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RESEARCH ARTICLE

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Impact of number of shipping lines on ports' charges and profits: A game-theoretic model

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Abstract

This study examines the effect of changes in the number of shipping lines on port charges and profits. A two-stage noncooperative game-theoretic model with two ports and multiple identical shipping lines is developed. In the first stage, the ports decide their container handling charges, and in the second stage, the shipping lines decide on the number of calls to make at each port, given the port charges. The model is then applied to the case of competition between two ports in New Zealand, the Port of Tauranga and the Port of Auckland, to derive managerial insights. Our study extends the literature on port competition by incorporating competition between shipping lines. We also demonstrate that a decrease in the number of shipping lines may force ports to increase their handling charges. Furthermore, we show that each shipping line ships more cargo than the industry optimal via the port with lower costs.

KEYWORDS

container shipping, Cournot competition, game theory, port competition, transport chain

1 | INTRODUCTION

Container shipping is critical to international trade and global economic development because container ports are the gateways for a country's foreign trade and a key interface between sea transport and land transport. Shipping lines are strategically important customers of container ports. Competition among neighboring ports has intensified in recent years as shipping companies can easily change their ports of call (Cullinane & Song, 2006; Song et al., 2016). A shipping line's decision to switch from one port to another negatively impacts the former port but positively impacts the latter. The chosen port benefits from the additional cargo, which in turn benefits its hinterland economy. Conversely, the other port suffers reduced connectivity that drastically reduces its competitiveness and container throughput (Notteboom & Yap, 2012). The relationship between the Port of Tanjung Pelepas (Malaysia; PTP) and the Port of Singapore is the best example that illustrates the dependence of port operators on shipping lines (Bae et al., 2013). In 2000, the international container shipping company Maersk Line shifted its transshipment operations from the Port of Singapore to PTP, resulting in a dramatic drop of approximately 9.2% in the container throughput of the Port of Singapore in 2001, which was equivalent to approximately 1.57 million twenty-foot equivalent units (TEUs) (Tongzon, 2009). At the same time, PTP's throughput increased to approximately 1.63 million TEUs, nearly five times its earlier volume. Maersk Line's relocation sparked grave concern about the potential ripple effects of one shipping line's decisions on the related business decisions of other shipping lines (Kleywegt et al., 2002). Indeed, Maersk Line's decision to change its transshipment port of call led to similar decisions among other international carriers. In 2002, Evergreen Marine, a Taiwanese container shipping company, switched most of its container operations from the Port of Singapore to PTP, which amounted to 1–1.2 million TEUs of annual container throughput.

From the perspective of the shipping lines, competition is much more intense than before, and shippers have become more demanding and expect more frequent and reliable schedules and quicker service at lower rates (Notteboom, 2006). Therefore, the number of mergers and acquisitions among shipping lines has increased to respond to this intense competition. The consolidation wave was started by AP Moller–Maersk with the purchase of P&O Nedlloyd in August 2005 and Hamburg Süd in 2017 (Meersman et al., 2018). In 2016, COSCO Shipping, a former Chinese state-owned shipping company, merged with China Shipping, also a former Chinese state-owned shipping conglomerate, to form a new company—China COSCO Shipping—which later acquired OOCL, a Hong-Kong based container shipping company, in 2018. In 2016, CMA CGM, a French container shipping company, acquired NOL, Southeast Asia's largest container shipping company. In 2017, the three largest Japanese container shipping companies, NYK, MOL, and K Line, merged to form one company—Ocean Network Express. These mergers and acquisitions resulted in the top 20 shipping companies controlling 89% of the global market share in 2020, compared with 54% in 1999, 41.6% in 1992, and 26% in 1980 (Alphaliner, 2020; Notteboom, 2008).

The current trend of mergers and acquisitions among international shipping lines reduces the number of shipping lines and makes port operators more dependent on shipping companies. Therefore, the following question arises: What is the effect of a change in the number of shipping lines on port charges and profits? Moreover, how does asymmetry in the sizes of shipping lines affect port charges and profits?

To address the above questions, we propose a game-theoretic model to study the behavior of two competing ports and multiple identical shipping lines. This study extends the work of Song et al. (2016) by incorporating competition between shipping lines. Specifically, we develop a noncooperative model that has the following characteristics: (i) there is Cournot competition among N identical shipping lines; (ii) two container ports compete for the same market; and (iii) in the first stage, each port decides its container handling charge and in the second stage, given the port charges, each shipping line makes its port of call decision (i.e., the number of calls at each port) to maximize its profit. Other shipping lines' port of call decisions are also considered as these decisions affect port congestion. The case of the PTP (Malaysia) and the Port of Singapore shows the strong dependence of port operators on shipping lines. New Zealand ports face a similar situation, as illustrated by the recent shifting of Maersk Line's Southern Star service from the Port of Auckland (POA; formally known as Ports of Auckland as it operates two separate facilities in Auckland; this study refers to it as POA for the sake of simplicity) to the Port of Tauranga (POT). The switch between the two largest container ports in New Zealand resulted in an annual revenue loss of nearly NZD 20 million for POA (Skeller, 2011). Therefore, we apply our model to the case of inter-port competition between POT and POA to derive managerial insights.

This study makes two main contributions. First, we build a two-stage non-cooperative Cournot competition game and derive its unique equilibrium. Our model provides a framework for analyzing Cournot competition among shipping lines. It illustrates that under a two-stage noncooperative Cournot game model, shipping lines only consider the negative effect of switching from one port to another on their own output and not the effect on total output. Hence, each shipping line tends to ship more cargo than the industry optimal via the port with lower costs. Second, the case study of port competition between POT and POA reveals an unexpecting and exciting result: the decrease in the number of competing shipping lines forces the competing ports to increase their handling charges. We also provide a theoretical explanation for this interesting result.

The remainder of this paper is organized as follows. Section 2 presents the literature review. Section 3 describes the noncooperative game-theoretic model of the study, while Section 4 presents the case study and discussion of the model. Section 5 concludes the research and outlines the directions for future research.

2 | LITERATURE REVIEW

Game theory is a well-known mathematical framework used to study the interactions between intelligent, rational decision-makers to achieve optimal payoffs (Myerson, 1999). Game theory is a powerful tool in port research (Ishii et al., 2013). Bobrovitch (1982) is the first to apply game theory to port competition by developing a Cournot model to capture competition between two ports. Later, Zan (1999) analyzes a Stackelberg game to examine the interactions among shippers, ocean carriers, and container port management policy.

In the research on port pricing decisions, Van Reeven (2010) uses Hotelling's model to analyze port competition. The study focuses on the vertical integration decision and incorporates competition between service providers. Another study on port pricing is that of Zhang et al. (2010), which applies a Bertrand game-theoretic model to analyze price competition between the Port of Hong Kong and the Port of Shenzhen. The authors propose three strategies to reduce the "price war" between the two ports: service differentiation, coopetition, and cost-leadership. Basso et al. (2017) further explore port pricing strategy in the presence of multiple shippers, but their model incorporates a single port and a monopolistic shipping company.

In their research on the pricing and investment problems of container ports, De Borger et al. (2008) develop a two-stage game theoretic model to study the interactions between optimal investment policies, pricing decisions, and the hinterland capacity of competing ports. In a later paper, Ishii et al. (2013) expands the model further by developing a non-cooperative game-theoretic model with stochastic demand to analyze the pricing decisions of two competing ports under different timings for port capacity expansion. The model is then applied to the case of competition between the Port of Kobe, Japan and the Port of Busan, South Korea. The authors suggest that ports should set lower port charges when demand elasticity is high and that a longer time interval between capacity investments increases the Nash equilibrium port rates.

Saeed and Larsen (2010) build a two-stage game model for analyzing the effects of cooperation among competing ports and explore possible coalitions among three container terminals at the Port of Karachi in Pakistan. In the first stage, the three terminals decide whether to join a coalition or operate independently. The second stage is the Bertrand model, where the resulting coalitions compete with nonmembers. The study concludes that the "grand coalition" is the only coalition that is stable for all players. However, the real winners in the game are the terminals at other ports that earn a higher payoff without participating in the coalition and end up playing the role of the "orthogonal free-rider."

While previous studies on port competition focus on ports' decisions, Bae et al. (2013) consider the decisions of both the ports and the shipping lines and show that shipping lines tend to make more calls at ports with larger capacities and cheaper rates and the effect of the price difference between two competing ports is greater when there is less port congestion.

Song et al. (2016) initially model ocean carriers' port of call and port pricing decisions in the context of a transportation chain by considering the ocean freight cost, feeder service cost, port charges, and hinterland transport cost simultaneously. Their game-theoretic model provides many insights into the competition between the Port of Southampton and the Port of Liverpool in the United Kingdom. The model only considers a single shipping line and does not address the competitive dynamics between shipping lines and the effect of a change in the number of shipping lines on port charges and profits. Therefore, the authors call for future research to expand their model to include multiple identical shipping lines.

