

1 Cell type-specific inference of differential expression in spatial
2 transcriptomics

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15 February, 22nd 2022

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Abstract

17 Spatial transcriptomics enables spatially resolved gene expression measurements at near
18 single-cell resolution. There is a pressing need for computational tools to enable the de-
19tection of genes that are differentially expressed (DE) within specific cell types across
20 tissue context. We show that current approaches cannot learn cell type-specific DE due to
21 changes in cell type composition across space and the fact that measurement units often
22 detect transcripts from more than one cell type. Here, we introduce a statistical method,
23 Cell type-Specific Inference of Differential Expression (C-SIDE), that identifies cell type-
24 specific patterns of differential gene expression while accounting for localization of other cell
25 types. We model spatial transcriptomics gene expression as an additive mixture across cell
26 types of general log-linear cell type-specific expression functions. This approach provides
27 a unified framework for defining and identifying gene expression changes in a wide-range
28 of relevant contexts: changes due to pathology, anatomical regions, physical proximity
29 to specific cell types, and cellular microenvironment. Furthermore, our approach enables
30 statistical inference across multiple samples and replicates when such data is available.
31 We demonstrate, through simulations and validation experiments on Slide-seq and MER-
32 FISH datasets, that our approach accurately identifies cell type-specific differential gene
33 expression and provides valid uncertainty quantification. Lastly, we apply our method to
34 characterize spatially-localized tissue changes in the context of disease. In an Alzheimer's
35 mouse model Slide-seq dataset, we identify plaque-dependent patterns of cellular immune
36 activity. We also find a putative interaction between tumor cells and myeloid immune cells
37 in a Slide-seq tumor dataset. We make our C-SIDE method publicly available as part of
38 the open source R package <https://github.com/dmcable/spacexpr>.

39

Introduction

40 Spatial transcriptomics technologies profile gene expression in parallel across hundreds or thousands
41 of genes across spatial measurement units, or *pixels* [1–9]. These technologies have the potential to
42 associate gene expression with cellular context such as spatial position, proximity to pathology, or

43 cell-to-cell interactions. Studying gene expression changes, termed *differential expression* (DE), within
44 tissue context has the potential to provide insight into principles of organization of complex tissues
45 and disorganization in disease and pathology [1, 10–13].

46 Current methods for addressing differential expression in spatial transcriptomics fall into two categories:
47 nonparametric and parametric methods. Nonparametric differential expression methods [14–17]
48 do not use constrained hypotheses about gene expression patterns, but rather fit general smooth spatial
49 patterns of gene expression. Some of these approaches do not take cell types into account [14, 15], while
50 others operate on individual cell types [17]. Discovering non-parametric differential gene expression
51 can be advantageous in order to generate diverse exploratory hypotheses. However, if covariates are
52 available, for example predefined anatomical regions, parametric approaches increase statistical power
53 substantially and provide directly interpretable parameter estimates. Specific differential expression
54 problems have been addressed with ad-hoc solutions such as detecting gene expression dependent on
55 cell-to-cell colocalization [18] or anatomical regions [10, 19], but no general parametric framework is
56 currently available. In contrast, general parametric frameworks have been widely applied across bulk
57 and single-cell RNA-sequencing (scRNA-seq) to test for differences in gene expression across cell type,
58 disease state, and developmental state, among other problems [20–22]. Furthermore, although multi-
59 sample, multi-replicate differential expression methods exist for bulk and single-cell RNA-seq [20–22],
60 no statistical framework accounting for technical and biological variation [23] across samples and replicates
61 has been established for the spatial setting.

62 An important challenge unaddressed by current spatial transcriptomics DE methods is accounting
63 for observations generated from cell type mixtures. In particular, sequencing-based, RNA-capture
64 spatial transcriptomics technologies, such as Visium [7], DBiT-seq [6], GeoMx [8], and Slide-seq [1, 2],
65 can capture multiple cell types on individual measurement pixels. The presence of cell type mixtures
66 complicates the estimation of *cell type-specific differential expression* (i.e. DE within a cell type of
67 interest) because different cell types have different gene expression profiles, independent of spatial
68 location [24, 25]. Although imaging-based spatial transcriptomics technologies, such as MERFISH [3],
69 seqFISH [5], ExSeq [9], and STARmap [4], have the potential to achieve single cell resolution, these
70 technologies may encounter mixing or contamination across cell types due to diffusion or imperfect
71 cellular segmentation [26]. Several methods [24, 27, 28] have been developed to identify cell type
72 proportions in spatial transcriptomics datasets. However, at present no method accounts for cell type
73 proportions in differential expression analysis. Here, we demonstrate how not accounting for cell type
74 proportions leads to biased estimates of differential gene expression due to confounding caused by cell
75 type proportion changes or contamination from other cell types.

76 In this work we introduce Cell type-Specific Inference of Differential Expression (C-SIDE), a general
77 parametric statistical method that estimates cell type-specific differential expression in the context of
78 cell type mixtures. The first step is to estimate cell type proportions on each pixel using a cell type-
79 annotated single-cell RNA-seq (scRNA-seq) reference [24]. Next, we fit a parametric model, using
80 predefined covariates such as spatial location or cellular microenvironment, that accounts for cell type
81 differences to obtain cell type-specific differential expression estimates and corresponding standard
82 errors. The model accounts for sampling noise, gene-specific overdispersion, multiple hypothesis test-
83 ing, and platform effects between the scRNA-seq reference and the spatial data. Furthermore, when
84 multiple experimental samples are available, the C-SIDE model permits statistical inference across
85 multiple samples and/or replicates to achieve more stable estimates of population-level differential
86 gene expression.

87 Using simulated and real spatial transcriptomics data, we show C-SIDE accurately estimates cell
88 type-specific differential expression while controlling for changes in cell type proportions and contam-
89 ination from other cell types. We also demonstrate how cell type mixture modelling increases power,
90 especially when single cell type measurements are rare. Furthermore, on Slide-seq and MERFISH
91 datasets, we demonstrate how C-SIDE's general parametric framework enables testing differential
92 gene expression for diverse hypotheses including spatial position or anatomical regions [29], cell-to-cell
93 interactions, cellular environment, or proximity to pathology. By associating gene expression changes
94 with particular cell types, we use C-SIDE to systematically link gene expression changes to cellular

95 context in pathological tissues such as Alzheimer's disease and cancer.

96 Results

97 Cell type-Specific Inference of Differential Expression learns cell type-specific 98 differential gene expression in the context of spatial transcriptomics cell type 99 mixtures

Here, we develop Cell type-Specific Inference of Differential Expression (C-SIDE), a statistical method for determining differential expression (DE) in spatial transcriptomics datasets (Figure 1a). C-SIDE inputs one or more experimental samples of spatial transcriptomics data, consisting of $Y_{i,j,g}$ as the observed RNA counts for pixel i , gene j , and experimental sample g . We then assume Poisson sampling so that,

$$Y_{i,j,g} \mid \lambda_{i,j,g} \sim \text{Poisson}(N_{i,g}\lambda_{i,j,g}), \quad (1)$$

with $\lambda_{i,j,g}$ the expected count and $N_{i,g}$ the total transcript count (e.g. total UMIs) for pixel i on experimental sample g . Accounting for platform effects and other sources of technical and natural variability, we assume $\lambda_{i,j,g}$ is a mixture of K cell type expression profiles, defined by,

$$\log(\lambda_{i,j,g}) = \log \left(\sum_{k=1}^K \beta_{i,k,g} \mu_{i,k,j,g} \right) + \gamma_{j,g} + \varepsilon_{i,j,g}, \quad (2)$$

100 with $\mu_{i,k,j,g}$ the cell type-specific expected gene expression rate for pixel i , gene j , experimental sample
101 g , and cell type k ; $\beta_{i,k,g}$ the proportion of cell type k contained in pixel i for experimental sample g ;
102 $\gamma_{j,g}$ a gene-specific random effect that accounts for platform variability; and $\varepsilon_{i,j,g}$ a random effect to
103 account for gene-specific overdispersion.

To account for cell type-specific differential expression, we model across pixel locations the log of the cell type-specific profiles $\mu_{i,k,j,g}$ as a linear combination of L covariates used to explain differential expression. Specifically, we assume that,

$$\log(\mu_{i,k,j,g}) = \alpha_{0,k,j,g} + \sum_{\ell=1}^L x_{i,\ell,g} \alpha_{\ell,k,j,g}. \quad (3)$$

104 Here, $\alpha_{0,k,j,g}$ represents the intercept term for gene j and cell type k in sample g , and $x_{i,\ell,g}$ represents
105 the ℓ 'th covariate, evaluated at pixel i in sample g . Similarly as in linear and generalized linear
106 models [30], x , also called the *design matrix*, represents predefined covariate(s) that explain differential
107 expression, and the corresponding coefficient(s) $\alpha_{\ell,k,j,g}$ each represent the DE effect size of covariate ℓ
108 for gene j in cell type k for sample g .

109 With this general framework we can describe any type of differential expression that can be pa-
110 rameterized with a log-linear model. Examples include (Figure 1b):

- 111 1. Differential expression between multiple regions. In this case, the tissue is manually segmented
112 into multiple regions (e.g. nodular and anterior cerebellum, Figure 3). Design matrix x contains
113 discrete categorical indicator variables representing membership in 2 or greater regions.
- 114 2. Differential expression due to cellular environment or state (special case of (1)). Pixels are
115 discretely classified into local environments based on the surrounding cells (e.g. stages in the
116 testes Slide-seq dataset, Figure 4).
- 117 3. Differential expression as a function of distance to a specific anatomical feature. In this case,
118 x is defined as the spatial position or distance to some feature (e.g. distance to midline in the
119 hypothalamus MERFISH dataset, Figure 4).

- 120 4. Cell-to-cell interactions. In this case, we define a cell-to-cell interaction as differential expression
- 121 within one cell type (A) due to co-localization with a second cell type (B) (e.g. immune cell
- 122 density in cancer, Figure 5). For this problem, x is the continuous density of cell type B .
- 123 5. Proximity to pathology. Similar to (4), except covariate x represents density of a pathological
- 124 feature (e.g. Alzheimer's A β plaque, Figure 4), rather than cell type density.
- 125 6. General spatial patterns (termed *nonparametric*). In this case, we define design matrix x to
- 126 be smooth basis functions [31], where linear combinations of these basis functions represent the
- 127 overall smooth gene expression function and can accommodate any smooth spatial pattern.

128 To estimate this complex model with a computationally tractable algorithm, we note that the gene
129 expression variability across cell types is large enough that, in the first step, we can assume $\mu_{i,k,j,g}$
130 does not vary with i and g and estimate β using a previously published algorithm [24]. Here, some
131 pixels are identified as single cell types while others as mixtures of multiple cell types. Fixing the
132 β estimates, we next use maximum likelihood estimation to estimate the cell type-specific DE coef-
133 ficients α with corresponding standard errors, allowing for false discovery rate-controlled hypotheses
134 testing (see *Methods* for details). Lastly, C-SIDE performs statistical inference across multiple repli-
135 cates and/or samples, accounting for biological and technical variation across replicates, to estimate
136 consensus population-level differential expression (*Methods*, Supplementary Figure 1).

137 Because ground truth cell type-specific DE is unknown in spatial transcriptomics data, we first
138 benchmarked C-SIDE's performance on a simulated spatial transcriptomics dataset in which gene
139 expression varied across two regions. Considering the challenging situation where two cell types, termed
140 *cell type A* and *cell type B*, are colocalized on pixels within a tissue, we simulated, using a single-nucleus
141 RNA-seq cerebellum dataset, spatial transcriptomics mixture pixels with known proportions of single
142 cells from two cell types known to spatially colocalize [32] (*Methods*, Figure 2a). Across two spatially-
143 defined regions, we varied both the true cell type-specific gene expression of cell types A and B as well
144 as the average cell type proportions of cell types A and B (Figure 2a, Supplementary Figure 2). We
145 compared C-SIDE against three alternative methods (see *Methods* for details): *Bulk*, bulk differential
146 expression (ignoring cell type); *Single*, single cell differential expression that approximates each cell
147 type mixture as a single cell type; and *Decompose*, a method that decomposes mixtures into single
148 cell types prior to computing differential expression. By varying cell type frequencies between the
149 two regions without introducing differential expression, we observed that C-SIDE correctly attributes
150 gene expression differences across regions to differences in cell type proportions rather than spatial
151 differential expression (Figure 2b, Supplementary Figure 2); in contrast, the Bulk method incorrectly
152 predicts spatial differential expression since it does not control for differences of cell type proportions
153 across regions.

154 Next, we simulated cell type-specific differential expression (DE) by varying the differential expres-
155 sion in cell type A while keeping cell type B constant across regions. Background DE in cell type A
156 contaminated estimates of differential expression in cell type B for all three alternatives models Bulk,
157 Decompose, and Single (Figure 2c, Supplementary Figure 2). In contrast, C-SIDE's joint model of cell
158 type mixtures and cell type-specific differential expression correctly identified differential expression in
159 cell type A, but not cell type B. Next, we verified that, under the null hypothesis of zero differential
160 expression, C-SIDE's false positive rate was accurately controlled, standard errors were accurately esti-
161 mated, and confidence intervals contained the ground truth DE (Figure 2d, Supplementary Figure 2).
162 Finally, when nonzero differential expression was simulated, C-SIDE achieved unbiased estimation of
163 cell type-specific differential expression (Figure 2e). We also found that the power of C-SIDE depends
164 on gene expression level, number of cells, and differential expression magnitude (Supplementary Figure
165 2). Thus, our simulations validate C-SIDE's ability to accurately estimate and test for cell type-specific
166 differential expression in the cases of asymmetric cell type proportions and contamination from other
167 cell types.

168 **C-SIDE accurately identifies cell type-specific differential expression in spa-
169 tial transcriptomics data**

170 To validate C-SIDE's ability to discover cell type-specific differential expression on spatial transcript-
171 omics data, we collected Slide-seqV2 data [2] (including one replicate sourced from a prior study [24])
172 for three cerebellum replicates. We identified a spatial map of cell types (Figure 3a), previously shown
173 to correspond to known cerebellum spatial architecture [24]. We used discrete localization in the ante-
174 rior lobule or nodulus regions (Figure 3b), a known axis of spatial gene expression variation within the
175 cerebellum [32], as a covariate and estimated cell type-specific DE across regions using C-SIDE (Fig-
176 ure 3c, Supplementary Figure 3, Supplementary Table 1). As experimental validation, we performed
177 hybridization chain reaction (HCR) on four genes identified to be differentially expressed in specific
178 cell types, and we observed high correspondence between C-SIDE's estimates of cell type-specific dif-
179 ferential expression and DE measurements from HCR data (Figure 3d, $R^2 = 0.89$). For example, we
180 examined *Aldoc* and *Plcb4*, two genes expressed in both Purkinje and Bergmann cell types, which are
181 known to spatially colocalize in the cerebellum and appear as mixtures on Slide-seq pixels [24]. C-
182 SIDE determined that both *Aldoc* (log2-fold-change = -4.24 , $p < 10^{-8}$) and *Plcb4* (log2-fold-change
183 = 1.93 , $p < 10^{-8}$) were differentially expressed in the Purkinje cell type, but not the Bergmann cell
184 type. Similarly, HCR images of *Aldoc* and *Plcb4* showed substantial differential expression within
185 Purkinje cells across the nodulus and anterior lobule, whereas expression within Bergmann cells was
186 relatively even across regions (Figure 3d–e). We conclude that C-SIDE can successfully identify cell
187 type-specific spatial differential expression in spatial transcriptomics tissues, even when multiple cell
188 types are spatially colocalized.

189 **C-SIDE solves a diverse array of differential gene expression problems in
190 spatial transcriptomics**

191 We next explored the effect of discrete cellular microenvironments on cell type-specific DE in the
192 mouse testes Slide-seq dataset [12]. C-SIDE's testes principal cell type assignments (Figure 4a) revealed
193 tubular structures corresponding to cross-sectional sampling of seminiferous tubules. Individual tubules
194 have distinct stages of spermatogonia development, grouped into four classes of stages I–III, IV–VI,
195 VII–VIII, and IX–XII, which were determined from the prior testes Slide-seq study using tubule-
196 level gene expression clustering [12] (Figure 4b). Given that each tubule stage represents a distinct
197 microenvironment along the testes developmental trajectory, we applied C-SIDE to identify genes
198 that were differentially expressed, for each cell type, across tubule stages (Supplementary Table 2).
199 Furthermore, C-SIDE identified genes expressed in a single tubule stage within a single cell type (Figure
200 4c) which are known drivers of cellular development across stages [12]. For instance, the gene *Tnp1*
201 was identified by C-SIDE as upregulated in the IX–XII stage within the elongating spermatid (ES)
202 cell type, in agreement with the known biological role of *Tnp1* in nuclear remodeling of elongating
203 spermatids at the late tubule stage [33] (Supplementary Figure 4). After identifying stage-specific genes
204 within each cell type, we additionally found that a majority of C-SIDE-identified stage-specific genes
205 followed cyclic patterns across stages, consistent with previously-characterized cyclic gene regulation
206 in what is referred to as the seminiferous epithelial cycle [34] (Supplementary Figure 4).

207 Next, we evaluated C-SIDE's ability to identify DE for cell types that primarily appear as mixtures
208 with other cell types, particularly the spermatocyte (SPC) cell type. According to C-SIDE cell type
209 assignments, SPC frequently co-mixes with the ES and round spermatid (RS) cell types, consistent
210 with the known colocalization of spermatocytes with spermatids from previous histological studies [35]
211 (Supplementary Figure 4). Due to C-SIDE's ability to learn DE from cell type mixtures, C-SIDE
212 obtained increased power for identifying differentially expressed genes compared to a DE method that
213 only uses single cell type pixels (see *Supplementary Methods* for details, Supplementary Figure 4),
214 especially for spermatocyte cell type (217 significant SPC DE genes discovered by C-SIDE vs. 1 DE
215 gene for the single cell method). In order to validate C-SIDE's determination that *Prss40* (log2-
216 fold-change = 1.72 , $p = 8 \cdot 10^{-5}$) and *Snx3* (log2-fold-change = 1.17 , $p < 10^{-8}$) were differentially

217 expressed, between stage I-III and stage IX-XII, specifically in the SPC cell type, we compared the
218 average gene expression for three categories of testes pixels: pixels containing spermatid cell types,
219 but not SPC (called S+, SPC-); pixels containing both spermatid and SPC cell types (S+, SPC+);
220 and pixels containing SPC but not spermatids (S-, SPC+) (Figure 4d). For both genes, differential
221 expression across stages was not observed in (S+, SPC-) pixels, indicating that the spermatid cell
222 types do not exhibit DE. However, (S+, SPC+) pixels are significantly differentially expressed across
223 stages, enabling C-SIDE to infer DE specifically in the SPC cell type. On the other hand, (S-, SPC+)
224 pixels, which include SPC single cells, are not significantly differentially expressed across regions, due
225 to their low sample size. Therefore, C-SIDE's ability to handle cell type mixtures uniquely enables
226 the discovery of differential expression, even in cell types that only appear as mixtures with other cell
227 types.

228 C-SIDE identifies spatial gene expression changes in imaging-based technologies

229 Next, we demonstrated the utility of C-SIDE on an imaging-based spatial transcriptomics dataset
230 (i.e. MERFISH) which achieves closer to single-cell resolution compared to capture-based spatial
231 transcriptomics technologies (e.g. Slide-seq, Visium), which contain frequent cell type mixtures [24].
232 To do so, we applied C-SIDE to a MERFISH dataset collected in the mouse hypothalamus. During
233 development, hypothalamic progenitors create radial projections out from the hypothalamic midline,
234 which are used as scaffolds for the migration of differentiating daughter cells [36]. Thus, we investigated
235 radial distance to the hypothalamus midline as a predictor of differential expression in hypothalamus
236 cell types. First, we assigned cell types and found them to be consistent with the prior MERFISH
237 hypothalamus study [11] (Figure 4e). Although C-SIDE mostly assigned single cell types to MERFISH
238 pixels, a non-negligible proportion (12.6% double cell type pixels out of $n = 3790$ total single and double
239 cell type pixels) of pixels were assigned as mixtures of more than one cell type. Next, we computed
240 midline distance as a covariate for C-SIDE (Figure 4f), and we next detected genes in hypothalamus
241 excitatory, inhibitory, and mature oligodendrocyte cell types whose expression depended either linearly
242 or quadratically on distance from the midline (Figure 4g, Supplementary Table 3-4). For instance,
243 *Slc18a2* (Figure 4h), identified by C-SIDE as differentially upregulated within inhibitory neurons near
244 the midline ($\log_2\text{fold-change} = 6.14$, $p < 10^{-8}$), is required for dopaminergic function in certain
245 inhibitory neuronal subtypes [37], which are known to localize near the hypothalamus midline [11].

246 C-SIDE enables discovery of A β plaque-dependent cell type-specific differential expression 247 in Alzheimer's disease

248 We next explored the use of pathological staining, in particular A β plaques, as a continuous covariate
249 for cell type-specific gene expression changes. To do so, we performed Slide-seqV2 on the hippocampal
250 region of a genetic mouse model of amyloidosis in Alzheimer's disease (AD) [38] (J20, $n = 4$ slices,
251 *Methods*). C-SIDE identified spatial maps of cell types (Figure 4i) which were consistent with past
252 characterizations of hippocampus cellular localization [24]. We collected paired A β plaque staining
253 images (Anti-Human A β Mouse IgG antibody, *Methods*) to quantify the A β plaque density to use
254 as a covariate for C-SIDE (Figure 4j, Supplementary Figure 5). We then used C-SIDE to identify
255 genes whose expression depended in a cell type-specific manner on A β plaque density (Figure 4k,
256 Supplementary Table 5). For instance, we found that *Gfap* was enriched in astrocytes colocalizing
257 with A β plaque (Figure 4l, Supplementary Figure 5, $\log_2\text{fold-change} = 1.35$, $p < 10^{-8}$), a result
258 corroborated by studies that have established the role of *Gfap* in attenuating the proliferation of A β
259 plaques [39]. C-SIDE additionally discovered upregulation in astrocytes of the *C4b* complement gene
260 ($\log_2\text{fold-change} = .85$, $p = 1 \cdot 10^{-4}$), which is involved in plaque-associated synaptic pruning in
261 Alzheimer's disease [40-42]. Moreover, several cathepsin proteases including *Ctsb* ($\log_2\text{fold-change} =$
262 1.65 , $p < 10^{-8}$), *Ctsd* ($\log_2\text{fold-change} = 1.30$, $p < 10^{-8}$) *Ctsl* ($\log_2\text{fold-change} = 1.96$, $p = 4 \cdot 10^{-6}$),
263 and *Ctsz* ($\log_2\text{fold-change} = 1.11$, $p = 3 \cdot 10^{-4}$) were determined to be differentially upregulated
264 in microglia around plaque, consistent with prior evidence that cathepsins are involved with amyloid

265 degradation in Alzheimer's disease [43] (Supplementary Figure 5). In microglia, we also identified
266 known homeostatic microglia markers [44–46] including *P2ry12* (log2-fold-change = -1.33 , $p < 10^{-8}$)
267 and *Cx3cr1* (log2-fold-change = -0.68 , $p = 3 \cdot 10^{-4}$) as downregulated in the presence of plaque. *Apoe*,
268 which is known to have $A\beta$ plaque-dependent upregulation within microglia [47], was also detected
269 as significant (log2-fold-change = 1.58 , $p < 10^{-8}$), although it did not pass default C-SIDE gene
270 filters (*Methods*) due to its four-fold higher expression in astrocytes than microglia. Finally, the
271 anti-inflammatory gene *Grn* was determined by C-SIDE to be upregulated in microglia near plaque
272 (log2-fold-change = 0.79 , $p = 6 \cdot 10^{-4}$), consistent with prior knowledge [48].

