 **Differential imaging and directional autocorrelation**

765 To quantify at the directional flows of the FtsZ filament pattern, we used a previously
766 developed automated image analysis protocol³⁵. In this routine, we first track the
767 growing ends of FtsZ filament bundles and then compute the directional autocorrelation
768 from these trajectories. The corresponding correlation coefficient is obtained by
769 computing the correlation of the angle between two consecutive displacements as a
770 function of an increasing time interval (Δt) and is a measure for the local directional
771 persistence of treadmilling trajectory i . The autocorrelation curves were best fitted
772 assuming a fast and slow decay, whose rates were extracted by fitting a two-phase
773 exponential decay $y = a1 * e^{(-b1*t)} + a2 * e^{(-b2*t)}$, where a1, a2 are the starting points
774 and b1, b2 are the fast and slow decay rates. The two half times of the two-phase

775 exponential decay were calculated from the respective decay rates (**Fig. 1c** and
776 **Supplementary Fig. 1c**).

777

778 **Fluorescence recovery after photo-bleaching (FRAP) analysis**

779 For FRAP experiments, a small area of the membrane was bleached with a high 488nm
780 laser intensity after the pattern has reached equilibrium. To obtain the recovery half-time
781 we used a Jython macro script for ImageJ (Image Processing School 8 Pilsen 2009) to fit
782 the fluorescence recovery with $I(t) = a(1 - e^{-bt})$, where $I(t)$ is the intensity value
783 corrected for photobleached effects. FRAP experiments were acquired with an exposure
784 time of 50msec and an acquisition time of one frame every 1 second (**Supplementary**
785 **Fig. 1d** and **Supplementary Fig. 3i**).

786

787 **Collision angle analysis from STED movies**

788 The angle of FtsZ filaments collision was measured using the Angle tool of ImageJ. Three
789 points, corresponding to the incoming FtsZ and the center point of the collision,
790 respectively, were manually set. The subsequent alignment (parallel or antiparallel) was
791 evaluated manually (**Supplementary Fig. 1g**).

792

793 **Quantification of GTP hydrolysis rate of FtsZ**

794 To measure the GTPase rate of FtsZ WT and L169R we used the commercially available
795 EnzCheck kit (ThermoFisher, E6646). The protein was buffer exchanged into phosphate
796 free SLB reaction buffer (150mM KCl, 50mM Tris, 5mM MgCl₂, pH 7.4). The proteins were
797 then diluted to 5μM together with the 20x reaction buffer of the kit, the SLB reaction
798 buffer, MSG and PNP. The reaction was incubated for 15 minutes to remove any traces of
799 free phosphate with the PNP. Subsequently 200μM GTP were added and the production
800 of free phosphate by FtsZ was measured. The appropriate controls (Buffer + GTP, Buffer
801 + FtsZ WT/L169R) were measured simultaneously and subtracted from the Buffer + FtsZ
802 WT/L169R + GTP curves. The rates are extracted from the slope of the initial linear
803 increase from 0-5 minutes (**Supplementary Fig. 4k**).

804

805 **Ring analysis**

806 **Ring density:** the number of rings in a given field of view were manually counted after
807 averaging every 40 seconds of a time lapse movie (**Fig. 2g**).

808 **Ring diameter and width:** First, to remove imaging noise fast intensity fluctuations, we
809 first calculated a walking average over five frames of a time-lapse movie. Next, we
810 obtained the fluorescence intensity profile across the ring diameter using and further
811 analyzed this profile by background intensity subtraction and fitting a double Gaussian
812 function with the Scipy module [2]. From this fit, the ring width can be obtained as the
813 full width at half maximum (FWHM) averaged over the two peaks. The outer and inner
814 diameters were computed as distances between the corresponding FWHM values. The
815 outer diameter of rings was reported and compared to simulations (**Fig. 2i**,
816 **Supplementary Fig. 3g**).

817 **Ring lifetimes** were obtained by first preparing a kymograph along the diameter of a
818 ring and then measuring the length of two parallel vertical lines corresponding to the
819 lifetime of the ring (**Fig. 2h**).

820

821 **Single filament analysis**

822 HS-AFM movies were imported to FiberApp⁵⁹ and FtsZ filaments were traced. To simplify
823 the tracking of FtsZ filaments, we used the A* pathfinding algorithm with 100 iterations.
824 Furthermore, we used the open contour type, and the parameters were: Alpha = 10, Beta
825 = 10, Gamma = 20, Kappa1 = 20, Kappa2 = 10. The fiber intensity was measured directly
826 on the analyzed frame. After fitting all filaments within one frame, we extracted the
827 persistence length (L_p) from calculating the mean-squared end-to-end distance $y^2 = 4 * L_p * [x - 2 * L_p * (1 - e^{-\frac{x}{2*L_p}})]$. The contour length was extracted by measuring the
828 average length distribution of all frames within a specific FtsZ density. The local
829 curvature was extracted directly from the xy-coordinates of the filaments, obtained from
830 Fiber app. Subsequently, we averaged the local curvatures of each filament to obtain one
831 curvature value per filament at different densities (**Fig. 3c, 4c-4e**).

833

834 **Numerical simulations**

835 The simulations of FtsZ filaments are based on the self-propelled worm-like chain
836 model³¹, extended for polymer chirality (**Fig. 2a**). A filament is represented by $N+1$ beads
837 with radius a_0 connected by N stiff bonds and chiral bending potentials. To minimize
838 friction between the polymers the bond length is chosen to be equal to a_0 , leading to

839 overlaps of neighboring beads and filament length $L_f = N a_0$. The overdamped equation of
 840 motion is therefore given by:

$$841 \qquad \qquad \qquad \gamma \frac{dr_i}{dt} = \nabla_i V + F_p^i + F_{k_B T}^i$$

842 where r_i are the coordinates of the beads, γ is the friction coefficient, $F_{k_B T}^i$ is the thermal
 843 noise force (modeled as white noise with zero mean and variance $4k_B T \gamma / \delta t$, where δt is
 844 the simulation timestep and $k_B T$ is the effective temperature), and V is the potential
 845 energy, which comprises of both intrafilament and inter-filament interactions:

$$V = V_{bond} + V_{bend} + V_{pair}$$

847 where the first term is a harmonic bond potential penalizing filament stretching

$$V_{bond}(r) = \frac{1}{2} k_{bond} \sum_{i=1}^{N-1} (\vec{r}_{i,i+1} - r_0)^2$$

849 while the second term is a harmonic bending potential penalizing filament bending

$$V_{bend}(\theta) = \frac{1}{2} k_{bend} \sum_{i=1}^{N-2} (\theta_i - \theta_0)^2$$

851 where $\vec{r}_{i,i+1}$ is the bond vector between the neighboring beads, θ_i is the angle between
 852 the neighboring bonds, r_0 and θ_0 represent the equilibrium bond length and rest angle
 853 (with k_{bond} and k_{bend} the corresponding spring constants). The existence of a non-zero rest
 854 angle implies an intrinsic and chiral curvature for individual filaments, while the k_{bond}
 855 parameter is used large enough to keep the bond length in the polymer constant. Finally,
 856 filaments interact via soft Lennard-Jones potential, which accounts for excluded volume
 857 interactions and potentially a middle range attraction

$$\Delta v_{pair}(r_{i,j}) = -(r_{i,j} - r_{cut}) \frac{\partial v_{pair}}{\partial r}(r_{cut})$$

861 where r_{ij} is the vector between positions of beads i and j (which can belong to any
 862 filament), ϵ is the depth of the potential well, σ is distance where V_{pair} is zero and r_{cut} is
 863 the cutoff distance of the potential. The potential is shifted by the subtraction of the value
 864 of the force at r_{cut} such that the force smoothly goes to zero at the cut-off. When

865 accounting only for repulsive interactions, we use r_{cut} consistent with the interaction
866 minimum $\sqrt{2}\sigma$. F_p^i is the active self-propulsion force that mimics the filament treadmilling
867 (acting tangentially along the bonds of the polymer):

$$868 \quad F_p^i = \gamma v_0 \begin{cases} \vec{r}_{i,i+1} + d\theta & \text{if } i = 1 \\ \vec{r}_{i-1,i} & \text{if } 1 < i < N \end{cases}$$

869 where $d\theta = \pi - \theta_0$ ensures the chiral self-propulsion of the first bead. The simulations
870 are performed in 2 dimensions and employ periodic boundary conditions.

871

872 **Details of the simulation setup & parameter exploration**

873 HOOMD-blue v2.9⁶⁰ was used to run the simulations, with in-house modifications of the
874 chiral worm-like chain model. Specifically, to ensure the chirality of the polymers, an
875 asymmetric bending potential was used, having a signed curvature (always calculated in
876 the direction from head to tail of the filament).