This study responds to the call of Song et al. (2016) to extend their work by including Cournot competition among multiple shipping lines. The next section describes the model in detail.

3 | THE NONCOOPERATIVE GAME-THEORETIC MODEL

Before presenting the model, we first list the notations used in the article below.

Indices

i the index of a shipping line; $i = 1, 2, \ldots, N$,

j the index of a port; $j = 1, 2$.

Parameters

a_j a positive coefficient that represents the congestion cost in dollars incurred by shipping lines when utilization at Port j reaches its effective capacity,

c_j the average cost per container ($/TEU) for a shipping line when it calls at Port j; c_j does not include the congestion cost at a port,

k_j the unit operating cost at Port j ($/TEU),

K_j the effective handling capacity at Port j (TEUs/year),

L_j the lower bound on the port unit container handling price,

m_j the unit handling capacity investment at Port j ($/TEU),

N the number of shipping lines,

p the unit shipment price of a container, which is the price charged by shipping lines to shippers ($/TEU) (it is assumed that the return shipment price is included in p and all shipping lines charge shippers the same price, p),

U_j the upper bound on the port unit container handling price,

V the annual total cargo volume shipped by all shipping lines via both ports (TEUs/year),

v^b the annual cargo volume shipped by each shipping line via both ports (TEUs/year); $V = N \cdot v^b$.

Decision variables

C_j the unit port congestion cost incurred by Shipping Line i at Port j ($/TEU),

F_{ij} the annual number of container lifts carried out at Port j ($j = 1, 2$) for Shipping Line i (TEUs/year),

F_j the annual total number of container lifts carried out at Port j ($j = 1, 2$) for all shipping lines (TEUs/year),

q_{ij} the fraction of ports of call made by Shipping Line i at Port j,

r the average number of port operations per container,

v_{ij} the annual cargo volume transported by Shipping Line i via Port j (TEUs/year),

v_j the annual cargo volume transported by all shipping lines via Port j (TEUs/year),

w_j the container handling charge ($/TEU) at Port j.

Functions

π_i^l Shipping Line i's profit function ($/year),

π_j Port j's profit function ($/year).

This study considers a two-stage noncooperative game-theoretic model consisting of N identical shipping lines ($i = 1, 2, \ldots, N$) and two container ports ($j = 1, 2$), where the two ports compete for the same market. In the first stage, each port determines its container handling charge, w_j ($/TEU), that maximizes its profit. In the second stage, each shipping line decides the value of q_{ij}, which is the fraction of the ports of call made by the vessels of Shipping Line i at

Port j, such that $0 \leq q_{ij} \leq 1$ and $q_{i2} = 1 - q_{i1}$, to maximize its profit by observing the port charges, congestion costs, mother vessel's fuel cost, and feeder vessel's fuel cost. We assume that all transportation is operated by the shipping lines (Song et al., 2016).

3.1 | General setup of the model

We define V as the annual total cargo volume transported by all N shipping lines via both ports (TEUs/year); V is assumed to be fixed. This assumption is in line with that of past studies (e.g., Bae et al., 2013; Song et al., 2016) because often there is no feasible alternative to maritime transportation. For example, even if both ports were to increase their handling charges, it is unlikely that the total volume of cargo transported will change drastically. We define v^b as the annual cargo volume transported by each shipping line via both ports (TEUs/year) and $V = N \cdot v^b$. We assume v^b to be equal, and its split between the two ports is proportional to the ports' decision on the splitting of ports of call by vessels, denoted by q_{ij}. This assumption is adopted from Bae et al. (2013). The values of q_{ij} can be the same as or different from each other depending on how the shipping lines interact with one another to optimize their profits, as illustrated in Figure 1. We assume that all shipping lines make their port-of-call decisions simultaneously (Cournot competition for N shipping lines).

Let v_{ij} be the annual cargo volume transported by Shipping Line i via Port j(TEUs/year), which is the decision related to q_{ij}:

$$v_{i1} = v^b \cdot q_{i1}, \tag{1}$$

$$v_{i2} = v^b \cdot q_{i2} = v^b \cdot (1 - q_{i1}). \tag{2}$$

We define v_j as the hinterland volume transported by all shipping lines via Port j per year (TEUs/year). Then the total volume transported by all shipping lines via both ports per year, V, can also be written as follows:

$$V = \sum_{j=1}^{2} v_j = v_1 + v_2 = \sum_{i=1}^{N} v_{i1} + \sum_{i=1}^{N} v_{i2}. \tag{3}$$

From Equations (1)–(3), we have:

$$v_1 = \sum_{i=1}^{N} v_{i1} = \sum_{i=1}^{N} v^b \cdot q_{i1}, \tag{4}$$

$$v_2 = \sum_{i=1}^{N} v_{i2} = \sum_{i=1}^{N} v^b \cdot q_{i2} = \sum_{i=1}^{N} v^b \cdot (1 - q_{i1}),$$

$$\text{or } v_2 = V - v_1 = V - \sum_{i=1}^{N} v^b \cdot q_{i1}. \tag{5}$$

3.2 | Congestion cost function

For ease of exposition, we focus on the head-haul direction of containerized cargo trade. For example, for an import-oriented country, the head-haul direction is the direction of the flow of import cargo, and the ports and vessels handle the same volume of container flow (export containers or empty containers) in the opposite direction. This approach is adopted from Song et al. (2016).

Specifically, an importing transshipment container is first unloaded from the mother vessel to the port and then loaded onto a feeder vessel from the port. After the feeder vessel reaches the destination port, the container is discharged and unpacked and becomes empty. This empty container may be reloaded with new goods for export or returned as an empty container; the returned container is then unloaded from the feeder vessel to the port and loaded onto a mother vessel from the port. Therefore, one transshipment container translates into a total of four port operations when considering its contribution to port congestion.

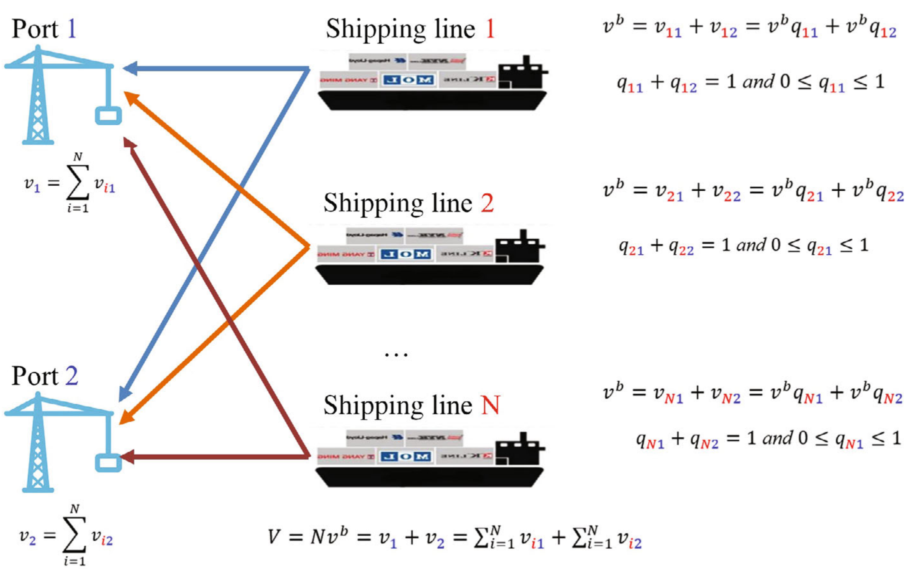

FIGURE 1 General setup of the model (The subscript numbers, letters in red, refer to the index of shipping lines. The subscript numbers, letters in blue, refer to the index of ports)

Likewise, an importing hinterland container is first unloaded from the vessel onto a truck or train. It is then discharged and unpacked at the destination. The empty container may be reloaded with new goods for export or returned as an empty container; the returned container is then unloaded from the truck or train onto the vessel. Therefore, one hinterland container translates into a total of two port operations when considering its contribution to port congestion. Let r denote the average number of port operations per container, where its value depends on the ratio of transshipment containers to hinterland containers.