273 C-SIDE discovers tumor-immune signaling in a mouse tumor model

274 Finally, we applied C-SIDE to identify genes with cell type-specific spatial differential expression in
275 a Slide-seq dataset of a *Kras*^{G12D/+} *Trp53*^{-/-} (KP) mouse tumor model [49, 50], where we analyzed a
276 single metastatic lung adenocarcinoma tumor deposit in the liver. We first used C-SIDE to generate
277 a spatial map of cell types and found several cell types within the tumor, including both tumor
278 cells and myeloid cells (Figure 5a). Next, we ran C-SIDE nonparametrically to discover arbitrary
279 smooth gene expression patterns (see *Supplementary Methods* for details, Supplementary Table 6). For
280 gene expression within the tumor cell type, this procedure identified three categories of genes: genes
281 with variable expression purely due to sampling noise rather than biology, genes exhibiting biological
282 variation partially explained by the spatial C-SIDE model, and genes exhibiting biological variation
283 not explained by the spatial model (Figure 5b, Supplementary Figure 6). We then hierarchically
284 clustered the C-SIDE fitted spatial patterns of significant differentially expressed genes within the
285 tumor cell type into seven clusters with distinct spatial patterns (Figure 5c, Supplementary Figure
286 6). We tested each cluster for gene set enrichment (see *Supplementary Methods* for details), and we
287 identified the *Myc* targets gene set as enriched in cluster 5 (7 out of 12 genes, $p = 2 \cdot 10^{-4}$, two-
288 sided binomial test, Supplementary Table 7, 1 significant gene set out of 50 tested), a cluster with a
289 spatial pattern of overexpression at the tumor boundary (Figure 5d). High expression of *Myc* target
290 genes is potentially indicative of an increased rate of proliferation [51] at the boundary, which has
291 been previously proposed as a correlate of tumor severity [52]. For example, the *Myc* target found
292 to have the most differential upregulation at the tumor boundary, *Kpnb1* (Supplementary Figure 6,
293 $p = 1 \cdot 10^{-5}$), has been previously been identified as an oncogene that drives cell proliferation and
294 suppresses apoptosis [53, 54].

295 Given the substantial variation in tumor cell spatial expression patterns, we next tested if such
296 variability could be explained by cell-to-cell interactions with immune cells, which have been shown to
297 influence tumor cell behavior in prior studies [55–57]. Using myeloid cell type density as the C-SIDE
298 covariate (Figure 5e), C-SIDE identified genes with immune cell density-dependent cell type-specific
299 differential expression (Figure 5f, Supplementary Table 8), including several genes that were also
300 discovered by our nonparametric procedure (Supplementary Figure 6). One of the genes with the
301 largest effects, *Ccl2* (log2-fold-change = 1.74 , $p < 10^{-8}$), is a chemotactic signaling molecule known
302 to attract myeloid cells [58, 59]. Furthermore, we tested C-SIDE's DE gene estimates for aggregate
303 effects across gene sets and found that the epithelial-mesenchymal transition (EMT) pathway was
304 significantly upregulated on average near immune cells (Figure 5f, Supplementary Figure 6, $p = 0.0011$,
305 permutation test (see *Methods*), 1 significant gene set out of 50 tested, Supplementary Table 7). C-
306 SIDE additionally identified *NfkB1* as upregulated in tumor cells in immune-rich regions (log2-fold-
307 change = 1.10 , $p = 1 \cdot 10^{-5}$), a gene that has been previously implicated in positively regulating the
308 EMT pathway of tumor cells [60, 61]. Moreover, the majority of tumor cells exhibiting a mesenchymal
309 phenotype were located in immune-rich regions (Figure 5g). Furthermore, morphological analysis and
310 annotation of an hematoxylin and eosin (H&E) stained adjacent section of the tumor demonstrated a
311 clear increase in the number of spindle-shaped tumor cells relative to polygonal-shaped tumor cells in
312 the immune rich-areas (Figure 5h). The collective morphological and gene expression changes suggest
313 a role for the immune microenvironment in influencing the epithelial-mesenchymal transition in this
314 tumor model [62]. Therefore, both exploratory nonparametric C-SIDE and more targeted immune

315 cell-dependent DE reveal biologically-relevant signatures of differential gene expression.

316 Discussion

317 Elucidating spatial sources of differential gene expression is a critical challenge for understanding bio-
318 logical mechanisms and disease with spatial transcriptomics. Here we introduced C-SIDE, a statistical
319 method to detect cell type-specific DE in spatial transcriptomics datasets. C-SIDE takes as input
320 one or more biologically-relevant covariates, such as spatial position or cell type colocalization, and
321 identifies genes, for each cell type, that significantly change their expression as a function of these
322 covariates. Tested on simulated spatial transcriptomics data, C-SIDE obtained unbiased estimation of
323 cell type-specific differential gene expression with a calibrated false positive rate, while other methods
324 were biased from changes in cell type proportion or contamination from other cell types. In the cerebel-
325 lum, we additionally used HCR experiments to validate C-SIDE's ability to identify cell type-specific
326 DE across regions. We further applied C-SIDE to a detect differential expression depending on tubular
327 microenvironment in the testes, midline distance in the MERFISH hypothalamus, and A β plaque den-
328 sity in the Alzheimer's model hippocampus. Finally, we applied both nonparametric and parametric
329 C-SIDE procedures in a mouse tumor model to discover an increase in tumor cells undergoing EMT
330 transition in immune-rich regions.

331 Several studies have established the importance of accounting for cell type mixtures in assigning
332 cell types in spatial transcriptomics data [24, 27, 28]. However, it remains a challenge to incorporate cell
333 type proportions into models of cell type-specific spatial differential gene expression. C-SIDE enables
334 such cell type-specific DE discovery by creating a statistical model of cell type-specific differential gene
335 expression in the presence of cell type mixtures. In this study, we demonstrated how other potential
336 solutions, such as bulk DE, approximation as single cell types, and decomposition into single cell types
337 can be confounded by cell type proportion changes and contamination from other cell types. C-SIDE
338 solves these issues by controlling for cell type proportions and jointly considering differential expression
339 within each cell type. Even in imaging-based spatial transcriptomics methods such as MERFISH that
340 mostly contain single cell type pixels, we detected some pixels with cell type mixtures, indicating
341 potential diffusion or imperfect cell segmentation [26]. To control for cell type proportions in DE
342 analysis, C-SIDE can estimate cell types directly or import cell type proportions from any cell type
343 mixture identification method [24, 27, 28].

344 C-SIDE provides a unified framework for detecting biologically-relevant differential expression in
345 spatial transcriptomics tissues along diverse array of axes including spatial distance, proximity to
346 pathology, cellular microenvironment, and cell-to-cell interactions. In settings without prior biological
347 hypotheses, C-SIDE may be run nonparametrically to discover general cell type-specific spatial gene
348 expression patterns. When using problem-specific knowledge to generate biologically-relevant DE
349 predictors, parametric C-SIDE efficiently detects DE genes along the parametric hypothesis axes. C-
350 SIDE can also be used to test among multiple models of differential expression, such as the linear and
351 quadratic models applied to the hypothalamus dataset. C-SIDE can also utilize multiple covariates in
352 a joint model of gene expression, such as spatial position and cell type colocalization, although more
353 complicated models require more data to fit accurately. Beyond individual samples, C-SIDE can also
354 perform differential expression statistical inference at the population level across multiple replicates
355 or biological samples, including modeling biological and technical variability in complex multi-sample,
356 multi-replicate experiments. Multi-replicate experiments, though more costly, produce more robust
357 DE estimates by reducing spurious discoveries of DE on single replicates.

358 One challenge for C-SIDE is obtaining sufficient DE detection statistical power, which we observed
359 can be hindered by low gene expression counts, small pixel number, or rare cell types. An advan-
360 tage of C-SIDE is that it increases its statistical power by including cell type mixture pixels in its
361 model. Ongoing technical improvements in spatial transcriptomics technologies [2] such as increased
362 gene expression counts, higher spatial resolution, and increased pixel number, have the potential to
363 dramatically increase the discovery rate of C-SIDE. Another limitation of C-SIDE is the requirement

364 of an annotated single-cell reference for reference-based identification of cell types in the cell type
365 assignment step. Although single-cell atlases are increasingly available for biological tissues, they may
366 contain missing cell types or substantial platform effects [24], and certain spatial transcriptomics tissues
367 may lack a corresponding single-cell reference.

368 We envision C-SIDE to be particularly powerful in the context of bridging cell type-specific gene
369 expression changes in pathology. Here, we demonstrate this in two contexts: one, wherein we leverage
370 histological features ($A\beta$ plaques) as a covariate, and two, wherein we nominate tumor-immune inter-
371 actions as a covariate. In the first, prior Alzheimer's disease (AD) studies have discovered candidate
372 genes for disease-relevance through GWAS [63], bulk RNA and protein differences between AD and
373 control samples [64], and single cell expression differences of disease associated cellular subtypes [41].
374 Here, with C-SIDE, we identify many genes previously identified by these methods including *Gfap*
375 in astrocytes [39] and *Apoe* in microglia [47]; furthermore, we take known disease-level associations
376 a step further towards mechanistic understanding by directly associating spatial plaque localization
377 with cell type-specific differential expression. For example, prior studies have established an associa-
378 tion between complement pathway activation in plaque-dense areas with synaptic pruning [40] and
379 neuronal degeneration [41] leading to cognitive decline. Using C-SIDE we provide evidence for the
380 upregulation of complement protein *C4b* specifically within plaque-localized astrocytes [65]. Thus,
381 amyloid plaques may trigger a cytokine-dependent signaling cascade that stimulates the expression of
382 complement genes in astrocytes, as supported by prior studies [42]. In contrast to *C4b* upregulation,
383 homeostatic microglia marker *P2ry12*, discovered by C-SIDE to be negatively plaque-associated within
384 microglia, has been shown to be downregulated in microglia in Alzheimer's disease (AD), a phenomena
385 associated with neuronal cell loss [44]. *P2ry12* is involved in early stage nucleotide-dependent acti-
386 vation of microglia and is downregulated in later stages of activated microglia [46]. We hypothesize
387 that plaque-dense areas in AD trigger microglia activation which downregulates homeostatic microglia
388 genes such as *P2ry12*. Lastly, the granulin gene (*Grn*), discovered by C-SIDE as upregulated in mi-
389 croglia near plaques, is an anti-inflammatory gene that attenuates microglia activation [66]. It has
390 been shown to be upregulated in plaque-localized microglia in AD [48] and to potentially have a role in
391 reducing plaque deposition and cognitive pathological effects in AD [67] and other pathological protein
392 aggregates [68].

393 Second, C-SIDE has the potential to elucidate tissue interactions driving system-level behavior in
394 complex tissues. For example, recent studies have characterized cell-to-cell interactions of immune
395 cells influencing the behavior of tumor cells [55–57]. Consistent with these studies, on a Slide-seq
396 dataset of a mouse tumor model, C-SIDE identified several genes whose expression within tumor cells
397 was upregulated near myeloid immune cells. We postulate that the tumor cells and myeloid cells
398 are involved in a synergistic feedback loop, driven by cell-to-cell signaling. For example, *Ccl2*, found
399 by C-SIDE to be upregulated in immune-adjacent tumor cells, is known to chemotactically recruit
400 myeloid cells and to induce pro-tumorigenic behavior, including growth, angiogenesis, and metastasis,
401 in myeloid cells [58, 59]. Another synergistic immune-tumor interaction identified by C-SIDE is the
402 myeloid-associated upregulation of the epithelial-mesenchymal transition (EMT) pathway, known to
403 be involved in tumor development and metastasis [62]. Although C-SIDE established an association
404 between immune cell colocalization and mesenchymal-like tumor cell state, conclusive establishment of
405 mechanism of causation requires future experimentation. Among other hypotheses, it is plausible that
406 myeloid cells induce tumor cells to undergo the EMT transition, potentially through the *NF- κ B* (also
407 identified as upregulated by C-SIDE) signaling pathway, as supported by other studies [55–57, 62].
408 Future work is necessary to characterize this phenomena across a broader cohort of samples and to
409 establish specific molecular mechanisms. Overall, these results highlight the power of combining the C-
410 SIDE framework with pathological measurements to understand cell type-specific responses to disease
411 and injury. We envision C-SIDE as a powerful framework for the systematic study of the impacts of
412 spatial and environmental context on cellular gene expression in spatial transcriptomics data.

413 Methods

414 C-SIDE model

415 Here, we describe Cell type-Specific Inference of Differential Expression (C-SIDE), a statistical method
416 for identifying differential expression (DE) in spatial transcriptomics data. Please first refer to the
417 overall definition of the C-SIDE model in equations (1), (2), and (3). Prior to fitting C-SIDE, the
418 *design matrix* x is predefined to contain *covariates*, variables on which gene expression is hypothesized
419 to depend such as spatial position or cellular microenvironment. Recall that $x_{i,\ell,g}$ represents the
420 ℓ 'th covariate, evaluated at pixel i in experimental sample g . For each covariate $x_{\cdot,\ell,g}$, there is a
421 corresponding coefficient $\alpha_{\ell,k,j,g}$, representing a gene expression change across pixels per unit change
422 of $x_{\cdot,\ell,g}$ within cell type k of experimental sample g . Next, recall from (2) random effects $\gamma_{j,g}$ and
423 $\varepsilon_{i,j,g}$, which we assume both follow normal distributions with mean 0 and standard deviations $\sigma_{\gamma,g}$
424 and $\sigma_{\varepsilon,j,g}$, respectively. We designed the overdispersion magnitude, $\sigma_{\varepsilon,j,g}$ to depend on gene j because
425 we found evidence that the overdispersion depends on gene j (Supplementary Figure 7), and modeling
426 gene-specific overdispersion is necessary for controlling the false-positive rate of C-SIDE.

427 Due to our finding that genes can exhibit DE in some but not all cell types (see e.g. Figure 3c),
428 C-SIDE generally does not assume that genes share DE patterns across cell types, allowing for the
429 discovery of cell type-specific DE. We also developed an option where DE can be assumed to be shared
430 across cell types (*Supplementary Methods*). We note that C-SIDE can be thought of as a modification
431 of the generalized linear model (GLM) [30] in which each cell type follows a cell type-specific log-linear
432 model before an additive mixture of all cell types is observed. See *Fitting the C-SIDE model* and
433 *Hypothesis testing* for C-SIDE model fitting and hypothesis testing, respectively.

434 Parameterization of the design matrix

435 For specific construction of design matrix x for each dataset, see *Cell type estimation and construction*
436 of *covariates*. Recall the specific examples of design matrix x presented in Figure 1b. In general, we
437 note that x can take on the following numerical forms:

- 438 1. Indicator variable. In this case, $x_{i,\ell,g}$ is always either 0 or 1. This represents differential expression
439 due to membership within a certain spatially-defined pixel set of interest. The coefficient $\alpha_{k,j,g}$
440 is interpreted as the log-ratio of gene expression between the two sets for cell type k and gene j
441 in experimental sample g .
- 442 2. Continuous variable. In this case, $x_{i,\ell,g}$ can take on continuous values representing, for example,
443 distance from some feature or density of some element. The coefficient $\alpha_{\ell,k,j,g}$ is interpreted
444 as the log-fold-change of gene expression per unit change in $x_{i,\ell,g}$ for cell type k and gene j in
445 sample g .
- 446 3. Multiple categories. In this case, we use x to encode membership to finitely many $L \geq 2$ sets.
447 For each $1 \leq \ell \leq L$, we define $x_{i,\ell,g}$ to be an indicator variable representing membership in set
448 ℓ for sample g . To achieve identifiability, the intercept is removed. The coefficient $\alpha_{\ell,k,j,g}$ is
449 interpreted as the average gene expression in set ℓ for cell type k and gene j . Cell type-specific
450 differential expression is determined by detecting changes in $\alpha_{\ell,k,j,g}$ across ℓ within cell type k
451 and sample g .
- 452 4. Nonparametric. In this case, we use x to represent L smooth basis functions, where linear
453 combinations of these basis functions represent the overall smooth gene expression function. By
454 default, we use thin plate spline basis functions, calculated using the `mgcv` package [31].

455 In all cases, we normalize each $x_{i,\ell,g}$ to range between 0 and 1. The problem is equivalent under linear
456 transformations of x , but this normalization helps with computational performance. The intercept
457 term, when used, is represented in x as a column of 1's.

458 **Fitting the C-SIDE model**

459 C-SIDE estimates the parameters of (1), (2), and (3) via maximum likelihood estimation. First, we
 460 note that all parameters and parameter relationships in the model are independent across samples, so
 461 we fit the model independently for each sample. We will return to the issue of population inference
 462 across multiple samples in *Statistical inference on multiple samples/replicates*. Next, the parameters
 463 of $\beta_{i,k}$ and γ_j are estimated by the RCTD algorithm as previously described [24]. We note that C-
 464 SIDE can also optionally import cell type proportions from external cell type proportion identification
 465 methods [27,28]. Here, some pixels are identified as single cell types while others as mixtures of multiple
 466 cell types. We can accurately estimate cell type proportions and platform effects without being aware
 467 of differential spatial gene expression because differential spatial gene expression is smaller than gene
 468 expression differences across cell types. After determining cell type proportions, C-SIDE estimates
 469 gene-specific overdispersion magnitude $\sigma_{\varepsilon,j,g}$ for each gene by maximum likelihood estimation (see
 470 *Supplementary Methods* for details). Finally, C-SIDE estimates the DE coefficients α by maximum
 471 likelihood estimation. For the final key step of estimating α , we use plugin estimates (denoted by $\hat{\cdot}$)
 472 of β , γ , and σ_{ε} . After we substitute (3) into (1) and (2), we obtain:

$$Y_{i,j,g} \mid \varepsilon_{i,j,g} \sim \text{Poisson} \left\{ N_{i,g} \exp \left[\log \left(\sum_{k=1}^K \hat{\beta}_{i,k,g} \exp \left(\alpha_{0,k,j,g} + \sum_{\ell=1}^L x_{i,\ell,g} \alpha_{\ell,k,j,g} \right) \right) + \hat{\gamma}_{j,g} + \varepsilon_{i,j,g} \right] \right\} \quad (4)$$

$$\varepsilon_{i,j,g} \sim \text{Normal}(0, \hat{\sigma}_{\varepsilon,j,g}^2), \quad (5)$$

473 We provide an algorithm for computing the maximum likelihood estimator of α , presented in the
 474 *Supplementary Methods*. Our likelihood optimization algorithm is a second-order, trust-region based
 475 optimization (see *Supplementary Methods* for details). In brief, we iteratively solve quadratic approx-
 476 imations of the log-likelihood, adaptively constraining the maximum parameter change at each step.
 477 Critically, the likelihood is independent for each gene j (and sample g), so separate genes are run in
 478 parallel in which case there are $K \times (L + 1)$ α parameters per gene and sample.

479 **Hypothesis testing**

480 In addition to estimating the vector $\alpha_{j,g}$ (dimensions $L + 1$ by K) for gene j and sample g , we
 481 can compute standard errors around $\alpha_{j,g}$. By asymptotic normality (see *Supplementary Methods* for
 482 details), we have approximately that (setting n to be the total number of pixels),

$$\sqrt{n}(\hat{\alpha}_{j,g} - \alpha_{j,g}) \sim \text{Normal}(0, I_{\alpha_{j,g}}^{-1}), \quad (6)$$

483 where $I_{\alpha_{j,g}}$ is the Fisher information of model (4), which is computed in the *Supplementary Methods*.
 484 Given this result, we can compute standard errors, confidence intervals, and hypothesis tests. As a
 485 consequence of (6), the standard error of $\alpha_{\ell,k,j,g}$, denoted $s_{\ell,k,j,g}$, is $\sqrt{(I_{\alpha_{j,g}}^{-1})_{\ell,k}/n}$.

486 First, we consider the case where we are interested in a single parameter, $\alpha_{\ell,k,j,g}$, for ℓ and g fixed
 487 and for each cell type k and gene j ; for example, $\alpha_{\ell,k,j,g}$ could represent the log-fold-change between
 488 two discrete regions. In this case, for each gene j , we compute the z-statistic, $z_{\ell,k,j,g} = \frac{\alpha_{\ell,k,j,g}}{s_{\ell,k,j,g}}$.
 489 Using a two-tailed z-test, we compute a p -value for the null hypothesis that $\alpha_{\ell,k,j,g} = 0$ as $p_{\ell,k,j,g} =$
 490 $2 * F(-|z_{\ell,k,j,g}|)$, where F is the distribution function of the standard Normal distribution. Finally,
 491 q -values are calculated across all genes within a cell type in order to control the false discovery rate
 492 using the Benjamini-Hochberg procedure [69]. We used a false discovery rate (FDR) of .01 (0.1 for
 493 nonparametric case) and a fold-change cutoff of 1.5 (N/A for nonparametric case). Additionally, for
 494 each cell type, genes were pre-filtered so that the expression within the cell type of interest had a total
 495 expression of at least 15 unique molecular identifiers (UMIs) over all pixels and at least 50% as large
 496 mean normalized expression as the expression within each other cell type.

497 For the multi-region case, we instead test for differences of pairs of parameters representing the
498 average expression within each region. As a result, p -values are scaled up due to multiple hypothesis
499 testing. We select genes which have significant differences between at least one pair of regions. For
500 other cases in which we are interested in multiple parameters, for example the nonparametric case, we
501 test each parameter individually and scale p -values due to multiple hypothesis testing.