877 Simulation parameters and results are reported in dimensionless form, where
878 length is measured in units of the effective bead diameter d (defined as the interaction
879 minimum $\sqrt{2}\sigma$), energies in units of the thermal energy $k_B T$, and time in units of the
880 filament rotation periods τ . The equations of motion were numerically integrated using
881 the Euler scheme with timestep $\delta t = 1.8 \cdot 10^{-5} \tau$. The dynamics of filaments was
882 governed mainly by two dimensionless numbers, namely the Peclet number

$$883 \quad Pe = \frac{F_p L_f^2}{k_B T}$$

884 and flexure number

$$885 \quad \mathcal{F} = \frac{F_p L_f^3}{k_{bend}}$$

886 Unless stated otherwise, the simulation parameters to model FtsZ WT were: $Pe = 900$, \mathcal{F}
887 = 40, $k_{bond} = 1000 k_B T/d^2$, $\gamma = 1$, $k_{bend} = 53.5 k_B T/rad^2$, bending angle $\theta_0 = 3.08$ rad, effective
888 temperature for the thermal noise force $0.215 k_B T$ and $r_{cut} = 1.7d$. Since the directional
889 motion of filament bundles are consistent with microscopic polar ordering of filaments
890 (**Fig. 1d**), we modelled filaments with length $L_f = 8 d$, with aspect ratio corresponding to
891 bundles with thickness of 5 FtsZ filaments³¹. To compare with experiment the reduced
892 simulation units were recalculated with constants $1d = 50\text{nm}$ (corresponding to filament
893 length FtsZ WT $L_f = 400 \mu\text{m}$ and filament curvature $3.48 \text{ rad}/\mu\text{m}$) and $\tau = 78.5 \text{ s}$

894 (considering a ring with diameter $1 \mu\text{m}$ and treadmilling speed $0.04 \mu\text{m/s}$). We define
895 packing fraction as $\phi = NN_f dr_0 / L^2$, where N_f refers to the number of filaments and L to
896 the box size. The simulated filament densities ranged from $\phi = 0.05$ to $\phi = 0.9$. The
897 parameter space was explored by varying N_f , k_{bend} , ϵ , and effective temperature $k_B T$ to
898 alter filament density, flexure number, attraction and Peclet number, respectively.
899 To check if our explored parameter space is reasonable, we performed a few sanity
900 checks. We computed the persistence length of filaments in our simulations from the
901 distribution of local filament curvatures in intermediate density⁵⁷. L_p was obtained by
902 fitting a Gaussian function to local curvature data and extracting its variance: $L_p = \frac{1}{\sigma^2 \Delta s}$,
903 where σ is the standard deviation of Gaussian function and Δs is the filament contour
904 spacing⁶¹. The resulting values $L_p = 6753 \text{ nm}$ for FtsZ WT (**Supplementary Fig. 3j**) and
905 $L_p = 11527 \text{ nm}$ for FtsZ L169R (**Supplementary Fig. 4m**) agrees well with our HS-AFM
906 analysis (**Fig. 4d**). We also checked the order of magnitude of Peclet numbers of FtsZ WT
907 in our simulations (**Supplementary Fig. 3k**) by comparing the single-filament
908 trajectories to experimental trajectories from TIRF data (**Fig. 1b**).

909 The simulations of active filaments were initialized in a nematic configuration
910 (each filament was placed on the lattice in a straight configuration randomly oriented
911 either left or right). The initial time needed for the equilibration of the system (4τ) is
912 discarded in the analysis. Parameter screening and subsequent simulations, which were
913 focused on the molecular detail (temporal interconversion of rings and bands in **Fig. 2c**,
914 topological defects in **Fig. 2e,f**, polarity sorting in **Fig. 4a,b**) were performed with a box
915 size $L = 42 d$, whereas all the reported large-scale patterns were simulated in the box of
916 size $L = 212 d$.

917 To get a well-mixed state of non-chiral filaments with lower F_p (**Fig. 4a,d**), the
918 system was initiated in low density ($\phi = 0.25$ and lower) on a lattice in a nematic
919 configuration with high thermal noise force ($Pe = 100$). The denser systems were
920 initialized with enlarged boxes of density $\phi = 0.25$, and after mixing of the filaments,
921 these systems were down-scaled and equilibrated in a stepwise manner, to reach a high-
922 density system with effectively random initial conditions.

923 The large-scale simulations (**Fig. 2b, 4d**) were run for 20τ , whereas the small
924 systems, where temporal interconversion of individual rings and bands was quantified
925 (**Fig. 2c**), were run for 40τ . Finally, polarity sorting simulations (**Fig. 4a,b**) were run for

926 10 τ and analyzed from the initial timestep. Each combination of parameters was run in
927 at least 10 repeats for small system sizes and 5 repeats for larger systems.

928

929 **Simulation analysis**

930 Unless stated otherwise, simulation frames were analyzed with frequency of 0.2 τ after
931 the equilibration. The average filament curvature (**Fig. 2d, 3b**) was computed by first
932 calculating the mean curvature of each filament and then averaging it over a single
933 snapshot of the system. The resulting distribution of the average curvatures consists of
934 pooled data over both multiple simulation time points and simulation repeats.

935 To automatically detect rings in the system the instantaneous centers of rotation were
936 computed for each filament in each analyzed snapshot. These points were subsequently
937 clustered using Freud library^{39,58} with distance threshold related to the density of the
938 system (cutoff defined as half of average distance between filaments: $\frac{L}{2\sqrt{N_f}}$). Clusters with
939 more than 10 members and normalized radius (given by radius of gyration / cluster size)
940 lower than 0.1 d were considered to be ring centers. To filter out unclosed rings, the
941 normalized polarity (given by norm of the vector summing the filament orientations /
942 cluster size) of filaments belonging to a ring was computed and had to be lower than
943 0.25 d (**Supplementary Fig. 3h**). Since stable rings didn't show significant overall
944 translation, we performed additional clustering of ring centers in time with distance
945 cutoff $3d$ to track rings in time and get a lifetime of the rings (estimated by subtraction of
946 last and first snapshots of each observed ring, **Fig. 2h, Supplementary Fig. 3c,d**). This
947 clustering was also used to remove falsely detected "rings" by removing all clusters of
948 size lower than 3 members (**Supplementary Fig. 3i**). All the cutoffs of this analysis were
949 chosen and validated by visual comparison to simulation movies in all studied densities.

950 To calculate the average density of rings in simulations the number of rings in a
951 snapshot was divided by the total area (**Fig. 2g, Supplementary Fig. 2c,d**). The diameter
952 of rings was estimated by calculating the average distance of filaments from the center of
953 the ring (**Fig. 2i, Supplementary Fig. 3e,f**).

954 The polarity alignment (**Fig. 5b, Supplementary Fig. 4d-h**) was analyzed by
955 computing all the neighboring beads (not belonging to the same filament, with the same
956 distance threshold r_{cut} used in the pair potential) in the system using Freud library and

957 classifying the neighbor relative orientation as either parallel or antiparallel based on
958 their angle (< 90 degrees = parallel, > 90 degrees = anti-parallel).

959

960 **Protein structure prediction**

961 Protein modeling was performed with AlphaFold-Multimer v2^{41,63} implemented in
962 Google Colab using the filament structure of *Staphylococcus aureus* FtsZ PDB 3VOB as
963 template.

964

965 **References**

- 966 1. Huber, L., Suzuki, R., Krüger, T., Frey, E. & Bausch, A. R. Emergence of coexisting
967 ordered states in active matter systems. *Science* **361**, 255–258 (2018).
- 968 2. Denk, J., Huber, L., Reithmann, E. & Frey, E. Active Curved Polymers Form Vortex
969 Patterns on Membranes. *Physical Review Letters* **116**, 178301 (2016).
- 970 3. Sciortino, A. & Bausch, A. R. Pattern formation and polarity sorting of driven actin
971 filaments on lipid membranes. *Proceedings of the National Academy of Sciences* **118**,
972 e2017047118 (2021).
- 973 4. Hsu, C.-P., Sciortino, A., Trobe, Y. A. de la & Bausch, A. R. Activity-induced polar
974 patterns of filaments gliding on a sphere. *Nature Communications* **13**, 2579 (2022).
- 975 5. Sumino, Y. *et al.* Large-scale vortex lattice emerging from collectively moving
976 microtubules. *Nature* (2012) doi:10.1038/nature10874.
- 977 6. Shaebani, M. R., Wysocki, A., Winkler, R. G., Gompper, G. & Rieger, H. Computational
978 models for active matter. *Nat Rev Phys* **2**, 181–199 (2020).
- 979 7. Gompper, G. *et al.* The 2020 motile active matter roadmap. *J. Phys.: Condens. Matter*
980 **32**, 193001 (2020).
- 981 8. Schaller, V., Weber, C., Semmrich, C., Frey, E. & Bausch, A. R. Polar patterns of driven
982 filaments. *Nature* **467**, 73–77 (2010).

983 9. Bugyi, B. & Carlier, M.-F. Control of actin filament treadmilling in cell motility. *Annual
984 review of biophysics* **39**, 449–70 (2010).

985 10. Alert, R. & Trepaut, X. Physical Models of Collective Cell Migration. *Annu. Rev. Condens.
986 Matter Phys.* **11**, 77–101 (2020).

987 11. Reymann, A.-C., Staniscia, F., Erzberger, A., Salbreux, G. & Grill, S. W. Cortical flow
988 aligns actin filaments to form a furrow. *eLife* **5**, (2016).

989 12. Balasubramaniam, L., Mège, R.-M. & Ladoux, B. Active nematics across scales from
990 cytoskeleton organization to tissue morphogenesis. *Current Opinion in Genetics &
991 Development* **73**, 101897 (2022).

992 13. Hannezo, E. & Heisenberg, C.-P. Mechanochemical Feedback Loops in Development
993 and Disease. *Cell* **178**, 12–25 (2019).

994 14. Marchetti, M. C. *et al.* Hydrodynamics of soft active matter. *Rev. Mod. Phys.* **85**, 1143–
995 1189 (2013).

996 15. Jülicher, F., Grill, S. W. & Salbreux, G. Hydrodynamic theory of active matter. *Rep. Prog.
997 Phys.* **81**, 076601 (2018).

998 16. Bowick, M. J., Fakhri, N., Marchetti, M. C. & Ramaswamy, S. Symmetry,
999 Thermodynamics, and Topology in Active Matter. *Phys. Rev. X* **12**, 010501 (2022).

1000 17. Fürthauer, S., Strempel, M., Grill, S. W. & Jülicher, F. Active chiral fluids. *The European
1001 physical journal. E, Soft matter* **35**, 89 (2012).

1002 18. Liebchen, B. & Levis, D. Chiral active matter. *Europhysics Letters* (2022)
1003 doi:10.1209/0295-5075/ac8f69.

1004 19. Shankar, S., Souslov, A., Bowick, M. J., Marchetti, M. C. & Vitelli, V. Topological active
1005 matter. *Nat Rev Phys* **4**, 380–398 (2022).

