

1 Active feature selection discovers minimal gene sets for classifying 2 cell types and disease states with single-cell mRNA-seq data

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10 Abstract

11 Sequencing costs currently prohibit the application of single-cell mRNA-seq to many biological
12 and clinical analyses. Targeted single-cell mRNA-sequencing reduces sequencing costs by
13 profiling reduced gene sets that capture biological information with a minimal number of genes.
14 Here, we introduce an active learning method (ActiveSVM) that identifies minimal but highly-
15 informative gene sets that enable the identification of cell-types, physiological states, and genetic
16 perturbations in single-cell data using a small number of genes. Our active feature selection proce-
17 dure generates minimal gene sets from single-cell data through an iterative cell-type classification
18 task where misclassified cells are examined at each round of analysis to identify maximally in-
19 formative genes through an ‘active’ support vector machine (ActiveSVM) classifier. By focusing
20 computational resources on misclassified cells, ActiveSVM scales to analyze data sets with over
21 a million single cells. We demonstrate that ActiveSVM feature selection identifies gene sets that
22 enable 90% cell-type classification accuracy across a variety of data sets including cell atlas and
23 disease characterization data sets. The method generalizes to reveal genes that respond to genetic
24 perturbations and to identify region specific gene expression patterns in spatial transcriptomics
25 data. The discovery of small but highly informative gene sets should enable substantial reductions
26 in the number of measurements necessary for application of single-cell mRNA-seq to clinical tests,
27 therapeutic discovery, and genetic screens.

28 Introduction

29 Single-cell mRNA-seq methods have scaled to allow routine transcriptome-scale profiling of thou-
30 sands of cells per experimental run. While single cell mRNA-seq approaches provide insights into
31 many different biological and biomedical problems, high sequencing costs prohibit the broad ap-
32 plication of single-cell mRNA-seq in many exploratory assays such as small molecule and genetic
33 screens, and in cost-sensitive clinical assays. The sequencing bottleneck has led to the develop-
34 ment of targeted mRNA-seq strategies that reduce sequencing costs, by up to 90%, by focusing
35 sequencing resources on highly informative genes for a given biological question or an analysis
36 [1, 2, 3, 4, 5, 6]. Commercial gene-targeting kits, for example, reduce sequencing costs through
37 selective amplification of specific transcripts using 1000 gene-targeting primers.

38 Targeted sequencing approaches require computational methods to identify highly informative
39 genes for specific biological questions, systems, or conditions. A range of computational ap-
40 proaches including differential gene expression analysis and PCA can be applied to identify highly
41 informative genes [1]. However, current methods for defining minimal gene sets are computa-
42 tionally expensive to apply to large single-cell mRNA-seq data sets and often require heuristic
43 user-defined thresholds for gene selection [7, 8]. As an example, computational approaches based
44 upon matrix factorization (PCA[9], NNMF[10]), are typically applied to complete data sets and so
45 are computationally intensive when data sets scale into the millions of cells [11, 12]. Further, gene
46 set selection after matrix factorization requires heuristic strategies for thresholding coefficients in
47 gene vectors extracted by PCA or NNMF, and then asking whether the selected genes retain core
48 biological information.

49 Here, inspired by active learning[13] approaches, we develop a computational method that selects
50 minimal gene sets capable of reliably identifying cell-types and transcriptional states in single-cell
51 mRNA-seq. Our method, ActiveSVM, constructs minimal gene sets by performing an iterative
52 support vector machine classification task [14, 15]. In ActiveSVM the minimal gene set grows
53 from an initial random seed. At each round, ActiveSVM classifies cells into classes that are
54 provided by unsupervised clustering of cell-states or by used-supplied experimental labels. The
55 ActiveSVM procedure analyzes cells that are misclassified with the current gene set, and, then,
56 identifies maximally informative genes that are added to the growing gene set to improve classi-
57 fication. Traditional active learning algorithms query an oracle for training examples that meet a
58 criteria [16]. Our ActiveSVM procedure actively queries the output of an SVM classifier for cells
59 that classify poorly, and then performs detailed analysis of the specific misclassified cells to select
60 maximally informative genes which are, then, added to a growing gene set. By focusing on a well-
61 defined classification task, we ensure that the gene sets discovered by ActiveSVM retain biological
62 information.

63 The central contribution of ActiveSVM is that the method can scale to large single-cell data sets
64 with more than one million cells. We demonstrate, for example, that ActiveSVM can analyze a
65 mouse brain data set with 1.3 million cells and requires only hours of computational time. Ac-

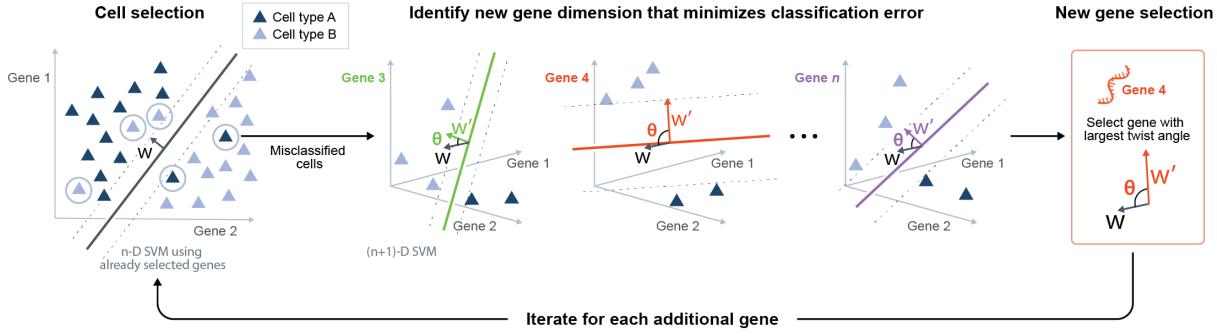


Figure 1: Description of ActiveSVM Feature Selection. At the n -th step, an n -D SVM using all already selected genes is trained to select a certain number of misclassified cells, which is the cell selection step. In the gene selection step, the least classifiable cells are taken as the training set. Based on this training set, $N-n$ ($n+1$)-D SVMs are trained, where n dimensions are the genes already selected and the last dimension is one of the previously unselected candidate genes. Then we would obtain $N-n$ weights w' corresponding to $N-n$ unselected genes as well as $N-n$ margin rotation angle θ between every w' and the original weight w of the n -D SVM. The gene with the maximum rotation of margin is selected for the next round.

66 ActiveSVM scales to large data sets because the procedure must only analyze the full-transcriptome
 67 of cells that classify poorly with the current gene set. As the procedure focuses computational
 68 resources on poorly classified cells, the method can be applied to large data sets to discover small
 69 sets of genes that can distinguish between cell-types at high accuracy. In addition to scaling, the
 70 classification paradigm generalizes to a range of single-cell data analysis tasks including the iden-
 71 tification of disease markers, genes that respond to Cas9 perturbation , and the identification of
 72 region specific genes in spatial transcriptomics .

73 To demonstrate the performance of ActiveSVM, we apply the method to a series of single-cell
 74 genomics data sets and analysis tasks. We identify minimal gene sets for cell-state classification in
 75 human peripheral blood mononuclear cells (PBMCs) [17], the megacell mouse brain data set [18],
 76 and the Tabula Muris mouse tissue survey[19]. We demonstrate application of ActiveSVM to iden-
 77 tify disease markers by analyzing a data set of healthy and multiple myeloma patient PBMCs [20].
 78 To highlight the generality of the method, we apply ActiveSVM to identify genes impacted by Cas9
 79 based gene-knock down in perturb-seq [21]. Further, we show that ActiveSVM can identify gene
 80 sets that mark specific spatial locations of a tissue through analysis of spatial transcriptomics data
 81 [22]. To benchmark the method, we compare the performance of the method to six conventional
 82 feature selection methods, showing that our method outperforms these methods in classification
 83 accuracy. Gene sets constructed by ActiveSVM are both small and highly efficient, for example,
 84 classifying human immune cell types within PMBCs using as few as 15 genes and classifying
 85 55 cell-states in Tabula Muris with < 150 genes. The gene sets we discover include both classi-
 86 cal markers and genes not previously established as canonical cell-state markers. Conceptually,
 87 ActiveSVM demonstrates how active sampling strategies can be applied to enable the scaling of
 88 algorithms to the large data sets generated single-cell genomics.

89 Method

90 We developed a computational method based on support vector machine (SVM) classifier to iden-
91 tify compact gene sets that distinguish cell-states in single-cell data. In the conventional Sequential
92 Feature Selection (SFS) [14], features are selected one-by-one in a greedy strategy to optimize an
93 objective function. Here, we develop an active SVM (ActiveSVM) feature selection method, where
94 we only analyze the subset of incorrectly classified cells at the current step and then select the new
95 gene features based upon those cells. This active learning strategy enables the efficient computa-
96 tion of small gene sets across large data sets by minimizing the total number of cells and genes that
97 are analyzed.

98 ActiveSVM proceeds through rounds of classification and gene selection based on a set of cell la-
99 bels. Cell labels can be derived from unsupervised analysis, experimental meta-data, or biological
100 knowledge of cell-type marker genes. A common work-flow in single-cell mRNA-seq experiments
101 defines a series of cell-states or cell-types using unsupervised clustering of cells [23, 24]. There-
102 fore, we developed our method to accept as input the cell-state labels that are typically derived
103 from unsupervised clustering. We, then, utilize the cell-state labels to identify a minimal set of
104 marker genes that can retain the separation between cell-states with a minimal set of gene features.
105 We note that our method can also accept user supplied cell-type labels as input if a user seeks to
106 identify new genes that separate cell-states based upon biologically curated markers.

107 Our ActiveSVM procedure starts with an empty gene set, an empty cell set and a list of candidate
108 genes and cells. The algorithm iteratively selects genes and classifies cells using identified genes
109 by training a SVM model to classify the cell-types according to labels. The algorithm identifies
110 cells in the data set that classify poorly given the current gene set, and uses misclassified cells
111 to select additional genes to improve classification accuracy on the entire data set. We supply
112 ‘min-complexity’ and ‘min-cell versions’ of ActiveSVM algorithm. The min-complexity algo-
113 rithm samples a fixed number of misclassified cells and directly uses them as the cell set to select
114 the next gene. The min-cell algorithm re-uses the misclassified cells selected in previous iterations
115 to reduce the total number of required cells. The procedure are shown in Figure 1.

116 In the first iteration, the procedure initially constructs single-gene classifiers and adds the gene that
117 provides highest initial classification accuracy to the gene set. The algorithm, then, samples c cells
118 misclassified by the initial single-gene classifier out of the total set of N cells and adds them to the
119 cell set. The parameter c is determined by the user according to the nature of dataset and available
120 computational resources. The algorithm trains an SVM on the cell set using the current gene set,
121 which defines an SVM margin w that optimally separates cells into classes that are consistent with
122 labels on the cell set. Using the SVM classification, the algorithm identifies cells that have been
123 misclassified with the initial gene set. The algorithm, then, identifies genes that can be added to
124 the gene set to improve performance on the misclassified examples.

125 To identify maximally informative genes, we developed a gene selection strategy, Max Margin
126 Rotation (MMR), that evaluates all candidate genes and selects the gene that induces maximum

127 rotation of the margin w . The ActiveSVM algorithm continues iteration until a max gene number,
128 k , is reached. The max gene number k can be set as any integer smaller than M and can be set to
129 small values during exploratory analysis and to larger values for more exhaustive exploration of a
130 data set. The integrated algorithm is shown in Algorithm 1.

131 The most important feature of our ActiveSVM procedure is that the algorithm must never load an
132 entire data set into memory. At each step, the procedure performs classification of cells using a
133 minimal gene set, and then performs detailed (all genes) analysis of only a subset of misclassified
134 cells. Due to the design of the procedure, ActiveSVM can analyze large data sets that do not easily
135 fit in memory. In conventional SVM based feature selection, the user would first train an SVM
136 classifier on the complete data set and then select features according to the absolute values of the
137 components of weight w .[25]. We note that conventional feature selection procedures typically
138 apply classification accuracy for feature selection. Conventional SFS often selects features based
139 upon improvement in classification accuracy. We found empirically that MMR provides improved
140 classification results and so selected MMR as our gene selection strategy.

141 Based on the above outline of ActiveSVM, we can formalize the specific gene and cell selection
142 strategies into two defined rules. For notation, in single-cell gene expression data, we use $x_i^{(j)} \in \mathbb{R}$
143 to denote the measurement of the j -th gene of the i -th cell. We assume the classification labels
144 are given and consider a data-set $\{x_i, y_i\}_{i \in \{1, \dots, N\}}$ contains N cells with total M genes, where
145 $x_i = [x_i^{(j)}]_{j \in \{1, \dots, M\}}$ and $y_i \in \mathbb{Z}^N$ are labels. The labels could be binary or multi-class and can be
146 derived from clustering. We also denote the gene expression vector of i -th cell with part of genes
147 as $x_i^{(D)} = [x_i^{(j)}]_{j \in D}$, where $D \subset \{1, \dots, M\}$. And we use J and I to refer to the set of selected
148 genes and cell set.

149 We assume the SVM classifier notation of one observation is $h_{w,b}(x_i^{(D)}) = g(w^T x_i^{(D)} + b)$ for any
150 $i \in \{1, 2, \dots, N\}$ and $D \subset \{1, 2, \dots, M\}$ with respect to observation $x \in \mathbb{R}^{|D|}$, where $w \in \mathbb{R}^{|D|}$
151 and $b \in \mathbb{R}$ are parameters (the margin and bias respectively). Here, $g(z) = 1$ if $z \geq 0$, and
152 $g(z) = -1$ otherwise. And the loss function is Hinge Loss[26] $\text{loss}_i = \max\{0, 1 - y_i(w^T x_i^{(D)} + b)\}$,
153 where $y_i \in \mathbb{R}$ is the ground truth label of observation x_i .