502 Statistical inference on multiple samples/replicates

503 C-SIDE can be run on either one or multiple biological replicates and/or samples. In the case of
504 multiple replicates, we recall α_g and s_g are the differential expression and standard error for replicate
505 g , where $1 \leq g \leq G$, and $G > 1$ is the total number of replicates. We now consider testing for
506 differential expression across all replicates for covariate ℓ , cell type k , and gene j . In this case, we
507 assume that additional biological or technical variation across samples exists, such that each unknown
508 α_g is normally distributed around a population-level differential expression A , with standard deviation
509 τ :

$$\alpha_{\ell,k,j,g} \stackrel{i.i.d.}{\sim} \text{Normal}(A_{\ell,k,j}, \tau_{\ell,k,j}^2). \quad (7)$$

510 Under this assumption, and using (6) for the distribution of the observed single-sample estimates $\hat{\alpha}$,
511 we derive the following feasible generalized least squares estimator of A (see *Supplementary Methods*
512 for details),

$$\hat{A}_{\ell,k,j} := \frac{\sum_{g=1}^G (\hat{\alpha}_{\ell,k,j,g}) / (\hat{\tau}_{\ell,k,j}^2 + s_{\ell,k,j,g}^2)}{\sum_{g=1}^G 1 / (\hat{\tau}_{\ell,k,j}^2 + s_{\ell,k,j,g}^2)}. \quad (8)$$

513 Here, $\hat{\alpha}$ and s are obtained from C-SIDE estimates on individual samples (see (6)), whereas $\hat{\tau}^2$
514 represents the estimated variance across samples (Supplementary Figure 7). Please see the *Supplementary*
515 *Methods* for additional details such as the method of moments procedure [70] for estimating $\hat{\tau}_{\ell,k,j}^2$ and
516 the standard errors of A . Intuitively, our estimate of the population-level differential expression is a
517 variance-weighted sum over the DE estimates of individual replicates, and we note that our multiple-
518 replicate approach is similar to widely used meta-analysis methods [70, 71]. As we have obtained
519 estimates and standard errors of A , these are subsequently used in hypothesis testing for the hypoth-
520 esis that $A_{\ell,k,j} = 0$ in a manner identical to what is described above in *Hypothesis testing* for the
521 single replicate case. We also derived a version of this estimator for the case where there are multiple
522 biological samples and multiple replicates within each sample (*Supplementary Methods*).

523 Collection and preprocessing of scRNA-seq, spatial transcriptomics, amyloid 524 beta imaging, and HCR data

525 We collected four Alzheimer's Slide-seq mouse hippocampus sections [38] using the Slide-seqV2 pro-
526 tocol [2] (see *Supplementary Methods* for details) on a female 8.8 month old J20 Alzheimer's mouse
527 model [38]. We used three total Slide-seq mouse cerebellum sections, two collected using the Slide-
528 seqV2 protocol, and one section used from a previous study [24]. Recall that data from multiple sections
529 is integrated as described in *Multiple replicates*. The Slide-seq mouse testes and mouse cancer datasets
530 were used from recent previous studies [12, 49]. In particular, the tumor dataset represented a single
531 *Kras*^{G12D/+} *Trp53*^{-/-} (KP) mouse metastatic lung adenocarcinoma tumor deposit in the liver [50]. The
532 MERFISH hypothalamus dataset was obtained from a publicly available study [11]. To identify cell
533 types on these datasets, we utilized publicly available single-cell RNA-seq datasets for the testes [72],
534 hypothalamus [11], cerebellum [32], cancer [49], and Alzheimer's hippocampus datasets [73]. All these
535 scRNA-seq datasets have previously been annotated by cell type.

536 Slide-seq data was preprocessed using the Slide-seq tools pipeline [2]. For all spatial transcriptomics
537 datasets, the region of interest (ROI) was cropped prior to running C-SIDE, and spatial transcriptomic

538 spots were filtered to have a minimum of 100 UMIs. We used prior anatomical knowledge to crop the
539 ROI from an image of the total UMI counts per pixel across space, which in many cases allows
540 one to observe overall anatomical features. For example, in Slide-seq Alzheimer's hippocampus, the
541 somatosensory cortex was cropped out prior to analysis.

542 For the Alzheimer's dataset, in order to test for differential expression with respect to amyloid
543 plaques, we collected fluorescent images of DAPI and amyloid beta (A β), using IBL America Amyloid
544 Beta (N) (82E1) A β Anti-Human Mouse IgG MoAb on sections adjacent to the Slide-seq data. We
545 co-registered the DAPI image to the adjacent Slide-seq total UMI image using the `ManualAlignImages`
546 function from the `STutility` R package [74]. To calculate plaque density, plaque images were convolved
547 with an exponentially-decaying isotropic filter, using a threshold at the 0.9 quantile, and normalized
548 to be between 0 and 1. For each Slide-seq section, plaque density was defined as the average between
549 the plaque densities on the two adjacent amyloid sections.

550 For *in situ* RNA hybridization validation of cerebellum DE results, we collected hybridization chain
551 reaction (HCR) data on genes *Aldoc*, *Kcnd2*, *Mybpc1*, *Plcb4*, and *Tmem132c* (Supplementary Table
552 9) using a previously developed protocol [75]. We simultaneously collected cell type marker genes
553 of Bergmann (*Gdf10*), granule (*Gabra6*), and Purkinje (*Calb1*) cell types, markers that were sourced
554 from a prior cerebellum study [32]. Data from *Kcnd2* was removed due to the HCR fluorescent channel
555 failing to localize RNA molecules, but rather reflecting tissue autofluorescence. ROIs of nodular and
556 anterior regions were cropped, and background, defined as median signal, was subtracted. For this
557 data, DE was calculated as the log-fold-change, across ROIs, of average gene signal over the pixels
558 within the ROI containing cell type markers of a particular cell type. Pixels containing marker genes of
559 multiple cell types were removed. C-SIDE single-sample standard errors in Figure 3d were calculated
560 by modeling single-sample variance as the sum of the variance across samples and variance representing
561 uncertainty around the population mean.

562 Cell type proportion estimation and construction of covariates

563 For each dataset, we constructed at least one covariate, an axis along which to test for DE. All
564 covariates were scaled linearly to have minimum 0 and maximum 1. For the cerebellum dataset, the
565 covariate was defined as an indicator variable representing membership within the nodular region (as
566 opposed to the anterior region). The nodular and anterior ROIs were annotated manually from the
567 total UMI image, and all other regions were removed. For the testes dataset, the covariate was a
568 discrete variable representing the cellular microenvironment of tubule stage, labels that were obtained
569 from tubule-level gene expression clustering from the previous Slide-seq testes study [12]. In that study
570 and here, tubules are categorized into 4 main stages according to tubule sub-stage groups of stage I–III,
571 IV–VI, VII–VIII, and IX–XII. For the cancer dataset, the covariate was chosen to be the density of
572 the myeloid cell type. Cell type density was calculated by convolving the cell type locations, weighted
573 by UMI number, with an exponential filter. For this dataset, we also ran C-SIDE nonparametrically.
574 For the Alzheimer's hippocampus dataset, the covariate was chosen to be the plaque density, defined
575 in Section *Collection and preprocessing*. For the MERFISH hypothalamus dataset, the covariate was
576 defined as distance to the midline, and we also considered quadratic functions of midline distance by
577 adding squared distance as an covariate. For the quadratic MERFISH C-SIDE model, we conducted
578 hypothesis testing on the quadratic coefficient. To estimate platform effects and cell type proportions,
579 RCTD was run on *full mode* for the testes dataset, and was run on *doublet mode* for all other datasets
580 with default parameters [24].

581 Validation with simulated gene expression dataset

582 We created a ground truth DE simulation to test C-SIDE on the challenging situation of mixtures
583 between two cell type layers. We tested C-SIDE on a dataset of cell type mixtures simulated from
584 the cerebellum single-nucleus RNA-seq dataset, which was also used as the reference for cell type
585 mapping. We restricted to Purkinje and Bergmann cell types, which are known to spatially colocalize.

586 In order to simulate a cell type mixture of cell types A (Purkinje) and B (Bergmann), we randomly
587 chose a cell from each cell type, and sampled a predefined number of UMIs from each cell (total
588 1,000). We defined two discrete spatial regions (Figure 1a), populated with A/B cell type mixtures.
589 We varied the mean cell type proportion difference across the two regions and also simulated the case
590 of cell type proportions evenly distributed across the two regions. Cell type-specific spatial differential
591 gene expression also was simulated across the two regions. To simulate cell type-specific differential
592 expression in the gene expression step of the simulation, we multiplicatively scaled the expected gene
593 counts within each cell of each cell type. An indicator variable for the two spatial bins was used as the
594 C-SIDE covariate.

595 Additional computational analysis

596 For confidence intervals on data points or groups of data points (Figure 4d, Figure 4g), we used the
597 predicted variance of data points from C-SIDE (see *Supplementary Methods* for details). Likewise, for
598 such analysis we used predicted counts from C-SIDE at each pixel (*Supplementary Methods*). For the
599 testes dataset, a cell type was considered to be present on a bead if the proportion of that cell type
600 was at least 0.25 (Figure 4d). Additionally, cell type and stage-specific marker genes were defined as
601 genes that had a fold-change of at least 1.5 within the cell type of interest compared to each other cell
602 type. We also required significant cell type-specific differential expression between the stage of interest
603 with all other stages (fold-change of at least 1.5, significance at the level of 0.001, Monte Carlo test on
604 Z-scores). Cyclic genes were defined as genes whose minimum expression within a cell type occurred
605 two tubule stages away from its maximum expression, up to log-space error of up to 0.25.

606 For nonparametric C-SIDE on the tumor dataset, we used hierarchical Ward clustering to cluster
607 quantile-normalized spatial gene expression patterns into 7 clusters. For gene set testing on the tumor
608 dataset, we tested the 50 hallmark gene sets from the MSigDB database [76] for aggregate effects
609 in C-SIDE differential expression estimates for the tumor cell type. For the nonparametric case, we
610 used a binomial test with multiple hypothesis correction to test for enrichment of any of the 7 spatial
611 clusters of C-SIDE-identified significant genes in any of the 50 gene sets. For the parametric case, we
612 used a permutation test on the average value of C-SIDE Z scores for a gene set. That is, we modified
613 an existing gene set enrichment procedure [77] by filtering for genes with a fold-change of at least 1.5
614 and using a two-sided permutation test rather than assuming normality. In both cases, we filtered
615 to gene sets with at least 5 genes and we used Benjamini-Hochberg procedure across all gene sets to
616 control the false discovery rate at 0.05. The proportion of variance not due to sampling noise (Figure
617 5b) was calculated by considering the difference between observed variance on normalized counts and
618 the expected variance due to Poisson sampling noise.

619 We considered and tested several simple alternative methods to C-SIDE, which represent general
620 classes of approaches. First, we considered a two-sample Z-test on single cells (defined as pixels with
621 cell type proportion at least 0.9). Additionally, we tested *Bulk* differential expression, which estimated
622 differential expression as the log-ratio of average normalized gene expression across two regions. The
623 *Single* method of differential expression rounded cell type mixtures to the nearest single cell type and
624 computed the log-ratio of gene expression of cells in that cell type. Finally, the *Decompose* method
625 of differential expression used a previously-developed method to compute expected gene expression
626 counts for each cell type [24], followed by computing the ratio of cell type-specific gene expression in
627 each region.

628 Implementation details

629 C-SIDE is publicly available as part of the R package <https://github.com/dmcable/spacexr>. The
630 quadratic program that arises in the C-SIDE optimization algorithm is solved using the *quadprog*
631 package in R [78]. Prior to conducting analysis on C-SIDE output, all ribosomal proteins and mi-
632tochondrial genes were filtered out. Additional parameters used for running C-SIDE are shown in
633 Supplementary Table 10. C-SIDE was tested on a Macintosh laptop computer with a 2.4 GHz Intel

634 Core i9 processor and 32GB of memory (we recommend at least 4GB of memory to run C-SIDE). For
635 example, we timed C-SIDE with four cores on one of the Slide-seq cerebellum replicates, containing
636 2,776 pixels across two regions, 5 cell types, and 4,812 genes. Under these conditions, C-SIDE ran in
637 13 minutes and 47 seconds (excluding the cell type assignment step in which computational efficiency
638 has been described previously [24]).

639 **Author Contributions**

640 D.M.C., R.A.I., and F.C. conceived the study; F.C., E.M., E.Z.M., and D.C. designed the Slide-seq,
641 antibody stain, and HCR experiments; E.M. generated the Slide-seq, antibody stain, and HCR data;
642 D.M.C., R.A.I., and F.C. developed the statistical methods; D.M.C., F.C., and R.A.I. designed the
643 analysis; D.M.C., S.Z., M.D., R.A.I., and F.C. analyzed the data; D.M.C., F.C., R.A.I., V.S., and H.C.
644 interpreted biological results; V.S. annotated the tumor H&E stain; D.M.C., F.C., and R.A.I. wrote
645 the manuscript; all authors read and approved the final manuscript.

646 **Acknowledgements**

647 We thank Luli Zou and Robert Stickels for providing valuable input on the analysis. We thank
648 Tongtong Zhao and Zachary Chiang for generously providing the cancer Slide-seq data and providing
649 helpful feedback. We thank Samuel Marsh for kindly providing mouse J20 Alzheimer's model samples.
650 We thank members of the Chen lab, Irizarry lab, and Macosko lab including Tushar Kamath for helpful
651 discussions and feedback. D.C. was supported by a Fannie and John Hertz Foundation Fellowship and
652 an NSF Graduate Research Fellowship. This work was supported by an NIH Early Independence
653 Award (DP5, 1DP5OD024583 to F.C.), the NHGRI (R01, R01HG010647 to F.C. and E.Z.M), as well
654 as the Burroughs Wellcome Fund, the Searle Scholars Award, and the Merkin Institute to F.C.. R.A.I.
655 was supported by NIH grants R35GM131802 and R01HG005220.

656 **Conflict of Interest Statement**

657 E.Z.M. and F.C. are listed as inventors on a patent application related to Slide-seq. F.C. is a paid
658 consultant for Celsius Therapeutics and Atlas Bio.

659 **Data Availability Statement**

660 Slide-seq V2 data generated for this study is available at the Broad Institute Single Cell Portal https://singlecell.broadinstitute.org/single_cell/study/SCP1663. Additional publicly available data
661 from other studies that was used for analysis is also included in this repository.
662

663 **Code Availability Statement**

664 C-SIDE is implemented in the open-source R package *spacexr*, with source code freely available at
665 <https://github.com/dmcable/spacexr>. Additional code used for analysis in this paper is available
666 at <https://github.com/dmcable/spacexr/tree/master/AnalysisC-SIDE>.

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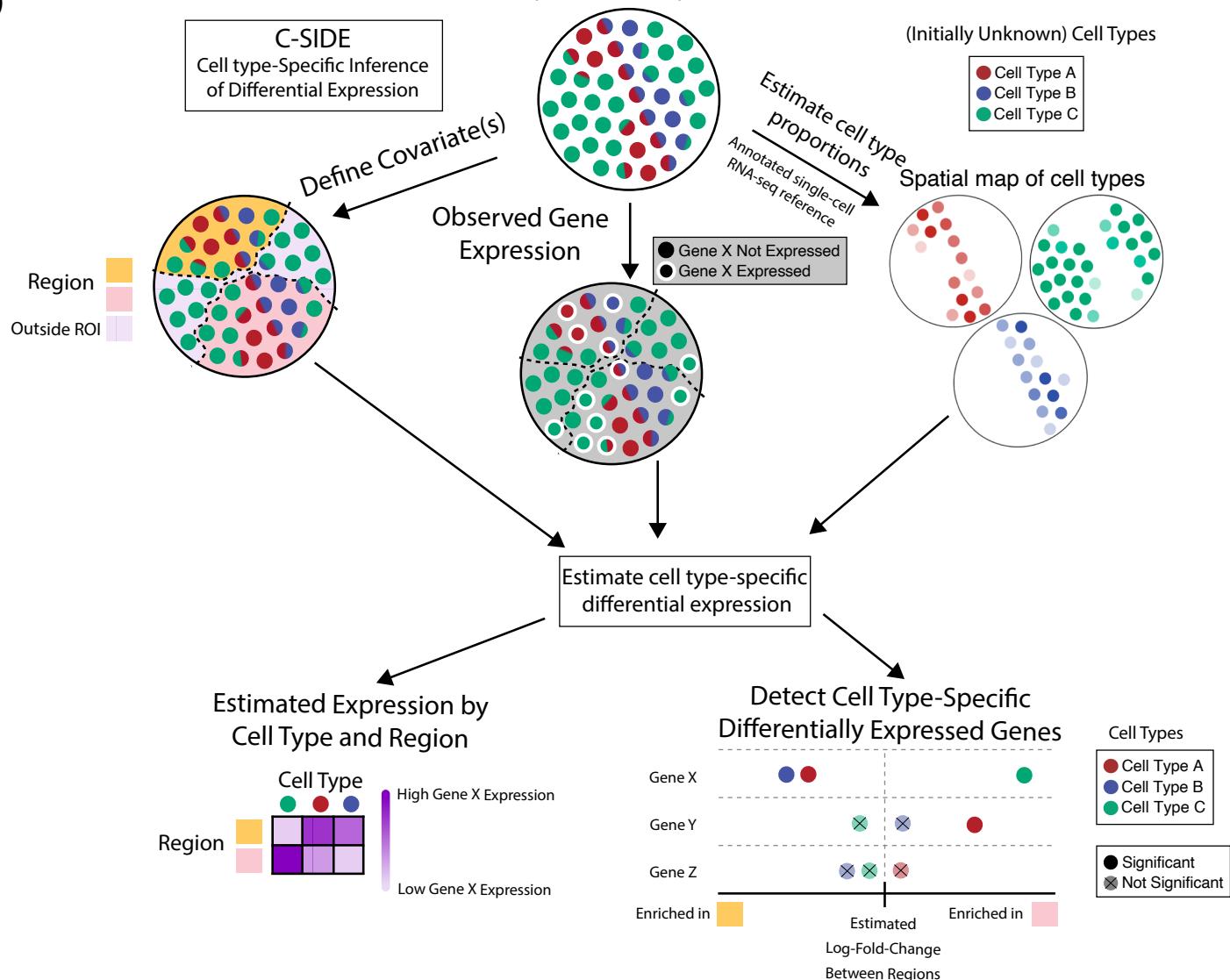
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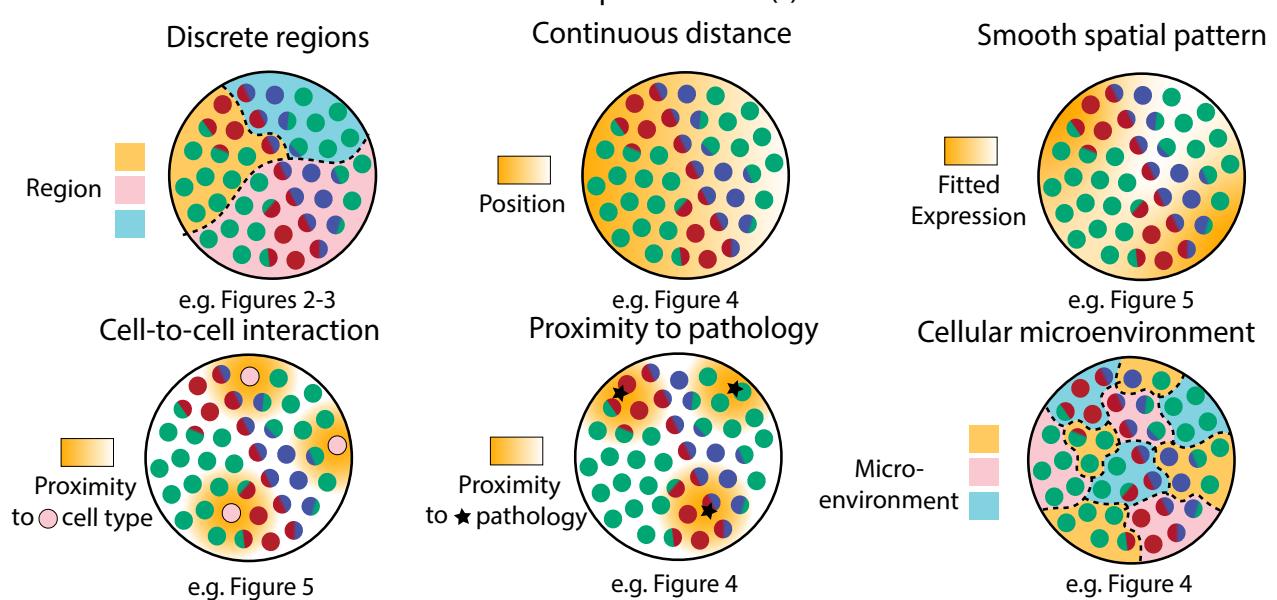
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840 **Figures**

a)



b)

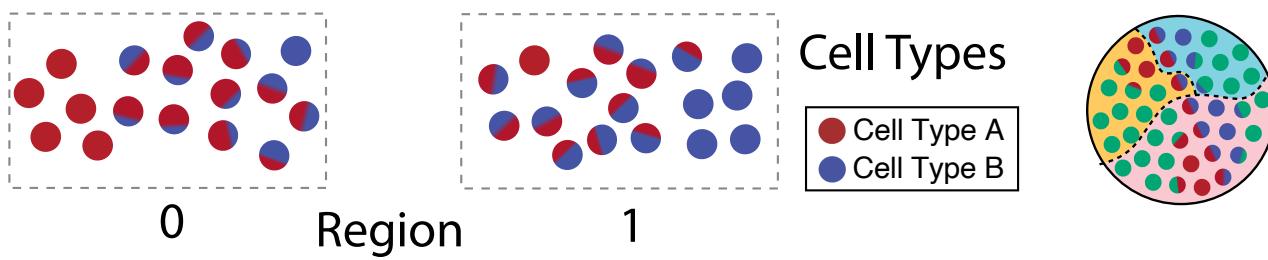


841 Figure 1: Cell type-Specific Inference of Differential Expression learns cell type-specific differential
842 expression from spatial transcriptomics data.

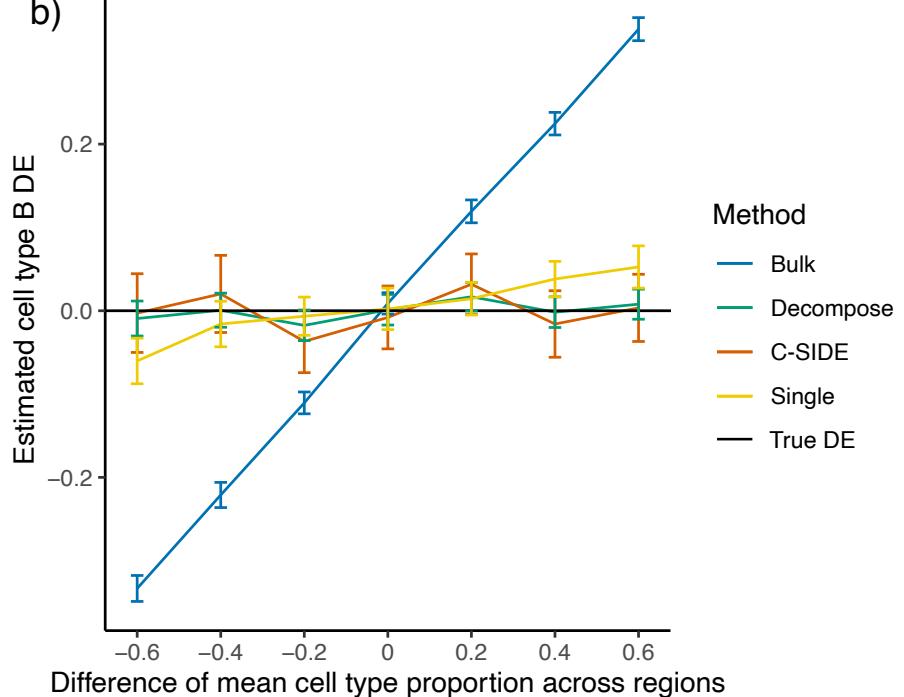
843 (a) Schematic of the C-SIDE Method. Top: C-SIDE inputs: a spatial transcriptomics dataset with
844 observed gene expression (potentially containing cell type mixtures) and a covariate for differential
845 expression. Middle: C-SIDE first assigns cell types to the spatial transcriptomics dataset, and
846 covariates are defined. Bottom: C-SIDE estimates cell type-specific gene expression along the
847 covariate axes.

