

# 1 Exploring the role of the outer 2 subventricular zone during cortical 3 folding through a physics-based 4 model

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10 **Abstract** The human brain has a highly complex structure both on the microscopic and  
11 macroscopic scales. Increasing evidence has emphasized the role of mechanical forces for cortical  
12 folding – a classical hallmark of the human brain. However, the link between cellular processes at  
13 the microscale and mechanical forces at the macroscale remains insufficiently understood.  
14 Recent findings suggest that an additional proliferating zone, the outer subventricular zone  
15 (OSVZ), is decisive for the particular size and complexity of the human cortex. To better  
16 understand how the OSVZ affects cortical folding, we establish a multifield computational model  
17 that couples cell proliferation and migration at the cell scale with growth and cortical folding at  
18 the organ scale by combining an advection-diffusion model with the theory of finite growth. We  
19 validate our model based on data from histologically stained sections of the human fetal brain.  
20 Finally, we address open questions regarding the role of the OSVZ for the formation of cortical  
21 folds. The presented framework not only improves our understanding of human brain  
22 development, but could eventually help diagnose and treat neuronal disorders arising from  
23 disruptions in cellular development and associated malformations of cortical development.

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## 25 **Introduction**

26 The brain is one of the most fascinating organs in the human body. Its complex structure on both  
27 micro- and macroscopic scales closely correlates with the unique cognitive abilities of humans.  
28 Cortical folding is one of the most important features of the human brain. Still, compared to other  
29 mammals, the human brain is neither the largest nor the most folded brain. However, relative  
30 to its size, it has the largest number of cortical neurons that connect with billions of neuronal  
31 synapses *Herculano-Houzel (2009)*. This fact attracted the attention of neuroscientists over the  
32 past few years to explore the source of these cells and how they develop in the early stages of  
33 brain development.

34 The number of brain cells is determined *in utero* through the proliferation process. Previous  
35 studies on different lissencephalic species such as mice have shown that cell division in the brain  
36 is confined to a small region near the cerebral ventricles *Hansen et al. (2010)*. However, in gyrencephalic  
37 species, this seems to be slightly different. Recent findings show that the human brain, for  
38 example, is characterized by two proliferation zones with two different types of progenitor cells.  
39 Both zones produce neurons that later migrate towards the outer brain surface and form the cor-

40 tex *Lui et al. (2011)*.

41 In rodents, progenitor cells around the ventricular zone (VZ) generate intermediate progenitor  
42 cells as their daughters, which accumulate above the ventricular zone and form a new layer called  
43 the subventricular zone (SVZ) *Noctor et al. (2002)*. In humans, there is an additional outer layer  
44 of the subventricular zone, often referred to as outer subventricular zone (OSVZ) *Hansen et al.*  
45 (*2010*); *Lui et al. (2011)*; *Noctor et al. (2007)*. This zone was first discovered in the monkey brain  
46 by Colette Dehay and his colleagues *Smart et al. (2002)*, and confirmed in the human brain by sev-  
47 eral following studies *Huttner and Kosodo (2005)*. The OSVZ seems to play a significant role in the  
48 proliferation process and affects the size and complexity of the human cortex. The evidence for  
49 this allegation is the wave of cortical neurogenesis that coincides with the cell division in the OSVZ  
50 *Lukaszewicz et al. (2005)*. At the macroscopic scale, the high proliferation in the OSVZ coincides  
51 with a significant tangential expansion of the cortical layers. The latter is an essential factor for  
52 the formation of cortical folds *Reillo et al. (2011)*. Still, it remains unknown, how exactly this prolif-  
53 eration process in the OSVZ affects gyration of the forming cortex. Different approaches have  
54 been used to understand the relation between cellular mechanisms at the microscopic scale and  
55 corticogenesis at the macroscopic scale. Genetic analyses and experimental studies using cell cul-  
56 ture models and brain organoids have given first valuable insights concerning the source of cells  
57 and their behavior *Hansen et al. (2010)*. Here, we intend to complement these studies by using a  
58 numerical approach to bridge the scales from the behavior of different progenitor cell types at the  
59 cell scale to the emergence of cortical folds at the tissue or organ scale.

60 From a mechanics point of view, forces that are generated due to cellular processes may act as  
61 a link to understand the underlying mechanisms behind cortical folding *Budday et al. (2015)*. Many  
62 previous studies tried to explain normal and abnormal cortical folding either from a purely biolog-  
63 ical or mechanical perspective. However, it will not be possible to capture the folding mechanism  
64 without considering both perspectives at the same time *Zarzor et al. (2021)*. In other words, to  
65 fully understand the physiological and pathological mechanisms underlying cortical folding in the  
66 developing human brain, we need to study the coupling between cellular processes and mechan-  
67 ical forces – to eventually assess how disruption of cellular processes affect the folding pattern  
68 and lead to malformations of cortical development *Guerrini et al. (2008)*; *Blumcke et al. (2021)*;  
69 *Llinares-Benadero and Borrell (2019)*.

70 To fill this knowledge gap, we establish a two-field computational model that accounts for both  
71 proliferating zones in the human brain. The first field in the model describes the cellular processes  
72 occurring during human brain development, where we use an advection-diffusion equation to  
73 mimic the migration in the subcortex and neuronal connectivity in the cortex *de Rooij and Kuhl*  
74 (*2018*); *Zarzor et al. (2021)*. We add two source terms to consider the division in both zones, VZ  
75 and OSVZ. Regarding the second field, we use the theory of finite growth. Finally, we validate the  
76 model through a comparison of the simulation results with histologically stained sections of the  
77 human fetal brain (HBS) with regard to the cell density distributions and surface morphology.

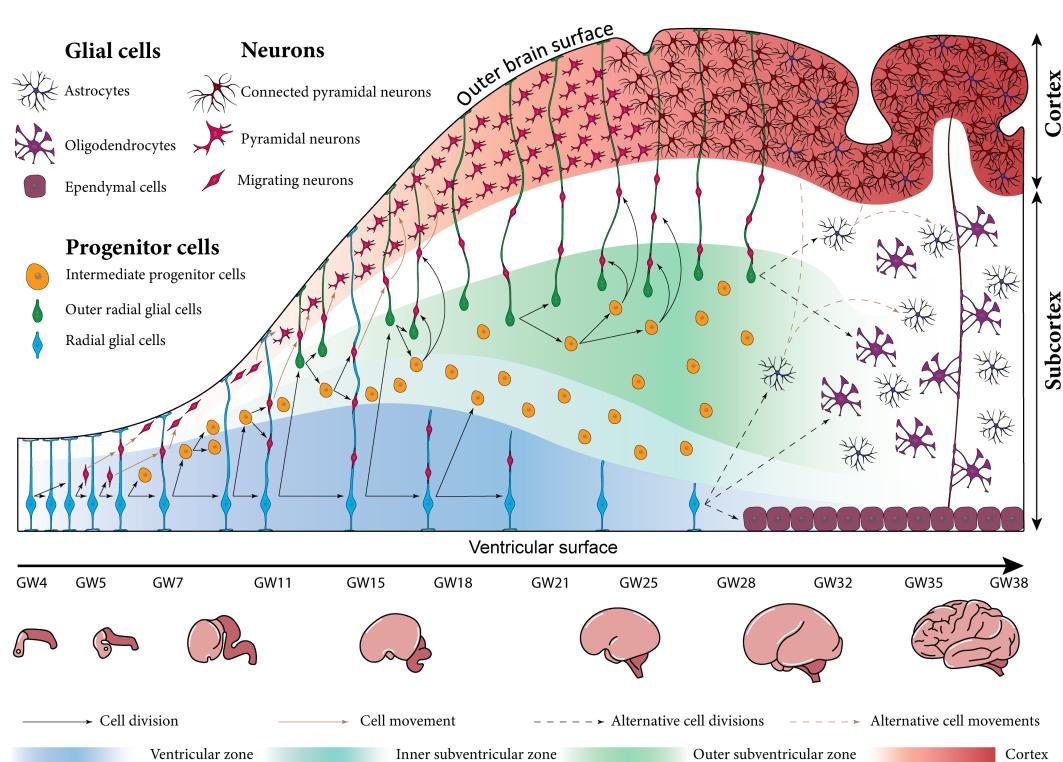
## 78 Cellular processes during brain development

79 The central cellular unit that plays a critical role in essential processes of brain development is a  
80 type of progenitor cells called radial glial cells (RGCs). In the early stage of brain development, when  
81 neurogenesis begins, the neuroepithelial cells transform into RGCs *Noctor et al. (2007)*. Around  
82 gestational week (GW) five, these cells locate near cerebral ventricles, in the ventricular zone (VZ),  
83 where they undergo interkinetic nuclear migration (INM). The associated symmetric division behav-  
84 ior leads to a significant increase in the number of RGCs and results in both increased thickness  
85 and surface area of the VZ *Blows (2003)*; *Fish et al. (2008)*; *Bystron et al. (2008)*. Subsequently, the  
86 cells switch to an asymmetric division behavior and generate intermediate progenitor cells (IPCs)  
87 *Noctor et al. (2004)*. The IPCs migrate to the subventricular zone (SVZ) where they proliferate and  
88 produce neurons, as illustrated in Figure 1 *Noctor et al. (2007)*; *Pebworth et al. (2021)*. Ultimately,

89 the majority of cortical neurons are produced by IPCs *Lui et al. (2011); Libé-Philippot and Vander-90 haeghen (2021)*.