154 Cell selection: identification of maximally informative cells

155 For the cell selection strategy, we simply choose cells with largest SVM classification loss. The
156 purpose of cell selection is to use the most maximally informative cells as a smaller training set to
157 select the next gene. In SVM classifier, samples separable in n -D are also separable in $(n+1)$ -D as
158 they are at least separated by the same boundary with zero at the $(n+1)$ -th dimension. Therefore,
159 to improve the accuracy with a new gene, we must only consider the misclassified cells. We
160 identify such cells through analysis of the dual form of the classical SVM classification problem.
161 After solving the primal optimization problem of soft margin SVM, we have the dual optimization
162 problem with a non-negative Lagrange multiplier $\alpha_i \in \mathbb{R}$ for each inequality constraint.[27]

$$\begin{aligned}
 \max_{\alpha} \quad & \sum_{i=1}^N \alpha_i - \frac{1}{2} \sum_{i_1, i_2=1}^N y_{i_1} y_{i_2} \alpha_{i_1} \alpha_{i_2} \langle x_{i_1}^{(J)}, x_{i_2}^{(J)} \rangle \\
 \text{s.t.} \quad & 0 \leq \alpha_i \leq C \\
 & \sum_{i=1}^N \alpha_i y_i = 0
 \end{aligned} \tag{1}$$

163 Here $x_i^{(J)}$ refers to the measurement of the i -th cell with all selected genes, and $C \in \mathbb{R}$ is a hyper-
 164 parameter we set to control the trade-offs between size of margin and margin violations when
 165 samples are non-separable.

166 We solve the optimal solution α^* and apply the Karush-Kuhn-Tucker(KKT) dual-complementarity
 167 conditions[28] to obtain the following results where $w \in \mathbb{R}^{|J|}$ and the intercept term $b \in \mathbb{R}$ are
 168 optimal.

$$\begin{aligned}
 \alpha_i^* = 0 \quad & \Rightarrow \quad y_i(w^T x_i^{(J)} + b) > 1 \\
 \alpha_i^* = C \quad & \Rightarrow \quad y_i(w^T x_i^{(J)} + b) < 1 \\
 0 < \alpha_i^* < C \quad & \Rightarrow \quad y_i(w^T x_i^{(J)} + b) = 1.
 \end{aligned} \tag{2}$$

169 Therefore, for each cell, the Lagrange multiplier α_i indicates whether the cell falls within the SVM
 170 margin defined by the vector w . $\alpha_i > 0$ means $y_i(w^T x_i + b) \leq 1$, i.e. cells are on or inside the
 171 SVM margin. Hence, we can directly select cells with $\alpha_i > 0$. In practice, we normally only select
 172 cells with $\alpha_i = C$, which indicates incorrectly classified cells.

173 Discussion of min-cell and min-complexity cell sampling strategies

174 Using this mathematical formulation, we develop two different versions of the ActiveSVM pro-
 175 cedure, the min-complexity strategy and min-cell strategy, for distinct goals. The min-complexity
 176 strategy minimizes the time and memory consumption when computational resources are restricted
 177 or where a user desires to reduce run-time. In the min-complexity strategy, a certain fixed number
 178 of cells is sampled among all misclassified cells and used as the cell set for gene selection in each
 179 iteration. Therefore, a small number of cells can be analyzed at each round and typically only few
 180 cells might be selected repeatedly.

181 In the min-cell strategy, to reduce the number of unique cells required, the misclassified cells
 182 already used in previous steps are given the highest priority to select again. Therefore, the min-cell
 183 strategy attempts to re-use cells across rounds of iteration and aims to minimize the total number
 184 of unique cells we acquire during the entire procedure. The min-cell strategy can be applied to

185 limit the number of cells required to perform the analysis in settings where cell acquisition might
186 be limiting including in the analysis of rare cell populations or in clinical data sets.

187 For the min-cell strategy, assume we select c cells for each iteration and there are $a+b$ misclassified
188 cells at the current iteration, where a cells have been used at least once in previous iterations while
189 b cells are new cells. If $a \geq c$, we do not need to add any new cells to current cell set. If $a < c$, we
190 sample $c - a$ cells among the b new cells. Then the algorithm uses the whole cell set for the next
191 gene selection step. When using the min-cell strategy, cells tend to be re-used many times and the
192 curve of number of unique cells we acquire converges to a fixed value along with the number of
193 genes we select. In experiments, the number of cells selected for each step, c , is a hyper-parameter
194 set by the user. Typically, the parameter can be set to a small number using the min-complexity
195 strategy, as a sufficient number of new cells is considered in the procedure. Selecting a small
196 number of cells each round reduces computational complexity. In the min-cell strategy it can be
197 advantageous to select a larger number of total cells to guarantee diversity of training cells while
198 still bounding the total number of cells used.

199 **Balancing cell-sampling across cell-classes**

200 In addition to the min-cell and min-complexity options, we also include two version of cell sam-
201 pling strategies. The first one is uniform, random sampling. Another option is cell ‘balanced’
202 sampling that can be applied to balance sampling across a series of cell classes. In the ‘balanced’
203 strategy, we sample a fixed number of cells from each cell class, and for classes with insuffi-
204 cient cells we sample all the cells in the class. Mathematically, assume there are Z classes and S is the
205 set of all misclassified cells this step. We should sample c' cells from a candidate cell set, S' , for
206 the current iteration. In min-complexity strategy, $c' = c$ and the candidate cell set, S' , should be
207 S itself. For the min-cell strategy, $c' = c - \min\{c, |I \cap S'|\}$, where I is the cell set before current
208 iteration, and the candidate cell set $S' = S \setminus I$. Assume $S' = \bigcup_{z=1}^Z S'_z$, where S'_z are the set of
209 cells in class z , and $|S'_z| \leq |S'_{(z+1)}|$ for any $z \in \{1, 2, \dots, Z-1\}$. We sample cells in order from
210 class 1 to class Z and denote P_z as the union set of all selected cells from all classes after class z .
211 Then, for class z , if $|S'_z| \leq (|S'| - |P_{z-1}|)/(Z - z + 1)$, we select all cells in S'_z . Otherwise, if
212 $|S'_z| > (|S'| - |P_{z-1}|)/(Z - z + 1)$, we randomly sample $(|S'| - |P_{z-1}|)/(Z - z + 1)$ cells in S'_z .
213 The procedure repeats for all classes and then we have P_Z as the cells we select at this iteration.

214 **Gene selection by maximizing margin rotation**

215 To select maximally informative genes at each round, we analyze misclassified cells and identify
216 genes that will induce the largest rotation of the classification margin. Our procedure is inspired
217 by the active learning method, Expected Model Change[16]. We quantify rotation of the margin
218 by calculating the twist angle induced in w when we add a new dimension (gene) to the classifier.
219 Assume J is the set of genes we have selected so far. Once we add a gene into the $|J|$ -dimensional

220 data space, the parameter w will have one more dimension. The rotation of margin measures how
221 much w twists after adding the new dimension compared with weight in the previous iteration.

222 Specifically, assume J is the set of genes we have selected so far. We derive the corresponding w
223 from the optimal solution α^* .[27] After solving the dual optimization problem (1), we have:

$$w = \sum_{i \in I} \alpha_i^* y_i x_i^{(J)}. \quad (3)$$

224 Then we pad w with zero to get a $|J + 1|$ -dimensional weight w_{padded} , whose first $|J|$ dimensions
225 is w and the $|J + 1|$ -th dimension is zero.

226 For each candidate gene j , we train a new $|J + 1|$ -dimensional SVM model and have weight
227 w_j , where $j \in \{1, \dots, M\} \setminus J$. That is to say, for candidate gene j , we solve the dual optimization
228 problem (4) and find a new optimal multiplier $\alpha^{*(j)}$. Note that we only use the selected cells here,
229 $i_1, i_2 \in I$.

$$\begin{aligned} \max_{\alpha} \quad & \sum_{i \in I} \alpha_i^{(j)} - \frac{1}{2} \sum_{i_1, i_2 \in I} y_{i_1} y_{i_2} \alpha_{i_1}^{(j)} \alpha_{i_2}^{(j)} \langle x_{i_1}^{(J \cup \{j\})}, x_{i_2}^{(J \cup \{j\})} \rangle \\ \text{s.t.} \quad & 0 \leq \alpha_i^{(j)} \leq C \\ & \sum_{i \in I} \alpha_i^{(j)} y_i = 0 \end{aligned} \quad (4)$$

230 Then we have w_j as shown in equation (5):

$$w_j = \sum_{i \in I} \alpha_i^{*(j)} y_i x_i^{(J \cup \{j\})} \quad (5)$$

231 The angle θ_j between w_j and w_{padded} is the expected angle the margin rotates, corresponding to
232 the j -th candidate gene. Then the j -th gene with largest angle θ_j will be selected. We measure the
233 angle between two vectors using cosine similarity[29]:

$$\vartheta_j = \arccos \cos \vartheta_j = \arccos \frac{\langle w_j, w_{padded} \rangle}{\| w_j \| \| w_{padded} \|} \quad (6)$$

234 Therefore, a new gene, which maximizes ϑ_j , is selected to maximize the expected model change.

235 For multi-class classification, the SVM is handled according to a one-vs-rest scheme[30], where a
236 separate classifier is fit for each class, against all other classes. Margin rotation is represented as

237 the sum of weight components in each class dimension. Hence with Z classes, we get Z weight
 238 components corresponding to Z one-vs-the-rest classification decision boundaries. Assume the
 239 weight component for class z of the previous $|J|$ -dimensional SVM model is $w^{(z)}$. Denote the
 240 $|J + 1|$ -dimensional weight after zero-padding of $w^{(z)}$ as $w_{padded}^{(z)}$ and the new $|J + 1|$ -dimensional
 241 weight component of class z with j -th gene as $w_j^{(z)}$, where $z \in 1, \dots, Z$. Then we have:

$$\vartheta_j^{(z)} = \arccos \cos \vartheta_j^{(z)} = \arccos \frac{\langle w_j^{(z)}, w_{padded}^{(z)} \rangle}{\| w_j^{(z)} \| \| w_{padded}^{(z)} \|} \quad (7)$$

$$\vartheta_j = \sum_{z=1}^Z \vartheta_j^{(z)} \quad (8)$$

Algorithm 1: Active Linear SVM Gene Selection

Input: $c, k \in \mathbb{N}, J = \emptyset$

Output: J

Randomly or ‘balanced’ select c cells $I \subset \{1, \dots, N\}, |I| = c$

Train a 1-D SVM model on training set I for each candidate gene: $\{h_{w,b}^{(j)}\}_{j \in \{1, \dots, M\}}$

$loss_j = \sum_{i \in I} \max\{0, 1 - y_i h_{w,b}^{(j)}(x_i^{(j)})\}$

Select one gene $j_0 \in \{1, \dots, M\}$ with lowest $loss_j$

$J = J \cup \{j_0\}$

repeat

 Optimize (1) and get optimal solution $\{\alpha_i^*\}_{i=1}^N$

 Get the set of misclassified cells $S \subset \{1, \dots, N\}$ with $\alpha_i^* = C$

if *min-complexity* **then**

 Randomly or ‘balanced’ select c cells $I \subset S$, where $|I| = c$;

else

if *min-cell* **then**

$c' = \min\{c, |I \cap S|\}$;

 Randomly or ‘balanced’ select $c - c'$ cells $I' \subset S \setminus I$, where $|I'| = c - c'$;

$I = I \cup I'$

end

end

$w = \sum_{i \in I} \alpha_i^* y_i x_i^{(J)}$

$w_{padded} = [w, 0]$

 For each $j \in \{1, \dots, M\} \setminus J$, optimize (4) and get optimal solution $\{\alpha_i^{*(j)}\}_{i \in I}$

$w_j = \sum_{i \in I} \alpha_i^{*(j)} y_i x_i^{(J \cup \{j\})}$

$\vartheta_j = \arccos \cos \vartheta_j = \arccos \frac{\langle w_j, w_{padded} \rangle}{\| w_j \| \| w_{padded} \|}$

 Select one gene $j^* \in \{1, \dots, M\} \setminus J$ with largest ϑ_j

$J = J \cup \{j^*\}$

until $|J| \geq k$

243 Memory complexity

244 One of the key contribution of ActiveSVM is that it significantly saves memory usage because
245 only a small part of data is used at each iteration. The entire dataset can be stored in disk and
246 the algorithm only loads two small matrices into memory, a $N \times |J|$ matrix of all cells with the
247 currently selected genes and a $|I| \times M$ matrix of the cell set with all genes. The memory complexity
248 is $\mathcal{O}(M + N)$ while the memory complexity of algorithms using the entire dataset should be at
249 least $\mathcal{O}(MN)$. The min-cell strategy minimizes the total number of unique cells acquired to reduce
250 the cost of data measurement, acquisition and storage.

251 Time complexity

252 The time complexity of the complete procedure depends primarily on the training of SVM. The
253 standard time complexity of SVM training is usually $\mathcal{O}(MN^2)$ [31, 32]. Assume that we plan to
254 select $k \in \mathbb{N}$ genes in total and use the cell set I_i of poorly classified cells at i -th iteration, where
255 $k, k^2 \ll M$ and $|I_i|, |I_i|^2 \ll N$ are constants. Then the computational complexity of ActiveSVM
256 is:

$$\mathcal{O}\left(\sum_{i=1}^k (i \cdot N^2 + (M - i) \cdot (i + 1) \cdot |I_i|^2)\right) \sim \mathcal{O}(N^2 + M).$$

257 The key reduction in total complexity occurs because each step is performed using N cells with
258 of order $k, k^2 \ll M$ genes or using order M genes with $|I_i|$ cells. Therefore, the polynomial
259 $\mathcal{O}(MN^2)$ is reduced to two separate steps that are individually $\mathcal{O}(N^2)$ and $\mathcal{O}(M)$.
260 And in practice, we implement ActiveSVM using the linear SVM library LIBLINEAR[33], whose
261 time complexity is $\mathcal{O}(MN)$. Therefore,

$$\mathcal{O}\left(\sum_{i=1}^k (i \cdot N + (M - i) \cdot (i + 1) \cdot |I_i|)\right) \sim \mathcal{O}(N + M),$$

262 and the corresponding time complexity of ActiveSVM with LIBLINEAR is $\mathcal{O}(M + N)$.
263 In the gene selection part, the margin rotation angles of all candidate genes can be computed in
264 parallel, which also accelerates the algorithm. The complexity provides a significant improvement
265 in marker gene selection methods especially for large-scale datasets.