Next, we define F_{ij} as the number of container lifts per year conducted at Port j ($j = 1, 2$) for Shipping Line i (TEUs/year). Let r be the average number of port operations per container. Hence, F_{ij} can be written as follows:

$$F_{i1} = r\, v_{i1} = rv^b \cdot q_{i1}, \tag{6}$$

$$F_{i2} = r\, v_{i2} = rv^b \cdot q_{i2} = rv^b \cdot (1 - q_{i1}). \tag{7}$$

Furthermore, let F_j be the total annual number of port operations of containers carried out at Port j ($j = 1, 2$) for all shipping lines (TEUs/year). Then, we have the following

$$F_1 = r\, v_1 = r \cdot \sum_{i=1}^{N} v^b \cdot q_{i1}, \tag{8}$$

$$F_2 = r\, v_2 = r\,(V - v_1) = r\left(V - \sum_{i=1}^{N} v^b \cdot q_{i1}\right). \tag{9}$$

Let C_j be the unit port congestion cost incurred by a shipping line at Port j (\$/TEU), which can be written as follows (De Borger & Van Dender, 2006):

$$C_j = a_j \cdot \left(\frac{F_j}{K_j}\right), \quad \text{for } j = 1, 2, \tag{10}$$

where a_j is a positive coefficient that represents the congestion cost in dollars when utilization at Port j reaches its effective capacity, and K_j is the effective handling capacity at Port j (TEUs/year). Generally, we have $F_j \le K_j$. Therefore, $\frac{F_j}{K_j}$ can be interpreted as Port j's utilization.

3.3 | Profit function of the shipping lines

The main purpose of this study is to determine the equilibrium value of q_{ij} for each shipping line, given the value of w_j set by the two ports in the first stage.

We need the following additional parameters to analyze the shipping lines' profit functions: c_j is the average cost per container (\$/TEU), which may comprise the feeder vessel and hinterland transportation costs; p is the unit shipment price, which is the price charged by shipping lines to shippers (\$/TEU). Similar to Song et al. (2016), we assume that the return shipment price is included in p and all shipping lines charge shippers the same price, p. The shipment price is fixed in our model as it is largely based on the shipping lines' available capacity and the total demand for shipping services, which are exogenous to our model. Hence, the profit function of Shipping Line i is given as follows:

$$\pi_i^l = \sum_{j=1}^{2} \left(\left(p - c_j - w_j\right) v_{ij} - C_j F_{ij} \right), \tag{11}$$

s.t.

$$0 \le q_{ij} \le 1, \tag{12}$$

$$q_{i2} = 1 - q_{i1}, \tag{13}$$

$$F_{ij} = r\, v_{ij} = rv^b q_{ij}. \tag{14}$$

The first term on the right-hand side of Equation (11) represents the annual total net revenue earned by Shipping Line i by transporting v_{ij} volume via Port j. The second term is the annual total port congestion cost paid by Shipping Line i for transporting v_{ij} volume via Port j.

By expanding the last term on the right-hand side of Equation (11) at Port j after substituting Equations (6), (7), and (10) into the two terms, they become:

$$-\frac{a_j r \left(\sum_{k=1}^{N} v^b q_{kj}\right) rv^b q_{i1}}{K_j}.$$

The total congestion cost at Port j depends on both the fraction of vessels' ports of call, q_{ij}, of Shipping Line i and the decisions made by all shipping lines, $\sum_{x=1}^{N} q_{xj}$. This shows that the decisions of different shipping lines are interrelated.

3.4 | Profit functions of the ports

To formulate the profit functions of the ports, we define k_j as the unit operating cost at Port j (\$/TEU), m_j as the unit handling capacity investment at Port j (\$/TEU), and L_j and U_j as the lower and upper bounds, respectively, on the port unit container handling price (Song et al., 2016). The profit functions ($j = 1, 2$) of the ports are adopted from Song et al. (2016) as follows:

$$\pi_j = \left(w_j - k_j\right) \cdot F_j - m_j \cdot K_j, \tag{15}$$

where $L_j \le w_j \le U_j$.

The first term on the right-hand side of Equation (15) represents the annual profit of Port j from handling F_j number of containers. The second term represents the annual total investment by Port j for capacity expansion to keep up with the growth in cargo throughput in the future. We examine the ports' capacity expansion decision in Section 5.3.

Summarizing the model, in the first stage, the two ports decide on their container handling prices to optimize Equation (15). In the second stage, N shipping lines decide on how to split the vessels' ports of call to optimize Equation (11). We use the backward induction method (Bae et al., 2013) to solve this game-theoretic model. We first start with the second stage to find the subgame Nash equilibrium fraction of ports of call, q_{ij}^*, of each shipping line. The equilibrium port-of-call decision, q_{ij}^*, is a function of the port prices, w_j. After that, the Nash equilibrium port container handling prices, w_j^*, can be obtained using equilibrium q_{ij}^* in the second stage. Last, we substitute w_j^* back into the equilibrium port of call, q_{ij}^*, to eliminate w_j from the result.

3.5 | Equilibrium outcome

Next, we solve the game starting with q_{ij}^*, as outlined above. The following lemma ensures the uniqueness of the equilibrium:

Lemma 1 *For the given port prices, w_1 and w_2, the profit function of Shipping Line i, π_i^l, is concave with respect to q_{i1} in the interval* [0, 1].

Lemma 1 indicates that for the given port prices, w_1 and w_2, $\frac{\partial \pi_i^l}{\partial q_{i1}}$ is a monotonic decreasing function in q_{i1} in the interval [0, 1]. Hence, there is a unique optimal solution of q_{i1}^* in the interval [0, 1]. Specifically, $q_{i1}^* = 0$ if $\frac{\partial \pi_i^l}{\partial q_{i1}} \leq 0$ for any $q_{i1} \in [0,1]$; $q_{i1}^* = 1$ if $\frac{\partial \pi_i^l}{\partial q_{i1}} \geq 0$ for any $q_{i1} \in [0,1]$; and $q_{i1}^* \in (0,1)$ otherwise.

Lemma 2 *For the given port prices, w_1 and w_2, each shipping line's Nash equilibrium fraction of ports of call, q_{ij}^*, is the same ($q_{1j}^* = q_{2j}^* = \ldots = q_{Nj}^*$) and given by*:

$$q_{i1}^* = \begin{cases} 0 & \text{if } D_1 < 0 \\ D_1 & \text{if } 0 \leq D_1 \leq 1 \\ 1 & \text{if } D_1 > 1 \end{cases},$$

where $D_1 = \frac{\left((N+1)a_2 r^2 v^b - \left(c_1 + w_1 - c_2 - w_2\right)K_2\right)K_1}{(N+1)r^2 v^b\left(a_1 K_2 + a_2 K_1\right)}$.

Using the Nash equilibrium port-of-call decision, q_{ij}^*, in Lemma 2, the Nash equilibrium port unit container handling prices, w_j^*, can then be derived in the first stage. Next, w_j^* is substituted back into the equilibrium port of call, q_{ij}^*, to eliminate $w_j$from the result. The main results of the game model are summarized in the following proposition.

Proposition 1 *The Nash equilibrium fraction of ports of call, q_{ij}^* ($q_{1j}^* = q_{2j}^* = \ldots = q_{Nj}^*$) and the Nash equilibrium port unit container handling prices, w_j^*, are given by (with $q_{i2}^* = 1 - q_{i1}^*$)* :

(i) *If $D_1 < 0$ then $q_{i1}^* = 0$; w_1^* and w_2^* are given by* $\left(w_1^*, w_2^*\right) = max\left\{ (w_1 w_2) | L_1 \leq w_1 \leq U_1, L_2 \leq w_2 \leq U_2, w_2 - w_1 < c_1 - c_2 - \frac{(N+1)a_2 r^2 v^b}{K_2} \right\}$.

(ii) *If $D_1 > 1$ then $q_{i1}^* = 1$; w_1^* and w_2^* are given by* $\left(w_1^*, w_2^*\right) = max\left\{ (w_1 w_2) | L_1 \leq w_1 \leq U_1, L_2 \leq w_2 \leq U_2, w_2 - w_1 > c_1 - c_2 + \frac{(N+1)a_2 r^2 v^b}{K_1} \right\}$.

(iii) *If $0 \leq D_1 \leq 1, L_1 \leq w_1 \leq U_1, L_2 \leq w_2 \leq U_2$, then q_{i1}^*, w_1^* and w_2^* are given by*

$$w_1^* = \frac{\left((c_2 - c_1 + 2k_1 + k_2) K_2 + 2(N+1)a_2 r^2 v^b\right) K_1 + a_1 r^2 v^b (N+1) K_2}{3K_1 K_2}, \tag{16}$$

$$w_2^* = \frac{\left((c_1 - c_2 + k_1 + 2k_2) K_2 + (N+1)a_2 r^2 v^b\right) K_1 + 2a_1 r^2 v^b (N+1) K_2}{3K_1 K_2}, \tag{17}$$

$$q_{i1}^* = \frac{\left((c_2 - c_1 + k_2 - k_1) K_2 + 2(N+1)a_2 r^2 v^b\right) K_1 + a_1 r^2 v^b (N+1) K_2}{3r^2 v^b (N+1)\left(a_1 K_2 + a_2 K_1\right)}. \tag{18}$$

Points (i) and (ii) of Proposition 1 represent the case where one of the two ports gains all the market share and the other gets nothing, which is unusual. If the shipping lines move all of their cargo via one port only, the hinterland transport congestion cost and port congestion cost increase significantly due to the capacity constraint, which in turn makes the decision uneconomical. Consequently, shipping lines must balance the vessels' ports of call between the two ports.