91 According to the radial unit hypothesis proposed by Pasko Rakic over thirty years ago, the ra-92 dial glial cell fibers organize the migration process, which starts around GW six *Nonaka-Kinoshita93 et al. (2013)*. He postulated that these fibers form a scaffold to guide neurons during their mi-94 gration from the proliferating zones to their final destination in the cortex, which forms the outer brain95 surface *Rakic (1988); Lui et al. (2011)*. In the gyrencephalic species, those fibers have a characteris-96 tic fan-like distribution *Borrell and Götz (2014); Nonaka-Kinoshita et al. (2013)*. However, it is still97 under debate whether this unique distribution is the cause or rather the result of cortical folding.98 Our recent computational analyses support the latter, i.e., that it is the result of cortical folding99 *Zarzor et al. (2021)*. The migration process synchronizes with a radial expansion of all brain layers.100 Still, the VZ does not expand remarkably as the IPCs move out to the SVZ. The migrated neurons101 organize themselves in the six-layered cortex in an inside-out sequence, where the early-born neu-102 rons occupy the inner layers *Gilmore and Herrup (1997)*. Until GW 23, the outer brain surface is103 still smooth, although the bottom four layers of the cortex are already filled with neurons *Shinmyo104 et al. (2017)*. The first brain folds begin to form around GW 25 as the cortical layer significantly ex-105 pands tangentially *Budday et al. (2015)*. Importantly, at GW 25, the cortical neuronal connectivity106 emerges and comes along with the horizontal elongation of neuronal dendrites *Takahashi et al.107 (2012)*.

108 While the processes summarized above are common among mammals, the human brain has109 some specific features that play a significant role in increasing the number of cortical neurons,



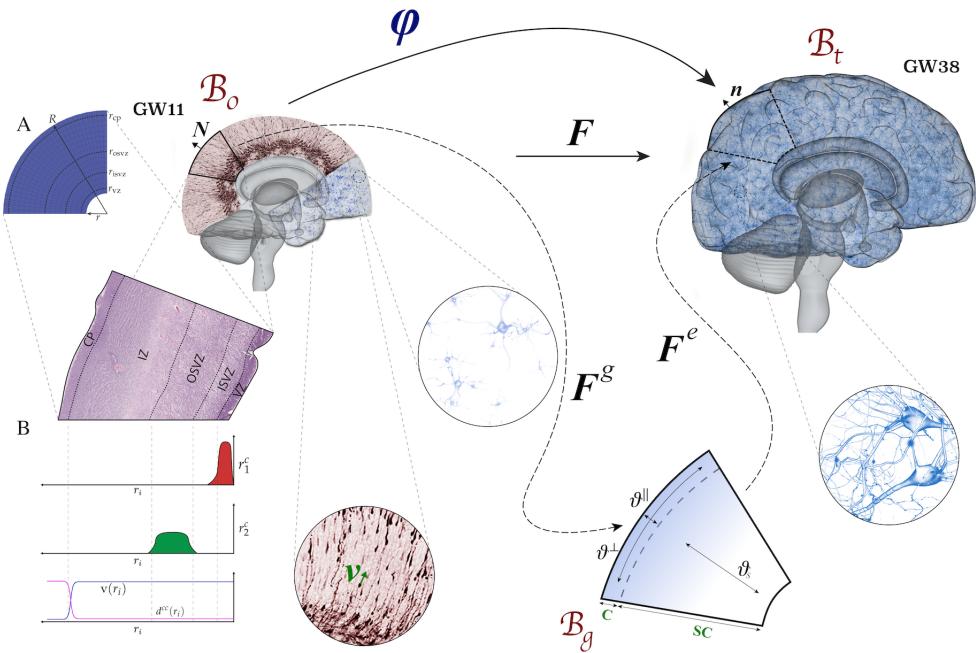
**Figure 1.** Schematic illustration of human brain development between gestational weeks 4 and 38 at the cellular scale (top) and the organ scale (bottom). In the early stage of development, the repetitive division of radial glial cells (RGCs) in the ventricular zone (VZ) significantly increases the total number brain cells. The newly born intermediate progenitor cells (IPCs) accumulate above the VZ and form a new layer called the inner subventricular zone (ISVZ). The outer radial glial cells (ORGs) that are produced around gestational week (GW) 11 form a new layer called the outer subventricular zone (OSVZ). The neurons generated from progenitor cells migrate along RGC fibers towards the cortex. Around GW 28, the migration process is almost finished, and the RGCs switch to produce different types of glial cells like astrocytes and oligodendrocytes.

110 and enhancing the complexity of cortical folds *Libé-Philippot and Vanderhaeghen (2021)*. At the  
111 beginning of the second trimester, around GW 11, the original RGCs switch from producing the IPCs  
112 to producing a special kind of cells that are found in all gyrencephalic species but are enriched in  
113 the human brain. The newly generated cells are similar to the RGCs in terms of shape and function,  
114 but unlike the original RGCs, they migrate to the outer layer of the subventricular zone (OSVZ) after  
115 they are born. Therefore, they are called outer radial glial-like cells (ORGs) *Lui et al. (2011); Fietz*  
116 *et al. (2010); Hansen et al. (2010); Reillo et al. (2011); Nonaka-Kinoshita et al. (2013)*. We would like  
117 to note that some literature refers to this type of cells as basal radial glial cells. While the original  
118 RGCs have a bipolar morphology with two processes – one extending to the cerebral ventricle and  
119 one to the outer cortical surface – the ORGs have a distinct unipolar structure with only a single  
120 process extending to the outer cortical surface *Hansen et al. (2010); Betizeau et al. (2013); Reillo*  
121 *et al. (2011); Nonaka-Kinoshita et al. (2013)*.

122 The OSVZ shows a significantly more pronounced radial expansion compared to the inner sub-  
123 ventricular zone (ISVZ) and VZ between GWs 11.5 and 32. The immediate reason causing this dif-  
124 ference is the characteristic division behavior of ORGs: they translocate rapidly in radial direction  
125 before they divide, which scientists refer to as "mitotic small translocation (MST)" *Fietz et al. (2010)*.  
126 Importantly, the MST behavior pushes the boundary of the OSVZ outward, which increases the ca-  
127 pacity of the OSVZ to produce new neurons. The IPCs have enough space to undergo multiple  
128 rounds of division before producing neurons, which increases the overall number of generated  
129 neurons *Kriegstein et al. (2006); Lui et al. (2011)*.

130 The ORGs, like RGCs, play an important role in the proliferation process: they divide symmet-  
131 rically and asymmetrically to produce further ORGs and IPCs *Libé-Philippot and Vanderhaeghen*  
132 *(2021)*. IPCs divide to generate a pair of neurons *Lui et al. (2011)*. According to previous studies,  
133 40% of produced neurons are generated by ORGs at GW 13, but this ratio increases to 60% by  
134 GW 14 and exceeds 75% by GW 15.5, then after GW 17, the ORGs become the only source of  
135 cortical neurons in the upper cortical layers *Hansen et al. (2010)*. Beside their role in increasing  
136 the number of neurons, the ORGs generate additional scaffolds that elongate to the outer brain  
137 surface and serve as paths for neuronal migration *Llinás-Benadero and Borrell (2019); Nonaka-*  
138 *Kinoshita et al. (2013)*. Compared to other mammals, the neurogenesis of the human cortex thus  
139 divides into two main stages. The first stage is characterized by migration along a continuous scaf-  
140 fold consisting of the RGC fibers, which run from the ventricular surface to the outer cortical layer  
141 around GW 15. During the second stage, the migration path switches to a discontinuous form. Af-  
142 ter GW 17, the RGC fibers run from the ventricular surface to the ISVZ, while ORG fibers run from  
143 the OSVZ to the outer brain surface, as demonstrated in Figure 1 *Nowakowski et al. (2016)*. Con-  
144 sequently, the migrating neuron follows a sinuous path through numerous radial fibers before it  
145 reaches its final location in the cortex *Lui et al. (2011)*. The newly generated scaffolds are not only  
146 important for neuronal migration, but also for the tangential expansion of the cortex. Previous  
147 studies show that a reduced number of ORGs leads to reduced tangential expansion. In these  
148 cases, the cortex is less folded or even lissencephalic *Poluch and Juliano (2015)*. In contrast, in-  
149 creasing the number of ORGs leads to more excessive folding *Florio et al. (2017); Borrell (2018)*.  
150 However, it is still unknown whether these effects are a result of the specific proliferation behavior  
151 of ORGs or the associated scaffold of ORG fibers. What is known, though, is that the existence of  
152 ORGs is a necessary but not sufficient condition for cortical folding *Llinás-Benadero and Borrell*  
153 *(2019)*.