266 ActiveSVM can incorporate cell labels derived from unsupervised analysis, 267 experimental conditions, or biological knowledge

268 The goal of ActiveSVM is to discover minimal gene sets for extracting biological information
269 from single-cell data sets. To define minimal gene sets, we apply a classification task in which
270 we find genes that enable a SVM classifier to distinguish single-cells with different labels (y_i).
271 In practice, explicit cell-type labels are often not known for a data set. An extremely common
272 work-flow in single-cell genomics applies Louvain clustering algorithms to identify cell classes
273 and visualizes these cell classes in UMAP or tSNE plots ([24, 23]. The cell clusters output by
274 clustering work-flows in commonly used single-cell analysis frameworks provide a natural set of
275 labels for down-stream analysis. In fact, ActiveSVM can, then, identify specific marker genes for
276 interpreting the identified cell-clusters and determining their biological identify. More broadly,
277 cell-class labels can be quite general including the identity of a genetic perturbation (Figure 6), the
278 spatial location of a cell (Figure 7). We can imagine the application of ActiveSVM to a broad set
279 of additional labels including membership to a differentiation trajectory or lineage tree [34].

280 Results

281 We test our ActiveSVM feature-selection method on four single-cell mRNA-seq datasets: a dataset
282 of peripheral blood mononuclear cells (PBMCs)[17], the megacell 1.3 million cell mouse brain
283 data set [18], the Tabula Muris mouse tissue survery dataset[19], and a multiple myeloma human
284 disease dataset [20]. Later, we demonstrate generalization of the strategy to additional types of
285 single-cell data analysis, including a perturb-seq dataset where genes impacted by Cas9 based
286 genetic perturbation, and a spatial transcriptomics dataset by seqFish+.

287 For each analysis, we show the classification accuracy of the test set along with the number of genes
288 we select. We also compare the classification performance to several widely-used feature selection
289 methods, including conventional SVM, correlation coefficient[35], mutual information[36], Chi-
290 square[37], feature importance by decision tree[38], and randomly sample genes, showing that
291 ActiveSVM obtains the highest accuracy. All of the comparison methods select genes one by one
292 and select a new gene with the largest score in terms of the corresponding evaluation functions
293 while using the same number of cells as our method. However, all methods randomly sample
294 cells at each iteration without an active learning approache. For perturb-seq and seqFish+ datasets,
295 we also show the accuracy performance of comparison methods, where the entire dataset is used.
296 Specifically, conventional SVM based feature selection also called naive SVM selects the gene
297 with largest weight component, which is the most popular SVM feature selection method. In our
298 application of ActiveSVM, we tested both the min-cell strategy and min-complexity strategies as
299 well as randomly sampling and 'balanced' sampling.

300 In each experiment, the data set was first pre-processed and normalized using standard single-

301 cell genomics strategies (See Data Pre-processing). The entire dataset was, then, randomly split
302 into training set with the size of 80% and test set with the size of 20%. For conventional and
303 ActiveSVM, we found the approximately optimal parameter by grid-search [39] across lists of
304 candidate values for some key parameters in the framework of 3-fold cross validation [40]. The
305 optimal parameters were fixed during all iterations. For the comparison methods, we use 3-fold
306 cross validation grid-search to obtain the optimal parameters at each single iteration. We also im-
307 plemented the algorithms called `min_complexity_cv` and `min_acquisition_cv` that apply
308 grid-search and cross validation for each single SVM trained in each iteration (see Code Availabil-
309 ity). The parameter setting details are shown in Parameters section.

310 In our evaluation, besides accuracy curves with proportion confidence interval[41], we also show
311 the distribution of gene markers we selected and the relation with classification target. The sub-
312 plots include the gene expression values on t-SNE projection, the mean of each class, histogram
313 distribution, violin plot, the correlation coefficient heatmap, etc.

314 To indicate the efficiency, we also recorded the run time, peak memory usage, and the total num-
315 ber of unique cells we used of ActiveSVM on these datasets. We used r5n.24xlarge[42], a type of
316 EC2 [43] virtual server instance on AWS[44], with 96 virtual central processing units (vCPU) and
317 768 GiB memory on Linux[45] system. For example, we selected 50 genes on the largest dataset,
318 mouse brain ‘megacell’ dataset, which contains 1306127 cells and 27998 genes, using ActiveSVM
319 and some other popular feature selected methods, including mutual information, feature impor-
320 tance by decision tree, and conventional SVM. The peak memory usage of ActiveSVM is 2111
321 MB while other methods all consume more than 78600 MB. The run time of the min-complexity
322 method is about 69 minutes and of the min-cell method is about 243 minutes. Each comparison
323 method takes more than 4 days on the same server machine. The run time and peak memory usage
324 of ActiveSVM on all six datasets are shown in Table 1. The ActiveSVM package used for the brain
325 megacell dataset only loads the selected genes and cells into memory at each iteration while other
326 two experiments called the package loading the entire dataset. Both packages are provided in Code
327 Availability Section.

Table 1: Run Time and Peak Memory Usage of ActiveSVM.

	matrix size (cells, genes)	min-complexity run time (s/gene)	min-cell run time (s/gene)	memory (MB)	unique cells (min-cell)
mouse megacell	(1306127, 27998)	4142/50	14580/50	2111	712
PBMC	(10194, 6915)	121/50	176/20	1325	298
Tabula Muris	(55656, 8661)	737/150	7701/100	1093	779
MM	(35159, 32527)	127/40	449/40	1616	445
seqFish	(913, 10000)	33/30	728/30	887	428
perturb-seq	(10895, 15976)	3424/50		9493	3827

328 Active feature selection on human PBMC data

329 To test the performance of ActiveSVM, we used the method to extract classifying gene subsets
330 for human PBMCs. We analyzed a single-cell transcriptional profiling data set for 10194 cells
331 [17] with 6915 genes. We used Louvain clustering [46] to identify T-cells, activated T/NK cells,
332 B-cells, and Monocytes (Figure 2(c)).

333 The min-cell strategy classified the 5 major cell-types at greater than 85% accuracy with as few as
334 15 total genes (Figure 2(a)) and the test accuracy of min-cell, with both randomly sampling and
335 'balanced' sampling, also reached much higher accuracy than the comparison methods.

336 A key benefit of the active learning strategy is that a relatively small fraction of the data set is
337 analyzed, so that the procedure can generate the gene sets while only analyzing 298 cells (Figure
338 2(d)). At each iteration, a specific number of misclassified cells ($c = 100$) are selected but the
339 total number of cells used does not increase in increments of 100, since some cells are repeatedly
340 misclassified and are thus repeatedly used for each iteration.

341 In addition to enabling cell-type classification of the data set, the ActiveSVM gene sets provide a
342 low-dimensional space in which to analyze the data. When we reduced our analysis to consider
343 only the top 100 genes selected by the ActiveSVM algorithm, we were able to generate a low-
344 dimensional representations of the cell population (t-SNE) that preserved critical structural features
345 of the data, including the distinct cell-type clusters (Figure 2(c)).

346 The procedure generates gene sets that contain known and novel markers, each plotted individu-
347 ally in a t-SNE grid (Figure 2(e)(f)). For instance, MS4A1 and CD79 are well-established B-cell
348 markers, and IL7R and CD3G are well-established T-cell markers. However, we also find genes
349 which are not commonly used as markers, but whose expression is cell-type specific. For instance,
350 we find highly monocyte-specific expression of FPR1, which encodes N-formylpeptide receptor,
351 which was recently discovered to be the receptor for plague effector proteins [47]. We also find
352 T-cell/NK-cell specific expression of a long noncoding RNA, LINC00861, whose function is un-
353 known but has been correlated with better patient outcome in lung adenocarcinoma [48]. The
354 marker genes are generally highly specific for individual cell types, but some mark multiple cell
355 types (i.e. MARCH1, which marks monocytes and B-cells).

356 Scaling of ActiveSVM feature selection to million cell dataset

357 To demonstrate the scaling of the ActiveSVM feature selection method to large single cell mRNA-
358 seq data sets, we applied the method to extract compact gene sets from the 10x genomics the
359 'megacell' demonstration data set [18]. The megacell dataset was collected by 10x genomics
360 as a scaling demonstration of their droplet scRNA-seq technology. The data set contains full
361 transcriptome mRNA-seq data for 1.3 million cells from the developing mouse brain profiled at

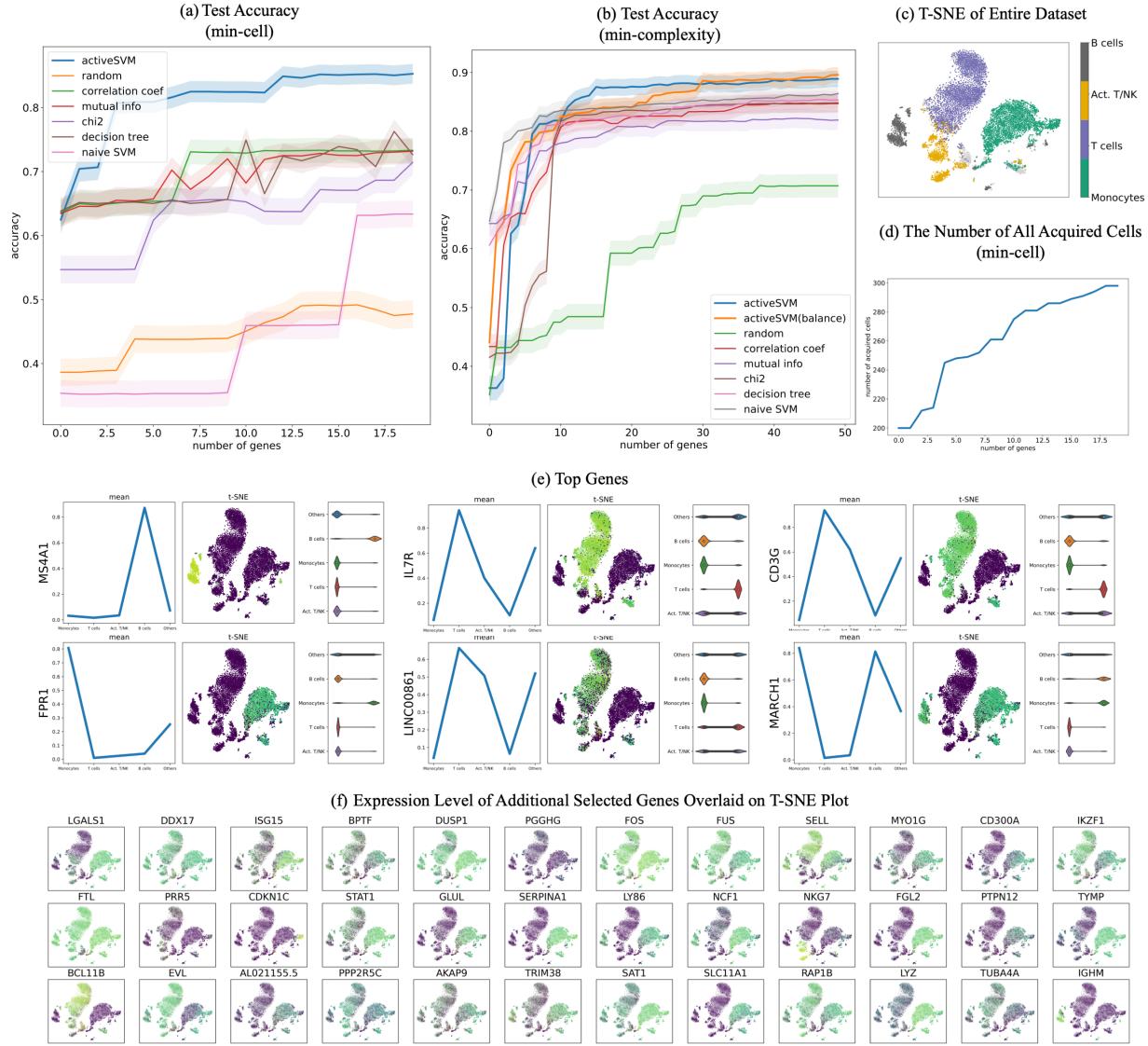


Figure 2: Gene selection and cell-type classification for PBMC dataset. (a) The test accuracy for min-cell strategy and a series of comparison classification strategies. The min-cell strategy selects $k = 20$ genes and select $c = 100$ cells each iteration with confidence interval estimates; (b) The test accuracy of min-complexity strategy that selects $k = 50$ genes using $c = 20$ cells each iteration; (c) The t-SNE plots of the entire filtered dataset; (d) The total number of unique cells used vs gene set size with the min-cell strategy; (e) plots showing the expression of several genes markers, including mean on classes, gene expression value on t-SNE projection, and violin plots; (f) expression level of additional selected genes overlaid on t-SNE plot.