848 (b) Example covariates for explaining differential expression with C-SIDE. Top: Segmentation into
849 multiple regions, continuous distance from some feature, or general smooth patterns (nonparamet-
850 ric). Bottom: density of interaction with another cell type or pathological feature or a discrete
851 covariate representing the cellular microenvironment.

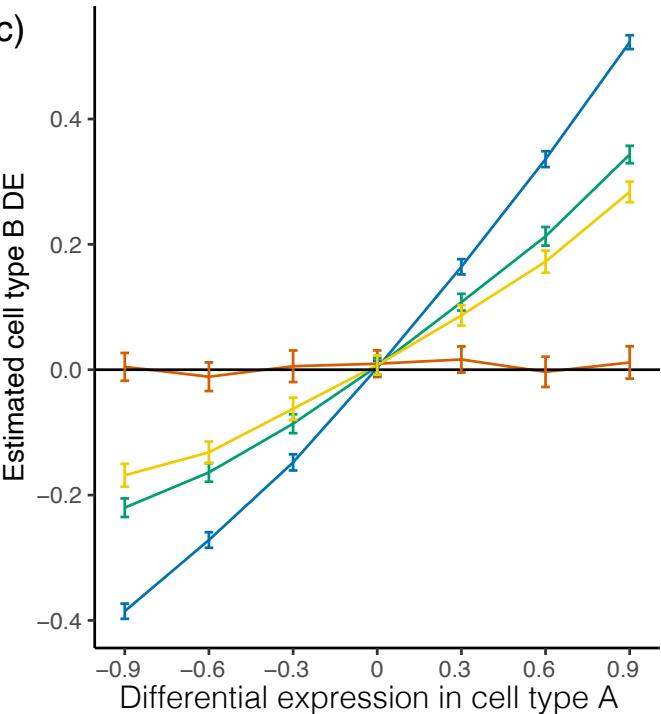
a)



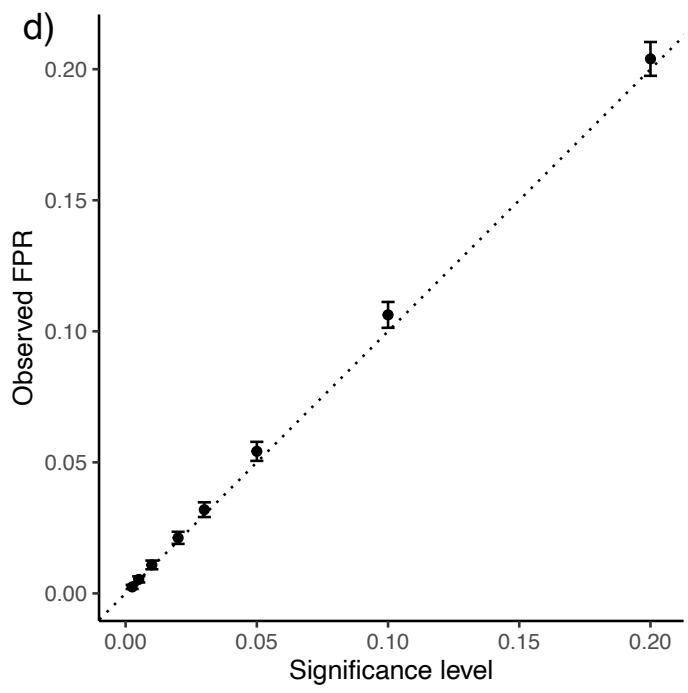
b)



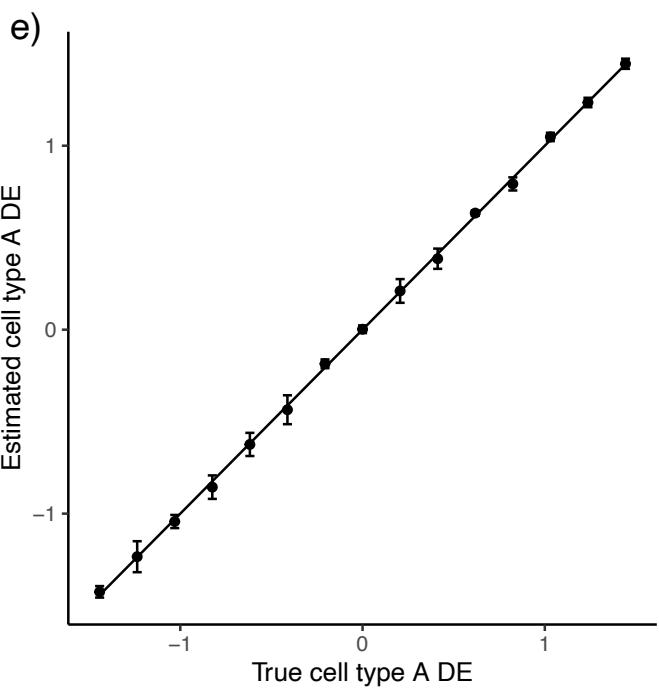
c)



d)



e)



852 Figure 2: C-SIDE provides unbiased estimates of cell type-specific differential expression in simu-
853 lated data.

854 All: C-SIDE was tested on a dataset of simulated mixtures of single cells from a single-nucleus
855 RNA-seq cerebellum dataset. Differential expression (DE) axes represent DE in log2-space of region 1
856 w.r.t. region 0.

857 (a) Pixels are grouped into two regions, and genes are simulated with ground truth DE across regions.
858 Each region contains pixels containing mixtures of various proportions between cell type A and
859 cell type B. The difference in average cell type proportion across regions is varied across simulation
860 conditions.

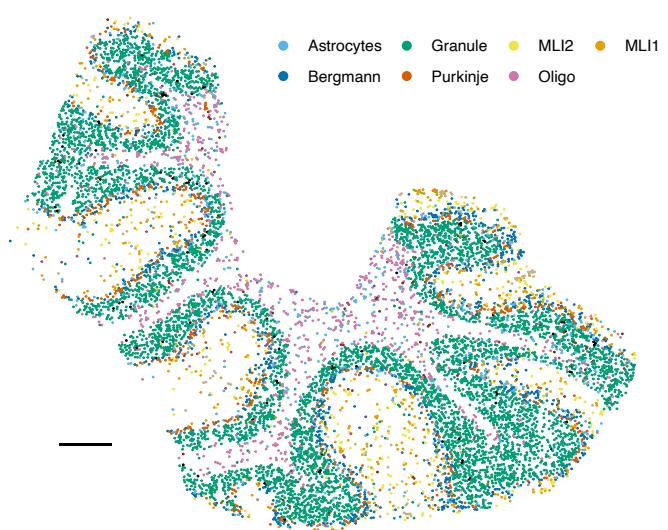
861 (b) Mean estimated cell type B *Astn2* DE (differential expression) across two regions as a function of
862 the difference in mean cell type proportion across regions. *Astn2* is simulated with ground truth 0
863 spatial DE, and an average of ($n = 100$) estimates is shown, along with standard errors. Black line
864 represents ground truth 0 DE (cell type B). Four methods are shown: *Bulk*, *Decompose*, *Single*,
865 and *C-SIDE* (see *Methods* for details).

866 (c) Same as (b) for *Nrxn3* cell type B differential gene expression as a function of DE in cell type A,
867 where *Nrxn3* is simulated to have DE within cell type A but no DE in cell type B.

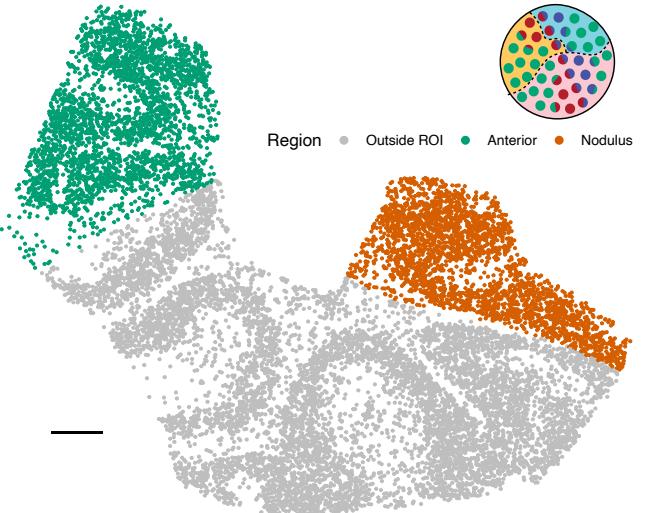
868 (d) For each significance level, C-SIDE's false positive rate (FPR), along with ground truth identity
869 line (s.e. shown, $n = 1500$, 15 genes, 100 replicates per gene).

870 (e) C-SIDE mean estimated cell type A differential expression vs. true cell type A differential expres-
871 sion (average over $n = 500$ replicates, s.e. shown). Ground truth identity line is shown, and one
872 gene is used for the simulation per DE condition (out of 15 total genes).

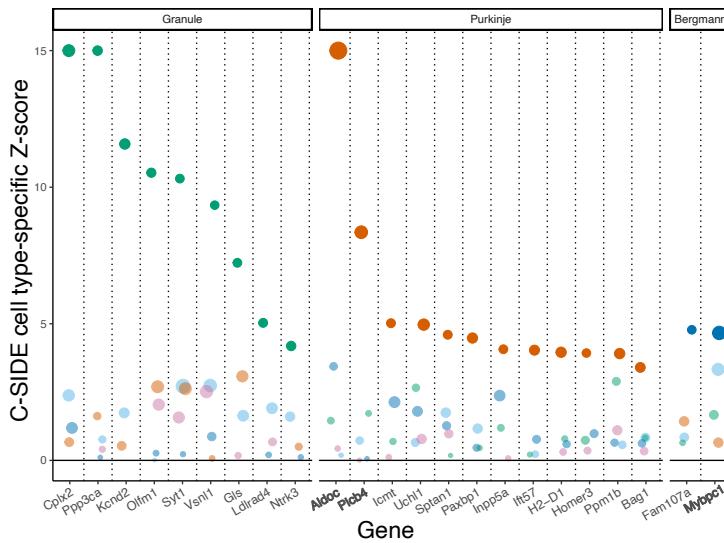
a)



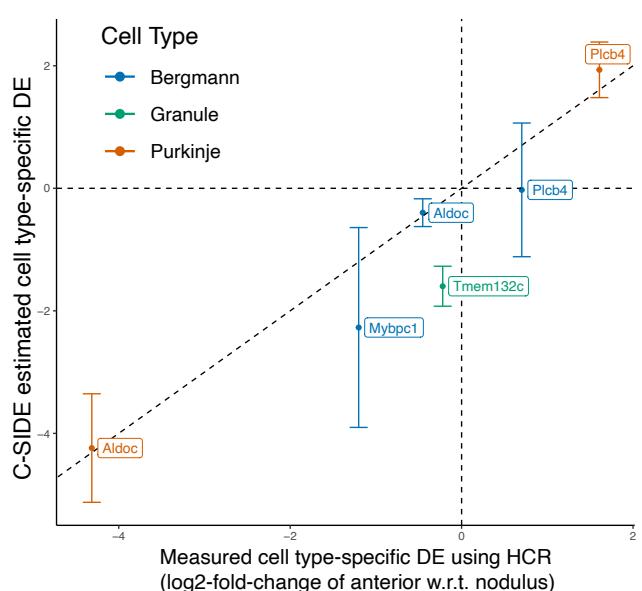
b)



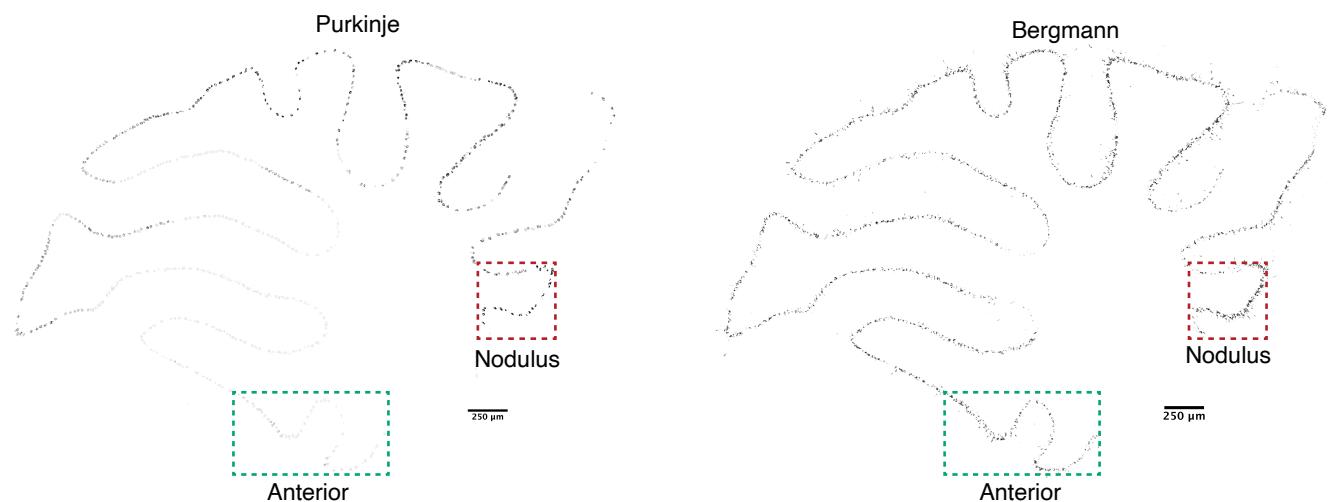
c)



d)



e)



873 Figure 3: C-SIDE’s estimated cell type-specific differential expression is validated by HCR-FISH.

874 (a) C-SIDE’s spatial map of cell type assignments in the cerebellum Slide-seq dataset. Out of 19 cell
875 types, the seven most common appear in the legend. Reproduced from [24]. Three total replicates
876 were used to fit C-SIDE.

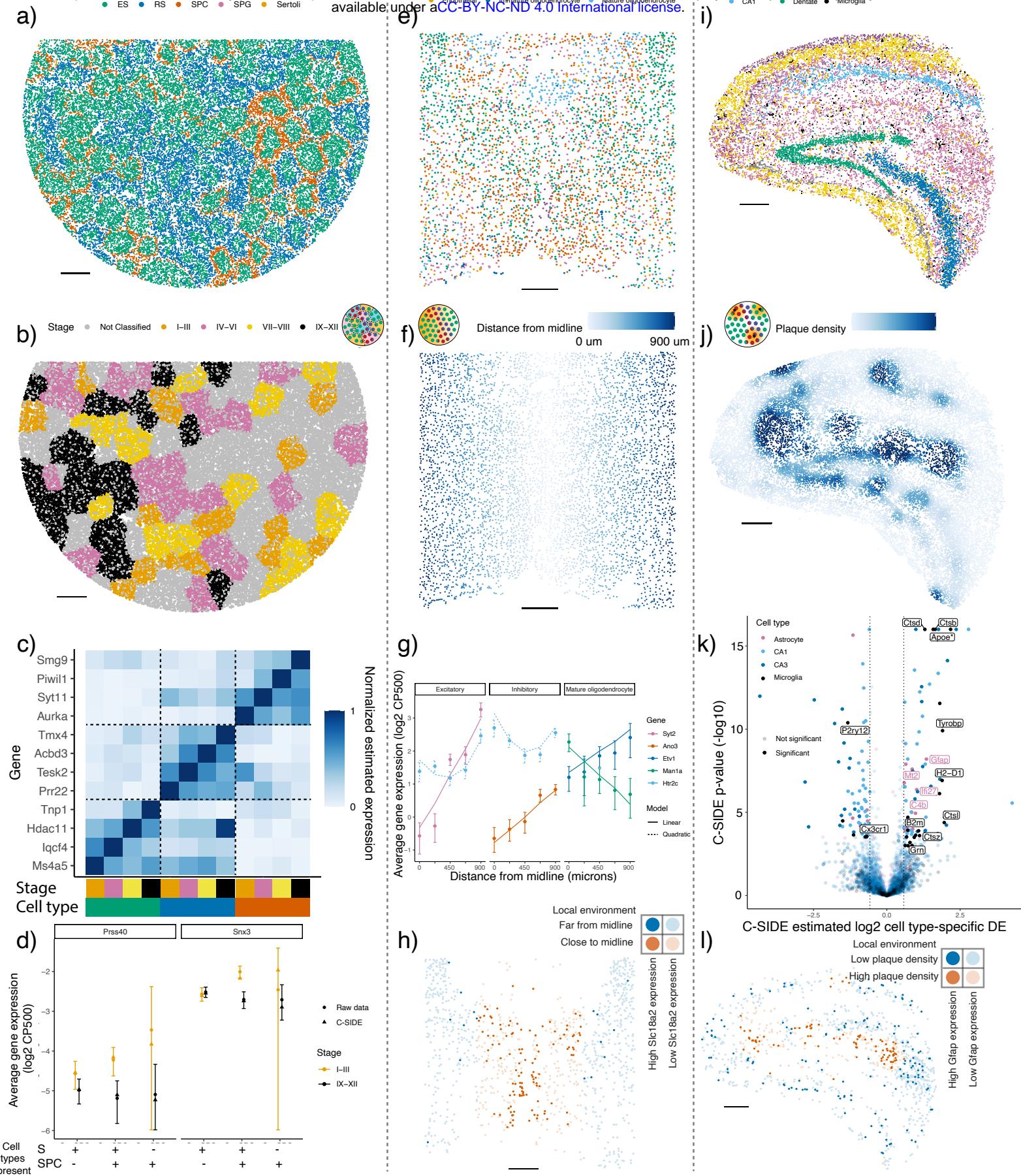
877 (b) Covariate used for C-SIDE, representing the anterior lobule region (green) and nodulus (red).
878 Schematic refers to the C-SIDE problem type outlined in Figure 1b.

879 (c) C-SIDE Z-score for testing for DE for each gene and for each cell type. Genes are grouped by cell
880 type with maximum estimated DE, and estimated DE magnitude appears as size of the points.
881 Bold genes appear below in HCR validation.

882 (d) Scatterplot of C-SIDE DE estimates vs. HCR measurements for cell type-specific log2 differential
883 expression. Positive values indicate gene expression enrichment in the anterior region. Error bars
884 represent C-SIDE confidence intervals for predicted DE on a new biological replicate. A dotted
885 identity line is shown, and cell types are colored.

886 (e) HCR images of *Aldoc* continuous gene expression. Only pixels with high cell type marker measure-
887 ments for Purkinje (left) and Bergmann (right) are shown. Regions of interest (ROIs) of nodulus
888 and anterior regions are outlined in green and red, respectively.

889 All scale bars 250 microns.



890 Figure 4: C-SIDE discovers cell type-specific differential expression in a diverse set of problems on
891 testes, Alzheimer's hippocampus, and hypothalamus datasets.

892 All panels: results of C-SIDE on the Slide-seqV2 testes (left column), MERFISH hypothalamus
893 (middle column), and Slide-seqV2 Alzheimer's hippocampus (right column). Schematics in b,f,j refer-
894 ence C-SIDE problem types (Figure 1b).

895 (a) C-SIDE's spatial map of cell type assignments in testes. All cell types are shown, and the most
896 common cell types appear in the legend.

897 (b) Covariate used for C-SIDE in testes: four discrete tubule stages.

898 (c) Cell type and tubule stage-specific genes identified by C-SIDE. C-SIDE estimated expression is
899 standardized between 0 and 1 for each gene. Columns represent C-SIDE estimates for each cell
900 type and tubule stage.

901 (d) Log2 average expression (in counts per 500 (CP500)) of pixels grouped based on tubule stage and
902 presence or absence of spermatid (S) cell types (defined as elongating spermatid (ES) or round
903 spermatid (RS)) and/or spermatocyte (SPC) cell type. Circles represent raw data averages while
904 triangles represent C-SIDE predictions, and error bars around circular points represent ± 1.96 s.d.
905 (Supplementary methods). Genes *Prss40* and *Snx3* are shown on left and right, respectively.

906 (e) Same as (a) for hypothalamus.

907 (f) Covariate used for C-SIDE in hypothalamus: continuous distance from midline.

908 (g) Log2 average expression (in counts per 500 (CP500)) of genes identified to be significantly differen-
909 tially expressed by C-SIDE for each of the excitatory, inhibitory, and mature oligodendrocyte cell
910 types. Single cell type pixels are binned according to distance from midline, and points represent
911 raw data averages while lines represents C-SIDE predictions and error bars around points represent
912 ± 1.96 s.d. (Supplementary methods).

913 (h) Spatial visualization of *Slc18a2*, whose expression within inhibitory neurons was identified by C-
914 SIDE to depend on midline distance. Red/blue represents inhibitory neurons close/far to midline,
915 respectively. Bold points inhibitory neurons expressing *Slc18a2* at a level of at least 10 counts per
916 500.

917 (i) Same as (a) for Alzheimer's hippocampus, where four total replicates were used to fit C-SIDE.

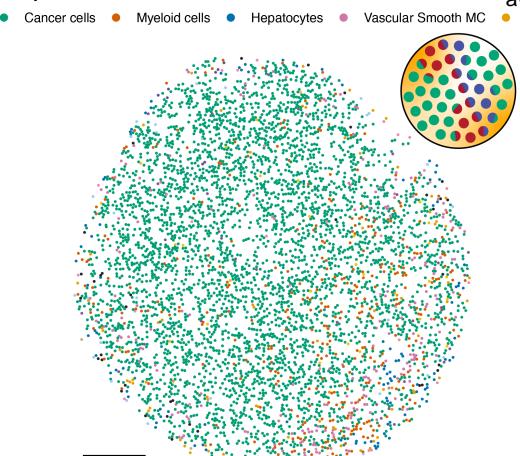
918 (j) Covariate used for C-SIDE in Alzheimer's hippocampus: continuous density of beta-amyloid (A β)
919 plaque.

920 (k) Volcano plot of C-SIDE differential expression results in log2-space, with positive values corre-
921 sponding to plaque-upregulated genes. Color represents cell type, and a subset of significant genes
922 are labeled. Dotted lines represents 1.5x fold-change cutoff used for C-SIDE. (*): *Apoe* didn't pass
923 default C-SIDE gene filters(Methods) because 4x higher expression in astrocytes than microglia.