154 Around GW 28, the migrating neurons occupy the first (top) cortical layer while the migration,  
155 and proliferation processes come to an end. After finishing their role during the neurogenesis  
156 stage, RGCs and ORGs switch to produce different types of glial cells, e.g., for astrocytes and  
157 oligodendrocytes *Schmeichel and Rakic (1979)*. Also, RGCs may convert later to ependymal cells  
158 that locate around the cerebral ventricles. The oligodendrocytes form myelin sheaths around neu-  
159 ronal axons, wherefore the subcortical layer gains its characteristic white color.



**Figure 2.** Kinematics of the multifield brain growth model. The reference configuration  $\mathcal{B}_0$  represents the initial state of the brain at gestational week (GW) 11. The spatial configuration  $\mathcal{B}_t$  represents the state of the brain at any time  $t$  during development. The stress-free (intermediate) growth configuration  $\mathcal{B}_g$  is inserted between reference and spatial configurations. (A) Simulation domain representing a part of the human brain's frontal lobe. (B) Distribution of model parameters ( $r_1^c, r_2^c, v$ , and  $d^{cc}$ ) along the brain's radial direction  $r_i$  from the ventricular surface to the outer cortical surface.

## 160 Computational model

161 To numerically study the effect of the ventricular zone (VZ) and the outer subventricular zone (OSVZ)  
 162 on the resulting folding pattern, we simulate human brain development by using the finite element  
 163 method. The influence of various factors on the emergence of cortical folds can be best shown on a  
 164 simple two-dimensional quarter-circular geometry *Darayi* (2021), as illustrated in Figure 2A. In the  
 165 following, we introduce the main equations describing the coupling between cellular mechanisms  
 166 in different proliferating zones and cortical folding, which we solve numerically.

## 167 Kinematics

168 To mathematically describe brain growth, we use the theory of nonlinear continuum mechanics  
 169 supplemented by the theory of finite growth. The initial state of the brain at an early stage of  
 170 development, around gestational week (GW) 11 is represented by the reference configuration  $\mathcal{B}_0$ .  
 171 The state of the brain at time  $t$  later during development is represented by the spatial configuration  
 172  $\mathcal{B}_t$ . The deformation map  $\mathbf{x} = \varphi(\mathbf{X}, t)$  maps a reference point  $\mathbf{X} \in \mathcal{B}_0 \subset \mathbb{R}^3$  to its new position  
 173  $\mathbf{x} \in \mathcal{B}_t \subset \mathbb{R}^3$  at a specific time  $t$ , as illustrated in Figure 2. The derivative of the deformation map  
 174 with respect to reference point position vector is called deformation gradient  $\mathbf{F} = \nabla_{\mathbf{X}} \varphi$ . The local  
 175 volume change of a volume element is described by the Jacobian  $J = \det \mathbf{F}$ .

176 Following the theory of finite growth, we introduce a stress-free configuration between refer-  
 177 ence and spatial configuration, the growth configuration  $\mathcal{B}_g$ . Accordingly, the deformation gradient  
 178 is multiplicatively decomposed into an elastic deformation tensor  $\mathbf{F}^e$  and a growth tensor  $\mathbf{F}^g$ , such  
 179 that,

$$180 \quad \mathbf{F} = \mathbf{F}^e \cdot \mathbf{F}^g. \quad (1)$$

181 The elastic deformation tensor describes the purely elastic deformation of the brain under the  
 182 effect of external forces or forces generated internally to preserve tissue continuity. On the other

183 hand, the growth tensor controls the amount and directions of unconstrained expansion. We note  
 184 that the elastic deformation tensor is reversible, while the growth tensor is not.

185 Besides the deformation map, we introduce the spatial cell density  $c(\mathbf{x}, t)$ , which is a scalar in-  
 186 dependent field that depends on the spatial point position and time. It represents the number of  
 187 cells per unit area *de Rooij and Kuhl (2018)*.

188 For the two unknown fields, the deformation and the cell density, we introduce appropriate  
 189 balance and constitutive equations in the following that then allow us to predict their evolution  
 190 in space and time through numerical simulations. In the following sections, we explain how we  
 191 mathematically describe the cellular processes and the mechanical problem. Then, we introduce  
 192 how those are linked through the growth problem.

### 193 Cell density problem

194 We formulate the balance equation of the cell density problem in such a way that we can math-  
 195 ematically describe the different cellular processes occurring at the microscopic scale. Temporal  
 196 changes in the cell density field are kept in balance by source and flux terms. The balance equation  
 197 given in the spatial configuration  $\mathcal{B}$ , follows as

$$198 \quad \frac{\dot{J}}{J} c + \dot{c} = -\nabla_{\mathbf{x}} \cdot [\hat{\mathbf{v}}(c, \mathbf{x}) c - \mathbf{d}^{cc}(\mathbf{x}) \cdot \nabla_{\mathbf{x}} c] + r_1^c(\mathbf{x}, s) + r_2^c(\mathbf{x}, s), \quad (2)$$

199 where the first flux term  $\hat{\mathbf{v}}(\mathbf{x}) c$  represents the migration in the subcortical plate, the second flux  
 200 term  $\mathbf{d}^{cc}(\mathbf{x}) \cdot \nabla_{\mathbf{x}} c$  represents the neuronal connectivity in the cortex, the first source term  $r_1^c(\mathbf{x}, s)$   
 201 represents cell proliferation in the VZ, and the second source term  $r_2^c(\mathbf{x}, s)$  cell proliferation in the  
 202 OSVZ. The migration velocity vector  $\hat{\mathbf{v}}(\mathbf{x})$  guides the cells along radial glial cell (RGC) fibers and  
 203 controls their speed,

$$204 \quad \hat{\mathbf{v}}(\mathbf{x}) = \mathcal{H}(c; \gamma) v(r_i) \mathbf{n} / \| \mathbf{n} \| . \quad (3)$$

205 The nonlinear regularized Heaviside function  $\mathcal{H}(c; \gamma)$  links the migration speed with the cell density  
 206 field, where  $\mathcal{H}(c; \gamma) = e^{\gamma[c - c_0]} / (1 + e^{\gamma[c - c_0]})$ . Accordingly, the cells start to migrate only when their den-  
 207 sity exceeds the critical threshold  $c_0$ . The value  $v$  specifies the maximum migration speed of each  
 208 individual cell in the domain. To ensure that this value vanishes smoothly at the cortex boundary  
 209  $r_{cp}$ , we formulate it as a function of the radial position  $r_i$ , as shown in Figure 2B, such that

$$210 \quad v(r_i) = v \left[ 1 - \frac{e^{10[r_i - r_{cp}]}}{1 + e^{10[r_i - r_{cp}]}} \right]. \quad (4)$$

211 The migration direction for each cell is determined by the norm vector  $\mathbf{n}$  that denotes the normal-  
 212 ized RGC fiber direction in the spatial configuration. After the cells reach the cortex, they diffuse  
 213 isotropically, where the diffusion tensor  $\mathbf{d}^{cc}(\mathbf{x}) = d^{cc}(r_i) \mathbf{I}$  with the diffusivity  $d^{cc}$  and the second or-  
 214 der unite tensor  $\mathbf{I}$  organizes this process. In addition, we introduce the diffusivity as a function of  
 215 the radial position  $r_i$  to act only in the cortex,

$$216 \quad d^{cc}(r_i) = d^{cc} \left[ \frac{e^{10[r_i - r_{cp}]}}{1 + e^{10[r_i - r_{cp}]}} \right]. \quad (5)$$

217 The first source term  $r_1^c$  represents the RGC proliferation in the VZ, as demonstrated in Figure  
 218 2B, and is given as

$$219 \quad r_1^c(\mathbf{x}, s) = G_{vz}^s(s) \left[ 1 - \frac{e^{50[r_i - r_{vz}]}}{1 + e^{50[r_i - r_{vz}]}} \right] \quad \text{with} \quad (6)$$

$$220 \quad G_{(\star)}^s(s) = G_{(\star)} - \begin{cases} (s - 1) G_{(\star)} & \text{if } s < 1.8 \\ 0.8 G_{(\star)} & \text{else} \end{cases} \quad (7)$$

221 where  $r_{vz}$  is the outer radial boundary of the VZ. By applying equation 7 for the VZ, we ensure that  
 222 the division rate decreases from its initial value  $G_{vz}$  to a smaller value with increasing maximum

223 stretch value  $s$  in the domain, i.e., with increasing gestational age. Besides the proliferation of  
 224 RGCs around cerebral ventricles in the VZ, the outer radial glial cells (ORGs) proliferate in the  
 225 OSVZ. To capture this effect, we add a second source term  $r_2^c$ , as demonstrated in Figure 2B. The  
 226 second source term is given as

$$227 \quad r_2^c(\mathbf{x}, s) = G_{\text{osvz}}^s(s) \left[ \frac{e^{50[r_i - r_{\text{isvz}}]}}{1 + e^{50[r_i - r_{\text{isvz}}]}} - \frac{e^{50[r_i - r_{\text{osvz}}(t)]}}{1 + e^{50[r_i - r_{\text{osvz}}(t)]}} \right], \quad (8)$$

228 where  $r_{\text{isvz}}$  is the outer radial boundary of the inner subventricular zone (ISVZ). To numerically cap-  
 229 ture the expansion of the OSVZ under the effect of mitotic small translocations (MST) of ORGs, we  
 230 formulate the outer radial boundary of the OSVZ as a function to time, such that,  
 231  $r_{\text{osvz}} = r_{\text{isvz}} + m_{\text{mst}} t$ , where  $m_{\text{mst}}$  is introduced as the MST factor. Again, we apply equation 7 for  
 232 the OSVZ, but in this case with the initial division rate  $G_{\text{osvz}}$ .