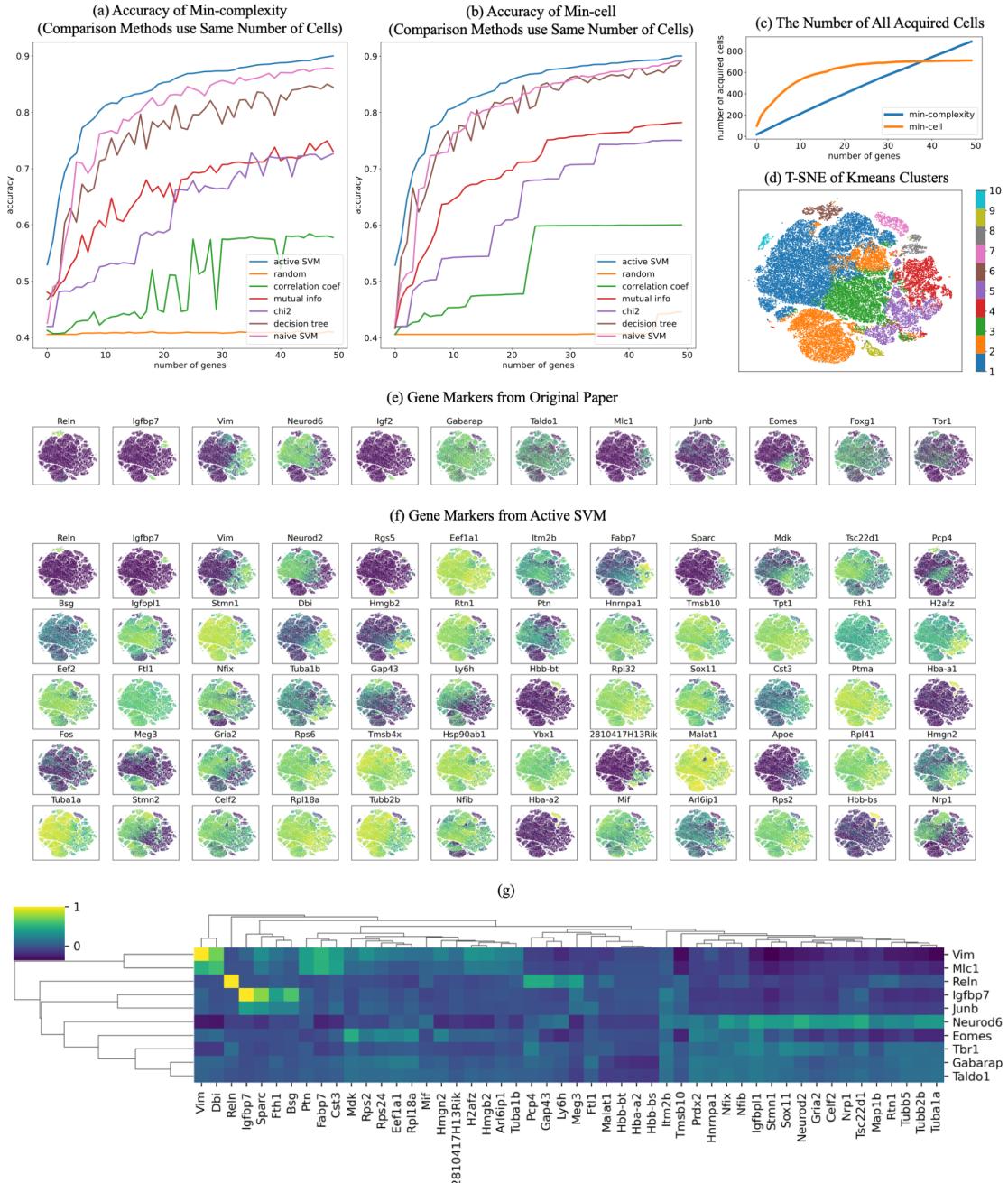


Figure 3: Scaling of ActiveSVM feature selection to 1.3 million cell mouse brain data set (a) The test accuracy of min-complexity strategy that selects 50 genes using 20 cells each iteration; (b) The test accuracy of min-cell strategy that selects 50 genes using 100 cells each iteration; (c) The total number of unique cells used vs gene set size with both the min-complexity and the min-cell strategy; (d) The t-SNE plots of the entire filtered dataset with 10 classes by k-means clustering; (e) expression level of the gene markers from previously published analysis overlaid on t-SNE plot; (f) expression level of the gene markers selected by ActiveSVM overlaid on t-SNE plot, where the first row are the genes that have similar distribution with gene markers from previously analysis and other genes are new markers correlated with the classification target. (g) Correlation matrix of literature markers (y-axis) from [12] versus ActiveSVM selected genes (x-axis).

362 embryonic day 18 (E18) [18]. The data set is one of the largest single cell mRNA-seq data sets
363 currently available. The size of the data set has been a challenge for data analysis, and a previous
364 analysis paper was published that developed sub-sampling methods that extract marker genes and
365 cell-types by extracting sub-sets of the data set containing $\sim 100,000$ cells [12]. We applied our
366 ActiveSVM method to extract minimal genes sets for classifying the 10 classes of cells that were
367 extracted through k-means[49] clustering in the internal analysis of the data (Figure 3(a)(b)). The
368 min-complexity algorithm used 20 cells at each iteration and the min-cell algorithm selected 100
369 cells each loop. The min-cell algorithm acquired fewer unique cells, as cells are selected repeatedly
370 (Figure 3(c)). On this dataset, both algorithms use 'balanced' sampling for both min-complexity
371 and min-cell strategies. As the dataset is too large to produce t-SNE, we randomly sampled 30,000
372 cells and find the tSNE projection, which is shown with the input cell clusters in Figure 3(d).

373 While the size of the data set has presented challenges for conventional sampling methods, the
374 ActiveSVM algorithm must only acquire from memory a small number of genes or cells at each
375 round of analysis, and therefore, the method avoids computing across the entire 1.3 million cells
376 and $\sim 30,000$ gene data set. We found that it was possible to run ActiveSVM on a conventional
377 lap-top. For decreasing compute time, we analyzed the megacell data set on an AWS instance
378 r5n.24xlarge. On this instance, ActiveSVM ran in 69 minutes for the min-complexity strategy and
379 243 minutes for the min cell strategy. As a comparison, naive SVM required greater than four days
380 of computation to run on all 1.3 million cells on the same AWS instance (Table 1).

381 To provide a bench-marking for ActiveSVM, we instead compared the accuracy of ActiveSVM to
382 a data set where we allow ActiveSVM to run on the data set; we extract the number of analyzed
383 cells, and then provide this same number of cells to the other methods shown in figure 3(a)(b).
384 Applying the other methods to sub-sampled data, allowed us to extract the classification accuracy
385 as a bench-marking for ActiveSVM.

386 In addition to performing the classification task, the ActiveSVM procedure discovers gene sets that
387 achieve $\sim 90\%$ classification accuracy with only 50 genes. The procedure discovered a series of
388 cluster specific marker genes that extend prior analysis. For example, the analysis in [12] identified
389 marker genes through sub-sampling and prior biological literature. A set of genes identified previ-
390 ously is shown in Figure 3(e). The ActiveSVM analysis discovered several of the same markers as
391 the previous work (Reln, Vim, Igfbp7) (Figure 3(f)).

392 Further, ActiveSVM extended previous analysis by identifying additional markers that correlate
393 with the previously analysis as well as marker genes of additional cell states. The development
394 of radial glial cells, in particular, has been of intense recent interest because radial glial cells
395 are the stem cells of the neocortex in mouse and human [50]. Careful molecular analysis has
396 defined markers of radial glial cells including Vim. ActiveSVM identified a group of genes whose
397 expression correlates with Vim across the E18 mouse brain. Our analysis identified an additional
398 set of genes expressed in the same cell population as Vim including, Dbi (Diazepam Binding
399 Inhibitor, Acyl-CoA Binding Protein), Hmgb2, and Ptn. A correlation matrix (Figure 3g) showing
400 the correlation of ActiveSVM identified genes (x-axis) with literature markers (y-axis) discussed

401 in [12] reveals the existence of Vim correlated genes. The Vim genes were of interest because they
402 include additional transcription factors Hmgb2 [50] and also a core group of genes, Ptn and Fabp7
403 (also Brain Lipid Binding Protein), two components of a radial glia signaling network [51, 52, 50]
404 that has been identified as a core regulatory module supporting the proliferation and stem cell state
405 in the radial glial cell population.

406 The neural progenitor transcription factor Neurod6 marked a separate cell population that we iden-
407 tified to contain genes including Neurod2 (a transcription factor) and Sox11 (a transcription factor)
408 as well as glial transcription factors Nfib and Nfix and the receptor Gria2 (Glutamate Ionotropic
409 Receptor AMPA Type Subunit 2). The marker genes observed in Neurod6 expressing cells were
410 anti-correlated with the Vim correlated markers suggesting that ActiveSVM identified two distinct
411 regulatory modules. Structurally, the tubulin proteins Tuba1b and Tuba1a were expressed in Vim
412 and Neurod6 populations respectively. In addition to genes correlated or anti-correlated with exist-
413 ing markers, ActiveSVM identified markers of additional cell populations including Meg3, a long
414 non-coding RNA expressed in cluster 2.

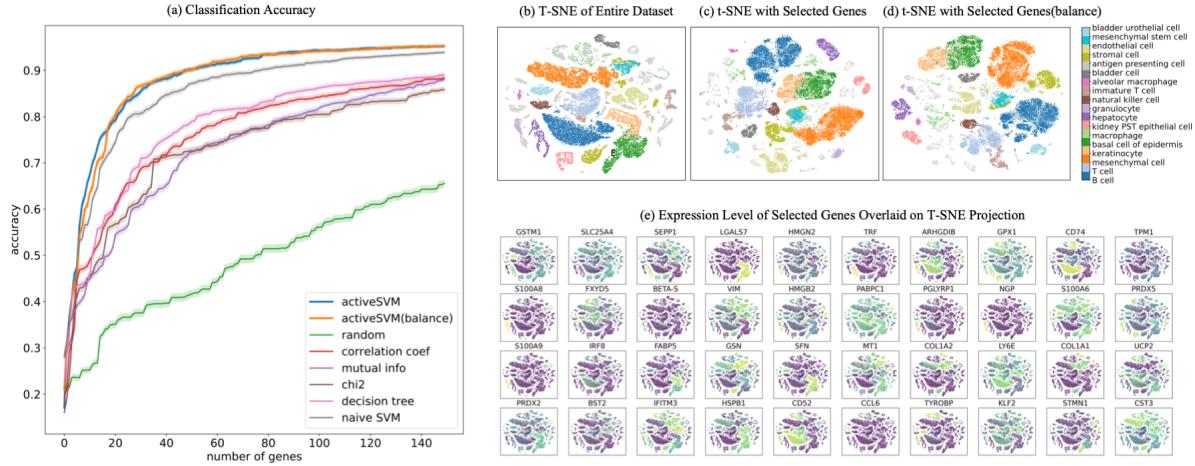
415 Broadly, the analysis of the ‘megacell’ mouse brain data set demonstrates that ActiveSVM scales
416 to analyze a large data set with > 1 million cells. The analysis of such large data sets has been chal-
417 lenging with conventional approaches that attempt to store the entire set in memory for analysis.
418 Previous analysis of the 10x megacell dataset found that sub-samples with greater than 100,000
419 cells would yield an out of memory error on a server node with 64 cores, a 2.6 GHz processor,
420 and 512 GB of RAM [12]. ActiveSVM iterates through analysis of cells and genes while focus-
421 ing computational resources on poorly classified cells, and so ActiveSVM does not load the entire
422 dataset into memory but can read cells and genes from disk as needed. Further, through iter-
423 ative analysis, ActiveSVM identifies known marker and regulatory genes, genes that correlate with
424 known markers as well as marker genes of additional cell populations that could provide a starting
425 point for future experimental investigations.

426 **Identifying gene sets for cell-type classification in the Tabula Muris tissue 427 survey**

428 In addition to analyzing a data set with a large number of total cells, we sought to benchmark
429 performance of ActiveSVM feature selection on a data set with a large number of distinct cell
430 types. We applied ActiveSVM to the Tabula Muris mouse tissue survey, a droplet-based scRNA-
431 sequencing data-set, that contains 55,656 single cells across 58 annotated cell types, and 12 major
432 tissues [19]. For each cell, 8,661 genes are measured. In our analysis, we used the supplied
433 cell-type labels, agnostic of tissue type. Thus, cells labeled ‘macrophage’ from the spleen are
434 considered to belong to the same class as cells labeled ‘macrophage’ from the mammary gland.

435 Even with a large number of cell types, ActiveSVM can construct gene sets that achieve high
436 accuracy ($> 90\%$), compared to other methods (Figure 4(a)(f)). To construct a gene set of size

A. Min-complexity Strategy



B. Min-cell Strategy

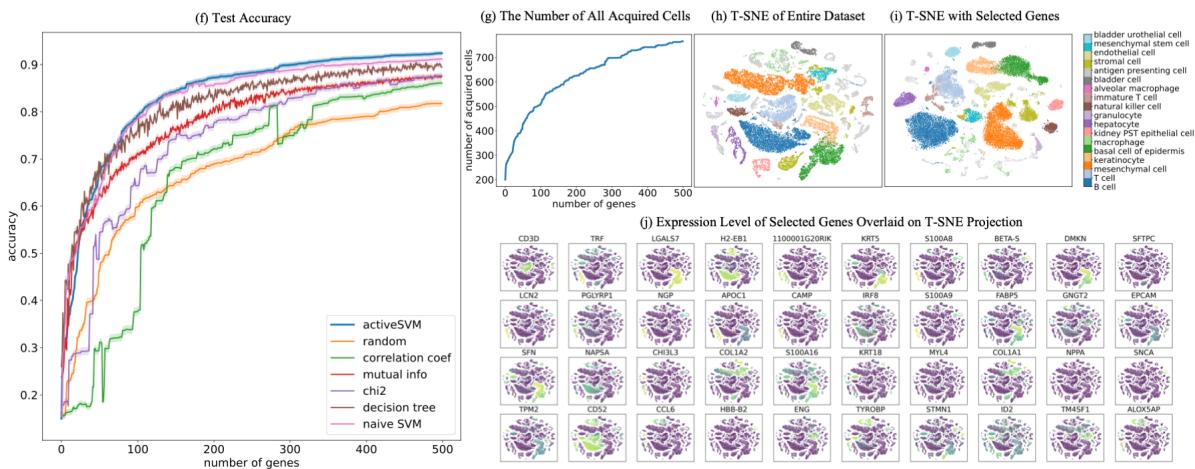


Figure 4: Minimal gene sets for cell-type classification in the Tabula Muris mouse tissue survey (A) Classification results of 150 genes selected using the min-complexity strategy with 20 cells each iteration. (B) 500 genes selected using the min-cell strategy with 200 cells per iteration. Results for standard and balanced strategy shown with comparison methods and confidence intervals. The subplots contain: classification accuracy vs gene set size using the min-complexity strategy (a) and min-cell strategy (f); the t-SNE plots of the entire filtered dataset (b)(h); the t-SNE plots of the gene set selected using min-complexity strategy with randomly sampling (c) and 'balanced' sampling (d), and gene set selected using the min-cell strategy (i); the expression level overlaid on t-SNE projection for genes selected by min-complexity (e) and by min-cell (j); and the total number of unique cells used vs gene set size with the min-cell strategy (g).

437 500, ActiveSVM feature selection used fewer than 800 unique cells (Figure 4(g)) or an average
438 of 14 cells per cell type. We were able to recreate the clustering patterns from the original data
439 (Figure 4(b)(h) when analyzing the cells within the low dimensional t-SNE space spanned by the
440 selected 150 genes (Figure 4(c)(d)) or 500 genes (Figure 4(i)).

441 Our approach allowed us to construct a set of marker genes able to identify mouse cell types across
442 disparate tissues. Even when analyzing a large number of cell types, we were able to identify highly
443 cell-type specific genes, such as CD3D, a well-established T-cell marker, or TRF (transferrin),
444 which is selectively secreted by hepatocytes[53], or LGALS7 (galectin-7), which is specific for
445 basal and differentiated cells of stratified epithelium [54]. However, given the functional overlap
446 between different cell types, the genes within our set include many that mark multiple cell types.
447 For instance, H2-EB1[55], a protein important in antigen presentation, is expressed in B-cells and
448 Macrophages, both of which are professional antigen presenting cells (APCs). Our analysis also
449 identified cell type-specific expression for a number of poorly studied genes, such as granulocyte-
450 and hepatocyte- specific expression of 1100001G20RIK (also known as Wdnm-like adipokine),
451 which has previously only been associated with adipocytes [56].