Point (iii) of Proposition 1 represents a more realistic scenario and provides the Nash equilibrium fraction of ports of call, q_{ij}^*, and the Nash equilibrium port unit container handling prices, w_j^*, to the non-cooperative game model. The three conditions in Proposition (iii), $0 \leq D_1 \leq 1$, $L_1 \leq w_1 \leq U_1, L_2 \leq w_2 \leq U_2$, can also be rewritten to explicitly show the relationships between the model parameters as follows:

$$\frac{\left((c_1 - c_2) K_1 + (3a_2 - a_1)(N+1) r^2 v^b\right) K_2 - a_2 r^2 v^b (N+1) K_1}{K_1 K_2} \leq k_2 - k_1 \leq \frac{\left((c_1 - c_2) K_2 + 2(N+1)a_2 r^2 v^b\right) K_1 - a_1 r^2 v^b (N+1) K_2}{K_1 K_2}, \tag{19}$$

$$L_1 \leq \frac{\left((c_2 - c_1 + 2k_1 + k_2) K_2 + 2(N+1)a_2 r^2 v^b\right) K_1 + a_1 r^2 v^b (N+1) K_2}{3K_1 K_2} \leq U_1, \tag{20}$$

$$L_2 \leq \frac{\left((c_1 - c_2 + k_1 + 2k_2) K_2 + (N+1)a_2 r^2 v^b\right) K_1 + 2a_1 r^2 v^b (N+1) K_2}{3K_1 K_2} \leq U_2. \tag{21}$$

Equations (19)–(21) can be obtained by substituting w_j with w_j^* in the three conditions, $0 \leq D_1 \leq 1, L_1 \leq w_1 \leq U_1, L_2 \leq w_2 \leq U_2$, and then simplifying them. These conditions define the existence of the Nash equilibrium such that each port receives its share of cargo. Condition (19) ensures that the operating costs of the two ports are not excessively different from each other. Conditions (20)–(21) state that the price bounds must not be too restrictive. Note that all of the analyses discussed hereafter refer to this interior solution (under conditions (19)–(21)).

Proposition 2 *Given the Nash equilibrium and conditions (19)–(21), the profit functions for the two ports and Shipping Line i are given by*:

$$\pi_1^* = \left(w_1^* - k_1\right) rv^b N q_{i1}^* - m_1 K_1,$$

$$\pi_2^* = \left(w_2^* - k_2\right) rv^b N \left(1 - q_{i1}^*\right) - m_2 K_2,$$

$$\pi_i^{l*} = \sum_{j=1}^{2} \left(\left(p - c_j - w_j^*\right) v_{ij} - \frac{a_j N \left(rv^b q_{ij}^*\right)^2}{K_j} \right), i = 1, \ldots, N,$$

where w_1^*, w_2^*, and q_{i1}^* are given in Equations (16), (17), and (18), respectively.

Next, we perform the sensitivity analysis of the equilibrium port handling charges and port-of-call decisions by the shipping lines. In particular, we are interested in determining the effect of the number of shipping lines on these decision variables.

Proposition 3 *Under conditions (19)–(21), the effect of the number of shipping lines, N, on the decision variables is given as follows*:

$$\frac{\partial w_1^*}{\partial N} < 0 \text{ and } \lim_{N\to\infty} \frac{\partial w_1^*}{\partial N} = 0;$$

$$\frac{\partial w_2^*}{\partial N} < 0 \text{ and } \lim_{N\to\infty} \frac{\partial w_2^*}{\partial N} = 0;$$

$$\lim_{N\to\infty} \frac{\partial q_{i1}^*}{\partial N} = 0.$$

Proposition 3 indicates that:

(i) Port 1's Nash equilibrium handling price, w_1^*, decreases with a decrease in N and converges to a finite number as N increases.
(ii) Port 2's Nash equilibrium handling price, w_2^*, decreases with a decrease in N and converges to a finite number as N increases.
(iii) Last, if $\frac{\partial q_{i1}^*}{\partial N} > (<) 0$, the Nash equilibrium fraction of the vessels' ports of call made by each shipping line at Port 1, q_{i1}^*, increases (decreases) with an increase (decrease) in N and converges to a finite number as N increases. We also find that if all of the costs at Port 2 are higher than the costs at Port 1, then $\frac{\partial q_{i1}^*}{\partial N} > 0$, which means that the equilibrium fraction of vessels' ports of call at Port 1 decreases as the number of shipping lines decreases. The inverse is also true.

Proposition 3 shows a surprising result: the equilibrium port handling charges w_1^* and w_2^* increase, while q_{i1}^* decreases (assuming Port 1 is the port with the lower cost) as the number of shipping lines, N, decreases. Intuitively, w_1^* and w_2^* are expected to increase only when there are fewer shipping lines in the market, as the shipping lines would have less bargaining power. Consequently, shipping lines would not be able to negotiate for favorable handling charges with the ports.

We start the analysis by understanding why q_{i1}^* decreases as the number of shipping lines decreases. To explain this, we adopt the argument of Tirole (1988), which explains why firms tend to produce more than the optimal industry output in an oligopoly with Cournot competition.

First, recall that each shipping line's profit function, represented by Equation (17), consists of three components: revenue, congestion costs, and other costs (including transportation costs and port charges). To easily understand the competitive dynamics among the shipping lines, we consider a simplified version of each shipping line profit's function below that consists of the three components.

$$\pi_i^l = R - \sum_{j=1}^{2} \left[C_j \cdot Q_{ij} + \left(t_j^g \cdot \sum_{k=1}^{N} Q_{kj} \right) \cdot Q_{ij} \right],$$

where, R is the total revenue of each shipping line for shipping Q_i ($Q_i = Q_{i1} + Q_{i2}$) number of containers;

C_j is the unit cost (excluding congestion cost) paid by the shipping line for shipping one container via Port j ($j = 1, 2$);

Q_{ij} is the number of containers shipped by Shipping Line i via Port j ($j = 1, 2$), $Q_i = Q_{i1} + Q_{i2}$; and.

t_j^g is a positive number that represents the congestion cost in dollars when utilization at Port j reaches its effective capacity.

According to the shipping line's profit function given above, shipping lines ship more cargo via the port that has lower costs, C_j (excluding the congestion cost), to earn more profit.

When the number of shipping lines in the market increases, each shipping line sends more cargo via the lower-cost Port 1 as the shipping lines only consider the negative effect of a higher coefficient of congestion cost ($t_j^g \cdot \sum_{i=1}^{N} Q_{ij}$) on their own output instead of the effect on the aggregate output. For example, consider a case where there is only one shipping line; the total number of shipping containers in the market is 100 000, and the optimal division of cargo between Port 1 and Port 2 is 51,000 and 49 000 containers, respectively. If the shipping line decides to send one more container to Port 1 instead of Port 2 (51 001 and 48 999 containers, respectively), that extra container increases the coefficient of $t_j^g \cdot \sum_{i=1}^{N} Q_{ij}$, for example, from $30/TEU to $30.1/TEU. The increase in $t_j^g \cdot \sum_{i=1}^{N} Q_{ij}$, in turn, has an adverse effect on the extra container and the remaining 51 000 containers, which means that the cost of sending one more container to Port 1 is $5130.1 ($30.1 \times 1 + 0.1 \times 51\ 000 = 5130.1$).

The same argument Is used for a case where there are two shipping lines in the market, and the optimal division of cargo for each shipping line between Port 1 and Port 2 is

25 500 and 24 500 $\left(25\ 500 = \frac{51\ 000}{2} \text{and } 24\ 500 = \frac{49\ 000}{2}\right)$ as now there are two shipping lines instead of one. If one of them sends one more container to Port 1 instead of Port 2 (25 501 and 24 499), that extra container will cost them only \$2580.1 (30.1×1+0.1×25 500 = 2580.1) instead of \$5130.1. Hence, each shipping line ships more cargo through Port 1 until the marginal cost of shipping the extra container via Port 1 equals the marginal cost of shipping via Port 2, which explains why the equilibrium fraction of ports of call, q_{i1}^*, increases in the original model as the number of shipping lines increases (similarly, it decreases as the number of shipping lines decreases).