### 233 Mechanical problem

234 To govern the mechanical problem, we use the balance of linear momentum given in the spatial  
 235 configuration  $\mathcal{B}_t$ ,

$$236 \quad \nabla_{\mathbf{x}} \cdot \boldsymbol{\sigma} = \mathbf{0} \quad \text{with} \quad \boldsymbol{\sigma} = \boldsymbol{\sigma}(\mathbf{F}^e), \quad (9)$$

237 where  $\nabla_{\mathbf{x}}$  is the spatial gradient operator and  $\boldsymbol{\sigma}$  is the Cauchy stress tensor formulated in terms  
 238 of the elastic deformation tensor. The Cauchy stress is computed by deriving the strain energy  
 239 function  $\psi_g$  with respect to elastic deformation tensor,

$$240 \quad \boldsymbol{\sigma}(\mathbf{F}^e) = \frac{1}{J^e} \frac{\partial \psi_g(\mathbf{F}^e)}{\partial \mathbf{F}^e} \cdot \mathbf{F}^{eT}, \quad (10)$$

241 where  $J^e = \det \mathbf{F}^e$ . The strain energy function describes the material behavior of brain tissue  
 242 mathematically. In our case, we consider a nonlinear hyperelastic material model as viscous ef-  
 243 fects, which have been observed for higher strain rates, become less relevant in the case of the  
 244 slow process of brain development occurring over the course of weeks and months. Our previous  
 245 studies have shown that the isotropic neo-Hookean constitutive best represents the material be-  
 246 havior of brain tissue during cortical folding **Budday et al. (2020)**. The corresponding strain energy  
 247 function  $\psi_g$  is given as

$$248 \quad \psi_g(\mathbf{F}^e) = \frac{1}{2} \lambda \ln^2(J^e) + \frac{1}{2} \mu(r_i) [\mathbf{F}^e : \mathbf{F}^e - 3 - 2\ln(J^e)], \quad (11)$$

249 where  $\mu$  and  $\lambda$  are the Lamé parameters. As there is a smooth transition from the cortex to the  
 250 subcortical plate with distinct mechanical parameters, we use the following function

$$251 \quad \mu(r_i) = \mu_s + \left[ [\mu_c - \mu_s] \times \left( \frac{e^{20[r_i - r_{cp}]} - 1}{1 + e^{20[r_i - r_{cp}]}} \right) \right]. \quad (12)$$

252 Our recent simulation study suggested that the cortical stiffness continuously changes during hu-  
 253 man brain development due to the changes in the local microstructure **Zarzor et al. (2021)**. Accord-  
 254 ingly, we formulate the cortical shear modulus  $\mu_c$  as a function of the cell density,

$$255 \quad \mu_c(c) = \begin{cases} \mu_{\infty} & \text{if } c \geq c_{\max}, \\ \mu_s + m_c(c - c_{\min}) & \text{if } c_{\max} > c > c_{\min}, \\ \mu_s & \text{if } c \leq c_{\min}. \end{cases} \quad (13)$$

256 It varies in the range  $\mu_c(c) \in [\mu_s, \mu_{\infty}]$ , while the subcortical shear modulus  $\mu_s$  remains constant.  
 257 The slope is defined as  $m_c = \mu_{\infty} - \mu_s / c_{\max} - c_{\min}$  and the stiffness ratio as  $\beta_{\mu} = \mu_{\infty} / \mu_s$ .

258 **Mechanical growth problem**

259 The growth tensor introduced in the **Kinematics** section is the key feature in our model that links  
 260 the cell density problem with the mechanical problem. As it controls the amount and direction of  
 261 growth, we need to consider how cellular processes affect the physiological growth behavior in  
 262 order to find an appropriate formulation. During cellular migration, the subcortical layers expand  
 263 isotropically. Then, under the effect of neuronal connectivity, the cortex grows – more pronounced  
 264 in circumferential than in radial direction – as illustrated in Figure 2. Thus, we introduce the growth  
 265 tensor as

266 
$$\mathbf{F}^g = \vartheta^\perp [\mathbf{I} - \mathbf{N} \otimes \mathbf{N}] + \vartheta^\parallel \mathbf{N} \otimes \mathbf{N}, \quad (14)$$

267 where  $\mathbf{N}$  is the norm vector in the reference configuration  $\mathcal{B}_0$  (it is linked to the spatial norm vector  
 268 through  $\mathbf{N} = \mathbf{F}^{-1} \cdot \mathbf{n}$ ), while  $\vartheta^\perp$  and  $\vartheta^\parallel$  denote the growth multipliers in circumferential and radial  
 269 direction, respectively. Those multipliers control the amount of growth as a function of the cell  
 270 density,

271 
$$\vartheta^\perp = [1 + \kappa^\perp(r_i)c]^\alpha \quad \text{and} \quad \vartheta^\parallel = [1 + \kappa^\parallel(r_i)c]^\alpha, \quad (15)$$

272 where  $\kappa^\perp$  and  $\kappa^\parallel$  are the growth factors in the circumferential and radial direction, respectively, and  
 273  $\alpha$  is the growth exponent. To ensure isotropic growth in the subcortical layers, we formulate those  
 274 factors as a function of the radius  $r_i$ , such that

275 
$$\kappa^\perp(r_i) = \kappa_s + \left[ \kappa_s \left[ \beta_\kappa - 1 \right] \left( \frac{e^{20[r_i - r_{cp}]}}{1 + e^{20[r_i - r_{cp}]}} \right) \right] \quad \text{and} \quad (16)$$

276 
$$\kappa^\parallel(r_i) = \kappa_s + \left[ \kappa_s \left[ \frac{1}{\beta_\kappa} - 1 \right] \left( \frac{e^{20[r_i - r_{cp}]}}{1 + e^{20[r_i - r_{cp}]}} \right) \right], \quad (17)$$

277 where  $\kappa_s$  is the growth factor in the subcortical layers, and  $\beta_\kappa$  is the growth ratio between  $\kappa^\perp$  and  
 278  $\kappa_s$  **Zarzor et al. (2021)**.

**Table 1.** Model parameters.

| Geometry parameters                   |              |                         | Cell density problem parameters |                                      |                |   |
|---------------------------------------|--------------|-------------------------|---------------------------------|--------------------------------------|----------------|---|
| parameter                             | value        | unit                    | parameter                       | value                                | unit           |   |
| Outer brain radius                    | $R$          | 2                       | mm                              | division rate in VZ                  | $G_{VZ}$       | [30-120] $\text{mm}^{-2} \text{d}^{-1}$ |
| inner brain radius                    | $r$          | 0.4                     | -                               | division rate in OSVZ                | $G_{OSVZ}$     | [10-30] $\text{mm}^{-2} \text{d}^{-1}$  |
| VZ radius                             | $r_{VZ}$     | 0.5                     | mm                              | migration speed                      | $v$            | 5 $\text{mm d}^{-1}$                    |
| ISVZ radius                           | $r_{ISVZ}$   | 0.8                     | mm                              | migration threshold                  | $c_0$          | 500 $\text{mm}^{-2}$                    |
| cortex radius                         | $r_{cp}$     | 1.8                     | mm                              | Heaviside exponent                   | $\gamma$       | 0.008 -                                 |
| MST factor                            | $m_{mst}$    | 0.02 $\text{mm d}^{-1}$ |                                 | diffusivity                          | $d^{cc}$       | 0.11 $\text{mm}^2 \text{d}^{-1}$        |
| Mechanical elastic problem parameters |              |                         |                                 | Mechanical growth problem parameters |                |   |
| parameter                             | value        | unit                    | parameter                       | value                                | unit           |   |
| cortex shear modulus                  | $\mu_\infty$ | 2.07                    | kPa                             | growth parameter                     | $\kappa_s$     | $4.07e^{-4}$ $\text{mm}^2$              |
| Poisson ratio                         | $\nu$        | 0.38                    | -                               | growth exponent                      | $\alpha$       | 1.65 -                                  |
| stiffness ratio                       | $\beta_\mu$  | 3,8                     | -                               | growth ratio                         | $\beta_\kappa$ | 1.5,3 -                                 |
| maximum threshold                     | $c_{max}$    | 700                     | $\text{mm}^{-2}$                |                                      |                |   |
| minimum threshold                     | $c_{min}$    | 200                     | $\text{mm}^{-2}$                |                                      |                |   |