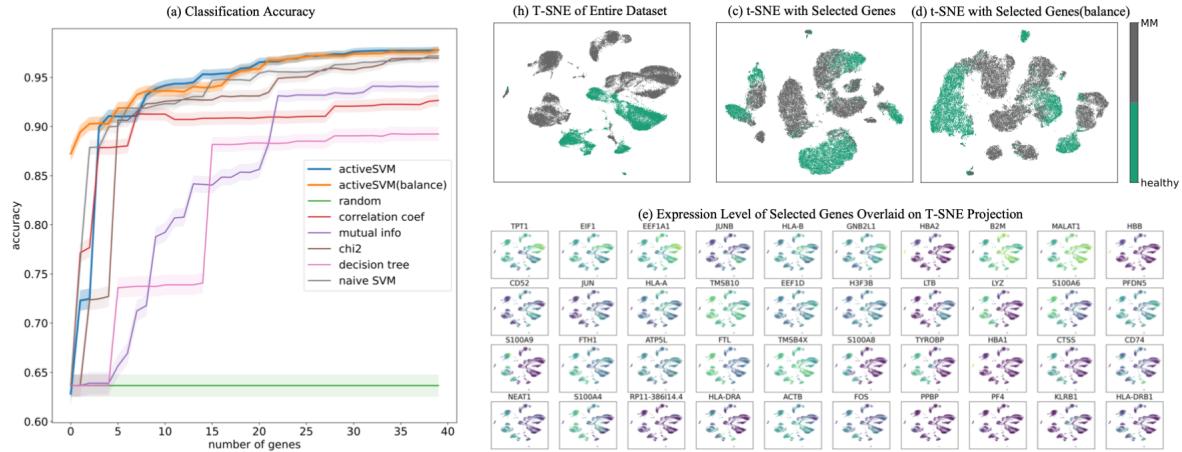
452 **Extraction of gene sets for classification of disease state in peripheral blood
453 cells from multiple myeloma patient samples**

454 To analyze ActiveSVM as a tool for the discovery of disease-specific markers, we used single-cell
455 data from peripheral blood immune cells collected from two healthy donors and four patients who
456 have been diagnosed with multiple myeloma (MM)[20]. MM is an incurable cancer of plasma
457 cells, known as myeloma cells, that over-proliferate in the bone marrow. Although myeloma cells
458 are typically the target of analysis because they are the causative agent of disease, peripherally
459 circulating immune cells also contain signatures of disease, including a depleted B-cell population
460 [57, 58], an increased myeloid-derived suppressor cell count [59], and T-cell immunosenescence
461 [60, 58].

462 We sought to further define transcriptional markers that distinguish healthy peripheral immune
463 cells from the cells of MM patients. We performed feature selection using heterogeneous popula-
464 tions of cells labeled only by disease state. The data set contains 35159 with 32527 genes (Table
465 1).

466 We compared the classification accuracy for ActiveSVM vs the other methods (Figure 5(a)(f)),
467 and found that ActiveSVM achieved high accuracy in a limited number of steps and consistently
468 outperformed the other methods. We tested ActiveSVM with two different cell sampling strategies,
469 randomly sampling, and 'balanced' sampling, in which equal numbers of cells from each cell type
470 are sampled to correct for artifacts due to different cell-type proportions between samples. We
471 noted that although the balanced approach gave higher classification accuracy at early iterations,
472 these differences are no longer apparent after selecting 20 genes (Figure 5(a)).

A. Min-complexity Strategy



B. Min-cell Strategy

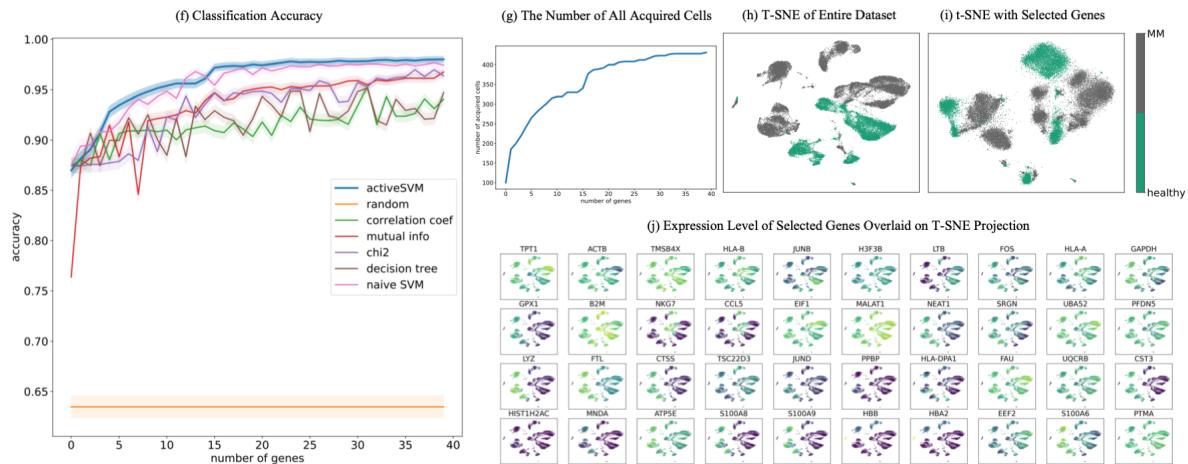


Figure 5: Gene set selection for healthy vs disease classification in multiple myeloma dataset. (A) classification results of 40 genes selected by min-complexity strategy using 20 cells each iteration. (B) 40 genes selected using Min-cell strategy with 100 cells per iteration. Results for standard and balanced strategy shown with comparison methods and confidence intervals. As in Figure 4, each sub-figure, sub-panels show the number of acquired cells per iteration, tSNE visualizations of using the complete data set, visualizations using only the ActiveSVM extracted data set, and marker genes identified by ActiveSVM.

473 Non-overlapping cell-type clusters were identified for healthy and MM cells in the original dataset
474 in t-SNE projections (Figure 5(b)(h)). The non-overlapping clusters are replicated in t-SNEs con-
475 structed from 40 genes selected using both the min-complexity strategy (Figure 5(c)(d)) and the
476 min-cell strategy (Figure 5(i)).

477 Analysis of the function of the genes identified by ActiveSVM revealed most regulate house-
478 keeping functions, suggesting that global shifts in translation and motility are disrupted in multiple
479 myeloma patients. Translation-associated markers include Eukaryotic Translation Initiation Factor
480 1 (EIF1), Eukaryotic Translation Elongation Factor 1 Alpha 1 (EEF1A1), and prefoldin subunit 5
481 (PFDN5). Motility associated genes include ACTB, putative anti-adhesion molecule CD52, and
482 actin-sequestering protein TMSB4X.

483 We also found both known and novel markers of MM within the peripheral blood immune cells.
484 Our analysis identified TPT1, previously associated with MM [61], and RACK1 (also known as
485 GNB2L1), a scaffolding protein that coordinates critical functions including cell motility, survival
486 and death, which is broadly upregulated in peripheral immune cells from MM patients. Although
487 this gene has been previously associated with myeloma cells [62], its regulation had not been re-
488 ported in peripherally circulating immune cells. Our ability to discover MM-specific genes within
489 peripheral immune cells suggests a broader use for discovering disease-specific genes across many
490 different types of pathologies.

491 Interestingly, the procedure also identifies multiple members of the S100 Calcium Binding Protein
492 Family (S100A8, S100A9 and S100A6, and S100A4) [63, 64, 65] as members of the genes sets that
493 separate MM vs healthy samples. The S100 protein family defines a module of genes that are asso-
494 ciated with the induction of stress response pathways. The expression of S100 genes is prognostic
495 for a number of diseases. Specifically, a recent study found that S100A4 expression correlates with
496 poor patient survival in multiple myeloma and that S100A8, and S100A9 are markers that correlate
497 with poor response of multiple myeloma patients to treatment with proteasome inhibitors and the
498 histone deacetylase inhibitor panobinostat [64]. The result demonstrates that ActiveSVM can
499 automatically define groups of genes that have clinical association with disease progression and
500 treatment outcome. The minimal gene sets generated by ActiveSVM could provide useful targeted
501 sequencing panels for a variety of clinical tasks.

502 **ActiveSVM identifies genes impacted by Cas9 based genetic perturbation**

503 The previous analyses above have demonstrated that ActiveSVM identifies minimal gene sets for
504 cell-state identification across a range of single-cell mRNA-seq data sets. We, next, demonstrate
505 that ActiveSVM provides a more general analysis tool with potential applications to a range of
506 single-cell genomics analysis tasks. To demonstrate generalization of ActiveSVM based gene set
507 selection across single-cell genomics tasks, we applied the method to identify marker genes in two
508 additional applications: perturb-seq and spatial transcriptomics.

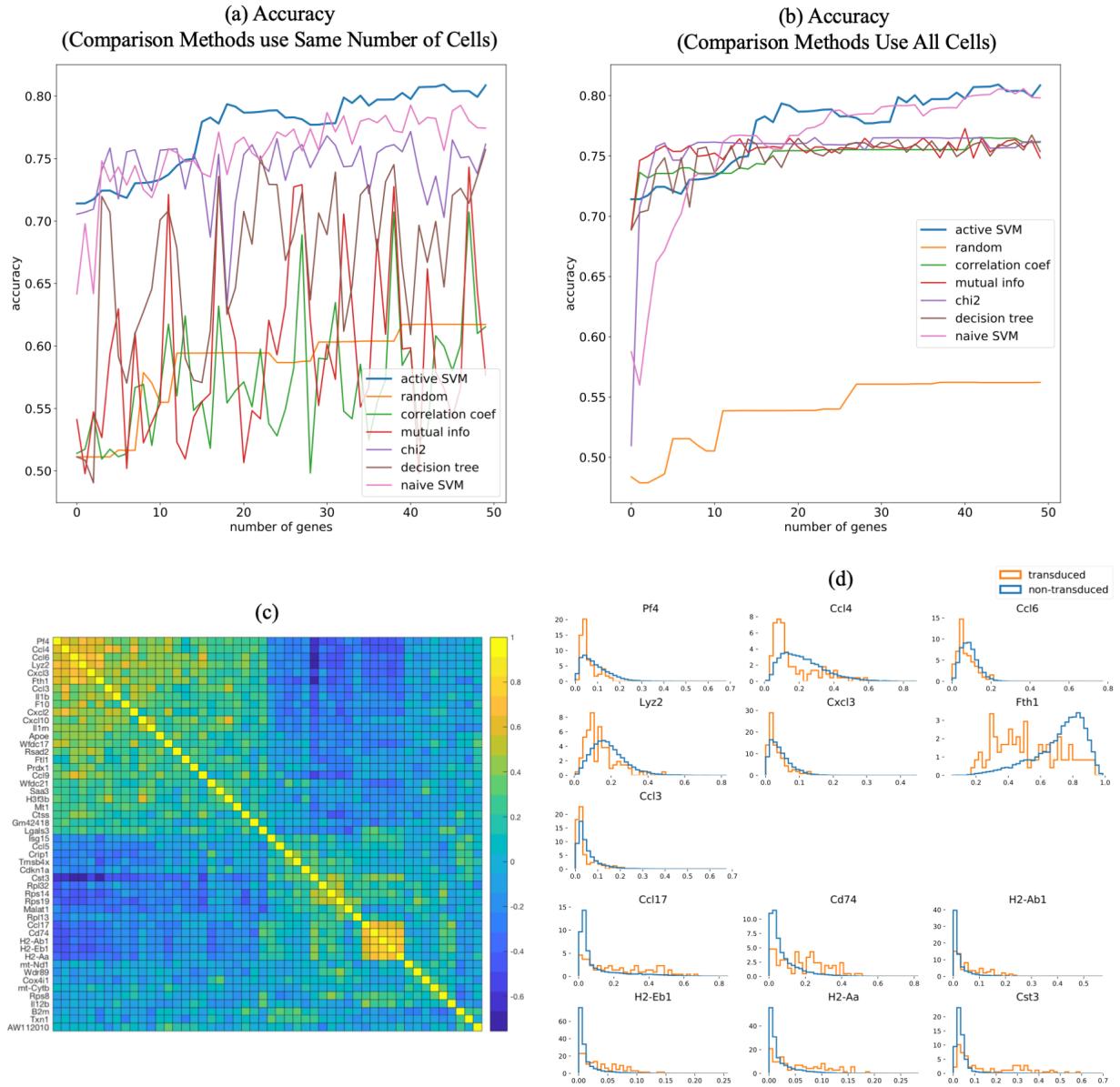


Figure 6: Application of ActiveSVM to identify genes expression changes following Cebp knock-down with perturb-seq The results of classification on perturb-seq data [21] where cells are labeled and classified as Cebp sgRNA transduced or not-transduced with a guide RNA. (a-b) accuracy of entire dataset with min-complexity strategy, where comparison methods use the same number of cells as ActiveSVM in (a) and use the entire dataset in (b). (c) correlation matrix showing pair-wise correlation coefficients for genes in Cebp perturbed cells. Correlation matrix identifies two gene modules. (d) Distributions of gene expression in Cebp sgRNA transduced (orange) or not transduced (blue) cells. Selected genes from modules in (c) shown and organized so that genes whose expression increases with Cebp perturbation are on top and repressed genes are on the bottom of the figure.

509 Perturb-seq is an experimental method for performing Cas9-based genetic screens with single-cell
510 mRNA-seq read-outs. In perturb-seq, cells are induced with libraries of guide RNA's that target the
511 Cas9 protein to cut and silence specific genes [21, 3]. Perturb-seq is performed in a pooled fashion
512 so that a pooled set of sgRNA molecules is delivered to a cell population. Individual cells stochas-
513 tically take-up specific guide RNAs, and the guide RNAs target Cas9 cuts and silences genes in the
514 genome. Following the perturbation experiment, single-cell mRNA-seq is applied to read both
515 the transcriptome of each cell and the identify of the delivered sgRNA through sequencing. The
516 advantage of the perturb-seq method is that many knock-out experiments can be performed simul-
517 taneously. However, a challenge is that noise impacts the measurement of guide RNA identify,
518 and, further, the cutting of the genome by the Cas9 molecule is not complete. Due to measurement
519 and experimental noise, identifying the impact of genetic perturbation on a cell population can be
520 challenging, and various methods have been developed to boost signal [3]. We applied ActiveSVM
521 to identify a minimal gene set as well as down-stream effects of gene knock-down in perturb-seq
522 data.