Additionally, the negative effect of the extra container being sent to Port 1 by each shipping line becomes smaller as the number of shipping lines increases, which means that it is more flexible for each shipping line to decide how many shipments to be transported via Port 1 and Port 2. This implies that the demand curves of the two ports become more elastic, as it is easier for their customers (shipping lines) to switch from Port 1 to Port 2 for the port of call. Hence, it is more profitable for the ports to lower their port charges because a 1% reduction in price leads to an increase of more than 1% in the volume of cargo handled. This explains why w_1^* and w_2^* increase as N decreases. The inverse is also true.

The above analysis explains why q_{i1}^* increases and converges to a finite number when N increases as the negative effect of the extra shipment transported via Port 1 becomes smaller. Therefore, the ports are obliged to lower their port charges because the demand curve becomes more elastic. Similarly, the ports increase their charges because the demand curve becomes more inelastic when fewer shipping lines are present.

While we have considered the effect of the change in the number of shipping lines on the equilibrium port handling charges, we have not confirmed that a merger of the two shipping lines would lead to a similar result. The merger would decrease the number of shipping lines and create an asymmetry between the shipping lines in terms of the annual cargo volume. Another question worth examining is: Does port competition drive the handling charge dynamics? If that is the case, it would be necessary to consider a merger between the two ports to form a single entity. These two scenarios are tested in the next section.

4 | COMPETITION EFFECT

4.1 | Merger among shipping lines

Next, we study the effect of a merger between M shipping lines on the equilibrium decisions, assuming $M < N$. In particular, we consider a scenario wherein M shipping lines merge to form a new shipping line with an annual cargo volume that is M times the original volume. Consequently, we have $N - M$ shipping lines, each with an annual volume of $V_s = \frac{V}{N}$, where $s = 1, 2, \ldots, N - M$, and the merged shipping line (denoted as m) with an annual volume of $V_m = \frac{MV}{N}$.

Our primary focus is to compare the equilibrium port handling charges in the symmetric case, w_1^* and w_2^*, with those after the merger, denoted as $\hat{w}_1^*$ and $\hat{w}_2^*$. Following the same logic as in the previous section, we obtain the results summarized in Proposition 4.

Proposition 4 *The difference in the equilibrium port handling charges after a merger of M shipping lines is given as follows*:

$$\hat{w}_1^* - w_1^* = \frac{r^2 v^b (2K_1 a_2 + K_2 a_1)(M-1)}{3K_1 K_2 (N - M + 1)} > 0,$$

$$\hat{w}_2^* - w_2^* = \frac{r^2 v^b (K_1 a_2 + 2K_2 a_1)(M-1)}{3K_1 K_2 (N - M + 1)} > 0,$$

$$\frac{\partial\left(\hat{w}_1^* - w_1^*\right)}{\partial M} = \frac{r^2 v^b (2K_1 a_2 + K_2 a_1)}{3K_1 K_2 (N - M + 1)^2} > 0,$$

and

$$\frac{\partial\left(\hat{w}_2^* - w_2^*\right)}{\partial M} = \frac{r^2 v^b (K_1 a_2 + 2K_2 a_1)}{3K_1 K_2 (N - M + 1)^2} > 0.$$

The above results confirm our intuition described in Section 3.5. The equilibrium port handling charges increase after a merger of shipping lines. Furthermore, as the consolidation among shipping lines increases (i.e., as M increases), the handling charges increase further.

4.2 | Merger among ports

Next, we consider symmetric shipping lines but assume that the ports maximize their joint profit functions when deciding on the handling charges. Our primary interest is to establish whether a decrease in the number of shipping lines leads to an increase in handling charges in the absence of competition between ports. Proposition 5 captures the sensitivity of the equilibrium port handling charges in this scenario.

Proposition 5 *The optimal handling charges of the monopoly port are independent of the number of shipping lines.*

We find that in the absence of competition, a change in the number of shipping lines does not affect port handling charges because the monopoly port always sets handling charges in such a way that extracts maximum profit. As the total container volume and shipment prices do not depend on the number of shipping lines in our model, the total profit of the shipping lines remains the same as N changes; hence, the optimal handling charge remains the same. Therefore, competition among ports is an important factor that drives the increase in handling charges as the number of shipping lines decreases.

Next, we validate our key findings with a case study and discuss the implications in more detail.

5 | CASE STUDY

This section introduces a case study that illustrates our model. We describe two competing ports to estimate the model parameters. Then, we consider the equilibrium outcomes with a focus on the effect of the number of shipping lines on the outcomes.

5.1 | Case description

POT is the largest port in New Zealand (NZ) and, in 2019, it handled a cargo volume of 12.5 million tons and container throughput of 1.2 million TEU. Additionally, POT is the only port in NZ that handles more than a million TEUs annually (Deloitte, 2020). POA is the second-largest container port in NZ and handled an estimated container volume of 939 680 TEUs in 2019. It was the first port in NZ to operate automated straddles (Deloitte, 2020). It was voted the "Best Port in Oceania" every year from 2016 to 2019 at the Asian Freight, Logistics, and Supply Chain Awards (Asia Cargo News, 2020). Table 1 shows a comparison of the facilities and capacities of POA and POT.

TABLE 1 Port facilities and capacity comparison (Deloitte, 2020)

Criteria	POA	POT
Port harbor type	Natural	Natural
Draught (m)	12.5	14.5
Port operating land (ha)	77	190
Container terminal area (ha)	34	75
Container wharf length (km)	1.0	.8
Quay cranes	5	8
Forklifts/stackers	14	0
Straddles	39	46
Rail connection	Yes	Yes
Throughput (1000 TEU)	939.7	1233.2
Container ship calls	864	888

We select the Southern Star service operated by Maersk Line (Maersk, 2017) as the deep-sea container shipping route to demonstrate our model's results, which can select between POT and POA as the port of call.

In Alternative 1, the vessels call at the following ports: PTP → Singapore → Brisbane → POT → Lyttelton → Port Otago → PTP, as shown in Figure 2.

In Alternative 2, POT is replaced by POA for the deep sea and feeder container routes; other ports remain the same as in Alternative 1. Hereafter, subscript 1 ($j = 1$) in the notation denotes POT, and subscript 2 denotes POA.

In the baseline scenario, the variable values are set according to Table 2.

5.2 | Effect of the number of shipping lines on port charges

To explore the effect of the number of shipping lines, N, on the decision variables w_1^*, w_2^* and q_{i1}^*, we vary N from 6 to 8, 10, 12, and 14. Figure 3 shows that the results are in line with Proposition 3.

In this case, POT is the port with lower costs, corresponding to Port 1 in our theoretical analysis, while POA corresponds to Port 2. In line with our theoretical predictions, fewer shipping lines lead to higher handling charges. Consequently, the profits of both ports increase while the total profit of all the shipping lines decreases as N decreases, as shown in Figure 4.

5.3 | Investment in capacity expansion

In our example, POT has a lower capacity than POA. Next, we investigate whether investing in additional capacity is

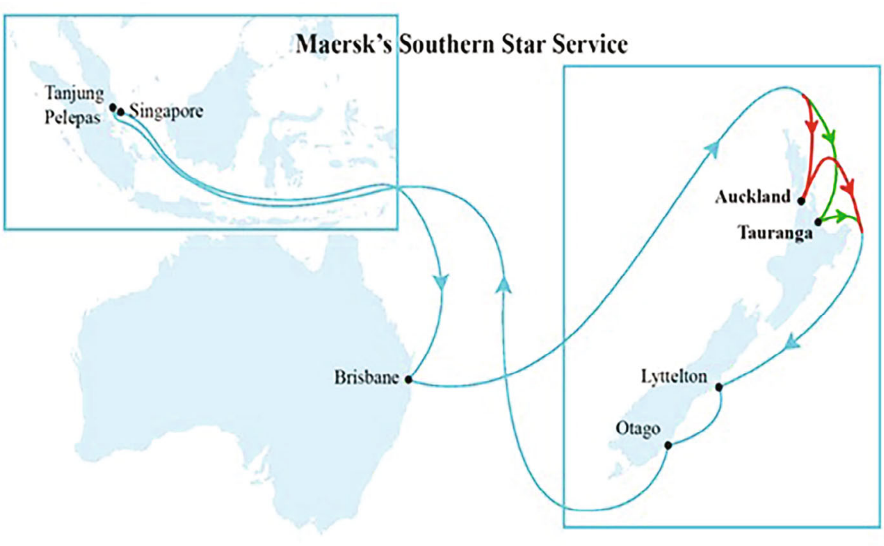

FIGURE 2 The deep-sea container route (Maersk, 2017) (The green line shows alternative 1, which means that POT is chosen as the port of call by Maersk's southern star service. The red line shows alternative 2, which means that POT is replaced by POA as the port of call on this service)