1006 20. McQuillen, R. & Xiao, J. Insights into the Structure, Function, and Dynamics of the
1007 Bacterial Cytokinetic FtsZ-Ring. *Annual Review of Biophysics* **49**, 309–341 (2020).

1008 21. Vaughan, S., Wickstead, B., Gull, K. & Addinall, S. G. Molecular evolution of FtsZ protein
1009 sequences encoded within the genomes of archaea, bacteria, and eukaryota. *J Mol Evol*
1010 **58**, 19–29 (2004).

1011 22. Wagstaff, J. M. *et al.* A Polymerization-Associated Structural Switch in FtsZ That
1012 Enables Treadmilling of Model Filaments. *mBio* **8**, (2017).

1013 23. Whitley, K. D. *et al.* FtsZ treadmilling is essential for Z-ring condensation and septal
1014 constriction initiation in *Bacillus subtilis* cell division. *Nature Communications* **12**,
1015 2448 (2021).

1016 24. Walker, B. E., Männik, J. & Männik, J. Transient Membrane-Linked FtsZ Assemblies
1017 Precede Z-Ring Formation in *Escherichia coli*. *Curr Biol* **30**, 499-508.e6 (2020).

1018 25. Thanedar, S. & Margolin, W. FtsZ exhibits rapid movement and oscillation waves in
1019 helix-like patterns in *Escherichia coli*. *Curr Biol* **14**, 1167–1173 (2004).

1020 26. Lan, G., Daniels, B. R., Dobrowsky, T. M., Wirtz, D. & Sun, S. X. Condensation of FtsZ
1021 filaments can drive bacterial cell division. *Proc Natl Acad Sci U S A* **106**, 121–126
1022 (2009).

1023 27. Coltharp, C., Buss, J., Plumer, T. M. & Xiao, J. Defining the rate-limiting processes of
1024 bacterial cytokinesis. *Proc Natl Acad Sci U S A* **113**, E1044–1053 (2016).

1025 28. Loose, M. & Mitchison, T. J. The bacterial cell division proteins ftsA and ftsZ self-
1026 organize into dynamic cytoskeletal patterns. *Nature Cell Biology* vol. 16 38–46
1027 Preprint at <https://doi.org/10.1038/ncb2885> (2014).

1028 29. Duman, Ö., Isele-Holder, R. E., Elgeti, J. & Gompper, G. Collective dynamics of self-
1029 propelled semiflexible filaments. *Soft Matter* **14**, 4483–4494 (2018).

1030 30. Moore, J. M., Thompson, T. N., Glaser, M. A. & Betterton, M. D. Collective motion of
1031 driven semiflexible filaments tuned by soft repulsion and stiffness. *Soft Matter* **16**,
1032 9436–9442 (2020).

1033 31. Isele-Holder, R. E., Elgeti, J. & Gompper, G. Self-propelled worm-like filaments:
1034 spontaneous spiral formation, structure, and dynamics. *Soft Matter* **11**, 7181–7190
1035 (2015).

1036 32. Guan, F. *et al.* Lateral interactions between protofilaments of the bacterial tubulin
1037 homolog FtsZ are essential for cell division. *Elife* **7**, e35578 (2018).

1038 33. Fu, G. *et al.* In vivo structure of the *E. coli* FtsZ-ring revealed by photoactivated
1039 localization microscopy (PALM). *PLoS One* **5**, e12682 (2010).

1040 34. Caldas, P. *et al.* Cooperative ordering of treadmilling filaments in cytoskeletal
1041 networks of FtsZ and its crosslinker ZapA. *Nature Communications* (2019)
1042 doi:10.1038/s41467-019-13702-4.

1043 35. Caldas, P., Radler, P., Sommer, C. & Loose, M. Chapter 8 Computational analysis of
1044 filament polymerization dynamics in cytoskeletal networks. in vol. 158 145–161
1045 (Academic Press, 2020).

1046 36. Ando, T. *et al.* A high-speed atomic force microscope for studying biological
1047 macromolecules. *Proc. Natl. Acad. Sci. U.S.A.* **98**, 12468–12472 (2001).

1048 37. Ando, T. High-speed atomic force microscopy. *Curr Opin Chem Biol* **51**, 105–112
1049 (2019).

1050 38. Haeusser, D. P., Rowlett, V. W. & Margolin, W. A mutation in *Escherichia coli* ftsZ
1051 bypasses the requirement for the essential division gene zipA and confers resistance
1052 to FtsZ assembly inhibitors by stabilizing protofilament bundling. *Mol Microbiol* **97**,
1053 988–1005.

1054 39. Krupka, M. *et al.* *Escherichia coli* FtsA forms lipid-bound minirings that antagonize
1055 lateral interactions between FtsZ protofilaments. *Nature communications* **8**, 15957
1056 (2017).

1057 40. Jumper, J. *et al.* Highly accurate protein structure prediction with AlphaFold. *Nature*
1058 **596**, 583–589 (2021).

1059 41. Mirdita, M. *et al.* ColabFold: making protein folding accessible to all. *Nature methods*
1060 **19**, 679–682 (2022).

1061 42. Elsen, N. L. *et al.* Mechanism of action of the cell-division inhibitor PC190723:
1062 modulation of FtsZ assembly cooperativity. *J Am Chem Soc* **134**, 12342–12345 (2012).

1063 43. Hussain, S. *et al.* MreB filaments align along greatest principal membrane curvature
1064 to orient cell wall synthesis. *Elife* **7**, e32471 (2018).

1065 44. Nierhaus, T. *et al.* Bacterial divisome protein FtsA forms curved antiparallel double
1066 filaments when binding to FtsN. *Nat Microbiol* **7**, 1686–1701 (2022).

1067 45. Rode, S., Elgeti, J. & Gompper, G. Chiral-filament self-assembly on curved manifolds.
1068 *Soft Matter* **16**, 10548–10557 (2020).

1069 46. Patra, P. *et al.* Collective migration reveals mechanical flexibility of malaria parasites.
1070 *Nature Physics* **18**, 586–594 (2022).

1071 47. Tan, T. H. *et al.* Odd dynamics of living chiral crystals. *Nature* **607**, 287–293 (2022).

1072 48. Duclos, G. *et al.* Spontaneous shear flow in confined cellular nematics. *Nature Physics*
1073 **14**, 728–732 (2018).

1074 49. Jauffred, L., Munk Vejborg, R., Korolev, K. S., Brown, S. & Oddershede, L. B. Chirality in
1075 microbial biofilms is mediated by close interactions between the cell surface and the
1076 substratum. *ISME J* **11**, 1688–1701 (2017).

1077 50. Tee, Y. H. *et al.* Cellular chirality arising from the self-organization of the actin
1078 cytoskeleton. *Nature Cell Biology* **17**, 445–457 (2015).

1079 51. Yang, X. *et al.* GTPase activity-coupled treadmilling of the bacterial tubulin FtsZ
1080 organizes septal cell wall synthesis. *Science* **355**, 744–747 (2017).

1081 52. Bisson-Filho, A. W. *et al.* Treadmilling by FtsZ filaments drives peptidoglycan
1082 synthesis and bacterial cell division. *Science* **355**, 739–743 (2017).

1083 53. Radler, P. *et al.* In vitro reconstitution of *Escherichia coli* divisome activation. *Nat
1084 Commun* **13**, 2635 (2022).

1085 54. Lim, K. *et al.* High-Speed AFM Reveals Molecular Dynamics of Human Influenza A
1086 Hemagglutinin and Its Interaction with Exosomes. *Nano Lett* **20**, 6320–6328 (2020).

1087 55. Lim, K. *et al.* Millisecond dynamic of SARS-CoV-2 spike and its interaction with ACE2
1088 receptor and small extracellular vesicles. *J Extracell Vesicles* **10**, e12170 (2021).

1089 56. Sajidah, E. S. *et al.* Spatiotemporal tracking of small extracellular vesicle nanotopology
1090 in response to physicochemical stresses revealed by HS-AFM. *J Extracell Vesicles* **11**,
1091 e12275 (2022).

1092 57. Donnert, G. *et al.* Macromolecular-scale resolution in biological fluorescence
1093 microscopy. *Proc. Natl. Acad. Sci. U.S.A.* **103**, 11440–11445 (2006).

1094 58. Schindelin, J. *et al.* Fiji: An open-source platform for biological-image analysis. *Nature
1095 Methods* Preprint at <https://doi.org/10.1038/nmeth.2019> (2012).

1096 59. Usov, I. & Mezzenga, R. FiberApp: An open-source software for tracking and analyzing
1097 polymers, filaments, biomacromolecules, and fibrous objects. *Macromolecules* **48**,
1098 1269–1280 (2015).

1099 60. Anderson, J. A., Glaser, J. & Glotzer, S. C. HOOMD-blue: A Python package for high-
1100 performance molecular dynamics and hard particle Monte Carlo simulations.
1101 *Computational Materials Science* **173**, 109363 (2020).

1102 61. Wisanpitayakorn, P., Mickolajczyk, K. J., Hancock, W. O., Vidali, L. & Tüzel, E.
1103 Measurement of the persistence length of cytoskeletal filaments using curvature
1104 distributions. *Biophysical Journal* **121**, 1813–1822 (2022).

1105 62. Ramasubramani, V. *et al.* freud: A software suite for high throughput analysis of
1106 particle simulation data. *Computer Physics Communications* **254**, 107275 (2020).

1107 63. Evans, R. *et al.* Protein complex prediction with AlphaFold-Multimer.
1108 <http://biorxiv.org/lookup/doi/10.1101/2021.10.04.463034> (2021)
1109 doi:10.1101/2021.10.04.463034.