523 We specifically applied ActiveSVM to analyze public data collected from mouse dendritic cells
524 with transcription factor knock-downs [21]. The experiment analyzed cells in which transcription
525 factors has been knocked-down using perturb-seq in mouse dendritic cells stimulated for 3 hours
526 with LPS, a signal that mimics bacterial infection.

527 To apply ActiveSVM to the data, we focused our analysis on knock-down of Cebp an pioneer
528 transcription factor. We pre-processed the data to identify cells induced with sgRNA against Cebp
529 and non-induced cells, and used transduced and non-transduced as our cell-labels. We applied Ac-
530 tiveSVM to select a minimal gene set that could classify transduced versus non-transduced cells.
531 ActiveSVM identified minimal gene sets (50 genes) that achieved 80% classification accuracy on
532 the Cebp sgRNA cell label. As we applied the class-balanced model to obtain the classification
533 accuracy and there are only about 20 transduced cells in test set, we show the accuracy on entire
534 dataset instead of test set. On this noisy dataset, ActiveSVM worked better than comparison meth-
535 ods with the condition that ActiveSVM only used a small subset of data while comparison methods
536 performed on the entire dataset (Figure 6(b)).

537 We found that the discovered gene set could be decomposed into two modules of correlated genes
538 (Figure 6c). Figure 6(c) shows a clustered correlation matrix for the 50 identified genes. Gene
539 expression distributions for cells in transduced vs non-transduced cells demonstrated that the mod-
540 ules represented two groups of genes. One group (including Pf4, Ccl4, Ccl6, Lyz2) was repressed
541 by Cebp knock-down, and the second gene group was activated by Cebp knock-down including
542 (Ccl17, Cd74, H2-Ab1) (Figure 6d).

543 In both cases, the identified gene sets contained known targets of Cebp, the perturbed transcription
544 factor. For example, Fth1 (ferritin, heavy polypeptide 1), Cst3, Tmsb4x, Lgals3, Ccl4, and Cd74
545 are all previously identified as direct binding targets of Cebp as determined by Chip-seq [66]. Since
546 Cebp knock-down leads to both up-regulation and down-regulation of genes, the results suggest
547 that the factor can play both activating and repressive roles consistent with prior literature [67].

548 Our analysis of the perturb-seq data set, therefore, demonstrates that ActiveSVM can be applied as
549 a useful tool for the identification of genes modulated by perturb-seq experiments. ActiveSVM can
550 return minimal genes sets that contain functional information. Moreover, perturb-seq has been a
551 main application of gene targeting approaches [3]. Therefore, ActiveSVM could provide a method
552 for identifying minimal gene sets that can be applied to increase the scale of perturb-seq data
553 collection.

554 **ActiveSVM defines region specific markers in spatial transcriptomics data**

555 Finally, to further demonstrate the generality of the ActiveSVM approach, we applied the pro-
556 cedure to identify minimal gene sets for classification of cells by spatial location in spatial tran-
557 scriptomics data. Spatial transcriptomics is an emerging method for measuring mRNA expression
558 within single cells while retaining spatial information and cellular proximity within a tissue. As an
559 example, in SeqFish+, an imaging based spatial transcriptomics method, cells are imaged in their
560 tissue environment, and mRNA transcripts are counted using single-molecule imaging of mRNA
561 spots [22]. In all spatial transcriptomics applications, a common goal is the identification of genes
562 that mark specific spatial locations within a tissue sample. Additionally, spatial imaging methods
563 are commonly limited by imaging time. While Seqfish+ can profile 10,000 mRNA molecules per
564 cell, the identification of reduced gene sets would reduce imaging time and throughput.

565 We applied ActiveSVM to identify genes associated with specific spatial locations in the mouse
566 brain. We used a seqFISH+ data set in which the authors profile 10,000 mRNA molecules in 7
567 fields of view (FOV) in the mouse brain [22]. Fields of view correspond to spatially distinct regions
568 of the mouse cortex as well as the sub-ventricular zone and chordid plexus. We used the spatial
569 location labels provided by [[22]] to identify seven different brain locations (Fields of view 1-5
570 corresponding to Cortex Layers 2/3 through Layer 6; FOV 6 is sub-ventricular zone, and FOV 7
571 is chordid plexus). Applying the spatial location labels as class labels, we applied ActiveSVM to
572 identify genes that could allow classification of single-cells by their location in one of the seven
573 classes and to define marker genes that correspond to specific spatial locations.

574 We identified gene sets of < 30 genes that enabled location classification with greater than 85% ac-
575 curacy with min-complexity strategy (Figure 7(a)). ActiveSVM used only 10 cell at each iteration
576 but worked better than comparison methods who performed on the entire dataset (Figure 7(b)).

577 In the spatial application, the result means that the ~ 30 genes are sufficient to classify single-cells
578 as belonging to one of the 7 spatial classes. In Figure 7, we show the mean expression of identified
579 genes across cortical fields of view corresponding to a sweep through cortical layers 2/3 through 6
580 as well as SVZ and CP. Our analysis identifies markers *Prex1* that are specific to the upper cortical
581 layers of the brain. *Efhd2*, a calcium binding protein linked to Alzheimer's disease and dementia,
582 was similarly expressed in lower cortical layers [68, 69]. Finally, *Pltp*, a Phospholipid transfer
583 protein, was localized to the chordid plexus. In Figure 7e, we show the spatial distribution of

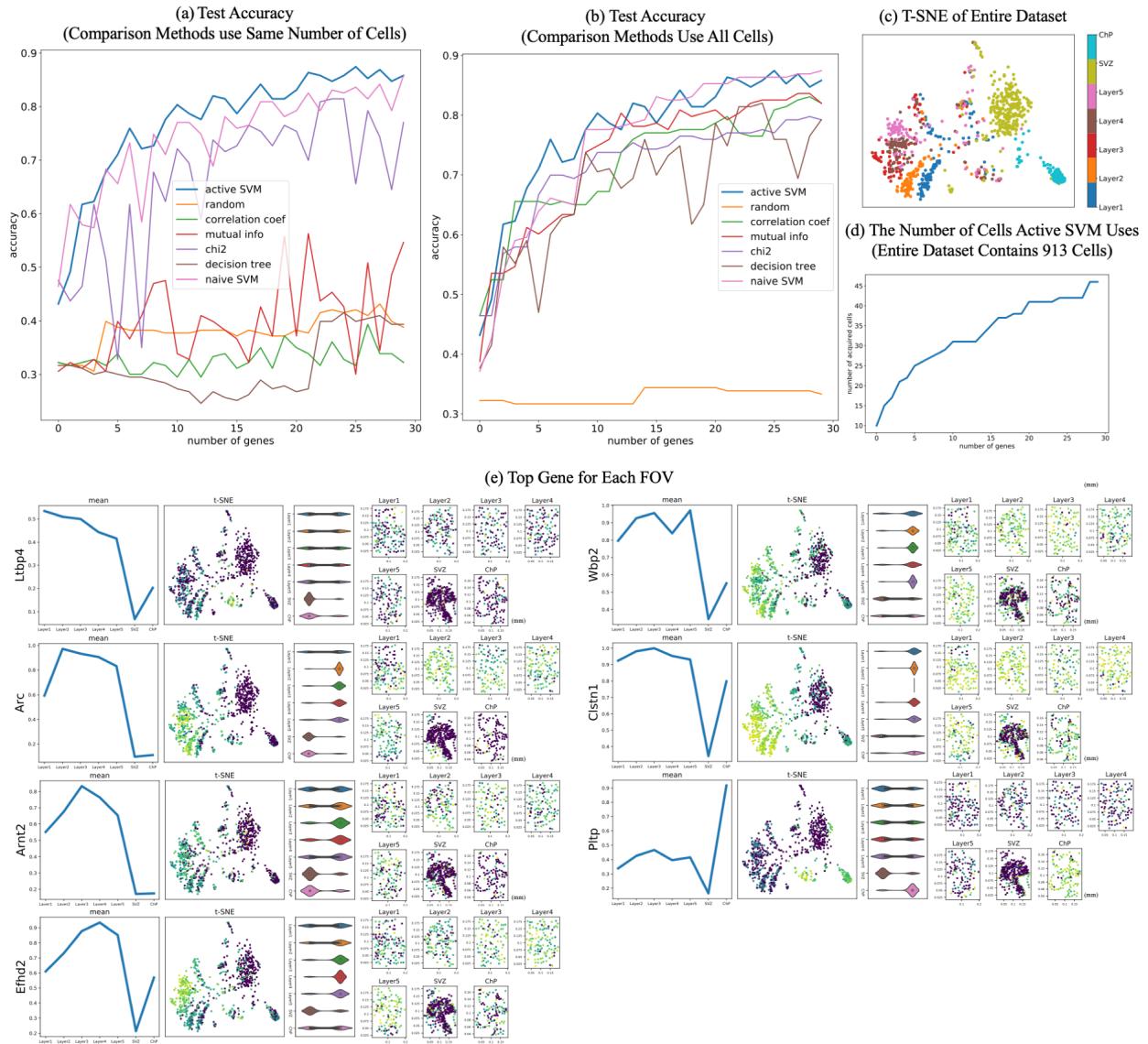


Figure 7: Application of ActiveSVM to identify region specific marker genes in the mouse brain with spatial transcriptomic data The results of classification where cells are labeled according to fields of view (FOV) in [22]. (a-b) test accuracy with min-complexity strategy, where comparison methods use the same number of cells as ActiveSVM in (a) and use the entire dataset in (b). Fields of view 1-5 correspond to 5 regions of the mouse cortex, additional fields of view are labeled SVZ (sub-ventricular zone) and ChP (chordid plexus). (c) tSNE of cell transcriptomes for all cells (d) number of cells used per iteration (e) Sample of identified genes where each sub-panel shows mean expression across FOV/brain regions for selected gene, a tSNE plot colored by expression of selected gene, a violin plot of single cell gene expression values for selected gene in FOV/brain region, and spatial plots of each field of view where dots represent cells in 2D imaging slice, cells are colored by intensity of selected gene and units are in millimeters.

584 these genes including their mean expression across regions, violin plots documenting expression
585 distribution, and renderings of the single-cells within the field of view and the relative expression
586 of each gene.

587 The spatial analysis demonstrates that a broad range of different experimental variables can be
588 applied as labels. In each case ActiveSVM discovers genes that allow classification of cells ac-
589 cording to labels and identifies interesting genes. Regional gene marker identification is a major
590 task in seqFish data analysis and ActiveSVM is able to identify genes enriched in different brain
591 regions automatically. Such spatial information could provide interesting new insights into disease
592 processes mediated by genes like Efhd2.

593 Discussion

594 In this paper, we introduce ActiveSVM as a feature selection procedure for discovering minimal
595 gene sets in large single-cell mRNA-seq datasets. ActiveSVM extracts minimal gene sets through
596 an iterative cell-state classification strategy. At each round the algorithm applies the current gene
597 set to identify cells that classify poorly. Through analysis of misclassified cells, the algorithm
598 identifies maximally informative genes to incorporate into the target gene set. The iterative,active
599 strategy reduces memory and computational costs by focusing resources on a highly informative
600 subset of cells within a larger data set. By focusing computational resources on misclassified cells,
601 the method can run on large data sets with more than one million cells. We demonstrate that
602 ActiveSVM is able to identify compact gene sets with tens to hundreds of genes that still enable
603 highly accurate cell-type classification. We demonstrate that the method can be applied to a variety
604 of different types of data set and single-cell analysis tasks including perturb-seq data analysis and
605 spatial transcriptomic marker gene analysis.

606 Conceptually, we refer to our strategy ‘active’ because it actively selects data examples (here cells)
607 at each iteration for detailed analysis . Our algorithm specifically selects cells that within the
608 margin of the SVM classifier, and uses these poorly classified cells to search for maximally infor-
609 mative genes (features). In traditional active learning strategies, an algorithm is typically called
610 active when it can directly query an oracle for data examples that meet a criteria [16, 70]. In the
611 tradition of active learning, our ActiveSVM procedure queries the SVM classifier for cells that
612 have been misclassified, and then expends computational resources to analyze all genes within that
613 limited subset of cells to discover informative genes. Thus, while our algorithm cannot query the
614 biological system directly for cells that meet a specific criteria, the algorithm queries the data set
615 itself for informative examples, and therefore we refer to it as ‘active’. Our current work focuses
616 on a single classification method, the support vector machine, as the computational engine. Active
617 learning methods can be applied more broadly to additional classification strategies like neural
618 network based classification as well as to additional types of analysis like data clustering and gene
619 regulatory network inference.

620 Our method also has some conceptual similarity to boosting methods [71, 72]. Boosting algorithms
621 (e.g AdaBoost) train a series of 'weak' learners for a classification tasks, and then combine these
622 weak classifiers to generate a strong classifier. In boosting a single weak learner may initially
623 obtain moderate performance on a task. The performance of weak learners is improved through
624 iterative training of additional learners and focusing their training on difficult data examples, for
625 example, misclassified examples. The boosting algorithm constructs a final, strong classifier by
626 combining the results of the ensemble of weak classifiers through a weighted majority vote. Our
627 method is distinct from conventional boosting, because we search for a minimal set of features
628 in our data that allows a single SVM classifier to achieve high-accuracy classification. However,
629 ActiveSVM feature selection shares conceptual ideas with boosting in that both methods focus
630 analysis on challenging examples and combine information to achieve strong classification from
631 initially weak classifiers.

632 ActiveSVM provides an iterative strategy for extracting a compact set of highly informative genes
633 from large single cell data sets. Biologically, recent work highlights the presence of low-dimensional
634 structure within the transcriptome [1]. Low-dimensional structure emerges in gene expression data
635 because cells modulate their physiological state through gene expression programs or modules that
636 contain large groups of genes. Since genes within transcriptional modules have highly correlated
637 expression, measurements performed on a small number of highly informative signature genes can
638 be sufficient to infer the state of a cell [73]. Low-dimensional structure can be exploited to decrease
639 measurement and analysis costs since a small fraction of the transcriptome must be measured to in-
640 fer cellular state. We developed ActiveSVM as a scalable strategy for extracting high information
641 content genes within a sharply defined task, cell-state classification.