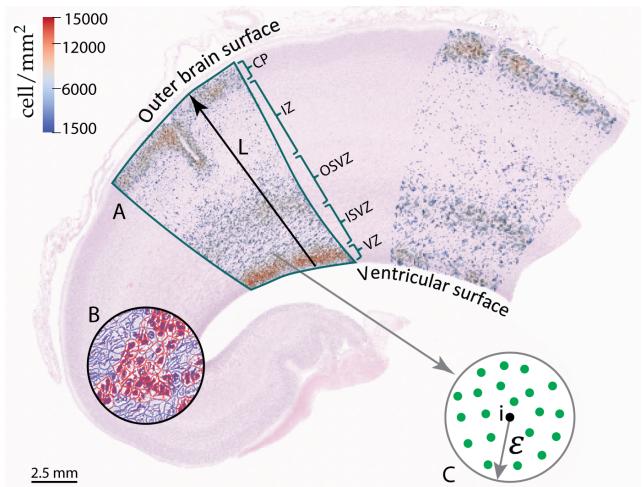
## 279 Model parameters

280 In this work, we will consider two different cases regarding the mechanical model: The first case  
281 considers a varying cortical stiffness (VS) as introduced in the **Mechanical elastic problem** section,  
282 while the second case assumes a constant cortical stiffness (CS), i.e.,  $\mu_c = \mu_\infty = \text{constant}$ . While our  
283 previous study had suggested that the simulations with varying cortical stiffness lead to morpholo-  
284 gies that better agree with those in the actual human brain *Zarzor et al. (2021)*, we still consider  
285 both cases in the following, VS and CS, as the situation might change when including the OSVZ and  
286 we aim to investigate corresponding interdependency effects. Table 1 summarizes the model pa-  
287 rameters that are used in the simulation. We have previously thoroughly studied the effect of the  
288 stiffness ratio on the resulting folding pattern *Zarzor et al. (2021)*. Here, we choose a stiffness ratio  
289 of 8 for the constant stiffness case and a ratio of 3 for the varying stiffness case. Those values led  
290 to the best agreement of simulation results with data from stained histological sections regarding  
291 the local gyration index (LGI) value and the thickness ratio between gyri and sulci. For more  
292 details, we refer to *Zarzor et al. (2021)*. We note that the tissue shows a stiffer behavior in the case  
293 of constant stiffness than in the case of varying stiffness for the same value of the stiffness ratio.  
294 For that reason, a higher stiffness ratio (lower stiffness in the subcortical layers since the final cor-  
295 tical stiffness  $\mu_\infty$  is constant in both cases) is required in the case of constant stiffness to achieve a  
296 similar level of folding.

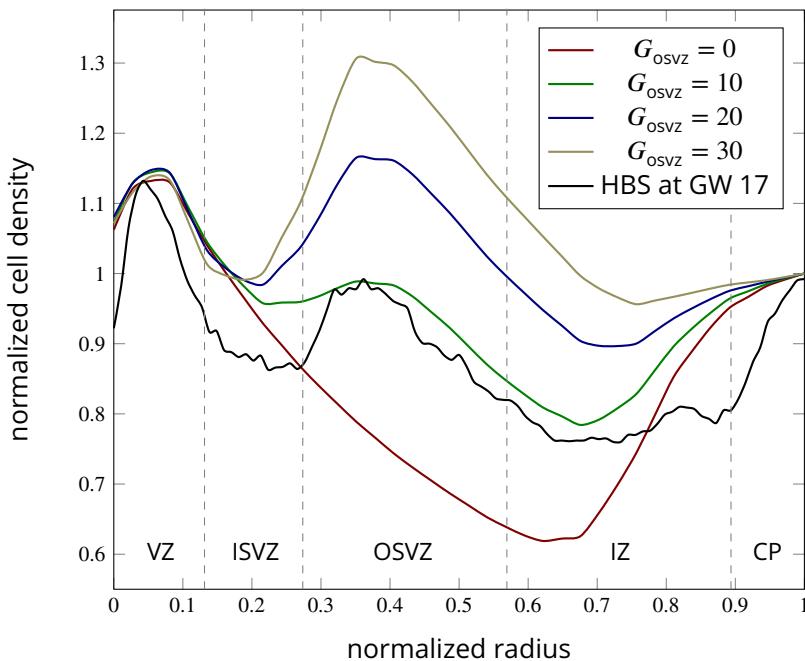
## 297 Model validation

298 We validate our computational model by comparing the simulation results with histologically stained  
299 sections of the human fetal brain (HBS). For details on the corresponding preparation and  
300 staining, we refer to *Zarzor et al. (2021)*. The sections belong to  
301 human fetuses aborted at gesta-  
302 tional weeks (GW) 17, 24, 30, and  
303 34. After defining the areas that  
304 are representative of the sim-  
305 ulation domain introduced in the  
306 previous section (see Figure 2A)  
307 for each GW, we detect and as-  
308 sess the cell density using the soft-  
309 ware *Qupath*. This study was ap-  
310 proved by the ethics review board  
311 of the University of Erlangen, and  
312 all procedures were conducted in  
313 accordance with the Declaration  
314 of Helsinki. In the following, we  
315 summarize the steps we followed  
316 to determine the cell density in  
317 human fetal brain sections (HBS).  
318 Since this analysis is intended to  
319 mainly serve as a means of com-  
320 parison between different gesta-  
321 tional weeks, a more detailed analysis using individual markers was not necessary at this stage.

323 (1) Identify regions of interest to which the analysis should be applied, as shown in Figure 3A.  
324 (2) Pre-process the annotated area to ensure better cell detection through image color transfer,  
325 contrast ratio modification, and stain vector settings.  
326 (3) Automatically detect the cells in the relevant area by using the "positive cell detection" com-  
327 mand in *Qupath*, which distinguishes between cell types according to the staining. Here, the



**Figure 3.** Part of the frontal lobe of a histologically stained section of the human fetal brain at gestational week (GW) 17. (A) Annotated area with final cell density distribution. (B) Example of cell detection by using *Qupath*: red cells depict neurons and blue cells glial cells. (C) Procedure to determine the cell density.



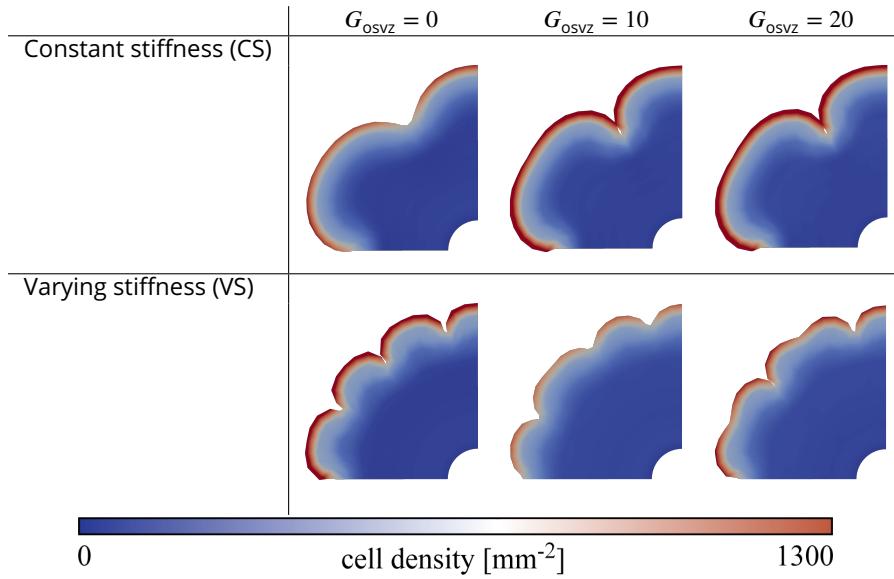
**Figure 4.** Evolution of the normalized cell density in the normalized radial direction from the ventricular surface to the outer cortical surface for numerical simulations and histologically stained human brain sections (HBS) at gestational week (GW) 17. The simulation results correspond to the time-varying cortical stiffness case with a stiffness ratio of 3, and a ventricular zone division rate of 120. The HBS results correspond to line L in Figure 3.

negative cells (blue) are mostly glial cells or extracellular matrix components, while positive cells (red) are mostly neurons or progenitor cells, as shown in Figure 3B. While this distinction between cell types might not be fully accurate, the results are satisfactory for our specific application.

- (4) Count the nearby detections in a small circle with an arbitrarily chosen radius  $\epsilon = 100 \mu\text{m}$  around each detected positive cell (i) to determine the corresponding cell density by dividing the number of nearby detections by the circular area, as demonstrated in Figure 3C.
- (5) Visualize the cell density, as illustrated in Figure 3A.