1110

1111 **Acknowledgements**

1112 This work was supported by the European Research Council through grant ERC 2015-
1113 StG-679239 by the Austrian Science Fund (FWF) StandAlone P34607 to M.L., and by the
1114 Kanazawa University WPI- NanoLSI Bio-SPM collaborative research program. Z.D. has
1115 received funding from Doctoral Programme of the Austrian Academy of Sciences (OeAW):
1116 [Grant agreement 26360]. We thank Jan Brugues (MPI CBG, Dresden, Germany), Andela
1117 Saric (ISTA, Klosterneuburg, Austria), Daniel Pearce (Uni Geneva, Switzerland) for
1118 valuable scientific input and comments on the manuscript. We are also thankful for the
1119 support by the Scientific Service Units (SSU) of IST Austria through resources provided
1120 by the Imaging and Optics Facility (IOF) and the Lab Support Facility (LSF).

1121

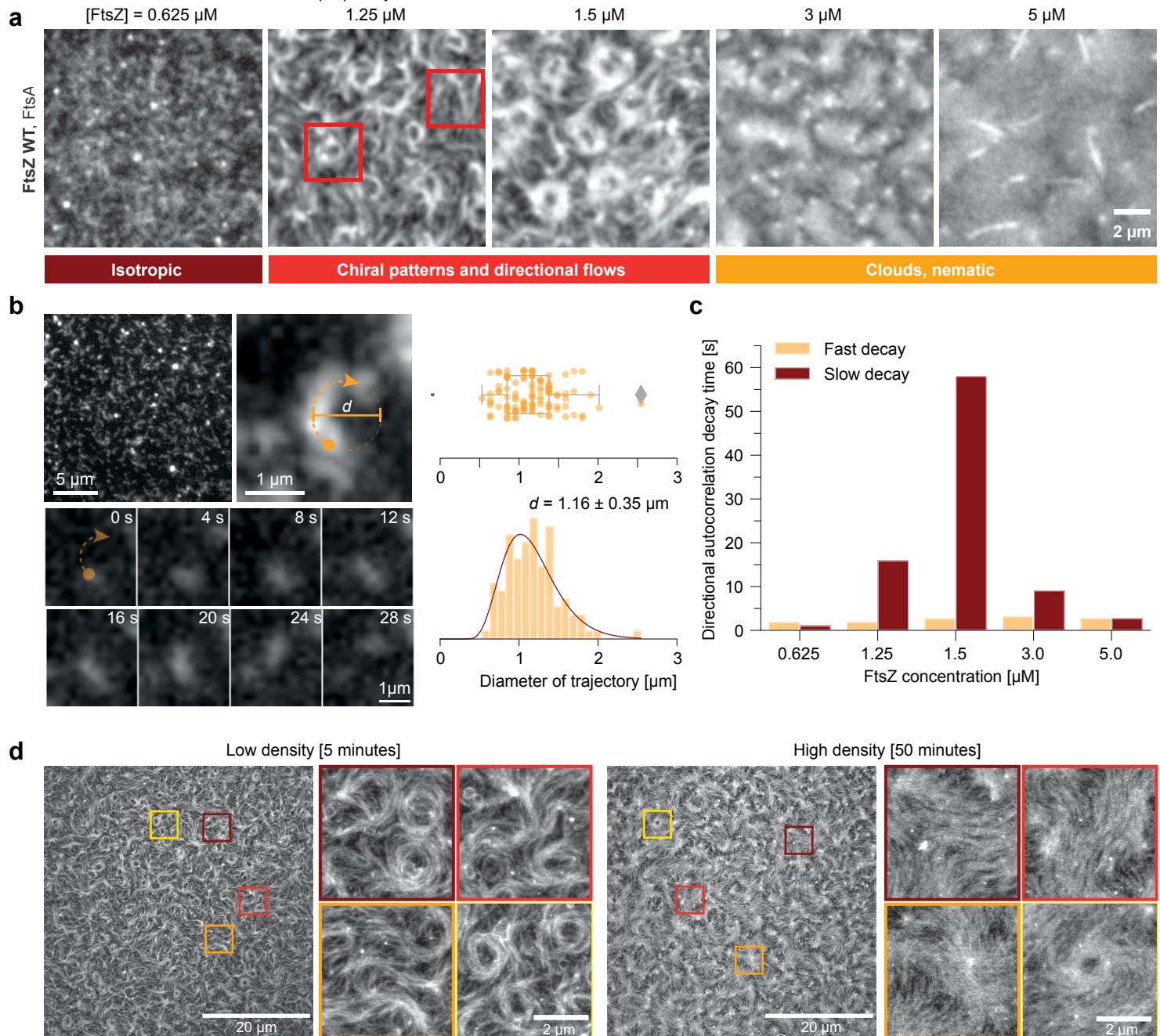
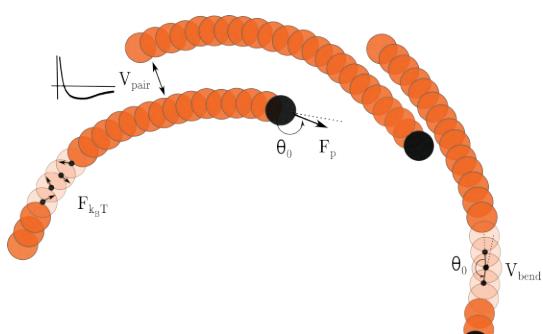
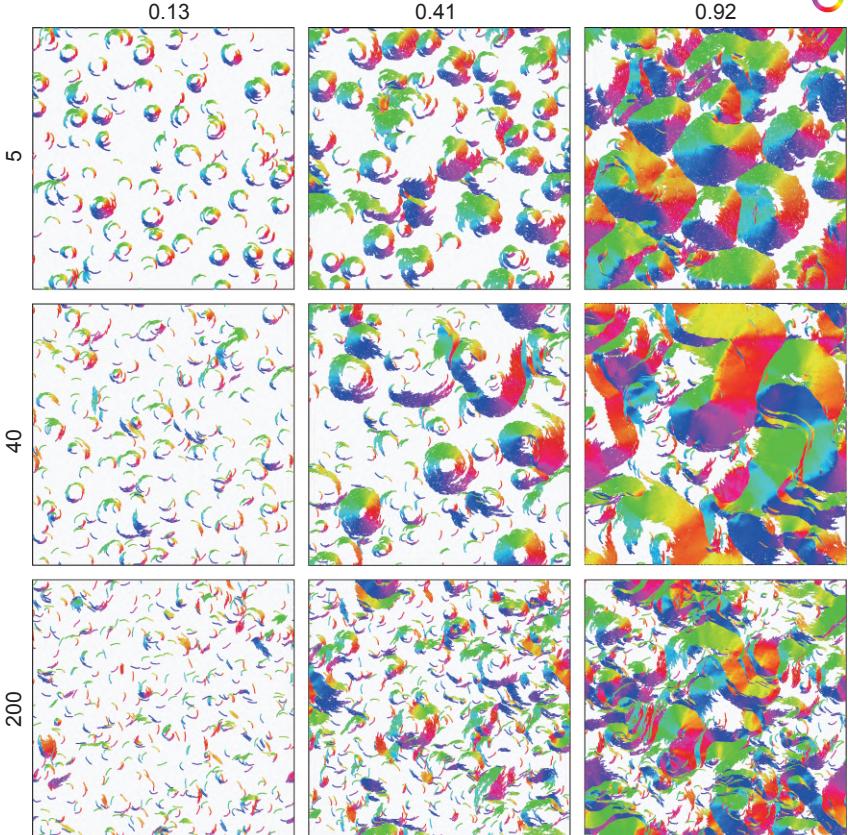