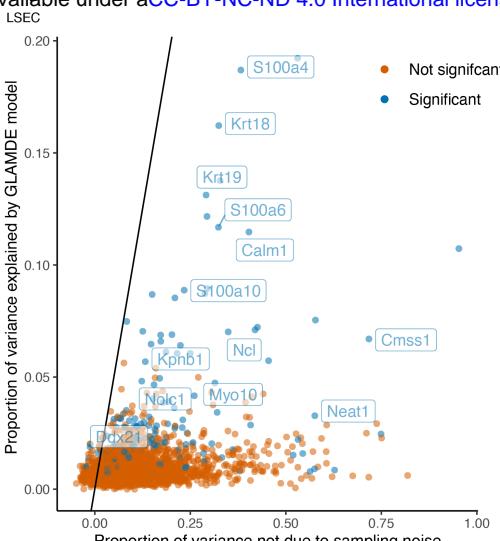
924 (l) Spatial visualization of *Gfap*, whose expression within astrocytes was identified by C-SIDE to
925 depend on plaque density. Red/blue represents the astrocytes in high/low plaque density areas,
926 respectively. Bold points represent astrocytes expressing *Gfap* at a level of at least 1 count per
927 500.

928 All scale bars 250 microns.

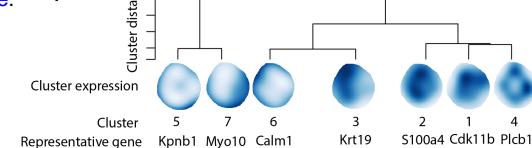
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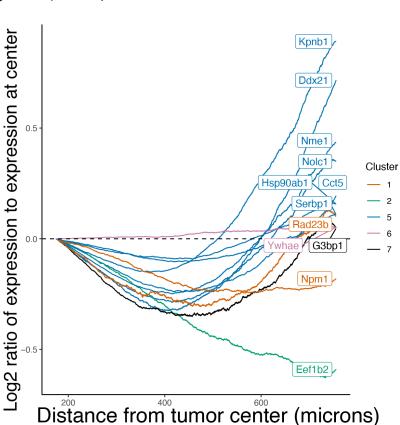
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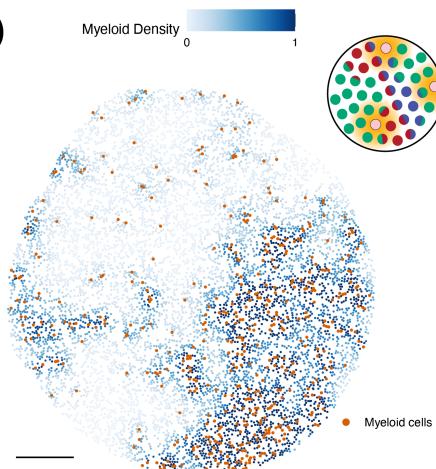
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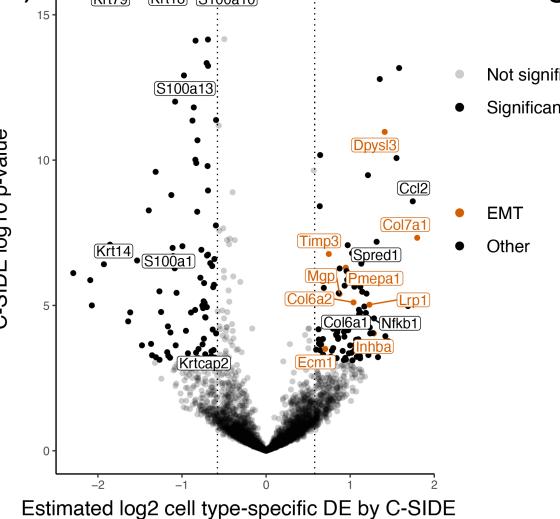
d)



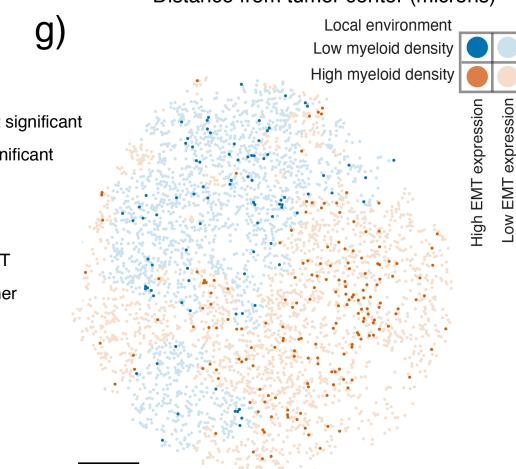
e)



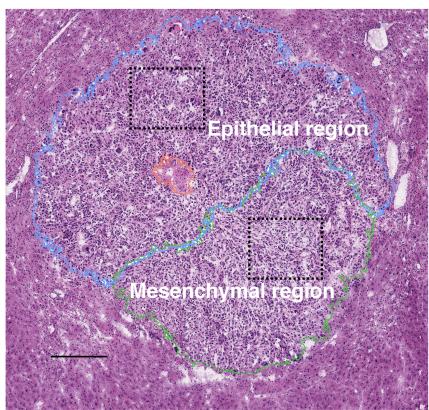
f)



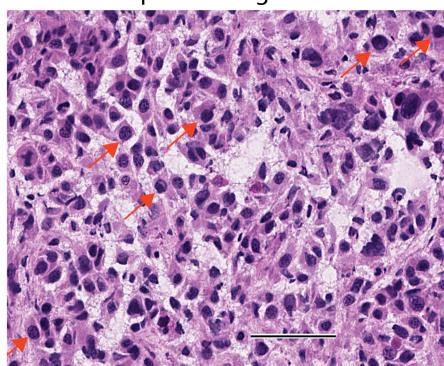
g)



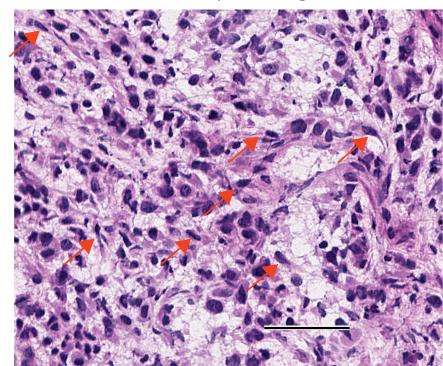
h)



Epithelial region



Mesenchymal region



929 Figure 5: C-SIDE enables the discovery of differentially expressed pathways in a *Kras*^{G12D/+}
930 *Trp53*^{+/−} (KP) mouse model.

931 All panels: C-SIDE was run on multiple cell types, but plots represent C-SIDE results on the tumor
932 cell type. Nonparametric C-SIDE results are shown in panels b–d, while parametric C-SIDE results are
933 shown in panels e–h.

934 (a) C-SIDE’s spatial map of cell type assignments. Out of 14 cell types, the five most common appear
935 in the legend.

936 (b) Scatter plot of C-SIDE R^2 and overdispersion (defined as proportion of variance not due to sam-
937 pling noise) for nonparametric C-SIDE results on the tumor cell type. Identity line is shown,
938 representing the maximum possible variance that could be explained by any model.

939 (c) Dendrogram of hierarchical clustering of ($n = 162$ significant genes) C-SIDE’s fitted smooth spa-
940 tial patterns at the resolution of 7 clusters. Each spatial plot represents the average fitted gene
941 expression patterns over the genes in each cluster.

942 (d) Moving average plot of C-SIDE fitted gene expression (normalized to expression at center) as a
943 function of distance from the center of the tumor for 12 genes in the *Myc* targets pathway identified
944 to be significantly spatially differentially expressed by C-SIDE.

945 (e) Covariate used for parametric C-SIDE: continuous density of myeloid cell types in the tumor.
946 Schematic refers to C-SIDE problem type (Figure 1b).

947 (f) Volcano plot of C-SIDE log2 differential expression results on the tumor cell type with positive
948 values representing upregulation in the presence of myeloid immune cells. A subset of significant
949 genes are labeled, and dotted lines represent 1.5x fold-change cutoff.

950 (g) Spatial plot of total expression in tumor cells of the 9 differentially expressed epithelial-mesenchymal
951 transition (EMT) genes identified by C-SIDE in (f). Red represents the tumor cells in high myeloid
952 density areas, whereas blue represents tumor cells in regions of low myeloid density. Bold points
953 represent tumor cells expressing these EMT genes at a level of at least 2.5 counts per 500.

954 (h) Hematoxylin and eosin (H&E) image of adjacent section of the tumor. Left: mesenchymal (green),
955 necrosis (red), and epithelial (blue) annotated tumor regions, with dotted boxes representing ep-
956 ithelial and mesenchymal areas of focus for the other two panels. Middle/right: enlarged images
957 of epithelial (middle) or mesenchymal (right) regions. Red arrows point to example tumor cells
958 with epithelial (middle) or mesenchymal (right) morphology.

959 All scale bars 250 microns, except for (h) middle/right, which has 50 micron scale bars.

960 Supplementary Methods

961 Introduction and model definition

962 We now revisit our Cell type-Specific Inference of Differential Expression (C-SIDE) model at an in-
 963 creased level of detail. Recall the following definition of the C-SIDE model, where for each pixel
 964 $i = 1, \dots, I$ in the spatial transcriptomics dataset, we denote the observed gene expression counts as
 965 $Y_{i,j,g}$ for each gene $j = 1, \dots, J$ and experimental sample $g = 1, \dots, G$:

$$Y_{i,j,g} \mid \lambda_{i,j,g} \sim \text{Poisson}(N_{i,g} \lambda_{i,j,g}) \quad (9)$$

$$\log(\lambda_{i,j,g}) = \log \left(\sum_{k=1}^K \beta_{i,k,g} \mu_{i,k,j,g} \right) + \gamma_{j,g} + \varepsilon_{i,j,g},$$

966 with $N_{i,g}$ the total transcript count or number of unique molecular identifies (UMIs) for pixel i and
 967 sample g , K the number of cell types present in our dataset, $\mu_{k,j,g}$ the mean gene expression profile for
 968 cell type k and gene j and sample g , $\beta_{i,k,g}$ the proportion of the contribution of cell type k to pixel i
 969 in sample g , $\gamma_{j,g}$ a gene-specific platform random effect, and $\varepsilon_{i,j,g}$ a random effect to account for other
 970 technical and biological sources of variation. We assume $\gamma_{j,g}$ and $\varepsilon_{i,j,g}$ both follow normal distributions
 971 with mean 0 and standard deviation $\sigma_{\gamma,g}$ and $\sigma_{\varepsilon,j,g}$, respectively. Lastly, $\mu_{i,k,j,g}$ represents the average
 972 gene expression of gene j in cell type k at pixel location i in sample g . We model $\mu_{i,k,j,g}$, for each gene
 973 j , each cell type k , and each sample g as depending log-linearly on several covariates, x :

$$\log(\mu_{i,k,j,g}) = \alpha_{2,0,k,j,g} + \sum_{\ell=1}^{L_1} x_{1,i,\ell,g} \alpha_{1,\ell,j,g} + \sum_{\ell=1}^{L_2} x_{2,i,\ell,g} \alpha_{2,\ell,k,j,g}. \quad (10)$$

974 More specifically, we split our covariates into two sets (of sizes L_1 and L_2). The first set, $x_{1,i,\ell,g}$, share
 975 coefficients across cell types, while the second set, $x_{2,i,\ell,g}$, has a different coefficient for each cell type.
 976 This notation is different from the presentation of C-SIDE in the main methods section, in which x_1
 977 was not present and no coefficients were shared across cell types. In practice, we do not typically
 978 assume that differential expression is shared across cell types (that is, $x_{1,i,\ell,g}$ is not used), but x_1 is
 979 included here as an optional feature. We have $x_{\cdot,i,\ell,g}$ representing the ℓ 'th covariate, evaluated at
 980 pixel i in sample g . In all cases, x is pre-determined to contain variables on which gene expression is
 981 hypothesized to depend.

982 For each covariate x , there is a corresponding coefficient α . More precisely, $\alpha_{1,\ell,j,g}$ represents a
 983 gene expression change per unit change of $x_{1,i,\ell,g}$ for gene j in sample g . Note that this coefficient is
 984 the same across all cell types. On the other hand, $\alpha_{2,\ell,k,j,g}$ represents a gene expression change per
 985 unit change of $x_{2,i,\ell,g}$ specific to cell type k in sample g . Finally, $\alpha_{2,0,k,j,g}$ represents the intercept
 986 term for gene j and cell type k in sample g . For ease of notation, we will sometimes use $\alpha_{1,k,j,g}$
 987 to equal $\alpha_{1,\ell,j,g}$ for all ℓ . Moreover, we will use α to refer to the joint vector of both α_1 and α_2 .
 988 The parameters α are estimated by C-SIDE by maximum likelihood. C-SIDE also obtains standard
 989 errors for each coefficient α . These standard errors are subsequently used for confidence intervals and
 990 hypothesis testing.

991 Maximum Likelihood Estimation

992 C-SIDE estimates the parameters of (9) via maximum likelihood estimation. First, we note that all
 993 parameters in the model are independent across samples. As such, we fit the model independently
 994 for each sample, and we now drop the subscript of sample g for notational convenience. We will
 995 return to the issue of integrating results across multiple samples in *Multiple replicates*. First, the
 996 parameters $\beta_{i,k}$ and γ_j are estimated by the RCTD algorithm as previously described [24]. We can
 997 accurately estimate cell type proportions and platform effects without being aware of differential spatial

998 gene expression because differential spatial gene expression is smaller than gene expression differences
 999 across cell types. After identifying cell types, C-SIDE estimates gene-specific overdispersion $\sigma_{\varepsilon,j}$ for
 1000 each gene by maximum likelihood estimation (see *Fitting the overdispersion parameter*). Finally, C-
 1001 SIDE estimates the parameters $\alpha_{1,\ell,j}$ and $\alpha_{2,\ell,k,j}$ by maximum likelihood estimation. For the final key
 1002 step of estimating α , we use plugin estimates (denoted by $\hat{\cdot}$) of $\beta_{i,k}$, γ_j , and σ_{ε} . After we substitute
 1003 (10) into (9), we obtain:

$$Y_{i,j} \mid \varepsilon_{i,j} \sim \text{Poisson} \left\{ N_i \exp \left[\log \left(\sum_{k=1}^K \hat{\beta}_{i,k} \exp \left(\alpha_{2,0,k,j} + \sum_{\ell=1}^{L_1} x_{1,i,\ell} \alpha_{1,\ell,j} + \sum_{\ell=1}^{L_2} x_{2,i,\ell} \alpha_{2,\ell,k,j} \right) \right) + \hat{\gamma}_j + \varepsilon_{i,j} \right] \right\} \quad (11)$$

$$\varepsilon_{i,j} \sim \text{Normal}(0, \hat{\sigma}_{\varepsilon,j}^2), \quad (12)$$

1004 Now, we provide an algorithm for computing the maximum likelihood estimator of α . Our likelihood
 1005 optimization algorithm is a second-order, trust-region based optimization. In brief, we iteratively
 1006 solve quadratic approximations of the log-likelihood, adaptively constraining the maximum parameter
 1007 change at each step. Critically, the likelihood is independent for each gene, so separate genes can be
 1008 run in parallel.

1009 Now, we consider the computation of the maximum likelihood estimator (MLE) of α for the likeli-
 1010 hood $\mathcal{L}(\alpha)$ of observing Y_i for $1 \leq i \leq I$, using the assumption that measurements on separate pixels
 1011 are independent. We define the predicted counts at pixel i as $\bar{\lambda}_i(\alpha)$, where,

$$\log(\bar{\lambda}_i(\alpha)) := \log \left(N_i \sum_{k=1}^K \hat{\beta}_{i,k} \mu_{i,k} \right) + \hat{\gamma}. \quad (13)$$

1012 From now on, we will drop the constant term $\hat{\gamma}$, as it can be equivalently factored into the μ intercept
 1013 term. Next, we can use (9) to compute the likelihood of the C-SIDE model,

$$\mathcal{L}(\alpha) = \sum_{i=1}^I \log P(Y_i \mid \bar{\lambda}_i(\alpha)) = \sum_{i=1}^I \log Q_{Y_i}(\bar{\lambda}_i(\alpha)), \quad (14)$$

1014 where we have introduced the function Q to represent the probability, under our Poisson-log-normal
 1015 sampling model, of observing Y_i counts given predicted counts $\lambda_i(\alpha)$,

$$Q_{\ell}(\lambda) \equiv \int_{-\infty}^{\infty} p_{\sigma_{\varepsilon}}(z) e^{-\lambda e^z} \frac{(e^z \lambda)^{\ell}}{\ell!} dz, \quad (15)$$

1016 where $p_{\sigma_{\varepsilon}}$ is the normal distribution pdf with standard deviation σ_{ε} . To optimize our likelihood, we
 1017 develop a second-order trust-region optimization method [79], in which sequential quadratic approx-
 1018 imations are optimized within a trust region, whose size is determined adaptively. To do so, we first
 1019 initialize α as α_0 , which is set to 0 for intercept terms, and -5 for non-intercept terms. Additionally,
 1020 we initialize the trust-region width, δ , as $\delta_0 = 0.1$. At step $n + 1$ of the algorithm, with previous
 1021 parameters α_n and δ_n , we make the following quadratic Taylor approximation, $\tilde{\mathcal{L}}_n$ to \mathcal{L} ,

$$-\mathcal{L}(\alpha) \approx -\tilde{\mathcal{L}}_n := -\mathcal{L}(\alpha_n) + b(\alpha_n)^T(\alpha - \alpha_n) + \frac{1}{2}(\alpha - \alpha_n)^T A(\alpha_n)(\alpha - \alpha_n), \quad (16)$$

1022 where b and A represent the gradient and Hessian of $-\mathcal{L}$, respectively, which are computed below. Next,
 1023 we define α_n^* as the solution to the following optimization problem of this quadratic approximation
 1024 over the trust region:

$$\begin{aligned} \min_{\alpha} \quad & b(\alpha_n)^T(\alpha - \alpha_n) + \frac{1}{2}(\alpha - \alpha_n)^T A(\alpha_n)(\alpha - \alpha_n) \\ \text{s.t.} \quad & |\alpha_j - \alpha_{n,j}| \leq \delta_n \quad \text{for } 1 \leq j \leq \text{length}(\alpha) \end{aligned} \quad (17)$$

1025 This quadratic program is solved using the `quadprog` package in R [78]. Next, we define α_{n+1} as:

$$\alpha_{n+1} := \begin{cases} \alpha_n^*, & \mathcal{L}(\alpha_n^*) - \mathcal{L}(\alpha_n) \geq \gamma(\tilde{\mathcal{L}}_n(\alpha_n^*) - \tilde{\mathcal{L}}_n(\alpha_n)) \\ \alpha_n, & \mathcal{L}(\alpha_n^*) - \mathcal{L}(\alpha_n) < \gamma(\tilde{\mathcal{L}}_n(\alpha_n^*) - \tilde{\mathcal{L}}_n(\alpha_n)), \end{cases} \quad (18)$$

1026 where $\gamma = 0.1$. Additionally, the trust region is updated as:

$$\delta_{n+1} := \begin{cases} \beta_{\text{succ}}\delta_n, & \mathcal{L}(\alpha_n^*) - \mathcal{L}(\alpha_n) \geq \gamma(\tilde{\mathcal{L}}_n(\alpha_n^*) - \tilde{\mathcal{L}}_n(\alpha_n)) \\ \beta_{\text{fail}}\delta_n, & \mathcal{L}(\alpha_n^*) - \mathcal{L}(\alpha_n) < \gamma(\tilde{\mathcal{L}}_n(\alpha_n^*) - \tilde{\mathcal{L}}_n(\alpha_n)), \end{cases} \quad (19)$$

1027 where $\beta_{\text{succ}} = 1.1$ and $\beta_{\text{fail}} = 0.5$, which, along with γ , were chosen by a combination of using standard
1028 parameter choices [79] and ensuring efficient and stable convergence to local minima. Intuitively, the
1029 quadratic approximation $\tilde{\mathcal{L}}_n$ will only be accurate within a local region, and the trust region is intended
1030 to empirically approximate that region. In order to test whether our local approximation is accurate,
1031 we check whether the predicted gain in log-likelihood, $\tilde{\mathcal{L}}_n(\alpha_n^*) - \tilde{\mathcal{L}}_n(\alpha_n)$, is close to the true gain in
1032 log-likelihood, $\mathcal{L}(\alpha_n^*) - \mathcal{L}(\alpha_n)$, within a factor of γ . If the local approximation is indeed accurate, the
1033 algorithm takes a step, and the trust region is allowed to grow. If not, the algorithm stays put, and the
1034 trust region shrinks. This prevents the algorithm from diverging due to poor quadratic approximations.
1035 This procedure is repeated until convergence (see *Stopping conditions and convergence*).

1036 Gradient and Hessian

1037 In this section, we will derive an expression for the gradient and hessian of $-\mathcal{L}(\alpha)$. First, we can
1038 calculate the gradient as,

$$\begin{aligned} b(\alpha) = -\nabla L(\alpha) &= -\sum_{i=1}^I \nabla \log Q_{Y_i}(\bar{\lambda}_i(\alpha)) \\ &= -\sum_{i=1}^I \frac{Q'_{Y_i}(\bar{\lambda}_i(\alpha))}{Q_{Y_i}(\bar{\lambda}_i(\alpha))} \nabla \bar{\lambda}_i(\alpha). \end{aligned} \quad (20)$$

1039 Additionally, we have the Hessian,

$$\begin{aligned} A(\alpha) = \text{Hess}(-L(\alpha)) &= -\sum_{i=1}^I \nabla \left(\frac{Q'_{Y_i}(\bar{\lambda}_i(\alpha))}{Q_{Y_i}(\bar{\lambda}_i(\alpha))} \right) (\nabla \bar{\lambda}_i(\alpha))^T - \sum_{i=1}^I \left(\frac{Q'_{Y_i}(\bar{\lambda}_i(\alpha))}{Q_{Y_i}(\bar{\lambda}_i(\alpha))} \right) \nabla^2 \bar{\lambda}_i(\alpha) \\ &= -\sum_{i=1}^I \left(\frac{Q''_{Y_i}(\bar{\lambda}_i(\alpha))}{Q_{Y_i}(\bar{\lambda}_i(\alpha))} - \left(\frac{Q'_{Y_i}(\bar{\lambda}_i(\alpha))}{Q_{Y_i}(\bar{\lambda}_i(\alpha))} \right)^2 \right) (\nabla \bar{\lambda}_i(\alpha)) (\nabla \bar{\lambda}_i(\alpha))^T \\ &\quad - \sum_{i=1}^I \left(\frac{Q'_{Y_i}(\bar{\lambda}_i(\alpha))}{Q_{Y_i}(\bar{\lambda}_i(\alpha))} \right) \nabla^2 \bar{\lambda}_i(\alpha). \end{aligned} \quad (21)$$

1040 We recall the procedure for computing Q and its derivatives as previously described [24]. What remains
1041 is to calculate explicit expressions for $\bar{\lambda}$ and its derivatives, which we do now. From (10) and (15), we
1042 recall the definition of $\bar{\lambda}_i(\alpha)$:

$$\bar{\lambda}_i(\alpha) = N_i \sum_{k=1}^K \hat{\beta}_{i,k} \exp \left(\sum_{\ell=1}^{L_2} x_{2,i,\ell} \alpha_{2,\ell,k} + \sum_{\ell=1}^{L_1} x_{1,i,\ell} \alpha_{1,\ell} \right). \quad (22)$$

1043 Next, we calculate the gradient of $\bar{\lambda}$ with respect to α_1 and α_2 separately:

$$\begin{aligned}\nabla_{\alpha_1} \bar{\lambda}_i(\alpha) &= N_i \sum_{k=1}^K \hat{\beta}_{i,k} \exp \left(\sum_{\ell=1}^{L_2} x_{2,i,\ell} \alpha_{2,\ell,k} + \sum_{\ell=1}^{L_1} x_{1,i,\ell} \alpha_{1,\ell} \right) x_{1,i} = \bar{\lambda}_i(\alpha) x_{1,i}, \\ \nabla_{\alpha_2^{(k)}} \bar{\lambda}_i(\alpha) &= N_i \hat{\beta}_{i,k} \exp \left(\sum_{\ell=1}^{L_2} x_{2,i,\ell} \alpha_{2,\ell,k} + \sum_{\ell=1}^{L_1} x_{1,i,\ell} \alpha_{1,\ell} \right) x_{2,i} = \bar{\lambda}_i^{(k)}(\alpha) x_{2,i},\end{aligned}\quad (23)$$

1044 where we have defined $\bar{\lambda}_i^{(k)}(\alpha) = N_i \hat{\beta}_{i,k} \exp \left(\sum_{\ell=1}^{L_2} x_{2,i,\ell} \alpha_{2,\ell,k} + \sum_{\ell=1}^{L_1} x_{1,i,\ell} \alpha_{1,\ell} \right)$. Next, we can com-
1045 pute the second derivatives:

$$\nabla_{\alpha_1} \nabla_{\alpha_1} \bar{\lambda}_i(\alpha) = \bar{\lambda}_i(\alpha) x_{1,i} x_{1,i}^T, \quad \nabla_{\alpha_2^{(k)}} \nabla_{\alpha_1} \bar{\lambda}_i(\alpha) = \bar{\lambda}_i^{(k)}(\alpha) x_{1,i} x_{2,i}^T, \quad (24)$$

$$\nabla_{\alpha_2^{(k)}} \nabla_{\alpha_2^{(k')}} \bar{\lambda}_i(\alpha) = \bar{\lambda}_i^{(k)}(\alpha) x_{2,i} x_{2,i}^T \mathbb{I}[k = k']. \quad (25)$$

1046 Finally, notice that all the above expressions, including $\bar{\lambda}_i$ and $\bar{\lambda}_i^{(k)}$ across all pixels i , can be computed
1047 efficiently using matrix multiplications. Lastly, the Fisher information is computed as a scaled version
1048 of the Hessian (see *Justification of consistency and asymptotic normality*).