Figure 3 shows the cell density distribution around GW 17 and demonstrates the densely packed ventricular zone (VZ) due to the high proliferation rate of radial glial cells (RGCs), with about 13 600 cells/mm<sup>2</sup>. In addition, we can locate the other zones introduced in Figure 1. The outer subventricular zone (OSVZ) shows a higher cell density (approximately 9550 cell/mm<sup>2</sup>) than both the inner subventricular (ISVZ) and the intermediate zone (IZ). This zone is composed of several types of cells, the original intermediate progenitor cells (IPCs) that migrated from the VZ, migrated neurons, ORGCs that produce more IPCs, and newborn neurons that are produced through the IPCs' asymmetric division. Our analysis of the HBS (see Figure 3) also shows that the OSVZ is 1.5 times thicker than the ISVZ, even though it did not emerge at an earlier stage of development (the ISVZ emerges around GW 7 and the OSVZ around GW 11). This implies that the thickness of the OSVZ increases with time. The IZ is characterized by a low cell density with about 1600 cells/mm<sup>2</sup>. Still, it is a transit area for the migrating neurons. The higher cell density in the ISVZ 4800 cells/mm<sup>2</sup> corroborates the presence of another type of cell besides migrating neurons, i.e. IPCs. The migration process in GW 17 is still ongoing – the cortex is not yet fully developed and filled with neurons with only about 12 000 cells/mm<sup>2</sup>. Therefore, at this stage of development, the VZ still has the highest cell density in the brain.

Figure 4 compares the cell density distribution along a line from the ventricular surface to the outer brain surface in the human fetal brain at GW 17 (indicated by line L in Figure 3) with different



**Figure 5.** Final folding patterns at gestational week (GW) 36 for different values of the division rate in the outer subventricular zone (OSVZ)  $G_{osvz}$  for the constant (CS, top) and time-varying (VS, bottom) cortical stiffness cases. The remaining parameters are fixed as follows: division rate in the ventricular zone  $G_{vz} = 120$ , and stiffness ratio  $\beta_\mu = 8$  for CS and 3 for VS.

354 simulation results. For better comparability and to avoid differences in the dimensions between  
355 the HBS and simulation domain, we normalize the domain's radius according to the extension from  
356 the ventricular to the outer brain surface in the HBS. In addition, we normalize the cell density with  
357 respect to their maximum value in the cortex. According to our previous work **Zarzor et al. (2021)**,  
358 we include a varying stiffness in the cortex during human brain development in our simulations,  
359 adopt a stiffness ratio of 3, and a division rate in the VZ of 120.

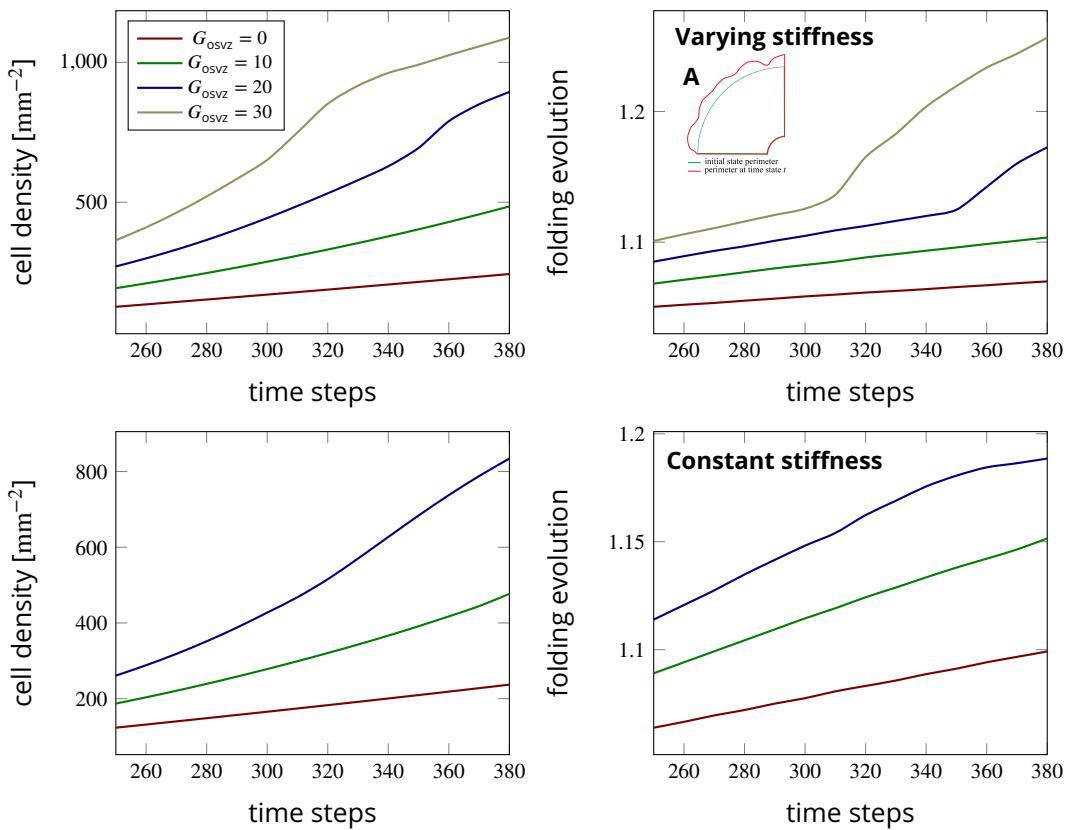
360 The simulation results for an initial division rate in the OSVZ of  $G_{osvz} = 10$  well capture the trends  
361 observed in the HBS. The cell density shows a first local peak representing the VZ for a normalized  
362 radius of approximately 0.05, which is an accurate prediction. It then gradually decreases to reach  
363 its first local minimum in the ISVZ for a normalized radius of 0.3. This effect is less pronounced  
364 in the simulations than in the actual human brain. The curves start to rise again in the OSVZ to  
365 reach the second peak for a normalized radius of 0.4. Again, the simulation results capture this  
366 peak quite accurately. The second local minimum represents the IZ for a normalized radius of 0.7,  
367 while the third peak represents the cortex. The simulation results for  $G_{osvz} = 20$  and 30 result in  
368 a higher cell density in the OSVZ than in the actual fetal human brain. In contrast, the curve for  
369  $G_{osvz} = 0$  (i.e., without including the effect of the OSVZ) shows a significantly decreased cell density  
370 in the OSVZ and IZ.

### 371 **Results and discussion**

372 In this section, we apply our computational model to answer some of the major questions regarding  
373 the role of the outer subventricular zone (OSVZ) for cortical folding during human brain development.  
374 The model parameters used here are introduced in the **Model parameters** section. As  
375 we mentioned above, we consider both cases constant (CS) and varying cortical stiffness (VS).

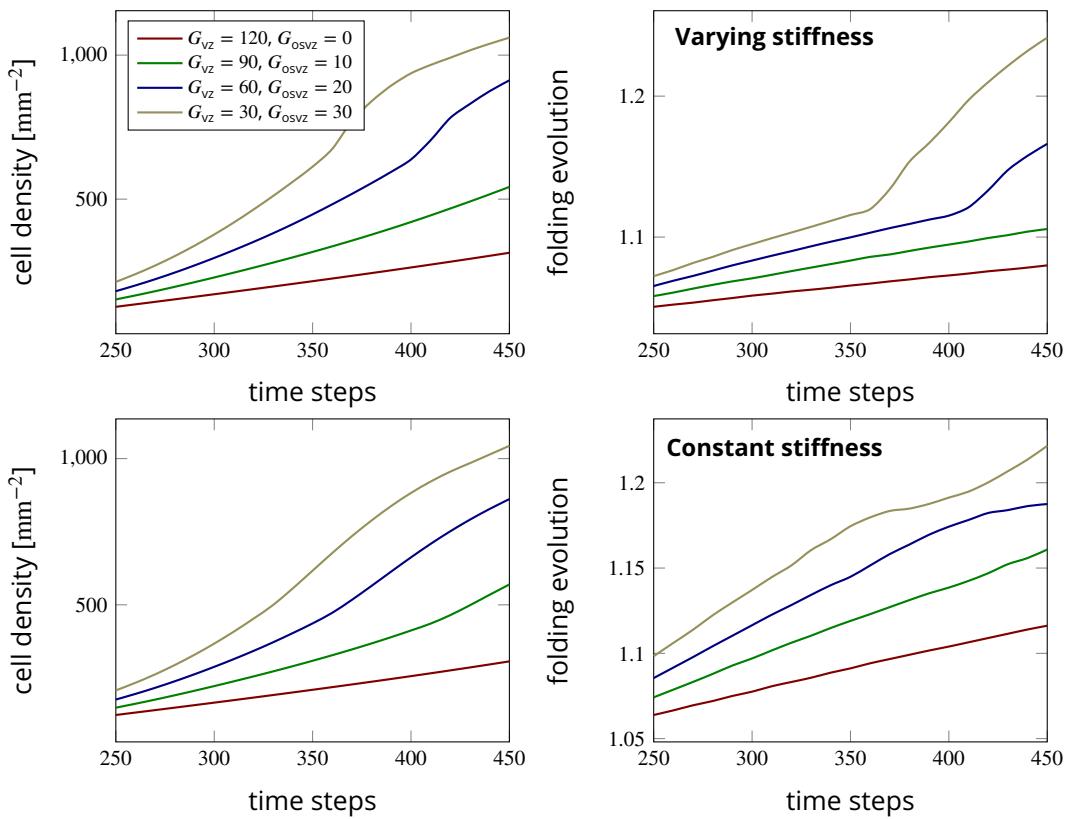
### 376 **How does cell proliferation in the OSVZ affect cortical folding patterns?**

377 Many previous studies have tried to address this point **Hansen et al. (2010)**. However, the experimental  
378 approach does not give a real answer to this question. In our model, we can apply different  
379 values of the division rate in OSVZ ( $G_{osvz}$ ) to show how the ORGCs proliferation reflects on cortical  
380 folding. Figure 5 shows the folding patterns emerging at gestational week (GW) 36 for both varying



**Figure 6.** Temporal evolution of the maximum cell density and the folding evolution (the current outer perimeter divided by the initial perimeter as indicated in subfigure A) at a constant division rate in the ventricular zone (VZ)  $G_{VZ} = 120$  and different initial division rates in the outer subventricular zone (OSVZ)  $G_{OSVZ}$ . The results in the top row correspond to the varying cortical stiffness case with a stiffness ratio of 3. The results in the bottom row correspond to the constant cortical stiffness case with a stiffness ratio of 8.