Figure 1

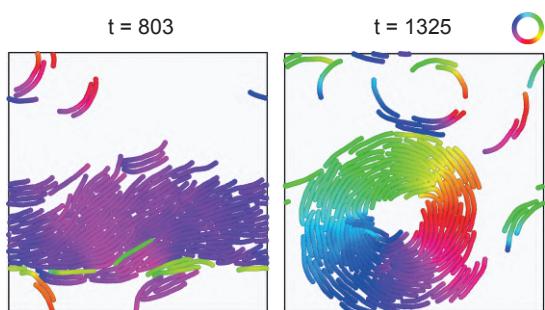
a



b

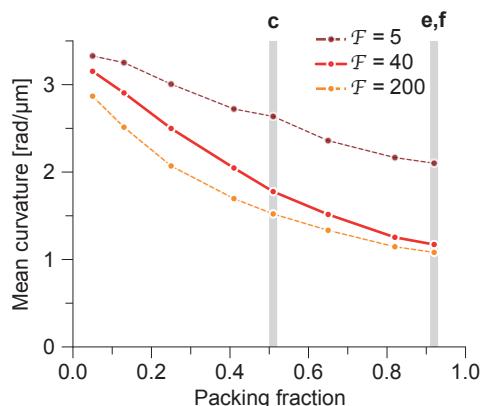


c



Filament flexibility F_{FlsZ}

d



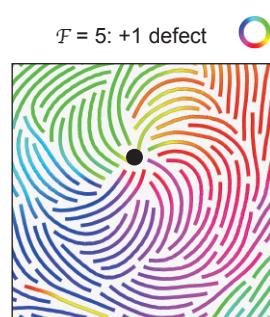
c

$F = 5$

$F = 40$

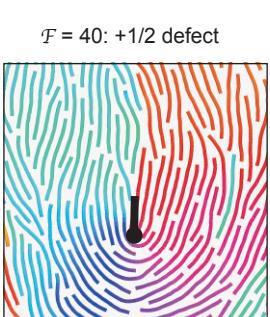
$F = 200$

e



$F = 5: +1/2$ defect

f

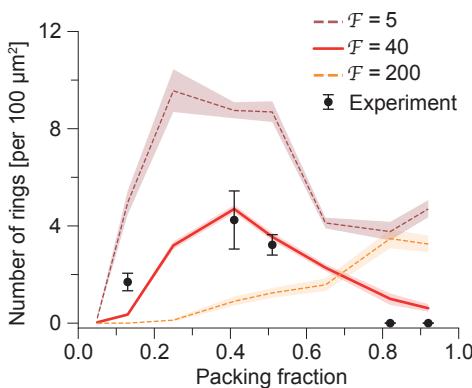


$F = 40: -1/2$ defect

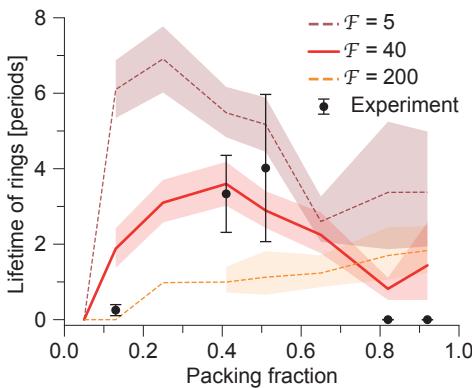
-1/2 defect

○

g



h



i

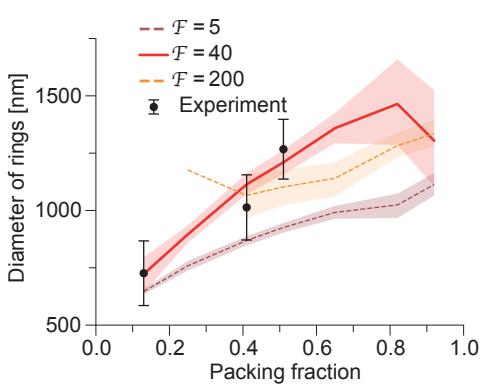


Figure 2

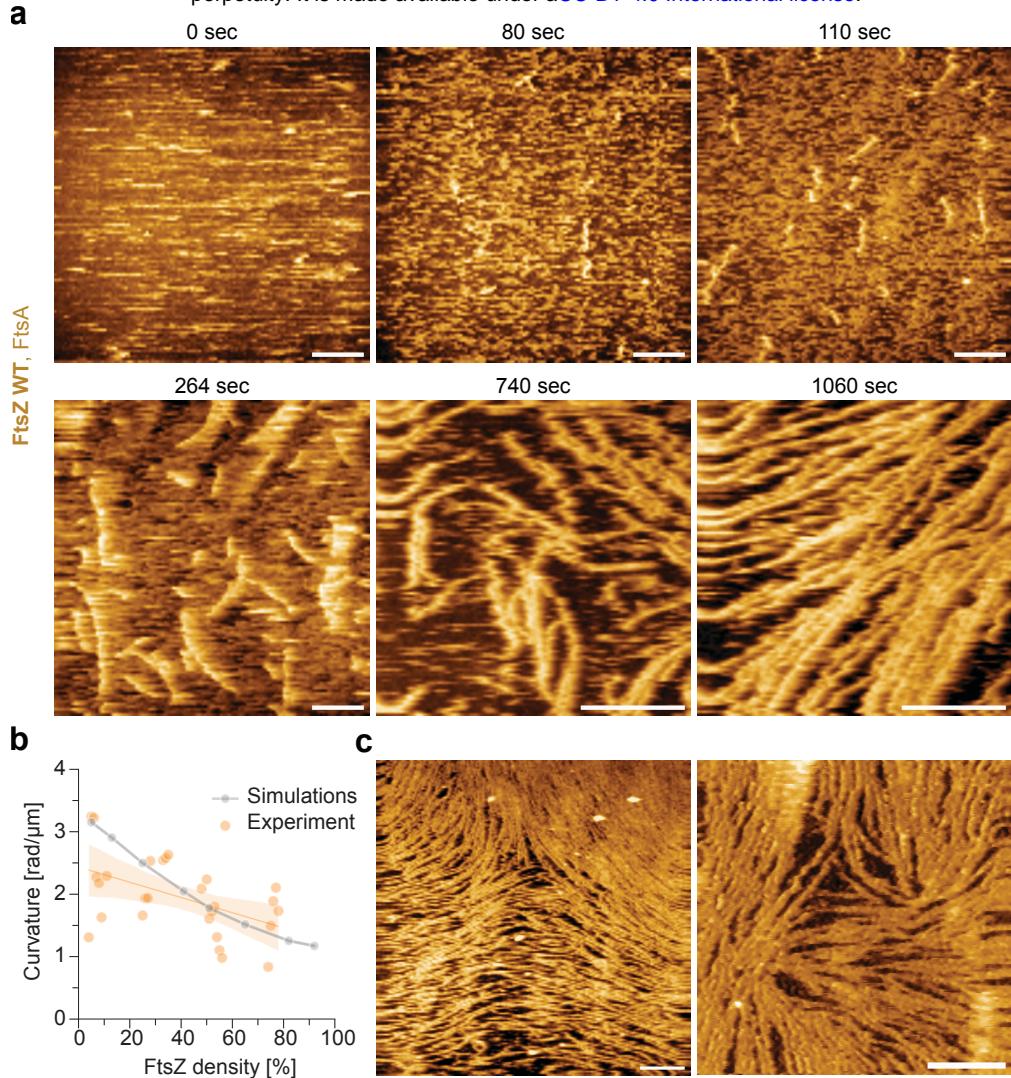
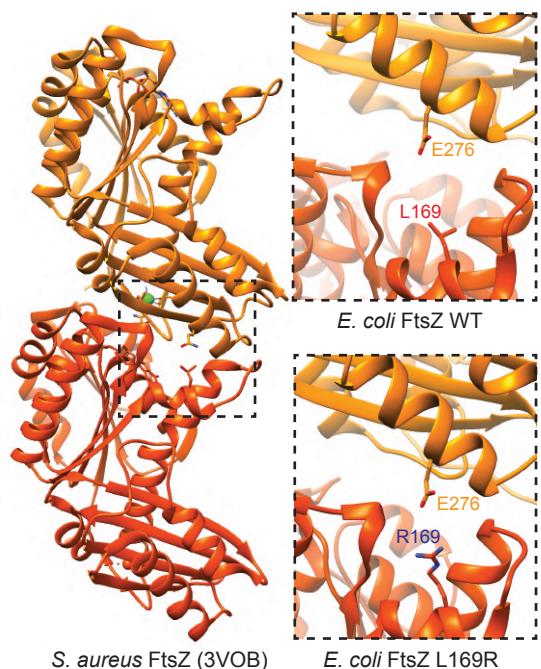
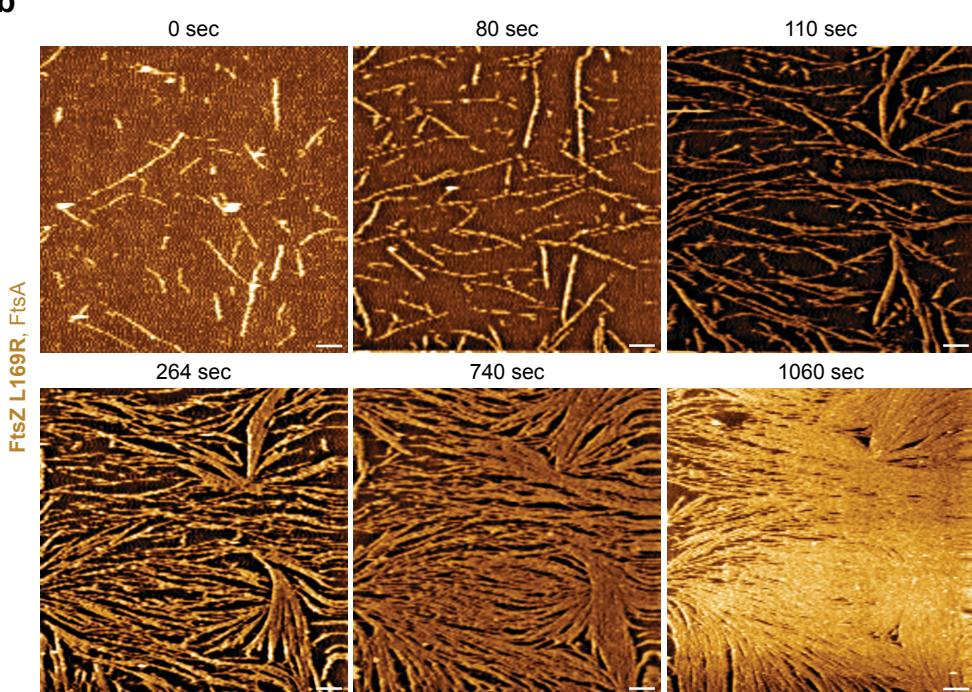