642 In ActiveSVM we apply an active learning strategy to reduce the computational and memory re-
643 quirements for analyzing single-cell data sets by focusing computational resources on 'difficult to
644 classify' cells. In the future, active learning strategies could be applied directly at the point of
645 measurement. In genomics measurement resources often limit the scale of data acquisition. In
646 future work we aim to develop strategies that can improve the on-line acquisition of single-cell
647 data. Active strategies could be implemented at the point of measurement by only sequencing or
648 imaging the content of cells that meet a criteria. Even more broadly, it might be possible to in-
649 duce a biological system to generate highly informative examples through designed experimental
650 perturbation [74].

651 Data Availability

652 All data used in the paper has been previously published. The PBMC Single-cell RNA-seq data
653 have been deposited in the Short Read Archive under accession number SRP073767 by the authors
654 of [17]. Data are also available at <http://support.10xgenomics.com/single-cell/datasets>.

655 The original Tabula Muris dataset is available at https://figshare.com/projects/Tabula_Muris_Trans

656 [criptomic_characterization_of_20_organs_and_tissues_from_Mus_musculus_at_single_cell_resolutio](#)
657 [n/27733](#).

658 The original multiple myeloma PBMC data, containing 2 healthy donors and 4 multiple myeloma
659 donors, is available at https://figshare.com/articles/dataset/PopAlign_Data/11837097/3.

660 The 10x genomics Megacell data set is available at <http://support.10xgenomics.com/single-cell/datasets>.

662 The perturb-seq data set [21] is available at <https://www.ncbi.nlm.nih.gov/geo/query/acc.cgi?acc=GSM2396856>

664 The spatial transcriptomics data [22] is available <https://github.com/CaiGroup/seqFISH-PLUS>.

665 **Code Availability**

666 Our method is integrated as a install-able Python package called activeSVC. The installation
667 instructions and user guidance are shown at <https://pypi.org/project/activeSVC>. The source codes of
668 activeSVC and some demo examples are publicly available on GitHub at https://github.com/xqc_hen/activeSVC.

670 The Python package provides six callable functions: (1) *min_complexity*; (2) *min_acquisition*,
671 the min-cell strategy; (3) *min_complexity_cv*, which use cross validation[40] and grid-search[39]
672 to train the best SVM estimator at each iteration; (4) *min_acquisition_cv*, the min-cell strategy
673 with cross validation and grid-search; (5) *min_complexity_h5py*, for large h5py[75] data files, it
674 only loads the part of data, the rows and columns of selected genes and cells, instead of loading the
675 entire dataset into memory; (4) *min_acquisition_h5py*, is similar with *min_complexity_h5py* but
676 uses min-cell strategy. All include the algorithm for both randomly and 'balanced' sampling. We
677 implement the SVM classifier with the LinearSVC package from scikit-learn[76] library, which
678 is implemented in term of LIBLINEAR[33]. And we use parfor[77] package to parallelize for-
679 loops to accelerate algorithm for large datasets. There are three hyper-parameters to set: balance
680 (boolean), num_features (int), and num_samples (int), to identify the sampling strategy, the number
681 of genes to select, and the number of cells each iteration.

682 In the GitHub project, we use the PBMC dataset[17] and Tabula Muris dataset[19] as examples
683 to show the procedure and its performance of *min_complexity* and *min_acquisition*. We also
684 have the test examples of *min_complexity_cv* and *min_acquisition_cv* on PBMC dataset and the
685 demo projects of *min_complexity_h5py* and *min_acquisition_h5py* on 1.3 millions mouse brain
686 'megacell' dataset[18]. The notebooks contain downloading dataset, preprocessing, and selecting
687 genes with our method. Besides, we created Google Colaboratory project for these two examples
688 that PBMC demo is at <https://colab.research.google.com/drive/16h8hsnJ3ukTWAPnCB581dwj-n>

689 N5oopyM?usp=sharing, Tabula Muris demo is at <https://colab.research.google.com/drive/1SLehI>
690 KIQqpjK6BzEKc9m0y3uJ_LBqRzA?usp=sharing, and PBMC cross-validation demo is at <https://colab.research.google.com/drive/1fhQ8GD3NyzB3w0vof9WimXK6BLqDNuDC?usp=sharing>.
691

692 Experiments

693 Data Pre-processing

694 We found that ActiveSVM was able to achieve high-performance across pre-processing strategies.
695 For single-cell mRNA-seq column normalization (12) was important for removing artifacts due
696 to cell to cell variability in mRNA capture. However, additional algorithm was not sensitive to
697 additional pre-processing steps.

698 PBMC, Tabula Muris, Multiple-Myeloma

699 These three data sets were pre-processed for a prior publication [20] via column normalizaiton.
700 In each experiment, we removed the columns and rows where all values are zero. Then, gene
701 expression matrices were first columns normalized and log transformed. For a cell j , each gene
702 x_{ij} (gene i in cell j) is first normalized as $\tilde{g}_{ij} = \frac{g_{ij}}{\sum_{i=1}^n g_{ij}}$ where n is the number of genes in the
703 transcriptome.

704 Mega-cell data set, perturb-seq, spatial transcriptomics

705 For these data sets, we removed the columns and rows where all values are zero. Then we did
706 l^2 -normalization along each cell to scale input cell vectors individually to unit squared-norm.

707 Parameters

708 Here we provide the algorithm parameters we used for ActiveSVM in Table 2,3,4. Besides the
709 training set and test set, there are 15 user-defined hyper-parameters in ActiveSVM, five of which
710 are about the feature selection procedure and the other ten are commonly-used parameters for
711 linear SVM classifier. The detailed description about all parameters of ActiveSVM are detailed
712 described in the integrated package page <https://pypi.org/project/activeSVC/>.

713 As for comparison methods, correlation coefficient, mutual information, and chi-squared methods

714 don't have specific parameters to set. We implemented them with scikit-learn[76] package 'Selc-
 715 tKBest'. For feature importance scores from decision tree and naive SVM, we did grid-search on
 716 key parameters based on 3-fold cross validation at each step. The parameters of decision tree are
 717 *criterion* and *min_samples_leaf* and of naive SVM are *tol* and *C*.

Table 2: Parameters of ActiveSVM (PBMC and mouse megacell datasets).

	PBMC (min-complexity)	PBMC (min-cell)	mouse megacell (min-complexity)	mouse megacell (min-cell)
<i>num_features</i>	50	20	50	50
<i>num_samples</i>	20	100	20	100
<i>init_features</i>	1	1	1	1
<i>init_samples</i>	20	200	20	100
<i>balance</i>	True/False	True	True	True
<i>penalty</i>	'l2'	'l2'	'l2'	'l2'
<i>loss</i>	squared_hinge	squared_hinge	squared_hinge	squared_hinge
<i>dual</i>	True	True	True	True
<i>tol</i>	1e-4	1e-4	1e-4	1e-4
<i>C</i>	1.0	1.0	1.0	1.0
<i>fit_intercept</i>	True	True	True	True
<i>intercept_scaling</i>	1	1	1	1
<i>class_weight</i>	None	None	'balanced'	'balanced'
<i>random_state</i>	None	None	None	None
<i>max_iter</i>	1000	1000	1000	1000

718 Confidence Intervals

719 Confidence intervals were estimated using a proportion confidence interval[41] as interval =
 720 $z\sqrt{\frac{\epsilon*(1-\epsilon)}{n}}$ where $z = 1.96$ for 95% confidence and n is the number of cells and ϵ the observed
 721 error.

722 References

723 [1] G. Heimberg, R. Bhatnagar, H. El-Samad, and M. Thomson, "Low dimensionality in gene
 724 expression data enables the accurate extraction of transcriptional programs from shallow se-
 725 quencing," *Cell systems*, vol. 2, no. 4, pp. 239–250, 2016.

726 [2] H. C. Fan, G. K. Fu, and S. P. Fodor, "Combinatorial labeling of single cells for gene expres-
 727 sion cytometry," *Science*, vol. 347, no. 6222, 2015.

Table 3: Parameters of ActiveSVM (Tabula Muris and MM datasets).

	Tabula Muris (min-complexity)	Tabula Muris (min-cell)	MM (min-complexity)	MM (min-cell)
<i>num_features</i>	150	500	40	40
<i>num_samples</i>	20	200	20	100
<i>init_features</i>	1	1	1	1
<i>init_samples</i>	20	200	20	100
<i>balance</i>	True/False	False	True/False	False
<i>penalty</i>	'l2'	'l2'	'l2'	'l2'
<i>loss</i>	squared_hinge	squared_hinge	squared_hinge	squared_hinge
<i>dual</i>	True	True	True	True
<i>tol</i>	1e-4	1e-4	1e-4	1e-4
<i>C</i>	1.0	1.0	1.0	1.0
<i>fit_intercept</i>	True	True	True	True
<i>intercept_scaling</i>	1	1	1	1
<i>class_weight</i>	None	None	None	None
<i>random_state</i>	None	None	None	None
<i>max_iter</i>	1000	1000	1000	1000

Table 4: Parameters of ActiveSVM (perturb-seq and seqFish datasets).

	perturb-seq (min-complexity)	seqFish (min-complexity)
<i>num_features</i>	50	30
<i>num_samples</i>	500	10
<i>init_features</i>	1	1
<i>init_samples</i>	1000	10
<i>balance</i>	True	False
<i>penalty</i>	'l2'	'l2'
<i>loss</i>	squared_hinge	squared_hinge
<i>dual</i>	True	True
<i>tol</i>	1e-6	1
<i>C</i>	1.0	10
<i>fit_intercept</i>	True	True
<i>intercept_scaling</i>	1	1
<i>class_weight</i>	'balanced'	None
<i>random_state</i>	None	None
<i>max_iter</i>	1,000,000	100,000

728 [3] J. M. Replogle, T. M. Norman, A. Xu, J. A. Hussmann, J. Chen, J. Z. Cogan, E. J. Meer, J. M.
729 Terry, D. P. Riordan, N. Srinivas, *et al.*, “Combinatorial single-cell crispr screens by direct
730 guide rna capture and targeted sequencing,” *Nature biotechnology*, vol. 38, no. 8, pp. 954–
731 961, 2020.

732 [4] J. L. Marshall, B. R. Doughty, V. Subramanian, P. Guckelberger, Q. Wang, L. M. Chen, S. G.
733 Rodrigues, K. Zhang, C. P. Fulco, J. Nasser, *et al.*, “Hypr-seq: Single-cell quantification of
734 chosen rnas via hybridization and sequencing of dna probes,” *Proceedings of the National
735 Academy of Sciences*, vol. 117, no. 52, pp. 33404–33413, 2020.

736 [5] G. M. Sheynkman, K. S. Tuttle, F. Laval, E. Tseng, J. G. Underwood, L. Yu, D. Dong,
737 M. L. Smith, R. Sebra, L. Willems, *et al.*, “Orf capture-seq as a versatile method for targeted
738 identification of full-length isoforms,” *Nature communications*, vol. 11, no. 1, pp. 1–12, 2020.

739 [6] K. A. Riemondy, M. Ransom, C. Alderman, A. E. Gillen, R. Fu, J. Finlay-Schultz, G. D.
740 Kirkpatrick, J. Di Paola, P. Kabos, C. A. Sartorius, *et al.*, “Recovery and analysis of tran-
741 scriptome subsets from pooled single-cell rna-seq libraries,” *Nucleic acids research*, vol. 47,
742 no. 4, pp. e20–e20, 2019.

743 [7] C. Delaney, A. Schnell, L. V. Cammarata, A. Yao-Smith, A. Regev, V. K. Kuchroo, and
744 M. Singer, “Combinatorial prediction of marker panels from single-cell transcriptomic data,”
745 *Mol. Syst. Biol.*, vol. 15, p. e9005, Oct. 2019.

746 [8] F. Wang, S. Liang, T. Kumar, N. Navin, and K. Chen, “Scmarker: ab initio marker selec-
747 tion for single cell transcriptome profiling,” *PLoS computational biology*, vol. 15, no. 10,
748 p. e1007445, 2019.

749 [9] H. Hotelling, “Analysis of a complex of statistical variables into principal components.,”
750 *Journal of educational psychology*, vol. 24, no. 6, p. 417, 1933.

751 [10] D. D. Lee and H. S. Seung, “Learning the parts of objects by non-negative matrix factoriza-
752 tion,” *Nature*, vol. 401, no. 6755, pp. 788–791, 1999.

753 [11] D. Song, K. A. Li, Z. Hemminger, R. Wollman, and J. J. Li, “scpnmf: sparse gene encoding
754 of single cells to facilitate gene selection for targeted gene profiling,” *bioRxiv*, 2021.

755 [12] A. Bhaduri, T. J. Nowakowski, A. A. Pollen, and A. R. Kriegstein, “Identification of cell
756 types in a mouse brain single-cell atlas using low sampling coverage,” *BMC biology*, vol. 16,
757 no. 1, pp. 1–10, 2018.

758 [13] R. M. Felder and R. Brent, “Active learning: An introduction,” *ASQ higher education brief*,
759 vol. 2, no. 4, pp. 1–5, 2009.

760 [14] T. Rückstieß, C. Osendorfer, and P. van der Smagt, “Sequential feature selection for classi-
761 fication,” in *Australasian Joint Conference on Artificial Intelligence*, pp. 132–141, Springer,
762 2011.

763 [15] W. S. Noble, “What is a support vector machine?,” *Nature biotechnology*, vol. 24, no. 12,
764 pp. 1565–1567, 2006.

765 [16] B. Settles, “Active learning literature survey,” 2009.

766 [17] G. X. Zheng, J. M. Terry, P. Belgrader, P. Ryvkin, Z. W. Bent, R. Wilson, S. B. Ziraldo, T. D.
767 Wheeler, G. P. McDermott, J. Zhu, *et al.*, “Massively parallel digital transcriptional profiling
768 of single cells,” *Nature communications*, vol. 8, no. 1, pp. 1–12, 2017.

769 [18] X. Genomics, “1.3 million brain cells from e18 mice,” *CC BY*, vol. 4, 2017.

770 [19] T. M. Consortium *et al.*, “Single-cell transcriptomics of 20 mouse organs creates a tabula
771 muris.,” *Nature*, vol. 562, no. 7727, p. 367, 2018.