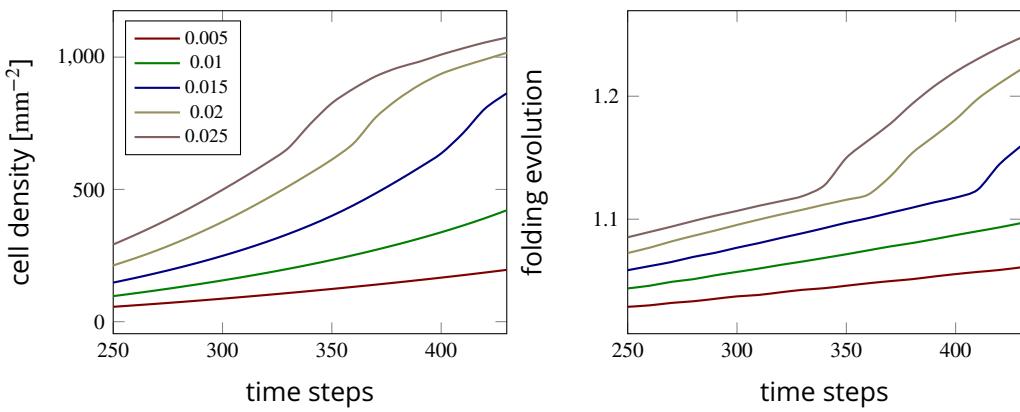
381 (VS) and constant (CS) stiffness cases and different values of the initial division rate in the OSVZ  
 382  $G_{OSVZ}$ . In the case of CS, the sulcus becomes deeper with increasing division rate. In the case of VS,  
 383 there is a more noticeable change in the folding patterns depending on the division rate  $G_{OSVZ}$ . In  
 384 general, the distance between neighboring sulci decreases with increasing  $G_{OSVZ}$ . In addition, we  
 385 observe period-doubling patterns emerge, which are most pronounced at a value of  $G_{OSVZ} = 20$ .  
 386 This indicates that the proliferation in the OSVZ enhances secondary mechanical instabilities and  
 387 leads to more complex folding patterns. Besides the direct relation between the proliferation in the  
 388 OSVZ and the folding morphology, there are indirect effects and other aspects concerning outer  
 389 radial glial cell (ORG) proliferation, which will be discussed in more detail in the following sections.  
 390 In the case of CS, the sulcus becomes deeper with increasing division rate. In the case of VS, there  
 391 is a more noticeable change in the folding patterns depending on the division rate  $G_{OSVZ}$ . In general,  
 392 the distance between neighboring sulci decreases with increasing  $G_{OSVZ}$ . In addition, we observe  
 393 period-doubling patterns emerge, which are most pronounced at a value of  $G_{OSVZ} = 20$ . This indi-  
 394 cates that the proliferation in the OSVZ enhances secondary mechanical instabilities and leads to  
 395 more complex folding patterns. Besides the direct relation between the proliferation in the OSVZ  
 396 and the folding morphology, there are indirect effects and other aspects concerning outer radial  
 397 glial cell (ORG) proliferation, which will be discussed in more detail in the following section.



**Figure 7.** Temporal evolution of the maximum cell density and the folding evolution for different division rates in the ventricular zone  $G_{VZ}$  and outer subventricular zone  $G_{OSVZ}$ . The results in the top row correspond to the time-varying cortical stiffness case with a stiffness ratio of 3. The results in the bottom row correspond to the constant cortical stiffness case with a stiffness ratio of 8.

**398 How does cell proliferation in the OSVZ affect the cell density and folding evolution?**

**399** After we have seen the effect of cell proliferation in the OSVZ on the final folding pattern, we investi-  
**400** giate its effect on the evolution of both cell density and folding morphology. Figure 6 shows the  
**401** temporal evolution of the maximum cell density in the domain and the folding evolution between  
**402** time steps 260 and 380 for different initial division rates in the OSVZ  $G_{OSVZ}$  and a constant division  
**403** rate in the VZ  $G_{VZ} = 120$ . Again, we consider both cases constant and varying cortical stiffness. The  
**404** folding evolution value quantifies the ratio between the outer perimeter at time step  $t$  and the ini-  
**405** tial perimeter as demonstrated in Figure 6A. Increasing the initial division rate in the OSVZ leads  
**406** to a significant increase in both the cell density and folding evolution. Consequently, for the case  
**407** of  $G_{OSVZ} = 30$ , the cell density reaches the highest value of  $1100 \text{ mm}^{-2}$  corresponding to a folding  
**408** evolution of 1.25. For the case of  $G_{OSVZ} = 0$ , in contrast, the cell density does not even exceed the  
**409** migration threshold. The kink observable at time step 360 for the varying cortical stiffness case and  
**410** an initial division rate in the OSVZ  $G_{OSVZ} = 20$  indicates that the mechanical instability has occurred,  
**411** i.e., the first gyri and sulci start to appear. Comparing the cases  $G_{OSVZ} = 20$  and  $G_{OSVZ} = 30$  shows that  
**412** the instability occurs earlier with increasing initial division rate in the OSVZ. Thus, the ORGC prolif-  
**413** eration decreases the time required to reach the final folding pattern. In general, the differences  
**414** between the results for the constant and varying cortical stiffness cases are minor. However, the  
**415** curves for the varying cortical stiffness case rise faster than for the constant case. We note that  
**416** we could not generate results for the initial division rate in the OSVZ  $G_{OSVZ} = 30$  and the constant  
**417** cortical stiffness case due to numerical issues.



**Figure 8.** Temporal evolution of the maximum cell density and the folding evolution for different values of the mitotic small translocation (MST) factor. The results correspond to the varying cortical stiffness case with a stiffness ratio of 3, a division rate in the ventricular zone (VZ) of  $G_{VZ} = 30$ , and an initial division rate in the outer subventricular zone (OSVZ) of  $G_{OSVZ} = 30$ .

**419 Which proliferation zone is more influential for fetal human brain development?**

**420** To answer the question whether one of the proliferation zones (in the VZ or OSVZ) is more impor-  
**421** tant for cellular brain development and cortical folding, we have implemented four sets of division  
**422** rates for both the VZ and the OSVZ. The first set assumes a high division rate in the VZ  $G_{VZ}$  and a low  
**423** initial division rate in the OSVZ  $G_{OSVZ}$ . For the following parameter sets, we gradually decrease  $G_{VZ}$ ,  
**424** while we increase  $G_{OSVZ}$ . Figure 7 shows the corresponding results for the maximum cell density  
**425** in the domain and the folding evolution between time steps 250 and 450 for both cases constant  
**426** and varying cortical stiffness. Unexpectedly, the cell density for the set ( $G_{VZ} = 30$ ,  $G_{OSVZ} = 30$ ) rises  
**427** faster than the one for the set ( $G_{VZ} = 120$ ,  $G_{OSVZ} = 0$ ). Indeed, not only the cell density is affected  
**428** by increasing the division rate in the OSVZ (at the expense of decreasing it in the VZ), but also con-  
**429** volutions (cortical folds) appear earlier, as illustrated in the curve for the folding evolution (Figure  
**430** 7, right). Our results thus indicate an unproportionally strong effect of ORGC proliferation in the  
**431** OSVZ on cellular brain development and cortical folding. We attribute this observation to the the  
**432** larger volume occupied by the OSVZ compared to the VZ. Concerning the difference between the  
**433** results for the constant and varying cortical stiffness cases, the curves for a varying stiffness rise  
**434** faster than for a constant stiffness.

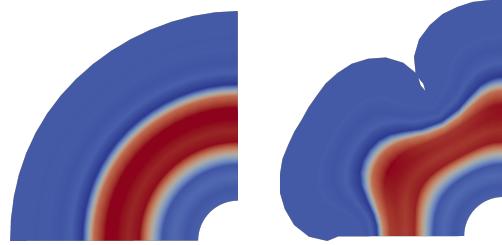
**435 How does the mitotic small translocation behavior of ORGCs affect brain develop-  
436 ment?**

**437** The final simulation parameter we would like to study is the MST factor that mimics the mitotic  
**438** small translocation behavior of ORGCs. For this parameter study, we limit ourselves to the varying  
**439** cortical stiffness case with a division rate in the VZ of  $G_{VZ} = 30$  and an initial division rate in the  
**440** OSVZ of  $G_{OSVZ} = 30$ . Figure 8 shows the temporal evolution of the maximum cell density and the  
**441** cortical folding evolution between time steps 250 and 430 for different values of the MST factor.  
**442** Our simulations show that with increasing MST factor, the value of the maximum cell density in  
**443** the domain increases exponentially. Since the MST factor incorporates the expansion of the OSVZ  
**444** with time, these results are consistent with the observation in the previous section showing an  
**445** increasing volume space of the OSVZ.