Figure 3

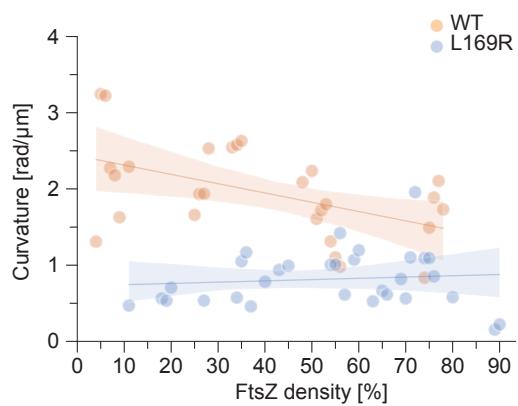
a



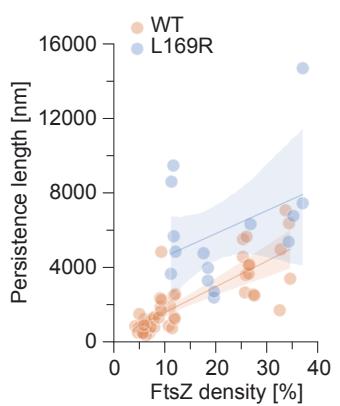
b



c



d



e

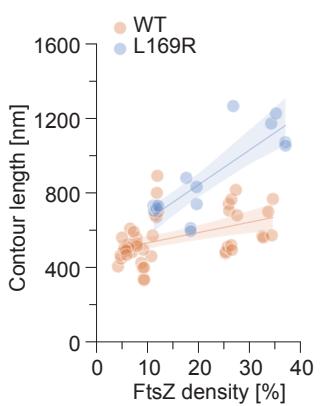
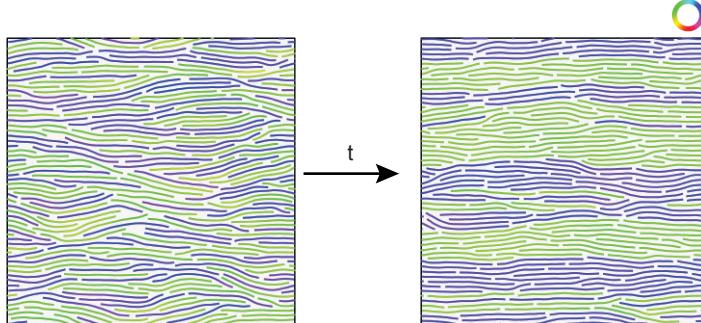
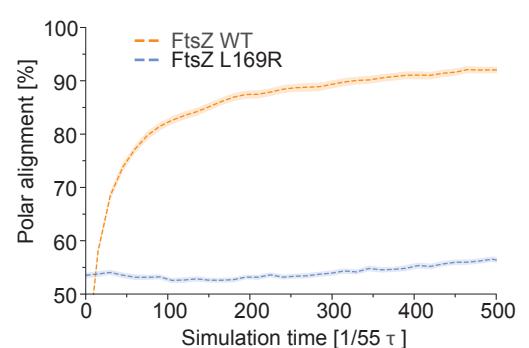