1049 Stopping conditions and convergence

1050 The algorithm stops when one of two conditions are satisfied: $\delta_n < \varepsilon_1$ or $\tilde{\mathcal{L}}_n(\alpha_n^*) - \tilde{\mathcal{L}}_n(\alpha_n) < \varepsilon_2$ for 6
1051 consecutive iterations. Default choices are $\varepsilon_1 = .001$ and $\varepsilon_2 = .00001$. Assume that the algorithm stops
1052 after $n-1$ iterations and arrives at solution α_n . Convergence is defined by considering the distance of α_n
1053 to the optimal solution of $\tilde{\mathcal{L}}_n$, which is the maximum step size of the next step of the algorithm. Since
1054 $\tilde{\mathcal{L}}_n$ is a quadratic function, its maximum can be calculated as $\alpha^* := \alpha_n - A(\alpha_n)^{-1}b(\alpha_n)$. Consequently,
1055 $\alpha_n - \alpha^* = A(\alpha_n)^{-1}b(\alpha_n)$. For each parameter $1 \leq i \leq \text{length}(\alpha)$, we define that parameter i has
1056 converged if $|\alpha_{n,i} - \alpha_i^*| \leq \varepsilon_3$, where $\varepsilon_3 = .01$. Intuitively, for all parameters i such that $|\alpha_{n,i} - \alpha_i^*| \leq \varepsilon_3$,
1057 these parameters will change by at most ε_3 in the next step of the algorithm. Note that it is possible
1058 for some parameters to converge while others do not. In the most common scenario, consider a case
1059 in which one cell type has very low gene expression in the gene of interest. In this case, it is possible
1060 that the parameter controlling the expression of this gene will diverge to $-\infty$. As such, this parameter
1061 doesn't have a practical effect on the model, but it should not prevent the other parameters (of cell
1062 types with higher expression) from converging. For each cell type, we filter out genes that did not
1063 converge for downstream analysis. In the multi-region case, for each cell type, we test for differential
1064 expression among the subset of regions that have converged.

1065 Fitting the overdispersion parameter

1066 Here, we describe the procedure for fitting the gene-dependent overdispersion parameter $\sigma_{\varepsilon,j}$. This is
1067 necessary because we found evidence that the overdispersion depends on gene j , and modeling gene-
1068 specific overdispersion is necessary for controlling the false-positive rate of C-SIDE. In order to fit
1069 a gene-dependent overdispersion parameter, we fit C-SIDE with initial overdispersion parameter σ_ε ,
1070 which is obtained from the cell type identification step. Next, we use the fitted parameters α and
1071 calculate the log-likelihood of C-SIDE for each possible choice of σ (out of a discrete set ranging from
1072 0.1 to 2). Because the log-normal distribution has a mean of $e^{\sigma^2/2}$, the C-SIDE predicted expression
1073 values $\bar{\lambda}$ are scaled by $e^{-\sigma^2/2}$ to maintain a consistent mean across different values of σ . In practice,
1074 this decision substantially increases the rate of convergence. After computing log-likelihood values for
1075 each σ , the best σ is chosen, and the parameters of C-SIDE are re-fit. This procedure is repeated until
1076 convergence at $\sigma = \sigma_{\varepsilon,j}$.

1077 **Predicted mean and variance of individual data pixel counts**

1078 After α is estimated, we can compute the predicted mean and variance of Y_i , given x_i , according to
 1079 the C-SIDE model. These predictions are used to check whether the observed behavior of data points
 1080 agrees with the predictions of the C-SIDE model. Rewriting (9),

$$Y_i | \alpha \sim \text{Poisson} \left\{ \text{Lognormal}(\bar{\lambda}_i(\alpha), \sigma_{\varepsilon,j}^2) \right\}. \quad (26)$$

1081 Using properties of the lognormal distribution, we can calculate the mean counts,

$$\mathbb{E}[Y_i | \alpha] = \bar{\lambda}_i(\alpha) e^{\sigma_{\varepsilon,j}^2/2}, \quad (27)$$

1082 as well as the variance of the counts, using the law of total variance,

$$\begin{aligned} \text{Var}[Y_i | \alpha] &= \mathbb{E}[\text{Var}[Y_i | \alpha, \varepsilon_i]] + \text{Var}_{\varepsilon_i}[\mathbb{E}[Y_i | \alpha, \varepsilon_i]] \\ &= \bar{\lambda}_i(\alpha) e^{\sigma_{\varepsilon,j}^2/2} + \bar{\lambda}_i(\alpha)^2 e^{\sigma_{\varepsilon,j}^2/2} (e^{\sigma_{\varepsilon,j}^2/2} - 1), \end{aligned} \quad (28)$$

1083 where the first part used the equivalence of the mean and variance of the Poisson distribution, and the
 1084 second part used the variance of the lognormal distribution.

1085 **Multiple replicates**

1086 In order to extend the hypothesis testing framework to the case of multiple replicates, we now recall
 1087 α_g and s_g to be the differential expression and standard error for replicate g , where $1 \leq g \leq G$, and
 1088 $G > 1$ is the total number of replicates. We will consider testing for differential expression for fixed
 1089 covariate ℓ , cell type k , and gene j . In this case, as later derived in (53), the observed estimate $\hat{\alpha}_{\cdot, \ell, k, j, g}$,
 1090 conditional on α , follows a univariate normal distribution with standard deviation $s_{\cdot, \ell, k, j, g}$:

$$\hat{\alpha}_{\cdot, \ell, k, j, g} | \alpha \sim \text{Normal}(\alpha_{\cdot, \ell, k, j, g}, s_{\cdot, \ell, k, j, g}). \quad (29)$$

1091 We further assume that additional biological and/or technical variation across samples exists, such
 1092 that each α_g is normally distributed around a population-level differential expression A , with standard
 1093 deviation τ :

$$\alpha_{\cdot, \ell, k, j, g} \sim_{\text{i.i.d.}} \text{Normal}(A_{\cdot, \ell, k, j}, \tau_{\cdot, \ell, k, j}^2) \quad (30)$$

1094 We estimate τ using the method of moments (second moment) on the observed estimate $\hat{\alpha}$, obtained
 1095 independently from each sample:

$$\begin{aligned} \mathbb{E}[V(\hat{\alpha}_{\cdot, \ell, k, j, 1}, \hat{\alpha}_{\cdot, \ell, k, j, 2}, \dots, \hat{\alpha}_{\cdot, \ell, k, j, G})] &= \\ \mathbb{E}[V((\hat{\alpha}_{\cdot, \ell, k, j, 1} - \alpha_{\cdot, \ell, k, j, 1}) + \alpha_{\cdot, \ell, k, j, 1}, (\hat{\alpha}_{\cdot, \ell, k, j, 2} - \alpha_{\cdot, \ell, k, j, 2}) + \alpha_{\cdot, \ell, k, j, 2}, \dots, \\ &(\hat{\alpha}_{\cdot, \ell, k, j, G} - \alpha_{\cdot, \ell, k, j, G}) + \alpha_{\cdot, \ell, k, j, G})] \\ &= \mathbb{E}[V((\hat{\alpha}_{\cdot, \ell, k, j, 1} - \alpha_{\cdot, \ell, k, j, 1}), (\hat{\alpha}_{\cdot, \ell, k, j, 2} - \alpha_{\cdot, \ell, k, j, 2}), \dots, (\hat{\alpha}_{\cdot, \ell, k, j, G} - \alpha_{\cdot, \ell, k, j, G}))] \\ &\quad + \mathbb{E}[V(\alpha_{\cdot, \ell, k, j, 1}, \alpha_{\cdot, \ell, k, j, 2}, \dots, \alpha_{\cdot, \ell, k, j, G})] \end{aligned} \quad (31)$$

1096 Here, the second step utilizes the independence of $\hat{\alpha} - \alpha$ and α . Additionally, we use the finite sample
 1097 variance function V to denote $V(x_1, x_2, \dots, x_G) = \frac{1}{G-1} \sum_{g=1}^G (x_g - \bar{x})^2$, which is an unbiased estimator
 1098 of the variance of x if x_g is an i.i.d. random variable. Consequently, the second term above equals
 1099 $\tau_{\cdot, \ell, k, j}^2$. Additionally, since $\hat{\alpha} - \alpha$ is mean 0, we can use the fact that for mean 0 variables y that are
 1100 coordinate-wise independent, $\mathbb{E}[V(y_1, y_2, \dots, y_G)] = \frac{1}{G} \sum_{g=1}^G \text{Var}(y_g)$. Applying this fact to the first
 1101 term, we obtain,

$$\mathbb{E}[V(\hat{\alpha}_{\cdot,\ell,k,j,1}, \hat{\alpha}_{\cdot,\ell,k,j,2}, \dots, \hat{\alpha}_{\cdot,\ell,k,j,G})] = \mathbb{E}\left[\frac{1}{G} \sum_{g=1}^G (\hat{\alpha}_{\cdot,\ell,k,j,g} - \alpha_{\cdot,\ell,k,j,g})^2\right] + \tau_{\cdot,\ell,k,j}^2 \quad (32)$$

$$= \frac{1}{G} \sum_{g=1}^G s_{\cdot,\ell,k,j,g}^2 + \tau_{\cdot,\ell,k,j}^2, \quad (33)$$

1102 where we have used the C-SIDE standard errors s^2 to estimate the variance of $\hat{\alpha} - \alpha$. Consequently,
1103 we obtain the following method of moments estimator of τ^2 :

$$\hat{\tau}_{\cdot,\ell,k,j}^2 := V(\hat{\alpha}_{\cdot,\ell,k,j,1}, \hat{\alpha}_{\cdot,\ell,k,j,2}, \dots, \hat{\alpha}_{\cdot,\ell,k,j,G}) - \frac{1}{G} \sum_{g=1}^G s_{\cdot,\ell,k,j,g}^2 \quad (34)$$

1104 Given the above analysis, the estimator is the unbiased method of moments estimator. Since we know
1105 that τ^2 is nonnegative, we next modify our estimator to an estimator that dominates the original:

$$\hat{\tau}_{\cdot,\ell,k,j}^2 := \max\left(\left[V(\hat{\alpha}_{\cdot,\ell,k,j,1}, \hat{\alpha}_{\cdot,\ell,k,j,2}, \dots, \hat{\alpha}_{\cdot,\ell,k,j,G}) - \frac{1}{G} \sum_{g=1}^G s_{\cdot,\ell,k,j,g}^2\right], 0\right). \quad (35)$$

1106 We note that the above method of moments estimator (and our overall approach) is similar to the
1107 widely used DerSimonian-Laird method in meta-analysis [70, 71]. After utilizing the estimate of τ^2 ,
1108 we can now compute the estimate and standard error of A , as follows. Given equations, (29) and (30),
1109 we have that $\hat{\alpha}_{\cdot,\ell,k,j,g}$ is distributed independently for $1 \leq g \leq G$ as:

$$\hat{\alpha}_{\cdot,\ell,k,j,g} \sim \text{Normal}(A_{\cdot,\ell,k,j}, \tau_{\cdot,\ell,k,j}^2 + s_{\cdot,\ell,k,j,g}^2). \quad (36)$$

1110 By the Gauss-Markov theorem for Generalized Least Squares, the best (i.e. minimum variance) unbi-
1111 ased estimator of A is:

$$\hat{A}_{\cdot,\ell,k,j} := \frac{\sum_{g=1}^G (\hat{\alpha}_{\cdot,\ell,k,j,g}) / (\tau_{\cdot,\ell,k,j}^2 + s_{\cdot,\ell,k,j,g}^2)}{\sum_{g=1}^G 1 / (\tau_{\cdot,\ell,k,j}^2 + s_{\cdot,\ell,k,j,g}^2)}. \quad (37)$$

1112 We further plugin our estimate $\hat{\tau}^2$ for τ^2 , which is an approach called feasible generalized least squares:

$$\hat{A}_{\cdot,\ell,k,j} := \frac{\sum_{g=1}^G (\hat{\alpha}_{\cdot,\ell,k,j,g}) / (\hat{\tau}_{\cdot,\ell,k,j}^2 + s_{\cdot,\ell,k,j,g}^2)}{\sum_{g=1}^G 1 / (\hat{\tau}_{\cdot,\ell,k,j}^2 + s_{\cdot,\ell,k,j,g}^2)}. \quad (38)$$

1113 Finally, the feasible estimate of variance of this estimator (also by the Gauss-Markov theorem) is:

$$\text{Var}(\hat{A}_{\cdot,\ell,k,j}) = \frac{1}{\sum_{g=1}^G 1 / (\hat{\tau}_{\cdot,\ell,k,j}^2 + s_{\cdot,\ell,k,j,g}^2)}. \quad (39)$$

1114 Multiple samples and replicates

1115 After developing a hypothesis testing framework for the case of multiple replicates, we now consider the
1116 extension of this framework to the more complicated study design of multiple biological samples (M
1117 samples) with multiple replicates per sample (G_m replicates per sample). In this case, we now model
1118 α for each sample $1 \leq m \leq M$ and each replicate $1 \leq g \leq G_m$ as normally distributed, independently
1119 for each replicate, with standard deviation τ , as follows,

$$\alpha_{\cdot,\ell,k,j,m,g} \sim \text{Normal}(A_{\cdot,\ell,k,j} + \delta_{\cdot,\ell,k,j,m}, \tau_{\cdot,\ell,k,j}^2), \quad (40)$$

1120 where δ represents a sample-specific random effect which is itself normally distributed with standard
1121 deviation Δ ,

$$\delta_{\cdot,\ell,k,j,m} \sim_{\text{i.i.d.}} \text{Normal}(0, \Delta_{\cdot,\ell,k,j}^2). \quad (41)$$

1122 Notice that for fixed sample m , conditional on δ , our problem is identical to the multiple replicate
1123 case above, given a population-mean of $A_{\cdot,\ell,k,j} + \delta_{\cdot,\ell,k,j,m}$. Using this reasoning, we take as an estimate
1124 of τ^2 the average, across samples, of the estimates of τ^2 in (35). As we have utilized the variance
1125 within each sample to obtain an estimate of τ , we will next use the variance across samples to estimate
1126 Δ . We take (38) and (39) as the value and variance (conditional on δ) respectively of the following
1127 unbiased estimate E of $A_{\cdot,\ell,k,j} + \delta_{\cdot,\ell,k,j,m}$, which represents the differential expression within sample
1128 m ,

$$E_{\cdot,\ell,k,j,m} := \frac{\sum_{g=1}^{G_m} (\hat{\alpha}_{\cdot,\ell,k,j,m,g}) / (\hat{\tau}_{\cdot,\ell,k,j}^2 + s_{\cdot,\ell,k,j,m,g}^2)}{\sum_{g=1}^{G_m} 1 / (\hat{\tau}_{\cdot,\ell,k,j}^2 + s_{\cdot,\ell,k,j,m,g}^2)}. \quad (42)$$

1129 Given that $E_{\cdot,\ell,k,j,m}$ is an unbiased estimate of $A_{\cdot,\ell,k,j} + \delta_{\cdot,\ell,k,j,m}$, we recognize that our problem has
1130 been reduced to the original multiple replicates problem (addressed above), where α has been replaced
1131 with $A + \delta$, τ has been replaced with Δ , $\hat{\alpha}$ has been replaced by E , and s^2 has been replaced by what
1132 we define as S^2 , the conditional (on δ) variance of E given in (39),

$$S_{\cdot,\ell,k,j,m}^2 := \frac{1}{\sum_{g=1}^{G_m} 1 / (\hat{\tau}_{\cdot,\ell,k,j}^2 + s_{\cdot,\ell,k,j,m,g}^2)}. \quad (43)$$

1133 As a result of this observation, we can apply a similar derivation as that of (35) to obtain the following
1134 method of moments estimate of Δ ,

$$\hat{\Delta}_{\cdot,\ell,k,j}^2 := \max \left(\left[\text{Var}(E_{\cdot,\ell,k,j,1}, E_{\cdot,\ell,k,j,2}, \dots, E_{\cdot,\ell,k,j,M}) - \frac{1}{M} \sum_{m=1}^M S_{\cdot,\ell,k,j,m}^2 \right], 0 \right). \quad (44)$$

1135 Continuing our parallel to our previous result, we use the feasible Gauss-Markov estimator of A derived
1136 in in (38) and (39),

$$\hat{A}_{\cdot,\ell,k,j} := \frac{\sum_{m=1}^M (E_{\cdot,\ell,k,j,m}) / (\hat{\Delta}_{\cdot,\ell,k,j}^2 + S_{\cdot,\ell,k,j,m}^2)}{\sum_{m=1}^M 1 / (\hat{\Delta}_{\cdot,\ell,k,j}^2 + S_{\cdot,\ell,k,j,m}^2)}. \quad (45)$$

1137 Moreover, using (39), the feasible estimate of variance of this estimator is,

$$\text{Var}(\hat{A}_{\cdot,\ell,k,j}) = \frac{1}{\sum_{m=1}^M 1 / (\hat{\Delta}_{\cdot,\ell,k,j}^2 + S_{\cdot,\ell,k,j,m}^2)}. \quad (46)$$

1138 Therefore, we have derived estimators of population-level differential expression in the case of multiple
1139 replicates or multiple samples with multiple replicates.

1140 **Justification of consistency and asymptotic normality of maximum likelihood
1141 estimator of α**

1142 Since each gene and each sample analyzed independently, we drop the notation of gene j and sample
1143 g . First, we consider the joint distribution of all the variables in our model: x_i , β_i , and Y_i . We recall
1144 that x_i and Y_i are observed, and we assume that these variables are generated i.i.d. for each pixel
1145 ($1 \leq i \leq n$, with $n := I$):

$$Z_i := (x_i, \beta_i, Y_i) \stackrel{\text{i.i.d.}}{\sim} P_\alpha, \quad (47)$$

1146 where Z_i represents the joint random variable and $P_\alpha(Z_i) = Q(x_i, \beta_i)P_\alpha(Y_i | x, \beta)$. Here, Q represents
 1147 the joint distribution, across pixels, of cell type proportions and covariates, which we assume does
 1148 not depend on α . As estimation of α does not depend on this term, we will ignore this term. The
 1149 conditional distribution $P_\alpha(Y_i | x, \beta)$ is precisely the probabilistic model specified by C-SIDE in (9).
 1150 For this analysis, we treat β as observed and do not consider the uncertainty around the estimation
 1151 of β , as errors in the estimation of β are expected to be small and independent across pixels.

1152 Due to the specification of C-SIDE, assuming that the columns of x are linearly independent,
 1153 identifiability is satisfied. That is, $P_\alpha \neq P_{\alpha'}$ for any other pair of distinct parameters α and α' . It
 1154 follows from standard asymptotic theory results [80] (using additional regularity conditions including
 1155 Lipschitz continuity of second derivatives and local convexity of the C-SIDE log-likelihood within a
 1156 bounded region) that if we let $\hat{\alpha}_n$ be the MLE estimator on n pixels, then asymptotic consistency
 1157 holds:

$$\hat{\alpha}_n \xrightarrow{\text{a.s.}} \alpha. \quad (48)$$

1158 In addition to consistency, asymptotic normality holds as $n \rightarrow \infty$ [80]:

$$\sqrt{n}(\hat{\alpha}_n - \alpha) \xrightarrow{\text{d}} \mathcal{N}(0, I_\alpha^{-1}), \quad (49)$$

1159 where I_α is defined to be the Fisher information, which can be represented as,

$$I_\alpha = -\mathbb{E}_\alpha[\nabla^2 \log P_\alpha(Y_i | x, \beta)]. \quad (50)$$

1160 In our case, we will use the observed Fisher information \hat{I}_α to estimate the Fisher information:

$$\hat{I}_\alpha := -\frac{1}{n} \sum_{i=1}^n \nabla^2 \log P_\alpha(Y_i | x, \beta) = \frac{1}{n} A(\alpha), \quad (51)$$

1161 where $A(\alpha)$, defined in (16), is the Hessian of the C-SIDE log-likelihood function. Substituting the
 1162 Hessian into the equation (49) above, we conclude that approximately for large n ,

$$(\hat{\alpha}_n - \alpha) \sim \mathcal{N}(0, A(\alpha)^{-1}). \quad (52)$$

1163 Next, for a fixed individual cell type k , gene j , sample g , and covariate ℓ , the distribution of
 1164 $\hat{\alpha}_{\cdot, \ell, k, j, g}$ follows a univariate normal distribution with standard deviation $s_{\cdot, \ell, k, j, g}$. According to (49),
 1165 if we define s as $s_{\cdot, \ell, k, j, g} = \sqrt{(I_{\alpha_{j,g}}^{-1})_{\ell, k} / n}$, we conclude that,

$$\hat{\alpha}_{\cdot, \ell, k, j, g} \mid \alpha \sim \text{Normal}(\alpha_{\cdot, \ell, k, j, g}, s_{\cdot, \ell, k, j, g}). \quad (53)$$

1166 Thus, we have derived the asymptotic distribution of $\hat{\alpha}$, allowing us to compute confidence intervals
 1167 and perform statistical inference.