**446 Is the OSVZ affected by cortical folding?**

**447** After we have discussed the effect of the OSVZ on the formation of cortical folds, we will now discuss  
**448** the opposite – the effect of cortical folding on the OSVZ. In the intermediate stage of human brain  
**449** development, before cortical folds emerge, the OSVZ has a constant thickness throughout the  
**450** entire domain, as demonstrated in Figure 9, left, where the OSVZ appears in red color. However,

451 according to our simulations, this quickly changes after the first folds start to emerge. The thickness  
452 of the OSVZ starts to vary and becomes thicker beneath gyri and thinner beneath sulci, as shown in  
453 Figure 9, right. This result is con-  
454 sistent with what was previously  
455 observed in the human brain **Kos-**  
456 **tović et al. (2002)**. While it is to  
457 date not clear whether this phe-  
458 nomenon is rather the cause or the  
459 result of cortical folding, our study  
460 clearly supports the latter. From  
461 a mechanical perspective, it seems  
462 that the forces generated due to  
463 the underlying cellular mechanism,  
464 do not only fold the cortical layer  
465 but also lead to undulations in the  
466 deeper zones. Still, the deeper subcortical layers (i.e ISVZ and VZ) remain equally smooth as the  
467 ventricular surface.



**Figure 9.** The effect of cortical folding on the OSVZ. While the OSVZ has a constant thickness before cortical folds emerge (left), it later becomes thicker beneath gyri than beneath sulci (right).

#### 468 **Are cortical folds affected by regional proliferation variations in the OSVZ?**

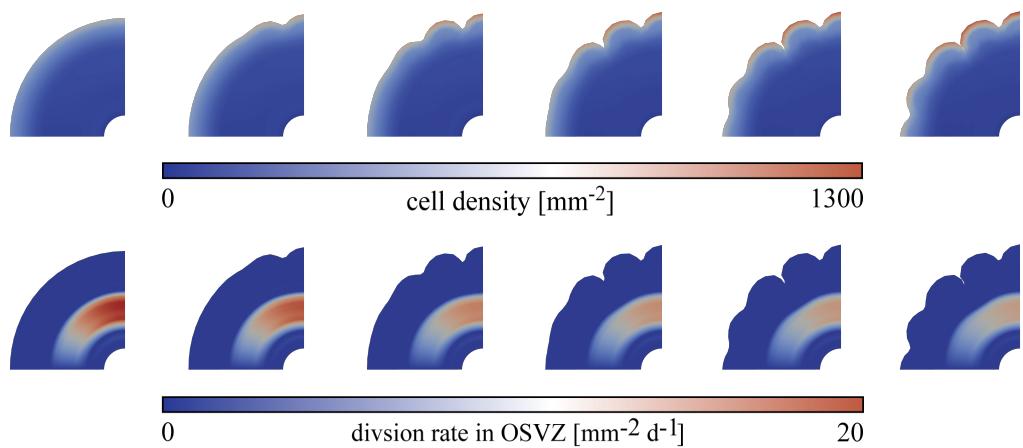
469 Previous studies have emphasized the existence of variations in the ORGC proliferation rate in  
470 different brain regions **Hansen et al. (2010)**. Some of those have suggested that the proliferation  
471 rate is higher beneath gyri than beneath sulci **Borrell (2018)**. To mimic this phenomenon, we apply  
472 a varying division rate by reformulating equation 8,

$$473 \quad r_2^c(\mathbf{x}, s) = G_{\text{osvz}}^s(s) \langle \sin(\theta) \rangle \left[ \frac{e^{50[r_i - r_{\text{isvz}}]}}{1 + e^{50[r_i - r_{\text{isvz}}]}} - \frac{e^{50[r_i - r_{\text{osvz}}(t)]}}{1 + e^{50[r_i - r_{\text{osvz}}(t)]}} \right], \quad (18)$$

474 where  $\langle \rangle$  are the Macaulay brackets, and  $\theta$  is the angle between the x-axis and the point position  
475 vector. Figure 10 shows the development of cortical folds between time steps 325 and 450 for  
476 the varying cortical stiffness case, a division rate in the VZ  $G_{\text{vz}} = 120$  and an initial division rate  
477 in the OSVZ of  $G_{\text{osvz}} = 20$ . The top row highlights the cell density distribution and the bottom  
478 row the locally varying division rate in the OSVZ. We observe that the cortex layer undergoes great  
479 surface expansion and the cortical folds indeed develop faster above the regions of higher division  
480 rates. Furthermore, the folds become more complex and have a higher cell density compared to  
481 regions nearby. However, the gyri and sulci are distributed equally between the different regions  
482 – regardless of the division rate. Still, sulci deepen faster above the regions with higher division  
483 rates. This simulation result is consistent with the previously found remarkable surface expansion  
484 above the regions with higher proliferation in the OSVZ **Llinares-Benadero and Borrell (2019)**.

#### 485 **Conclusion**

486 In this work, we have made use of a computational model for brain growth to provide insights  
487 into the role of the outer subventricular zone (OSVZ) – a unique additional proliferating zone in  
488 humans – during cortical folding in the developing brain. By using computational tools, we have  
489 addressed different open questions with the aim of valuably supplementing classical experimental  
490 approaches. Our simulations have systematically demonstrated how the proliferation in the OSVZ  
491 enhances the complexity of cortical folding patterns. An important decisive factor controlling the  
492 emerging folding pattern is the tissue stiffness ratio between the cortex and the subcortical plate.  
493 As the cortical stiffness appears to depend on the cell density, this is directly related to the cell pro-  
494 liferation in deeper zones. Our results show that the existence of the OSVZ particularly triggers the  
495 emergence of secondary mechanical instabilities leading to more complex folding patterns. Fur-  
496 thermore, the proliferation of outer radial glial cells (ORGs) reduces the time required to induce  
497 the mechanical instability and thus cortical folding. Interestingly, our simulation results suggest



**Figure 10.** The development of cortical folds between time steps 325 and 450 for the varying cortical stiffness case and initial division rate in the VZ  $G_{VZ} = 120$  and varying division rates in the OSVZ with an initial value of 20. The top row shows the cell density distribution, and bottom row shows the actual division rate in the OSVZ.

498 that the generated mechanical forces not only ‘fold’ the cortex but also deeper subcortical zones  
499 including the OSVZ, which becomes thicker beneath gyri and thinner beneath sulci as a result of  
500 cortical folding. In turn, we did not find any relation between regionally varying ORGCs prolifera-  
501 tion and the location of emerging sulci and gyri. Consequently, our physics-based analyses suggest  
502 that regional differences in the thickness of the OSVZ are rather a result of than a cause for cortical  
503 folding. Still, locally increased proliferation in the OSVZ leads to emergence of deeper sulci.

504 In conclusion, our physics-based computational modeling approach has proven valuable to  
505 predictively assess the links between cellular mechanisms during human brain development and  
506 cortical folding – the classical hallmark of the human cortex at the organ scale. It has allowed us  
507 to systematically assess the role of the OSVZ during human brain development and its effect on  
508 the cortical folding process. In the future, the computational framework can be used to not only  
509 better understand physiological brain development but also pathological processes – especially  
510 those involving abnormal cortical folding patterns. The computational model is able to shed new  
511 light on the interplay between the multiple processes at different scales and can help identify the  
512 main controlling parameters. However, it can only complement, not substitute, sophisticated ex-  
513 perimental approaches that are still needed to answer questions, e.g., regarding the functional  
514 difference between progenitor cell types and corresponding lineage decisions.

### 515 **Acknowledgements**

516 We would like to cordially thank Bettina Seydel for digitalizing the histological sections. In addition,  
517 we gratefully acknowledge the funding by the Deutsche Forschungsgemeinschaft (DFG, German  
518 Research Foundation) through the grant BU 3728/1-1 to SB.

### 519 **Author contributions statement**

520 S.B. and M.S.Z. conceptualized the study and developed the model. S.B. acquired funding. M.S.Z.  
521 implemented the computational model, performed the simulations, and analyzed the results. I.B.  
522 provided the human fetal brain sections and contributed to data analysis. M.S.Z. visualized the  
523 results and wrote the first draft. S.B supported visualization and oversaw the writing process. All  
524 authors discussed the results, and reviewed and edited the manuscript.

### 525 **Competing interests**

526 The authors declare no competing interests.

527 **Data availability**

528 The datasets and code generated and/or analyzed during the current study are available on GitHub:  
529 <https://github.com/SaeedZarzor/brain-development.git>

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