Figure 4

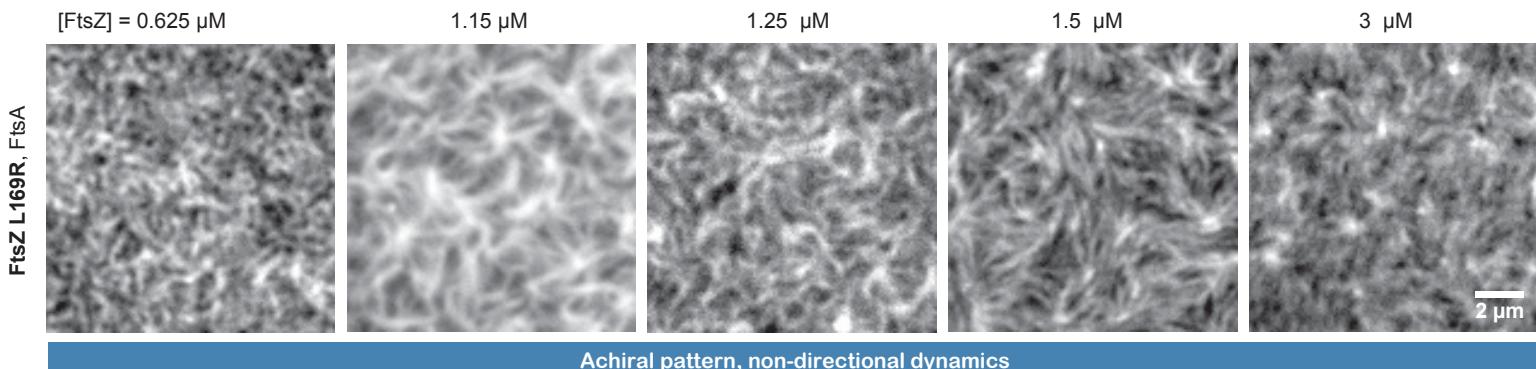
a



b



c



Achiral pattern, non-directional dynamics

d

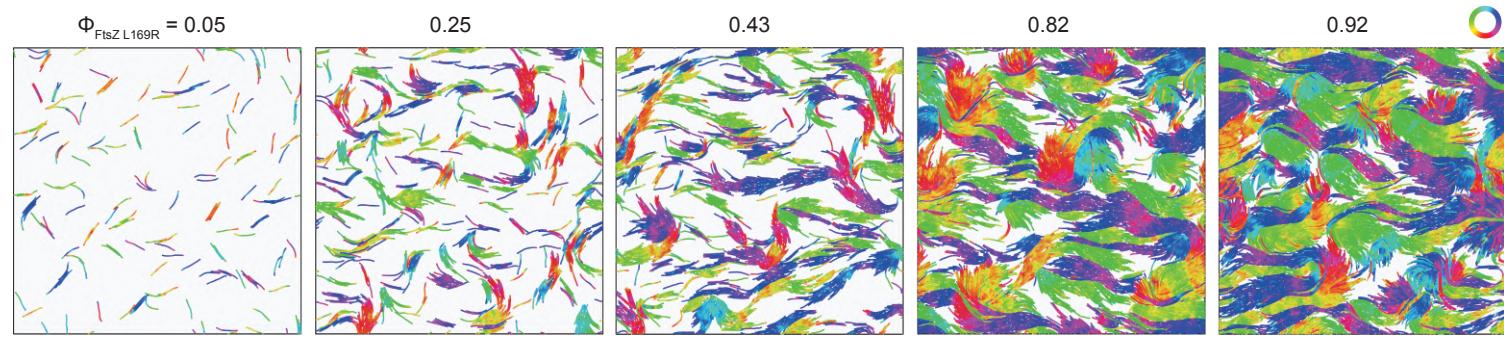
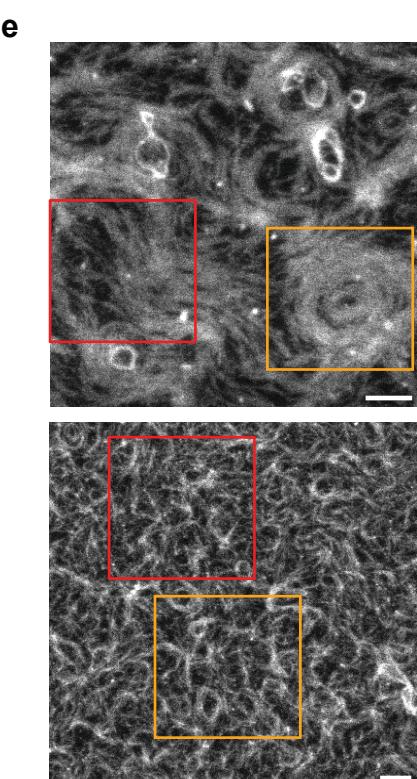
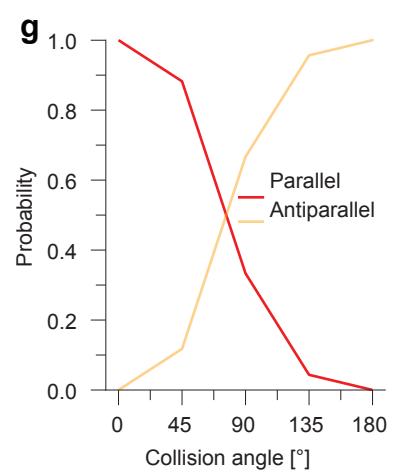
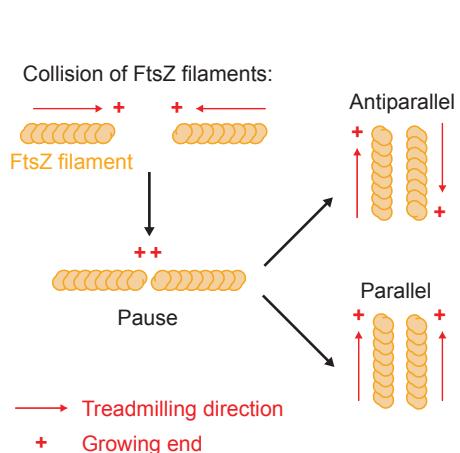
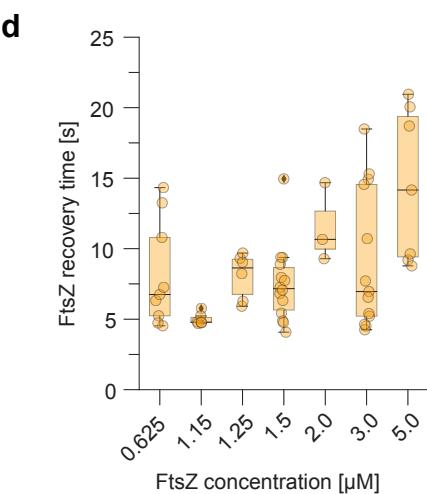
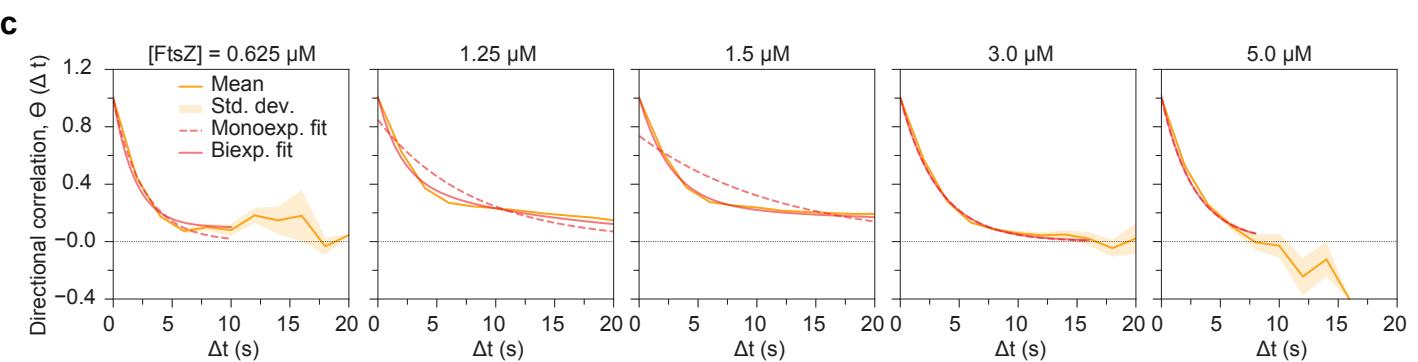
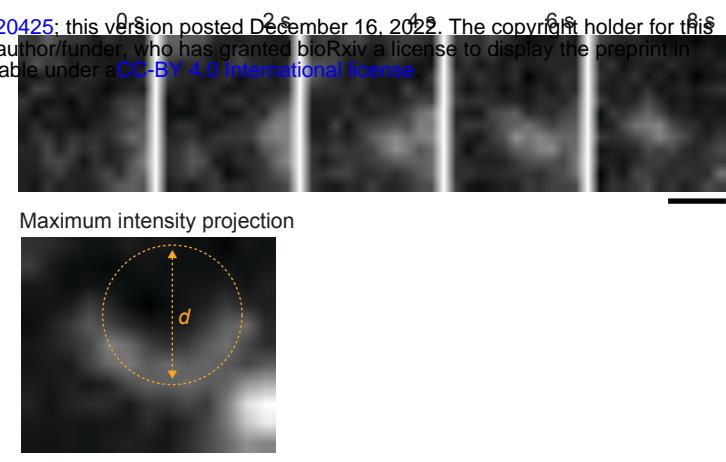
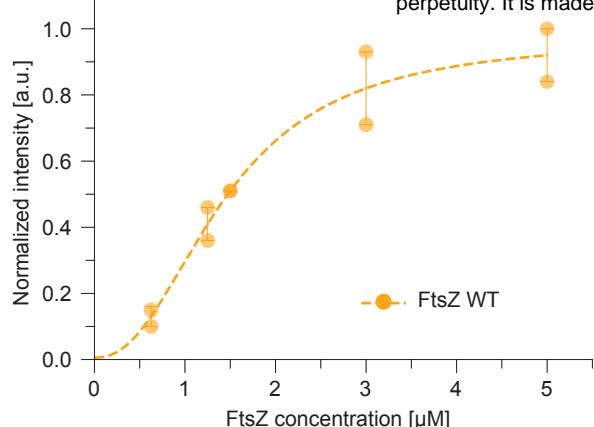
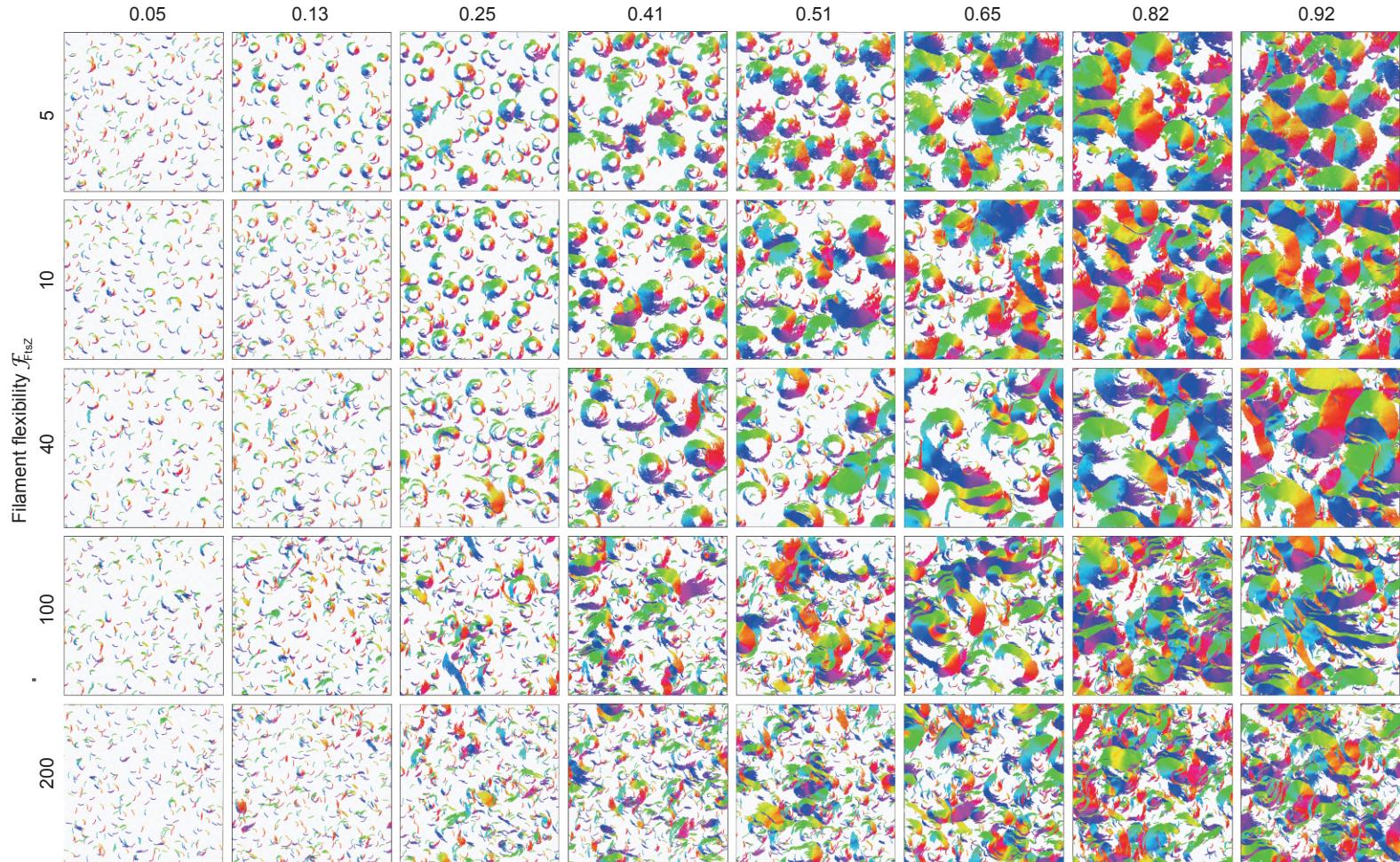