TABLE 2 Parameter estimation

Variable	Source
$V = 850\ 000$ TEUs/year	Freight Information Gathering System (Ministry of Transport, 2017)
$a_1 = a_2 = 60$ NZD	Hypothetical values
$K_1 = 1\ 240\ 000$ TEUs/year	$K_2 = 1\ 400\ 000$ TEUs/year (Deloitte, 2014)
$c_1 = 450$ NZD/TEU $c_2 = 463$ NZD/TEU	Hinterland cargoes are assumed to be in Hamilton. The cost of transportation is then NZD 450 for POT and NZD 463 for POA based on PwC's report.
$p = 2390$ NZD/TEU	The cost per container from Hamilton to Singapore based on PwC's report.
$m_1 = m_2 = 23$ TEUs/voyage	This parameter is calculated based on Deloitte's (2014) report which estimates that it will cost POT 398 million NZD to upgrade their handling capacity from 1.24 to 1.82 million TEUs for a period of 30 years. Therefore, $m_j = \frac{398 \cdot 10^6}{(1.82-1.24) \cdot 10^6 \cdot 30} \approx 23$. In this study, the unit handling capacity investment is assumed to be the same for both ports, similar to Song et al. (2016).
$k_1 = 33$ NZD/TEU $k_2 = 34$ NZD/TEU	According to Deloitte's (2014) report, the unit operating cost for one container throughput is around 67 NZDs/TEU for large ports in NZ. Since loading and unloading activities are counted separately in this study, the unit operating cost of POT and POA should lie somewhere around half of 67 NZDs/TEU. Moreover, it is well known that POT has a lower unit operating cost than POA.

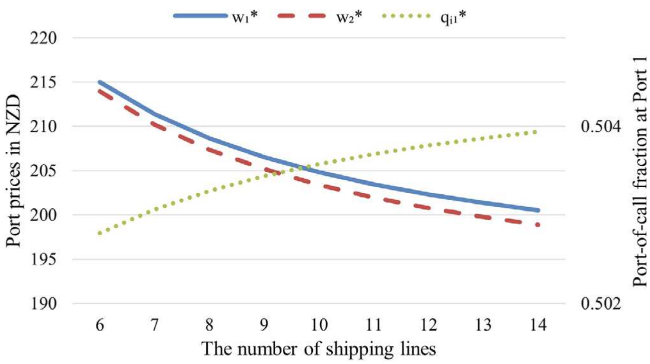

FIGURE 3 Effect of the number of shipping lines, N, on the decision variables

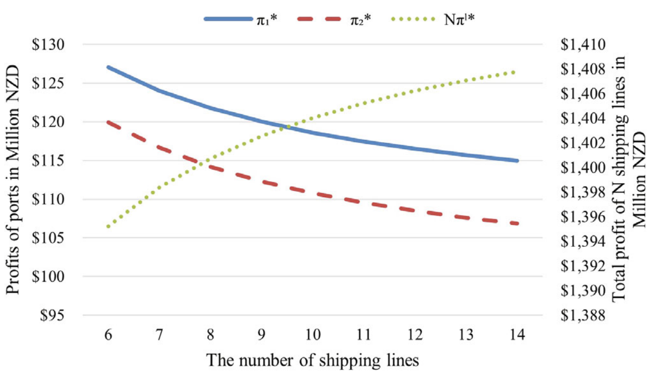

FIGURE 4 Effect of the number of shipping lines, N, on the profits of all of the shipping lines and the ports (profits are based on Proposition 2)

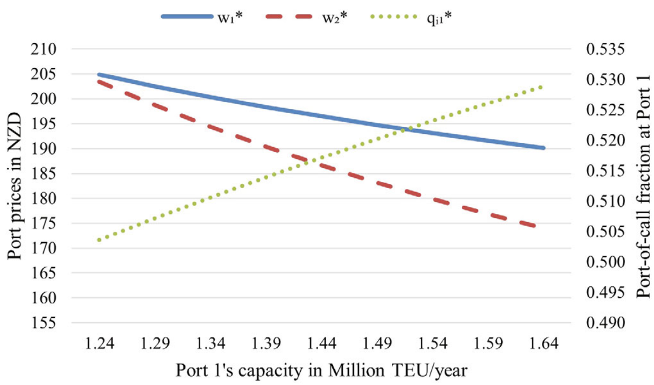

FIGURE 5 Effect of POT's capacity, K_1, on the decision variables

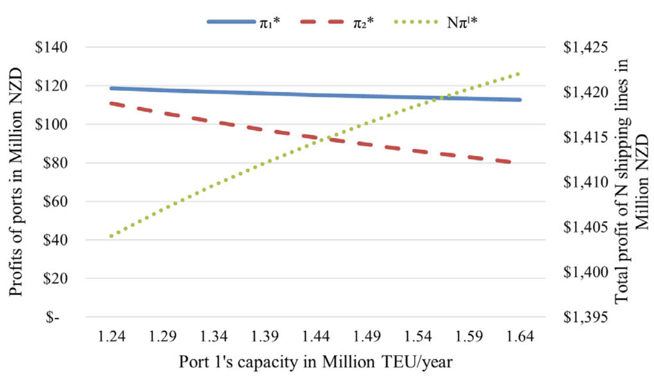

FIGURE 6 Effect of POT's capacity, K_1, on the profits of all of the shipping lines and the ports

beneficial for POT and how this may affect POA. The current capacity of POT is 1.24 million TEU/year, and Figure 5 illustrates the effect of POT's capacity expansion on the equilibrium handling charges and shipping lines' port-of-call decision. For this analysis, we fix the number of shipping lines at 10 ($N = 10$), although this parameter does not structurally change the outcome.

Although POT attracts more container traffic, as represented by the increasing q_{i1}^* line, the equilibrium handling charges decrease for both ports. In this case, the effect on POT's profits is unclear. Therefore, we construct the ports' profit functions using the same parameters, as shown in Figure 6.

We find that an increase in capacity benefits neither POT nor POA, although the adverse effect on POA's profit is profound. Because of the decrease in the equilibrium handling charges, only the shipping lines benefit from an increase in port capacity. Therefore, a port may not have a sufficient incentive for capacity expansion, even if its current capacity is less than the competitor's capacity, because an increased capacity decreases its congestion cost, thereby intensifying competition with the other port. As the competitor cannot feasibly add capacity in the short term, the only option for retaining the shipping lines is to lower the handling charges. This, in turn, makes the other port also decrease its charges. Ultimately, the profits of both ports decrease, as can be expected in a setting with more intense competition.

6 | CONCLUSION

This study is one of the first to examine the competitive dynamics between shipping lines and the effect of the number of shipping lines on port charges and profits. The study illustrates that under a two-stage noncooperative Cournot game model, shipping lines consider only the negative effect of the extra shipment transported via the port with lower costs (Port 1) on their own output instead of the effect on the aggregate output. Hence, each shipping line ships more cargo than the industry optimal via the port with lower costs. This insight can help small port operators (e.g., POT and POA) formulate their strategy to remain profitable with the trend of mergers and alliances among giant shipping lines.

Our study extends the literature on port competition by incorporating competition between multiple identical shipping lines. It is observed that competing ports set higher container handling charges when the number of shipping lines decreases. Similarly, ports increase handling charges when the number of shipping lines decreases.

Future research could be undertaken in the following directions. First, our model does not include the negotiation process for setting port charges between international shipping lines and container ports. In practice, mega shipping lines, such as Maersk, have immense bargaining power, which allows them to negotiate for a substantial discount in port charges because removing a port from the line's route would have a considerable negative impact on the port's profit. Second, the model is constructed based on an implicit assumption that the shipping lines bear the congestion costs. However, in practice, shipping lines may try to pass some or all of the congestion costs on to shippers by increasing their shipping charges for the cargo shipped through a congested port. This, in turn, may lead to shipping lines splitting their cargo between two ports.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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APPENDIXA

A.1 | PROOF OF LEMMA 1

Substituting Equations (1)–(10) into Equation (11), the profit function of Shipping Line i is given as follows:

$$\pi_i^l = (p - c_1 - - w_1)\, v^b q_{i1} - \frac{a_1 r\left(\sum_{k=1}^{N} v^b q_{k1}\right) r v^b q_{i1}}{K_1} + (p - c_2 - - w_2)\, v^b (1 - q_{i1}) - \frac{a_2 r\left(1 - \sum_{k=1}^{N} v^b q_{k1}\right) r v^b (1 - q_{i1})}{K_2}.$$

Taking the first partial derivative of the profit function of Shipping Line i with respect to q_{i1}, we obtain:

$$\frac{\partial \pi_i^l}{\partial q_{i1}} = (p - c_1 - w_1)\, v^b - \frac{a_1 \left(r v^b\right)^2 \left(q_{i1} + \sum_{k=1}^{N} q_{k1}\right)}{K_1} - (p - c_2 - w_2)\, v^b + \frac{a_2 \left(r v^b\right)^2 \left(1 - q_{i1} + 1 - \sum_{k=1}^{N} q_{k1}\right)}{K_2}.$$

Taking the second partial derivative with respect to q_{i1}, we obtain:

$$\frac{\partial^2 \pi_i^l}{\partial q_{i1}^2} = -\frac{2a_1 \left(r v^b\right)^2}{K_1} - \frac{2a_2 \left(r v^b\right)^2}{K_2} \leq 0.$$

As $\frac{\partial^2 \pi_i^l}{\partial q^2_{i1}} \leq 0$, the profit function of Shipping Line i, π_i^l, is concave with respect to q_{i1} in the interval $[0, 1]$.