Supplementary Experimental Methods

Animal Handling

All procedures involving animals at the Broad Institute were conducted in accordance with the US National Institutes of Health Guide for the Care and Use of Laboratory Animals under protocol number 0120-09-16.

Transcardial Perfusion

C57BL/6J mice were anesthetized by administration of isoflurane in a gas chamber flowing 3% isoflurane for 1 minute. Anesthesia was confirmed by checking for a negative tail pinch response. Animals were moved to a dissection tray and anesthesia was prolonged via a nose cone flowing 3% isoflurane for the duration of the procedure. Transcardial perfusions were performed with ice cold pH 7.4 HEPES buffer containing 110 mM NaCl, 10 mM HEPES, 25 mM glucose, 75 mM sucrose, 7.5 mM MgCl₂, and 2.5 mM KCl to remove blood from brain and other organs sampled. The appropriate organs were removed and frozen for 3 minutes in liquid nitrogen vapor and moved to -80C for long term storage.

Tissue Handling

Fresh frozen tissue was warmed to -20 C in a cryostat (Leica CM3050S) for 20 minutes prior to handling. Tissue was then mounted onto a cutting block with OCT and sliced at a 5° cutting angle at 10 µm thickness. Pucks were then placed on the cutting stage and tissue was maneuvered onto the pucks. The tissue was then melted onto the puck by moving the puck off the stage and placing a finger on the bottom side of the glass. The puck was then removed from the cryostat and placed into a 1.5 mL eppendorf tube. The sample library was then prepared as below. The remaining tissue was re-deposited at -80 C and stored for processing at a later date.

Puck preparation and sequencing

Pucks were prepared as described recently using barcoded beads synthesized in-house on an Akta Oligopilot 10 according to the updated Slide-seqV2 protocol [2]. Pucks were sequenced using a monobase-encoding sequencing-by-ligation approach also described in the updated protocol. We used slide-seq tools for alignment and processing of Slide-seq data.

Pucks were generated using one of two separate bead batches with the oligo sequences listed below:

Batch 1:

5'-
TTT_PC_GCCGTAATACGACTCACTATAGGGCTACACGACGCTTCCGATCTJJJJJJTCTTCAGCGTTCCGAGAJ
JJJJJTCNNNNNNNN25

Batch 2:

5'-

TTT_PC_GCCGTAATACGACTCACTATAGGGCTACACGACGCTTCCGATCTJJJJJJTCTTCAGCGTCCCGAGAJ
JJJJNNNNNNNVVT30

“PC” designates a photocleavable linker; “J” represents bases generated by split-pool barcoding, such that every oligo on a given bead has the same J bases; “N” represents bases generated by mixing, so every oligo on a given bead has different N bases; and “TX” represents a sequence of X thymidines. “V” represents bases which may contain A, C, G but not T.

Slide-seqV2 library preparation

RNA Hybridization:

Pucks in 1.5 mL tubes were immersed in 200 μ L of hybridization buffer (6x SSC with 2 U/ μ L Lucigen NxGen RNase inhibitor) for 15 minutes at room temperature to allow for binding of the RNA to the oligos on the beads.

First Strand Synthesis

Subsequently, first strand synthesis was performed by incubating the pucks in RT solution for 30 minutes at room temperature followed by 1.5 hours at 52 °C.

RT solution:

115 μ L H2O
40 μ L Maxima 5x RT Buffer (Thermofisher, EP0751)
20 μ L 10 mM dNTPs (NEB N0477L)
5 μ L RNase Inhibitor (Lucigen 30281)
10 μ L 50 μ M Template Switch Oligo (Qiagen #339414YCO0076714)
10 μ L Maxima H- RTase (Thermofisher, EP0751)

Tissue Digestion:

200 μ L of 2x tissue digestion buffer was then added directly to the RT solution and the mixture was incubated at 37 °C for 30 minutes.

2x tissue digestion buffer:

200 mM Tris-Cl pH 8
400 mM NaCl
4% SDS
10 mM EDTA
32 U/mL Proteinase K (NEB P8107S)

Second Strand Synthesis:

The solution was then pipetted up and down vigorously to remove beads from the surface, and the glass substrate was removed from the tube using forceps and discarded. 200 μ L of Wash Buffer was then added to the 400 μ L of tissue clearing and RT solution mix and the tube was then centrifuged for 2 minutes at 3000 RCF. The supernatant was then removed from the bead pellet, the beads were resuspended in 200 μ L of Wash Buffer, and were centrifuged again. This was repeated a total of three

times. The supernatant was then removed from the pellet. The beads were then resuspended in 200 μ L of Exol mix and incubated at 37 °C for 50 minutes.

Wash Buffer:

10 mM Tris pH 8.0
1 mM EDTA
0.01% Tween-20

Exol mix:

170 μ L H2O
20 μ L Exol buffer
10 μ L Exol (NEB M0568)

After Exol treatment the beads were centrifuged for 2 minutes at 3000 RCF. The supernatant was then removed from the bead pellet, the beads were resuspended in 200 μ L of Wash Buffer, and were centrifuged again. This was repeated a total of three times. The supernatant was then removed from the pellet. The pellet was then resuspended in 200 μ L of 0.1 N NaOH and incubated for 5 minutes at room temperature. To quench the reaction, 200 μ L of Wash Buffer was added and beads were centrifuged for 2 minutes at 3000 RCF. The supernatant was then removed from the bead pellet, the beads were resuspended in 200 μ L of Wash Buffer, and were centrifuged again. This was repeated a total of three times. Second Strand Synthesis was then performed on the beads by incubating the pellet in 200 μ L of Second Strand Mix at 37 °C for 1 hour.

Second Strand Synthesis mix:

133 μ L H2O
40 μ L Maxima 5x RT Buffer
20 μ L 10 mM dNTPs
2 μ L 1 mM dN-SMRT oligo
5 μ L Klenow Enzyme (NEB M0210)

After Second Strand Synthesis, 200 μ L of Wash Buffer was added and the beads were centrifuged for 2 minutes at 3000 RCF. The supernatant was then removed from the bead pellet, the beads were resuspended in 200 μ L of Wash Buffer, and were centrifuged again. This was repeated a total of three times.

Library Amplification:

200 μ L of water was then added to the bead pellet and the beads were centrifuged for 2 minutes at 3000 RCF. The supernatant was then removed from the bead pellet and the beads were resuspended in 50 μ L of library PCR mix and moved into a 200 μ L PCR strip tube. PCR was then performed as outlined below:

Library PCR mix:

22 μ L H2O
25 μ L of Terra Direct PCR mix Buffer (Takara Biosciences 639270)
1 μ L of Terra Polymerase (Takara Biosciences 639270)
1 μ L of 100 μ M Truseq PCR primer (IDT)
1 μ L of 100 μ M SMART PCR primer (IDT)

PCR program:

95 °C 3 minutes

4 cycles of:

98 °C 20 seconds

65 °C 45 seconds

72 °C 3 minutes

9 cycles of:

98 °C 20 seconds

67 °C 20 seconds

72 °C 3 minutes

Then:

72 °C 5 minutes

Hold at 4 °C

PCR cleanup and Nextera Tagmentation:

Samples were cleaned with Ampure XP (Beckman Coulter A63880) beads in accordance with manufacturer's instructions at a 0.6x bead/sample ratio (30 µL of beads to 50 µL of sample) and resuspended in 50 µL of water. The cleanup procedure was repeated, this time resuspending in a final volume of 10 µL. 1 µL of the library was quantified on an Agilent Bioanalyzer High sensitivity DNA chip (Agilent 5067-4626). Then, 600 pg of cDNA was taken from the PCR product and prepared into Illumina sequencing libraries through tagmentation using the Nextera XT kit (Illumina FC-131-1096).

Tagmentation was performed according to manufacturer's instructions and the library was amplified with primers Truseq5 and N700 series barcoded index primers. The PCR program was as follows:

PCR program:

72 °C for 3 minutes

95 °C for 30 seconds

12 cycles of:

95 °C for 10 seconds

55 °C for 30 seconds

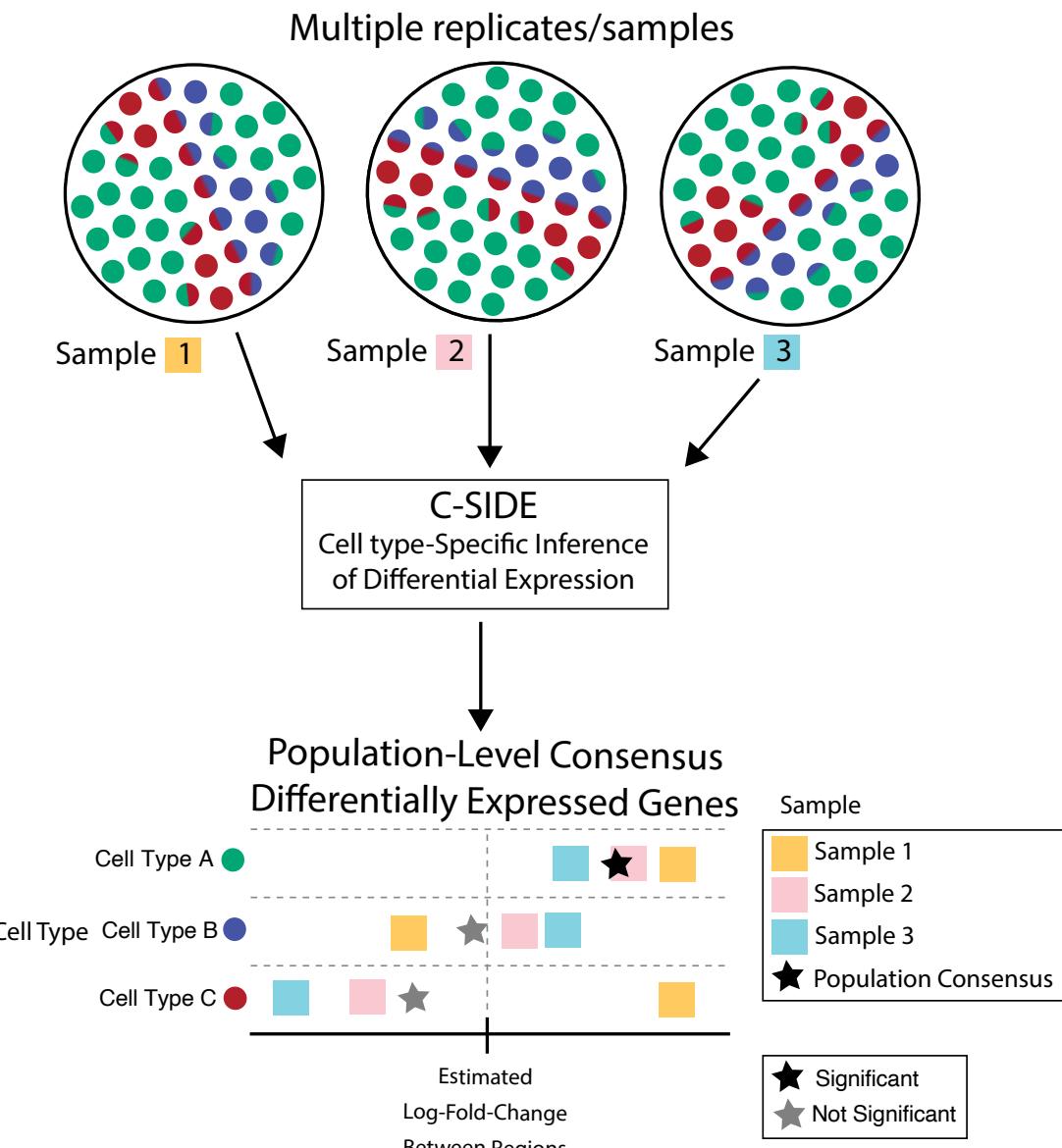
72 °C for 30 seconds

72 °C for 5 minutes

Hold at 4 °C

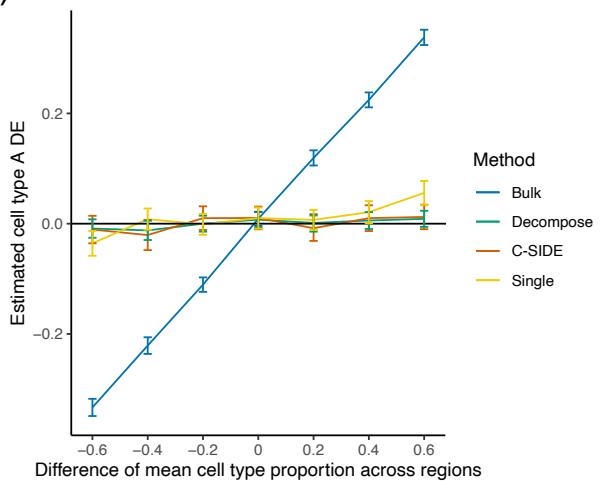
Samples were cleaned with Ampure XP (Beckman Coulter A63880) beads in accordance with manufacturer's instructions at a 0.6x bead/sample ratio (30 µL of beads to 50 µL of sample) and resuspended in 10 µL of water. 1 µL of the library was quantified on an Agilent Bioanalyzer High sensitivity DNA chip (Agilent 5067-4626). Finally, the library concentration was normalized to 4 nM for sequencing. Samples were sequenced on the Illumina NovaSeq S2 flowcell 100 cycle kit with 12 samples per run (6 samples per lane) with the read structure 44 bases Read 1, 8 bases i7 index read, 50 bases Read 2. Each puck received approximately 200-400 million reads, corresponding to 3,000-5,000 reads per bead.

¹¹⁶⁸ **Supplementary Figures**

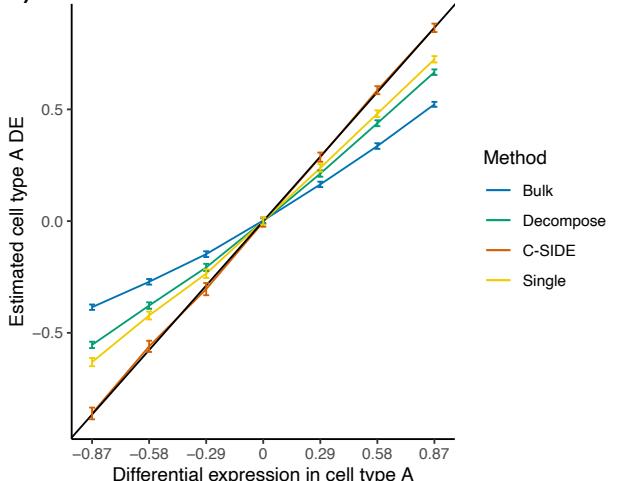


¹¹⁶⁹ Supplementary figure 1: C-SIDE can integrate results from multiple samples to form a robust
¹¹⁷⁰ estimate of population-level consensus differentially-expressed genes.

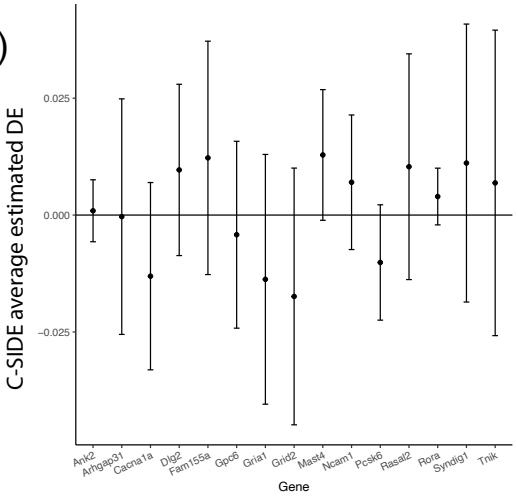
a)



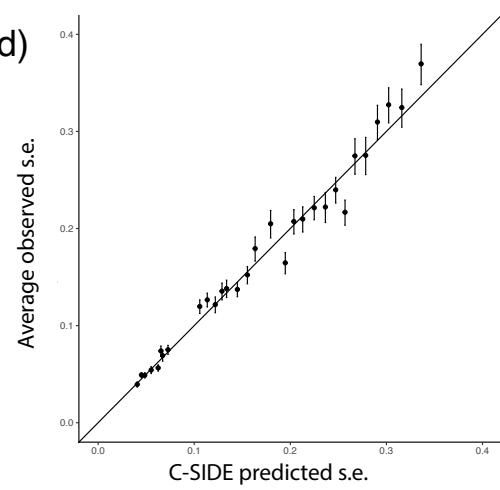
b)



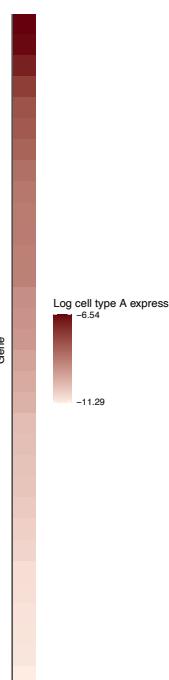
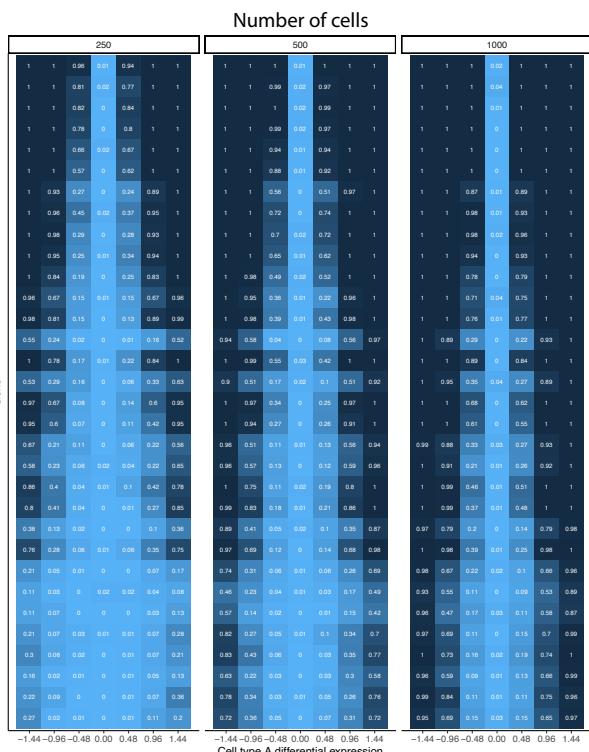
c)



d)



e)



1171 Supplementary figure 2: In simulated data, C-SIDE provides unbiased estimates of cell type-specific
1172 differential expression, with calibrated p —values.

1173 All: C-SIDE was tested on a dataset of simulated mixtures of single cells from a single-nucleus
1174 RNA-seq cerebellum dataset.

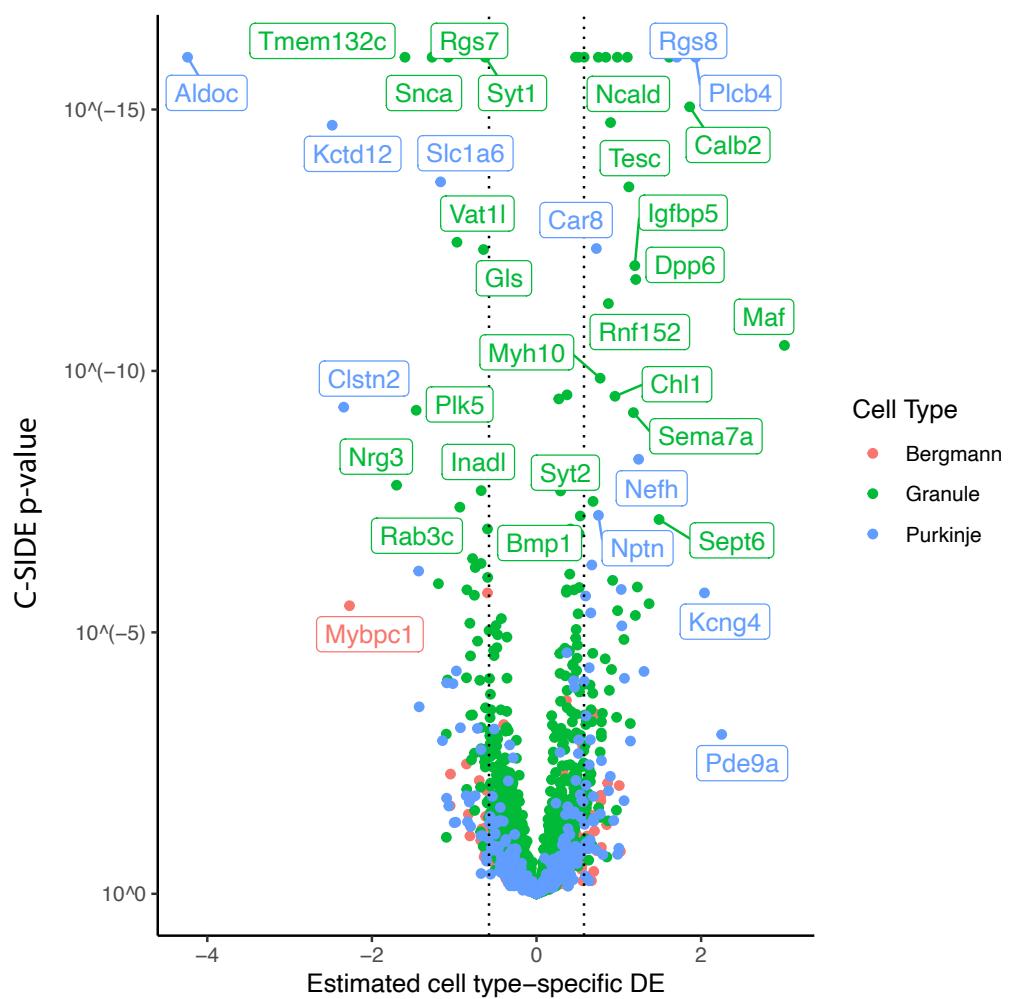
1175 (a) Mean estimated cell type A *Astn2* DE (differential expression) across two regions as a function of
1176 the difference in mean cell type proportion across regions. Ground truth 0 spatial DE is simulated,
1177 and average of ($n = 100$) estimates is shown, along with standard errors. Black line represents
1178 ground truth 0 DE (cell type B). Four methods are shown: *Bulk*, *Decompose*, *Single*, and *C-SIDE*
1179 (see *Methods* for details).

1180 (b) Same as (b) for *Nrxn3* cell type A differential gene expression as a function of DE in cell type A,
1181 where *Nrxn3* is simulated to have DE within cell type A but no DE in cell type B. Ground truth
1182 identity line shown.

1183 (c) C-SIDE mean estimated cell type B differential expression as a function of gene (average over
1184 $n = 500$ replicates, with confidence intervals shown). Ground truth line (0 DE) is shown, and each
1185 condition used a different gene (out of 15 total genes).