Figure 5

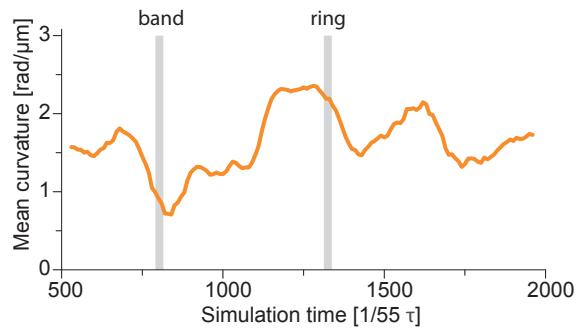


Supplementary Figure 1

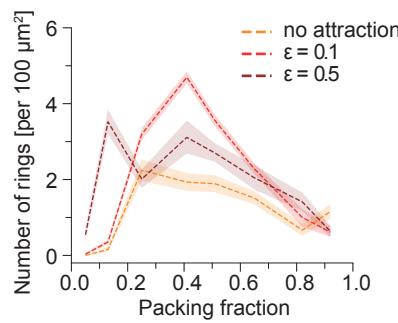
a



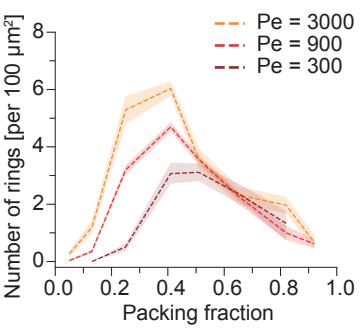
b



c

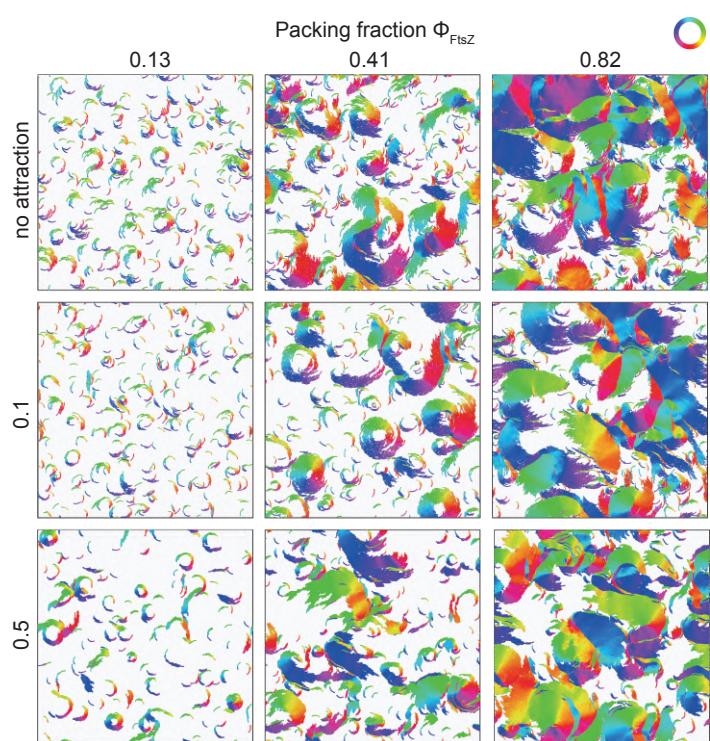


d

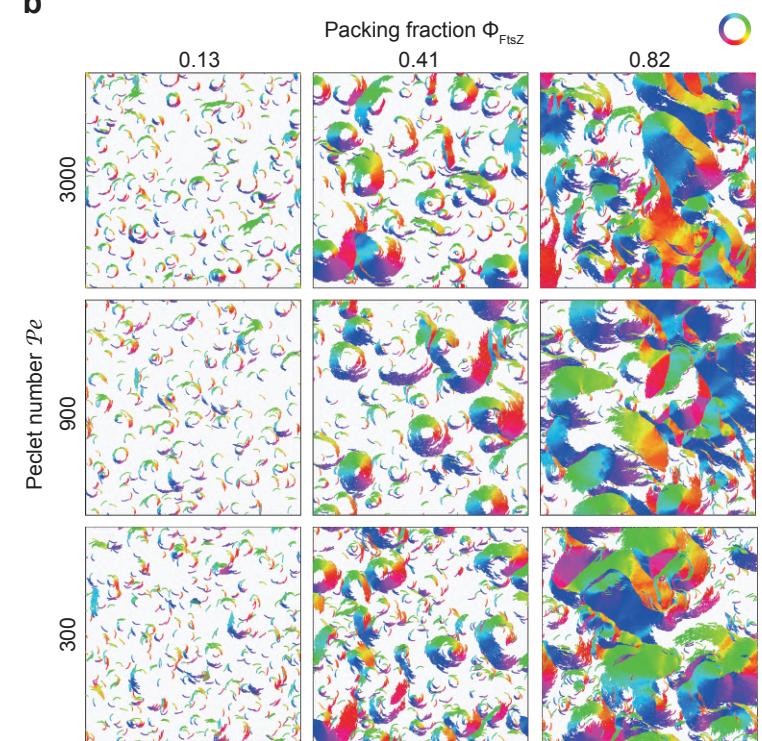


Supplementary Figure 2

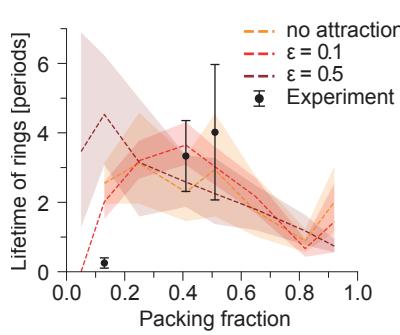
a



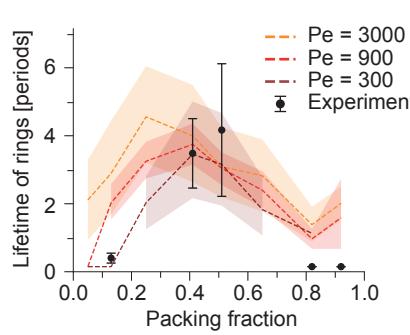
b



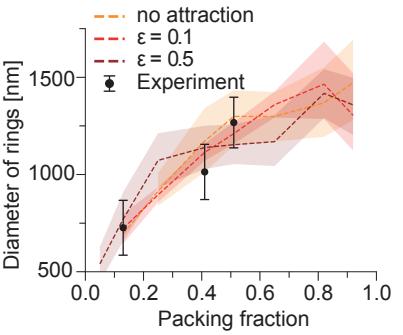
c



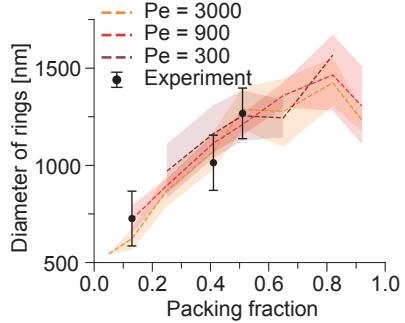
d



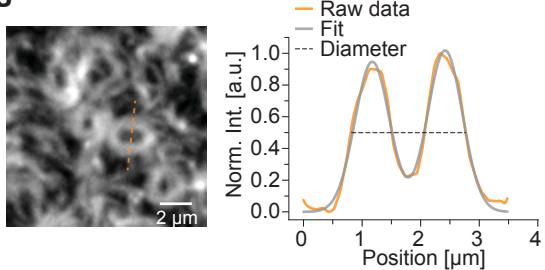
e



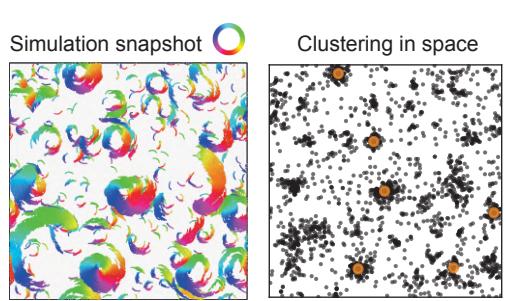
f



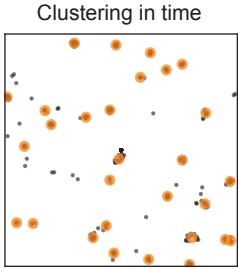
g



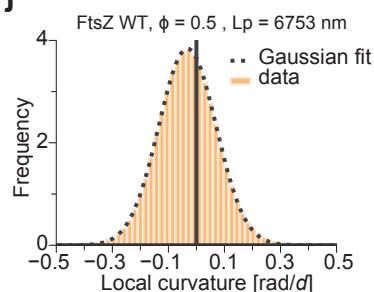
h



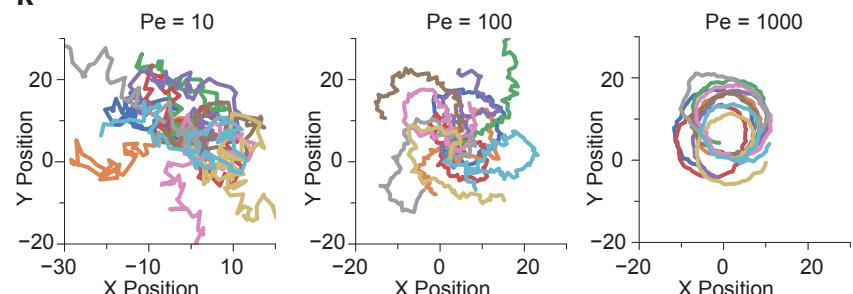
i



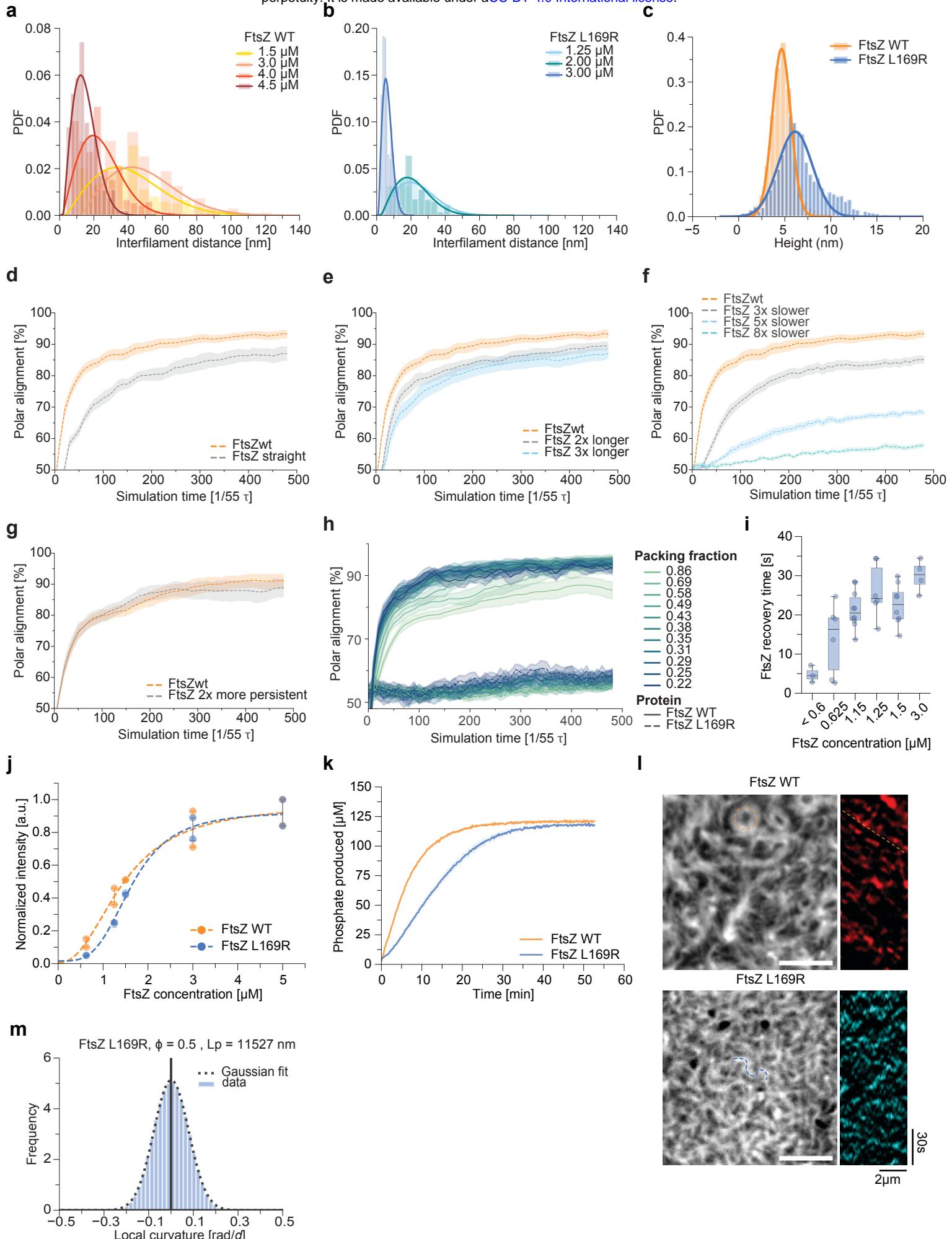
j



k



Supplementary Figure 3



Supplementary Figure 4