772 [20] S. Chen, P. Rivaud, J. H. Park, T. Tsou, E. Charles, J. R. Haliburton, F. Pichiorri, and
773 M. Thomson, “Dissecting heterogeneous cell populations across drug and disease conditions
774 with popalign,” *Proceedings of the National Academy of Sciences*, vol. 117, no. 46,
775 pp. 28784–28794, 2020.

776 [21] A. Dixit, O. Parnas, B. Li, J. Chen, C. P. Fulco, L. Jerby-Arnon, N. D. Marjanovic, D. Dionne,
777 T. Burks, R. Raychowdhury, *et al.*, “Perturb-seq: dissecting molecular circuits with scalable
778 single-cell rna profiling of pooled genetic screens,” *cell*, vol. 167, no. 7, pp. 1853–1866, 2016.

779 [22] C.-H. L. Eng, M. Lawson, Q. Zhu, R. Dries, N. Koulena, Y. Takei, J. Yun, C. Cronin, C. Karp,
780 G.-C. Yuan, *et al.*, “Transcriptome-scale super-resolved imaging in tissues by rna seqfish+,”
781 *Nature*, vol. 568, no. 7751, pp. 235–239, 2019.

782 [23] F. A. Wolf, P. Angerer, and F. J. Theis, “Scanpy: large-scale single-cell gene expression data
783 analysis,” *Genome biology*, vol. 19, no. 1, pp. 1–5, 2018.

784 [24] E. Z. Macosko, A. Basu, R. Satija, J. Nemesh, K. Shekhar, M. Goldman, I. Tirosh, A. R.
785 Bialas, N. Kamitaki, E. M. Martersteck, *et al.*, “Highly parallel genome-wide expression
786 profiling of individual cells using nanoliter droplets,” *Cell*, vol. 161, no. 5, pp. 1202–1214,
787 2015.

788 [25] Y.-W. Chang and C.-J. Lin, “Feature ranking using linear svm,” in *Proceedings of the Work-
789 shop on the Causation and Prediction Challenge at WCCI 2008* (I. Guyon, C. Aliferis,
790 G. Cooper, A. Elisseeff, J.-P. Pellet, P. Spirtes, and A. Statnikov, eds.), vol. 3 of *Proceed-
791 ings of Machine Learning Research*, (Hong Kong), pp. 53–64, PMLR, 03–04 Jun 2008.

792 [26] L. Rosasco, E. De Vito, A. Caponnetto, M. Piana, and A. Verri, “Are loss functions all the
793 same?,” *Neural computation*, vol. 16, no. 5, pp. 1063–1076, 2004.

794 [27] L. Bottou and C.-J. Lin, “Support vector machine solvers,” *Large scale kernel machines*,
795 vol. 3, no. 1, pp. 301–320, 2007.

796 [28] G. Gordon and R. Tibshirani, “Karush-kuhn-tucker conditions,” *Optimization*, vol. 10,
797 no. 725/36, p. 725, 2012.

798 [29] P. Xia, L. Zhang, and F. Li, “Learning similarity with cosine similarity ensemble,” *Informa-
799 tion Sciences*, vol. 307, pp. 39–52, 2015.

800 [30] C. M. Bishop, *Pattern recognition and machine learning*. springer, 2006.

801 [31] A. Abdiansah and R. Wardoyo, “Time complexity analysis of support vector machines (svm)
802 in libsvm,” *International journal computer and application*, vol. 128, no. 3, pp. 28–34, 2015.

803 [32] Scikit-learn, “1.4. support vector machines complexity.”

804 [33] R.-E. Fan, K.-W. Chang, C.-J. Hsieh, X.-R. Wang, and C.-J. Lin, “Liblinear: A library for
805 large linear classification,” *the Journal of machine Learning research*, vol. 9, pp. 1871–1874,
806 2008.

807 [34] K. Street, D. Risso, R. B. Fletcher, D. Das, J. Ngai, N. Yosef, E. Purdom, and S. Dudoit,
808 “Slingshot: cell lineage and pseudotime inference for single-cell transcriptomics,” *BMC ge-
809 nomics*, vol. 19, no. 1, pp. 1–16, 2018.

810 [35] R. Taylor, “Interpretation of the correlation coefficient: a basic review,” *Journal of diagnostic
811 medical sonography*, vol. 6, no. 1, pp. 35–39, 1990.

812 [36] J. R. Vergara and P. A. Estévez, “A review of feature selection methods based on mutual
813 information,” *Neural computing and applications*, vol. 24, no. 1, pp. 175–186, 2014.

814 [37] M. L. McHugh, “The chi-square test of independence,” *Biochemia medica: Biochemia med-
815 ica*, vol. 23, no. 2, pp. 143–149, 2013.

816 [38] S. R. Safavian and D. Landgrebe, “A survey of decision tree classifier methodology,” *IEEE
817 transactions on systems, man, and cybernetics*, vol. 21, no. 3, pp. 660–674, 1991.

818 [39] I. Syarif, A. Prugel-Bennett, and G. Wills, “Svm parameter optimization using grid search
819 and genetic algorithm to improve classification performance,” *Telkomnika*, vol. 14, no. 4,
820 p. 1502, 2016.

821 [40] S. Arlot and A. Celisse, “A survey of cross-validation procedures for model selection,” *Statis-
822 tics surveys*, vol. 4, pp. 40–79, 2010.

823 [41] L. D. Brown, T. T. Cai, and A. DasGupta, “Interval estimation for a binomial proportion,”
824 *Statistical science*, vol. 16, no. 2, pp. 101–133, 2001.

825 [42] Amazon, “Instance types.”

826 [43] Amazon, “Amazon elastic compute cloud documentation.”

827 [44] Amazon, “Aws innovate.”

828 [45] L. Torvalds, “Linux kernel.”

829 [46] V. D. Blondel, J.-L. Guillaume, R. Lambiotte, and E. Lefebvre, “Fast unfolding of communities in large networks,” *Journal of statistical mechanics: theory and experiment*, vol. 2008, no. 10, p. P10008, 2008.

832 [47] P. Osei-Owusu, T. M. Charlton, H. K. Kim, D. Missiakas, and O. Schneewind, “FPR1 is the plague receptor on host immune cells,” *Nature*, vol. 574, pp. 57–62, Oct. 2019.

834 [48] A. P. Sage, K. W. Ng, E. A. Marshall, G. L. Stewart, B. C. Minatel, K. S. S. Enfield, S. D. Martin, C. J. Brown, N. Abraham, and W. L. Lam, “Assessment of long non-coding RNA expression reveals novel mediators of the lung tumour immune response,” *Sci. Rep.*, vol. 10, p. 16945, Oct. 2020.

838 [49] A. Likas, N. Vlassis, and J. J. Verbeek, “The global k-means clustering algorithm,” *Pattern recognition*, vol. 36, no. 2, pp. 451–461, 2003.

840 [50] A. A. Pollen, T. J. Nowakowski, J. Chen, H. Retallack, C. Sandoval-Espinosa, C. R. Nicholas, J. Shuga, S. J. Liu, M. C. Oldham, A. Diaz, *et al.*, “Molecular identity of human outer radial glia during cortical development,” *Cell*, vol. 163, no. 1, pp. 55–67, 2015.

843 [51] T. E. Anthony, H. A. Mason, T. Gridley, G. Fishell, and N. Heintz, “Brain lipid-binding protein is a direct target of notch signaling in radial glial cells,” *Genes & development*, vol. 19, no. 9, pp. 1028–1033, 2005.

846 [52] M. G. Andrews, L. Subramanian, and A. R. Kriegstein, “mTOR signaling regulates the morphology and migration of outer radial glia in developing human cortex,” *Elife*, vol. 9, p. e58737, 2020.

849 [53] W. Guan, Z. Gao, C. Huang, M. Fang, H. Feng, S. Chen, M. Wang, J. Zhou, S. Hong, and C. Gao, “The diagnostic value of serum DSA-TRF in hepatocellular carcinoma,” *Glycoconj. J.*, vol. 37, pp. 231–240, Apr. 2020.

852 [54] T. Magnaldo, D. Fowlis, and M. Darmon, “Galectin-7, a marker of all types of stratified epithelia,” *Differentiation*, vol. 63, pp. 159–168, July 1998.

854 [55] M. J. Stables, S. Shah, E. B. Camon, R. C. Lovering, J. Newson, J. Bystrom, S. Farrow, and D. W. Gilroy, “Transcriptomic analyses of murine resolution-phase macrophages,” *Blood*, vol. 118, pp. e192–208, Dec. 2011.

857 [56] Y. Wu and C. M. Smas, “Wdnm1-like, a new adipokine with a role in MMP-2 activation,” *Am. J. Physiol. Endocrinol. Metab.*, vol. 295, pp. E205–15, July 2008.

859 [57] A. C. Rawstron, F. E. Davies, R. G. Owen, A. English, G. Pratt, J. A. Child, A. S. Jack, and G. J. Morgan, “B-lymphocyte suppression in multiple myeloma is a reversible phenomenon specific to normal b-cell progenitors and plasma cell precursors,” *British journal of haematology*, vol. 100, no. 1, pp. 176–183, 1998.

863 [58] R. J. P. de Magalhães, M.-B. Vidriales, B. Paiva, C. Fernandez-Gimenez, R. García-Sanz,
864 M.-V. Mateos, N. C. Gutierrez, Q. Lecrevisse, J. F. Blanco, J. Hernández, *et al.*, “Analysis
865 of the immune system of multiple myeloma patients achieving long-term disease control by
866 multidimensional flow cytometry,” *Haematologica*, vol. 98, no. 1, p. 79, 2013.

867 [59] E. Malek, M. de Lima, J. J. Letterio, B.-G. Kim, J. H. Finke, J. J. Driscoll, and S. A. Giralt,
868 “Myeloid-derived suppressor cells: The green light for myeloma immune escape,” *Blood
869 reviews*, vol. 30, no. 5, pp. 341–348, 2016.

870 [60] H. Suen, R. Brown, S. Yang, C. Weatherburn, P. J. Ho, N. Woodland, N. Nassif, P. Bar-
871 baro, C. Bryant, D. Hart, *et al.*, “Multiple myeloma causes clonal t-cell immunosenescence:
872 identification of potential novel targets for promoting tumour immunity and implications for
873 checkpoint blockade,” *Leukemia*, vol. 30, no. 8, pp. 1716–1724, 2016.

874 [61] F. Ge, L. Zhang, S.-C. Tao, K. Kitazato, Z.-P. Zhang, X.-E. Zhang, and L.-J. Bi, “Quantita-
875 tive proteomic analysis of tumor reversion in multiple myeloma cells,” *Journal of proteome
876 research*, vol. 10, no. 2, pp. 845–855, 2011.

877 [62] T. Xiao, W. Zhu, W. Huang, S.-S. Lu, X.-H. Li, Z.-Q. Xiao, and H. Yi, “Rack1 promotes
878 tumorigenicity of colon cancer by inducing cell autophagy,” *Cell death & disease*, vol. 9,
879 no. 12, pp. 1–13, 2018.

880 [63] C. Xia, Z. Braunstein, A. C. Toomey, J. Zhong, and X. Rao, “S100 proteins as an important
881 regulator of macrophage inflammation,” *Frontiers in immunology*, vol. 8, p. 1908, 2018.

882 [64] M. Liu, Y. Wang, J. J. Miettinen, R. Kumari, M. M. Majumder, C. Tierney, D. Bazou, A. Par-
883 sons, M. Suvela, J. Lievonen, *et al.*, “S100 calcium binding protein family members asso-
884 ciate with poor patient outcome and response to proteasome inhibition in multiple myeloma,”
885 *Frontiers in Cell and Developmental Biology*, p. 2261, 2021.

886 [65] T. Dobreva, D. Brown, J. H. Park, and M. Thomson, “Single cell profiling of capillary blood
887 enables out of clinic human immunity studies,” *Scientific reports*, vol. 10, no. 1, pp. 1–9,
888 2020.

889 [66] A. D. Rouillard, G. W. Gundersen, N. F. Fernandez, Z. Wang, C. D. Monteiro, M. G. Mc-
890 Dermott, and A. Ma’ayan, “The harmonizome: a collection of processed datasets gathered to
891 serve and mine knowledge about genes and proteins,” *Database*, vol. 2016, 2016.

892 [67] D. Pei and C. Shih, “Transcriptional activation and repression by cellular dna-binding protein
893 c/ebp,” *Journal of virology*, vol. 64, no. 4, pp. 1517–1522, 1990.

894 [68] I. E. Vega, “Efhd2, a protein linked to alzheimer’s disease and other neurological disorders,”
895 *Frontiers in neuroscience*, vol. 10, p. 150, 2016.

896 [69] E. Borger, A. Herrmann, D. A. Mann, T. Spires-Jones, and F. Gunn-Moore, “The calcium-
897 binding protein efhd2 modulates synapse formation in vitro and is linked to human dementia,”
898 *Journal of Neuropathology & Experimental Neurology*, vol. 73, no. 12, pp. 1166–1182, 2014.

899 [70] B. Settles, “From theories to queries: Active learning in practice,” in *Active learning and ex-*
900 *perimental design workshop in conjunction with AISTATS 2010*, pp. 1–18, JMLR Workshop
901 and Conference Proceedings, 2011.

902 [71] R. E. Schapire, “The boosting approach to machine learning: An overview,” *Nonlinear esti-*
903 *mation and classification*, pp. 149–171, 2003.

904 [72] R. E. Schapire, “A brief introduction to boosting,” in *Ijcai*, vol. 99, pp. 1401–1406, Citeseer,
905 1999.

906 [73] B. Cleary, L. Cong, A. Cheung, E. S. Lander, and A. Regev, “Efficient generation of transcrip-
907 *tomic profiles by random composite measurements,” *Cell*, vol. 171, no. 6, pp. 1424–1436,*
908 2017.

909 [74] J. Jiang, D. A. Sivak, and M. Thomson, “Active learning of spin network models,” *arXiv*
910 *preprint arXiv:1903.10474*, 2019.

911 [75] A. Collette, “Hdf5 for python.”

912 [76] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel,
913 P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher,
914 M. Perrot, and E. Duchesnay, “Scikit-learn: Machine learning in Python,” *Journal of Machine*
915 *Learning Research*, vol. 12, pp. 2825–2830, 2011.

916 [77] W. Pomp, “Python package - parfor.”