A.2 | PROOF OF LEMMA 2

The first partial derivative of the profit function of Shipping Line i with respect to q_{i1} can be written as follows:

$$\frac{\partial \pi_i^l}{\partial q_{i1}} = (p - c_1 - w_1)\, v^b - \frac{a_1 \left(r v^b\right)^2 \left(q_{i1} + \sum_{k=1}^{N} q_{k1}\right)}{K_1} - (p - c_2 - w_2)\, v^b + \frac{a_2 \left(r v^b\right)^2 \left(1 - q_{i1} + 1 - \sum_{k=1}^{N} q_{k1}\right)}{K_2}. \tag{A1}$$

There is a system of N equations like Equation (A1), each representing the first partial derivative of the profit function of Shipping Line i with respect to q_{i1}. To determine the Nash equilibrium fraction of the port of call for each shipping line, we set each equation equal to 0 and solve the system of N Equations (A1) simultaneously.

To solve the system of N Equations (A1), we add all of them as follows:

$$N (p - c_1 - w_1)\, v^b - \frac{a_1 \left(r v^b\right)^2 \left(\sum_{k=1}^{N} q_{k1} + N \sum_{k=1}^{N} q_{k1}\right)}{K_1} + N (p - c_2 - w_2)\, v^b - \frac{a_2 \left(r v^b\right)^2 \left(\left(1 - \sum_{k=1}^{N} q_{k1}\right) + N \left(1 - \sum_{k=1}^{N} q_{k1}\right)\right)}{K_2} = 0.$$

Solving for $\sum_{i=1}^{N} q_{i1}$, we obtain:

$$\sum_{i=1}^{N} q_{i1} = \frac{N\left((N+1) a_2 r^2 v^b - (c_1 + w_1 - c_2 - w_2) K_2\right)}{(N+1) r^2 v^b \left(\frac{a_1}{K_1} + \frac{a_2}{K_2}\right) K_2} = \frac{N\left((N+1) a_2 r^2 v^b - (c_1 + w_1 - c_2 - w_2) K_2\right) K_1}{(N+1) r^2 v^b \left(a_1 K_2 + a_2 K_1\right)}.$$

Substituting $\sum_{i=1}^{N} q_i$ back into Equation (A1) and solving for q_{i1}, we obtain:

$$q_{i1} = \frac{\left((N+1) a_2 r^2 v^b - (c_1 + w_1 - c_2 - w_2) K_2\right) K_1}{(N+1) r^2 v^b \left(a_1 K_2 + a_2 K_1\right)} \equiv D_1 \equiv q_{i1}^*, \forall i \in \{1, 2, \ldots, N\}. \tag{A2}$$

Equation (A2) indicates that each shipping line's Nash equilibrium port of call, q_{ij}, is equal to the others ($q_{1j}^* = q_{2j}^* = \ldots = q_{Nj}^*$) and given by D_1.

In contrast, according to Lemma 1, the first partial derivative of Shipping Line i's profit function with respect to q_{i1}, $\frac{\partial \pi_i^l}{\partial q_{i1}}$, is a monotonic decreasing function in q_{i1} in the interval $[0, 1]$. Hence, there is a unique, optimal solution, q_{i1}, in the interval $[0, 1]$. Specifically, $q_{i1}^* = 0$ if $\frac{\partial \pi_i^l}{\partial q_{i1}} < 0$ for any $q_{i1} \in [0, 1]$; $q_{i1}^* = 1$ if $\frac{\partial \pi_i^l}{\partial q_{i1}} > 0$ for any $q_{i1} \in [0, 1]$; and $q_{i1}^* \in [0, 1]$, otherwise.

Therefore, we conclude that the Nash equilibrium fraction of port of call of each shipping line, q_{ij}^*, is equal to the others ($q_{1j}^* = q_{2j}^* = \ldots = q_{Nj}^*$) and given by:

$$q_{i1}^* = \begin{cases} 0 & \text{if } D_1 < 0 \\ D_1 & \text{if } 0 \le D_1 \le 1 \\ 1 & \text{if } D_1 > 1 \end{cases}, \qquad \text{where}$$

$$D_1 = \frac{\left((N+1)a_2r^2v^b - (c_1 + w_1 - c_2 - w_2)K_2\right)K_1}{(N+1)r^2v^b\left(a_1K_2 + a_2K_1\right)}.$$

A.3 | PROOF OF PROPOSITION 1

With $D_1 = \frac{((N+1)a_2r^2v^b-(c_1+w_1-c_2-w_2)K_2)K_1}{(N+1)r^2v^b(a_1K_2+a_2K_1)}$ from Lemma 2, we observe the following condition:

(i) $D_1 < 0$ is equivalent to $w_2 - w_1 < c_1 - c_2 - \frac{(N+1)a_2r^2v^b}{K_2}$. Therefore, if $D_1 < 0$, then $q_{i1}^* = 0$ and $\left(w_1^*, w_2^*\right) = \max\left\{ (w_1 w_2) | L_1 \le w_1 \le U_1, L_2 \le w_2 \le U_2, w_2 - w_1 < c_1 - c_2 - \frac{(N+1)a_2r^2v^b}{K_2}\right\}$.

(ii) $D_1 > 1$ is equivalent to $w_2 - w_1 > c_1 - c_2 + \frac{(N+1)a_2r^2v^b}{K_1}$. Therefore, if $D_1 > 1$, then $q_{i1}^* = 1$ and $\left(w_1^*, w_2^*\right) = \max\left\{ (w_1 w_2) | L_1 \le w_1 \le U_1, L_2 \le w_2 \le U_2, w_2 - w_1 > c_1 - c_2 + \frac{(N+1)a_2r^2v^b}{K_1}\right\}$.

(iii) Under the condition $0 \le D_1 \le 1$, Port 1's profit function is given as follows:

$$\pi_1 = (w_1 - k_1)\, rv^bNq_{i1} - m_1K_1. \tag{A3}$$

Port 2's profit function is given as follows:

$$\pi_2 = (w_2 - k_2)\, rv^bN(1 - q_{i1}) - m_2K_2. \tag{A4}$$

Substituting q_{i1}^* for q_{i1} in Equations (A3) and (A4) and solving $\frac{\partial \pi_1}{\partial w_1} = \frac{\partial \pi_2}{\partial w_2} = 0$, we obtain $w_1^* = \frac{((c_2-c_1+2k_1+k_2)K_2+2(N+1)a_2r^2v^b)K_1+a_1r^2v^b(N+1)K_2}{3K_1K_2}$ and $w_2^* = \frac{((c_1-c_2+k_1+2k_2)K_2+(N+1)a_2r^2v^b)K_1+2a_1r^2v^b(N+1)K_2}{3K_1K_2}$. Last, by substituting w_j^* for w_j in $q_{i1}^* = D_1$, we obtain $q_{i1}^* = \frac{((c_2-c_1+k_2-k_1)K_2+2(N+1)a_2r^2v^b)K_1+a_1r^2v^b(N+1)K_2}{3r^2v^b(N+1)(a_1K_2+a_2K_1)}$.

A.4 | PROOF OF PROPOSITION 2

We substitute w_1^*, w_2^*, and q_{i1}^* as given in Equations (16), (17), and (18), and $q_{i2}^* = 1 - q_{i1}^*$ in the profit functions of Port 1, Port 2, and Shipping Line i as given in Equations (11) and (15).