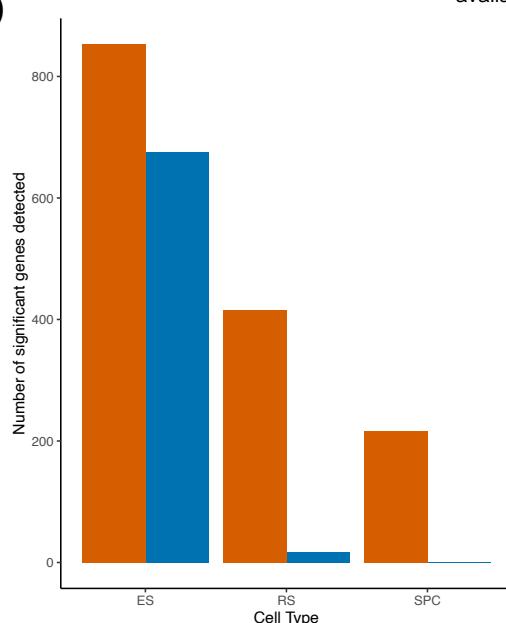
1186 (d) Average measured standard error of C-SIDE estimates for each bin of C-SIDE predicted standard
1187 error.

1188 (e) Statistical power (FPR = 0.01) as a function of gene (y-axis), cell type A DE (x-axis), and number
1189 of cells (table number). Genes are sorted by cell type A expression (shown on right in log2 counts
1190 per 1).

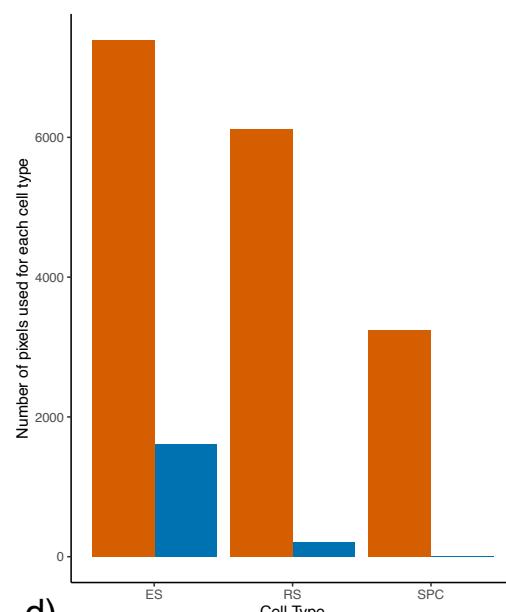


1191 Supplementary figure 3: Volcano plot of C-SIDE log2 differential expression results for cerebellum
1192 Slide-seq across three replicates, with positive values representing enrichment in the anterior region
1193 vs. the nodulus. Color represents cell type, and a subset of significant genes are labeled. Dotted lines
1194 represents C-SIDE fold-change cutoff at 1.5.

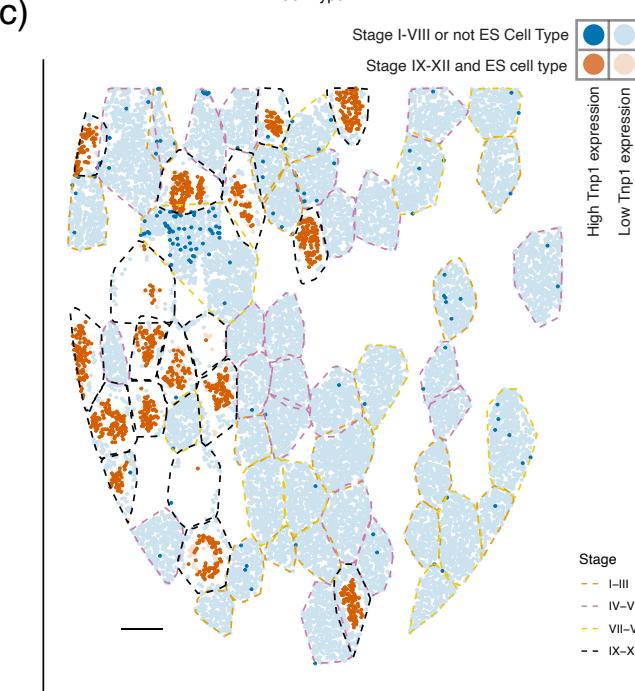
a)



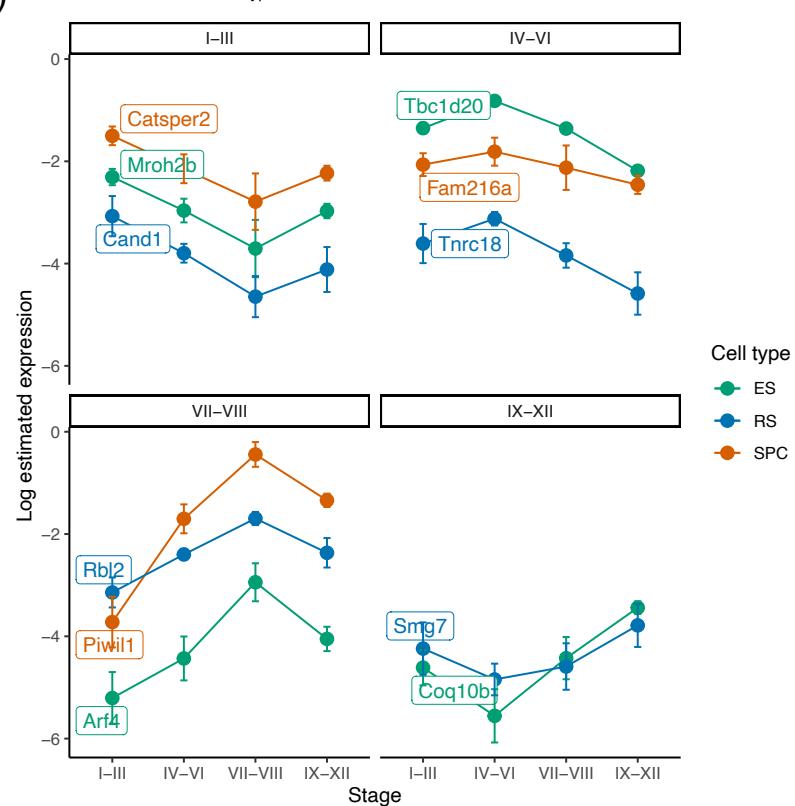
b)



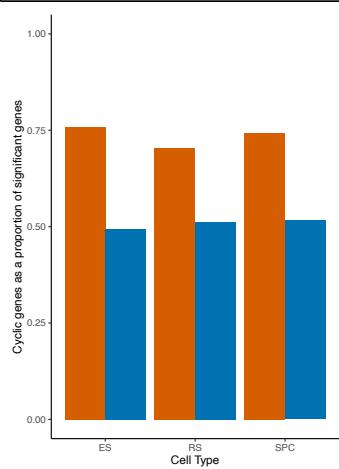
c)



d)



e)



1195 Supplementary figure 4: On the Slide-seq testes, C-SIDE achieves increased power in the presence
1196 of cell type mixtures to discover tubule stage-specific genes and cyclic genes.

1197 (a) Number of significant genes detected, for each cell type, by C-SIDE or the Z-test method.

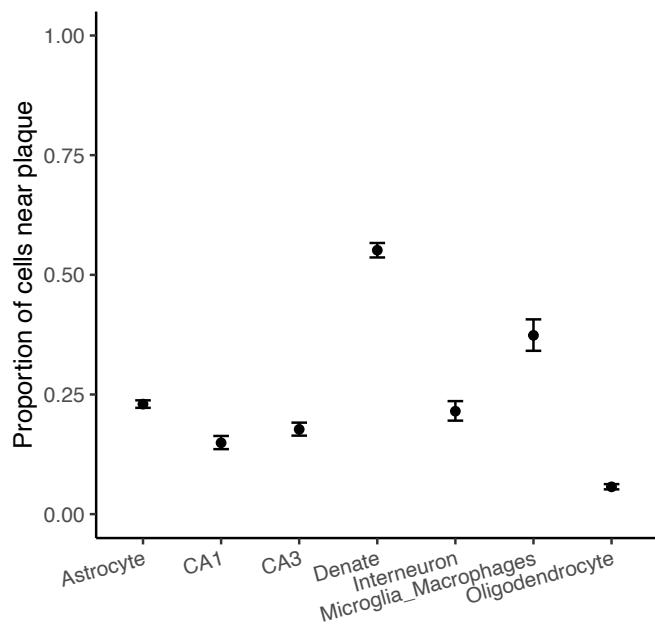
1198 (b) Number of pixels used, for each cell type, to fit the C-SIDE or Z-test model.

1199 (c) Spatial plot of *Tnp1*, a gene identified by C-SIDE to be differentially expressed in stage IX-XII of
1200 cell type ES. Red represents the pixels of cell type ES within stage IX-XII, whereas blue represents
1201 pixels of another cell type or region. Bold points represent pixels expressing *Tnp1* at a level of at
1202 least 7.5 counts per 500. Scale bar represents 250 microns.

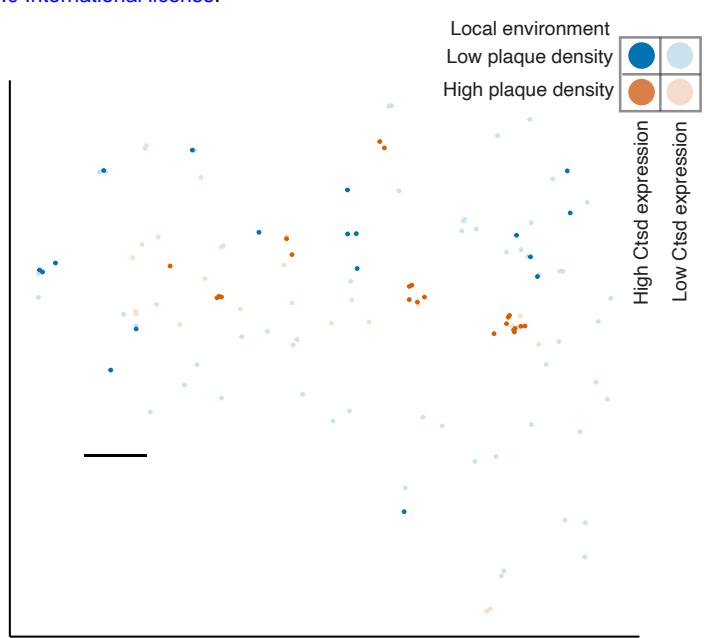
1203 (d) For each cell type, genes identified using C-SIDE results to be cyclic. Panels, indexed by tubule
1204 stage, contain cyclic genes whose peak estimated expression is at that stage. Error bars represent
1205 confidence intervals.

1206 (e) Proportion of genes categorized as cyclic (using C-SIDE fits), compared to proportion that would
1207 be categorized as cyclic if tubule stages were shuffled.

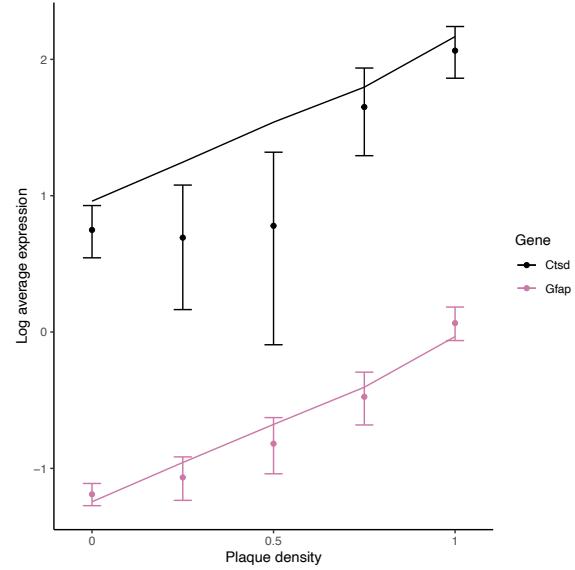
a)



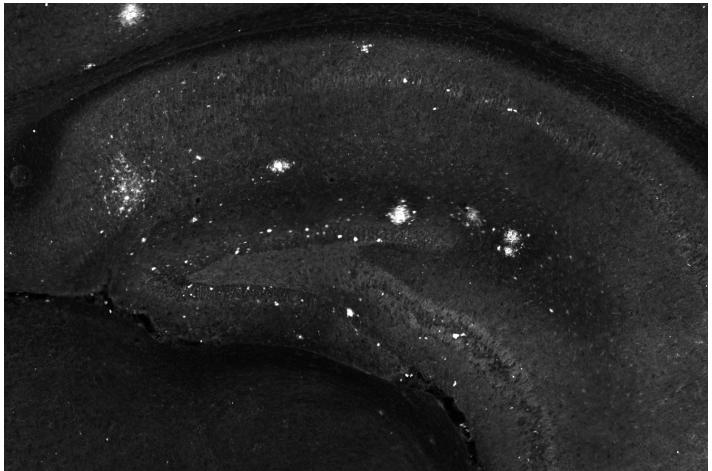
b)



c)



d)



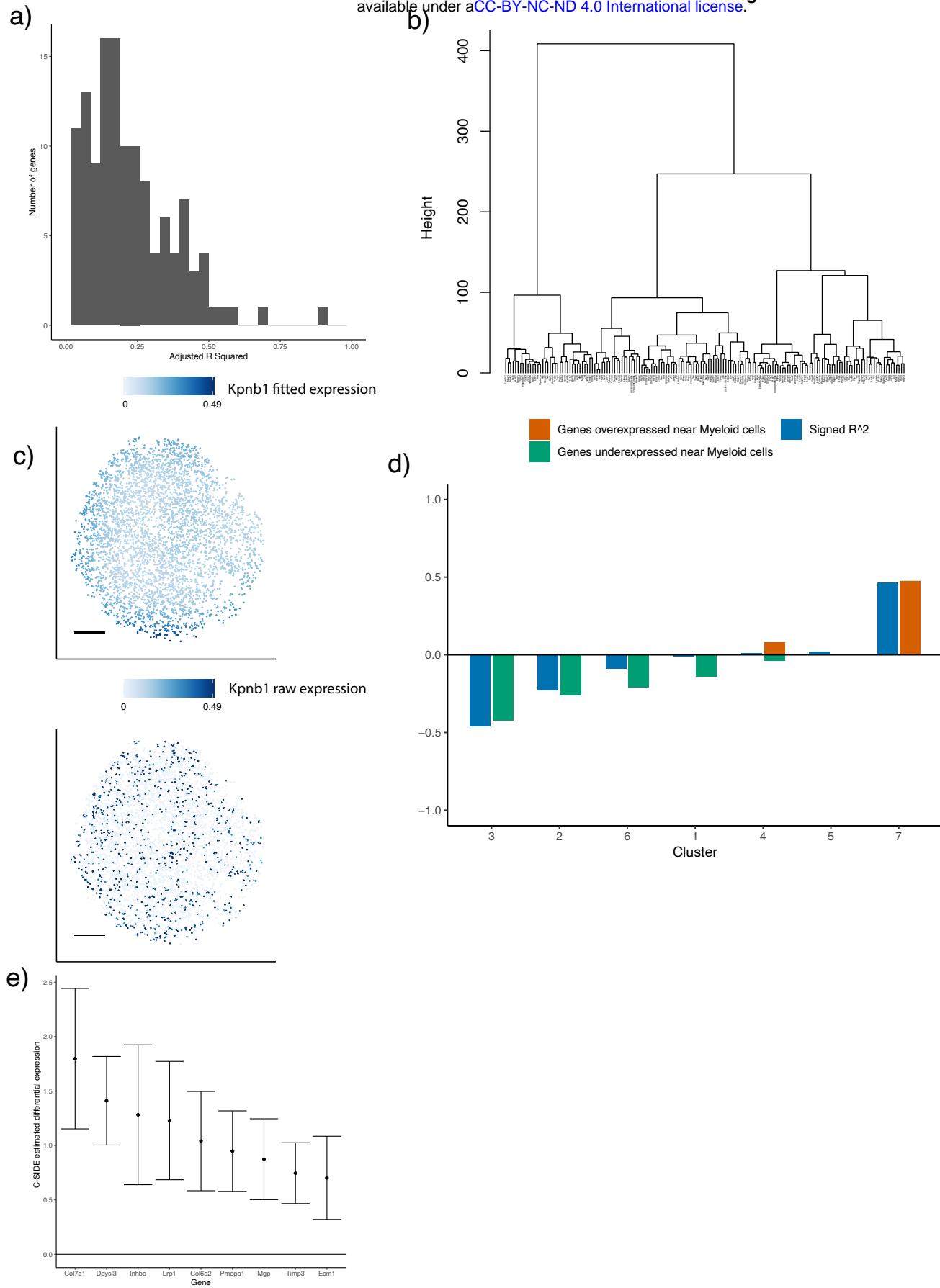
1208 Supplementary figure 5: On the Slide-seq Alzheimer's hippocampus, C-SIDE identifies genes whose
1209 expression depends on A β plaque density.

1210 (a) The proportion of cells, for each cell type, that localize in a high plaque density area.

1211 (b) Spatial visualization of *Ctsd*, whose expression within astrocytes was identified by C-SIDE to
1212 depend on plaque density. Red represents the astrocytes in high plaque density areas, whereas blue
1213 represents astrocytes in regions of low plaque density. Bold points represent astrocytes expressing
1214 *Ctsd* at a level of at least 3 counts per 500. Scale bar is 250 microns.

1215 (c) Log average expression of genes *Ctsd* and *Gfap*, which were identified to be significantly differ-
1216 entially expressed by C-SIDE for microglia/macrophages and astrocyte cell types, respectively.
1217 Single cell type pixels are binned according to plaque density, and points represent raw data aver-
1218 ages while lines represents C-SIDE predictions and error bars around points represent ± 1.96 s.d.
1219 (*Supplementary Methods*).

1220 (d) Antibody stain of A β plaque in adjacent hippocampus section. This image is subsequently trans-
1221 formed to calculate a covariate for C-SIDE.



1222 Supplementary figure 6: on the Slide-seq mouse tumor, C-SIDE identifies differentially expressed
1223 genes within tumor cells.

1224 (a) Histogram, across genes identified to be significantly DE within tumor cells by nonparametric C-
1225 SIDE, of adjusted R -squared, which is defined as the proportion of variance, not due to sampling
1226 noise, explained by the C-SIDE model.

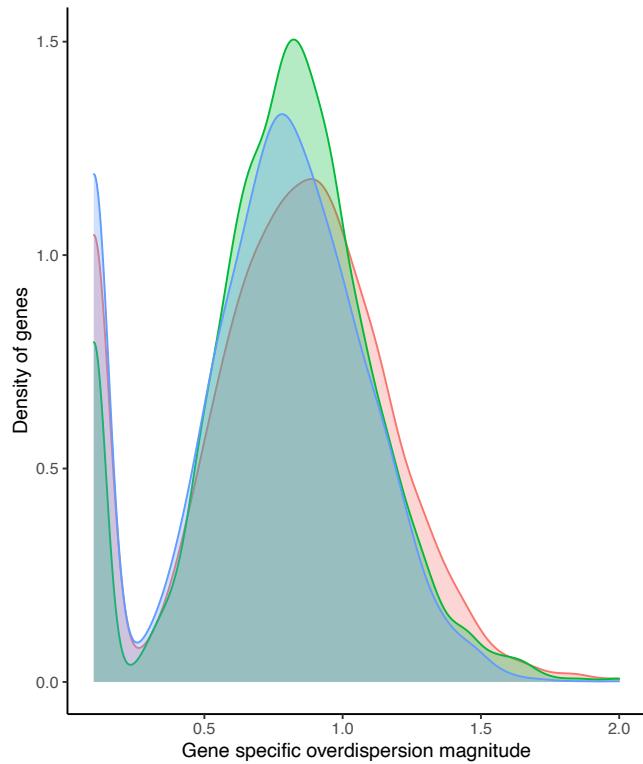
1227 (b) Dendrogram of hierarchical clustering of ($n = 162$ significant genes) C-SIDE's fitted smooth spatial
1228 patterns.

1229 (c) Spatial plot in tumor cells of *Kpnb1*, a *Myc*-target gene identified to be differentially expressed
1230 by nonparametric C-SIDE. Top shows C-SIDE fitted expression, while bottom shows observed
1231 expression in counts per 500. Scale bars are 250 microns.

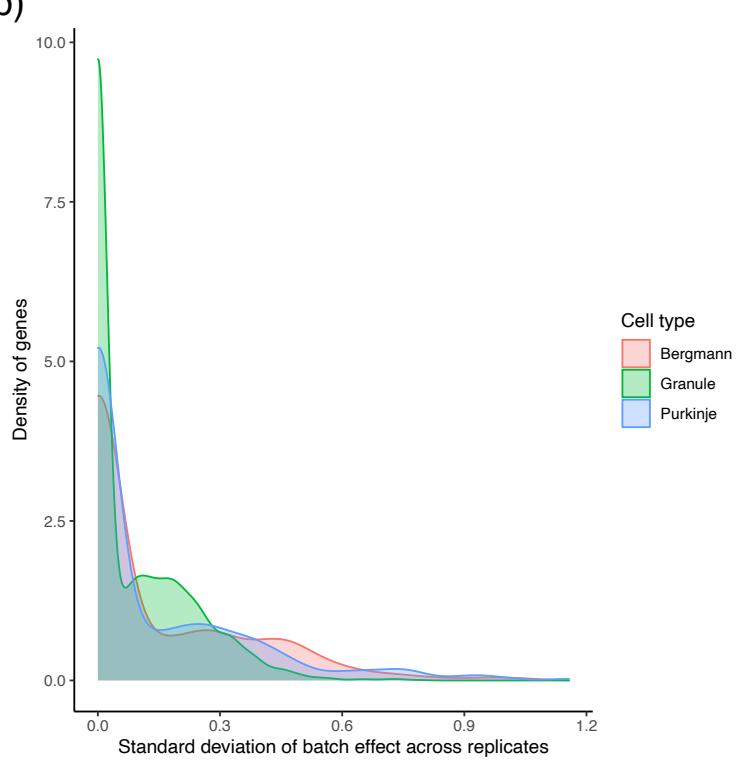
1232 (d) For each cluster of spatially-varying genes, the proportion of genes identified by hypothesis-driven
1233 C-SIDE to be over- or under-expressed near myeloid cells. This proportion is plotted alongside
1234 the squared correlation of the cluster to the density of myeloid cells.

1235 (e) C-SIDE estimated differential expression and 95% confidence intervals of 9 genes from the epithelial-
1236 mesenchymal transition (EMT) pathway identified to be significant.

a)



b)



1237 Supplementary Figure 7: C-SIDE estimated variance parameters on the Slide-seq cerebellum data.

1238 (a) Density plot, over genes, of overdispersion standard deviation, σ_ε , for each of three Slide-seq
1239 replicates.

1240 (b) Density plot, over genes, of C-SIDE estimated batch effect standard deviation, τ , for each of the
1241 Bergmann, granule, and Purkinje cerebellum cell types.