The profit function of Port 1 is given as follows:

$$\pi_1 = (w_1 - k_1)\, rv^bNq_{i1} - m_1K_1.$$

Substituting w_1^* and q_{i1}^* from Equations (16) and (18) for w_1 and q_{i1} in the above equation, the profit function of Port 1 becomes:

$$\pi_1^* = \left(w_1^* - k_1\right) rv^bNq_{i1}^* - m_1K_1.$$

Similarly, the profit function of Port 2 becomes:

$$\pi_2 = (w_2 - k_2)\, rv^bN(1 - q_{i1}) - m_2K_2.$$

Substituting w_2^* and q_{i1}^* for w_2 and q_{i1}, respectively, in the above equation, the profit function of Port 2 becomes:

$$\pi_2^* = \left(w_2^* - k_2\right) rv^bN\left(1 - q_{i1}^*\right) - m_2K_2.$$

The profit function of Shipping Line i is given as follows:

$$\pi_i^l = \sum_{j=1}^{2}\left(\left(p - c_j - w_j\right) v_{ij} - \frac{a_jr\left(\sum_{k=1}^{N} v^bq_{kj}\right) rv^bq_{i1}}{K_j}\right).$$

Substituting w_1^*, w_2^*, and q_{i1}^* for w_1, w_2, and q_{i1}, respectively, in the above equation, the profit function of Shipping Line i becomes:

$$\pi_i^{l*} = \sum_{j=1}^{2}\left(\left(p - c_j - w_j^*\right) v_{ij} - \frac{a_jN\left(rv^bq_{ij}^*\right)^2}{K_j}\right),$$

where w_1^*, w_2^*, and q_{i1}^* are given in Equations (16), (17), and (18) and $q_{i2}^* = 1 - q_{i1}^*$.

A.5 | PROOF OF PROPOSITION 3

The results are obtained by taking the first derivatives of w_1^*, w_2^*, and q_{i1}^* with respect to N and taking the limit of these derivatives with respect to N as N approaches infinity.

From Proposition 1, Port 1's Nash equilibrium container handling price, w_1^*, is given by $w_1^* = \frac{((c_2-c_1+2k_1+k_2)K_2+2(N+1)a_2r^2v^b)K_1+a_1r^2v^b(N+1)K_2}{3K_1K_2}$.

Note that v^b is a function of N, that is, $v^b = \frac{V}{N}$.

Therefore, taking the first derivative of w_1^* with respect to N and simplifying it, we obtain:

$$\frac{\partial w_1^*}{\partial N} = -\frac{r^2V\left(2a_2K_1 + a_1K_2\right)}{3K_1K_2N^2},$$

which means that $\frac{\partial w_1^*}{\partial N} < 0$.

Taking the limit of the first derivative of w_1^* with respect to N as N approaches infinity, we have: $\lim_{N\to\infty} \frac{\partial w_1^*}{\partial N} = 0$.

(i) Following the same steps as in Part (i), we obtain

$$\frac{\partial w_2^*}{\partial N} = -\frac{r^2V\left(a_2K_1 + 2a_1K_2\right)}{3K_1K_2N^2} < 0,$$

and $\lim_{N\to\infty} \frac{\partial w_2^*}{\partial N} = 0$.

(ii) From Proposition 1, the Nash equilibrium fraction of the vessels' port of call of each shipping line at Port 1, q_{i1}^*, is given by:

$$q_{i1}^{*}=\frac{\left((c_2-c_1+k_2-k_1)K_2+2(N+1)a_2r^2v^b\right)K_1+a_1r^2v^b(N+1)K_2}{3r^2v^b(N+1)(a_1K_2+a_2K_1)}.$$

Note that $v^b$is a function of N. Therefore, taking the first derivative of q_{i1}^* with respect to N and simplifying it, we obtain:

$$\frac{\partial q_{i1}^*}{\partial N}=\frac{K_1K_2(c_2-c_1+k_2-k_1)}{3(N+1)^2Vr^2(a_1K_2+a_2K_1)}.$$

Taking the limit of the first derivative of q_{i1}^* with respect to N as N approaches infinity, we have: $\lim_{N\to\infty}\frac{\partial q_{i1}^*}{\partial N}=0$.

A.6 | PROOF OF PROPOSITION 4

The profit function of Shipping Line $s\in\{1,2,\ldots,N-M\}$ is given by. $\widehat{\pi}_s^l=\left(p-c_1-\widehat{w}_1\right)v^b\widehat{q}_{s1}-\frac{a_1r\left(\sum_{k=1}^{N-M}v^b\widehat{q}_{k1}+Mv^b\widehat{q}_{M1}\right)rv^b\widehat{q}_{s1}}{K_1}+\left(p-c_2-\widehat{w}_2\right)v^b\left(1-\widehat{q}_{s1}\right)-\frac{a_2r\left(1-\sum_{k=1}^{N-M}v^b\widehat{q}_{k1}+Mv^b\left(1-\widehat{q}_{M1}\right)\right)rv^b\left(1-\widehat{q}_{k1}\right)}{K_2}$, where $\widehat{q}_{Mj}$ is the fraction of the vessels' port of call that the merged shipping line makes at Port j. The profit function of the merged shipping line is given by: $\widehat{\pi}_M^l=\left(p-c_1-\widehat{w}_1\right)v^b\widehat{q}_{M1}-\frac{a_1r\left(\sum_{k=1}^{N-M}v^b\widehat{q}_{k1}+Mv^b\widehat{q}_{M1}\right)rv^b\widehat{q}_{M1}}{K_1}+\left(p-c_2-\widehat{w}_2\right)v^b\left(1-\widehat{q}_{M1}\right)-\frac{a_2r\left(1-\sum_{k=1}^{N-M}v^b\widehat{q}_{k1}+Mv^b\left(1-\widehat{q}_{M1}\right)\right)rv^b\left(1-\widehat{q}_{M1}\right)}{K_2}$. Following the same steps as in Lemmas 1 and 2 (i.e., taking the partial derivative with respect to the port-of-call decision variables and solving the resulting system of equations), we obtain the following equilibrium values:

$$\widehat{q}_{s1}^*=\frac{\left(K_2\left(\widehat{w}_2-\widehat{w}_1+c_2-c_1\right)+a_2v^br^2(N-M+2)\right)K_1}{v^br^2(a_1K_2+a_2K_1)(N-M+2)},$$

$$\widehat{q}_{M1}^*=\frac{\left(K_2\left(\widehat{w}_2-\widehat{w}_1+c_2-c_1\right)+a_2v^br^2(N-M+2)M\right)K_1}{v^br^2(a_1K_2+a_2K_1)(N-M+2)M}.$$

Substituting these equilibrium values in the ports' objective function, we obtain the equilibrium port handling charges:

$$\widehat{w}_1^*=\frac{(2k_1+k_2+c_2-c_1)(N-M+1)K_1K_2+(a_1K_2+2a_2K_1)v^br^2N(N-M+2)}{3K_1K_2(N-M+1)},$$

$$\widehat{w}_2^*=\frac{(k_1+2k_2-c_2+c_1)(N-M+1)K_1K_2+(2a_1K_2+a_2K_1)v^br^2N(N-M+2)}{3K_1K_2(N-M+1)}.$$

Taking the difference between the equilibrium port handling charges and those under the symmetric case and simplifying the result, we obtain the following:

$$\widehat{w}_1^*-w_1^*=\frac{r^2v^b(2K_1a_2+K_2a_1)(M-1)}{3K_1K_2(N-M+1)}>0,$$

$$\widehat{w}_2^*-w_2^*=\frac{r^2v^b(K_1a_2+2K_2a_1)(M-1)}{3K_1K_2(N-M+1)}>0.$$

Taking a partial derivative of the above differences with respect to M and simplifying the result, we obtain the following:

$$\frac{\partial\left(\widehat{w}_1^*-w_1^*\right)}{\partial M}=\frac{r^2v^b(2K_1a_2+K_2a_1)}{3K_1K_2(N-M+1)^2}>0,$$

and

$$\frac{\partial\left(\widehat{w}_2^*-w_2^*\right)}{\partial M}=\frac{r^2v^b(K_1a_2+2K_2a_1)}{3K_1K_2(N-M+1)^2}>0.$$

A.7 | PROOF OF PROPOSITION 5

For this proof, we drop the indices $j=1,2$ that indicate the port as we only consider one port. The objective function of Shipping Line i is given as follows:

$$\tilde{\pi}_s^l=(p-c-\tilde{w})v^b-Crv^b=(p-c-\tilde{w})\frac{V}{N}-\frac{ar^2v^bV}{NK}.$$

The port's objective function is given by:

$$\tilde{\pi}=(\tilde{w}-k)rV-mK.$$

As $\tilde{\pi}$ increases with an increase in $\tilde{w}$, the port sets the highest possible $\tilde{w}$ such that $\tilde{\pi}_s^l\geq 0$. Solving $\tilde{\pi}_s^l=0$ for $\tilde{w}$, we find that $\tilde{w}^*=p-c-\frac{ar^2V}{K}$. Taking the partial derivative with respect to N, we obtain $\frac{\partial\tilde{w}^*}{\partial N}=0$.