

1 **Title:** Growth phase estimation for abundant bacterial populations sampled longitudinally from
2 human stool metagenomes.

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9

10 **ABSTRACT**

11 Longitudinal sampling of the stool has yielded important insights into the ecological dynamics of
12 the human gut microbiome. However, due to practical limitations, the most densely sampled
13 time series from the human gut are collected at a frequency of about once per day, while the
14 population doubling times for gut commensals are on the order of minutes-to-hours. Despite
15 this, much of the prior work on human gut microbiome time series modeling has, implicitly or
16 explicitly, assumed that day-to-day fluctuations in taxon abundances are related to population
17 growth or death rates, which is likely not the case. Here, we propose an alternative model of the
18 human gut as a continuous flow ecosystem at a dynamical steady state, where population
19 dynamics occur internally and the bacterial population sizes measured in stool represent an
20 endpoint of these internal dynamics. We formalize this idea as stochastic logistic growth of a
21 population held at a constant dilution rate. We show how this model provides a path toward
22 estimating the growth phases of gut bacterial populations *in situ*. We assess our model
23 predictions against densely-sampled human stool metagenomic time series data. Consistent
24 with our model, donors with slower defecation rates tended to harbor a larger proportion of taxa
25 in later growth phases, while faster defecation rates were associated with more taxa in earlier
26 growth phases. We discuss how these growth phase estimates may be used to better inform
27 metabolic modeling in flow-through ecosystems, like animal guts or industrial bioreactors.

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32 INTRODUCTION

33 The human gut microbiome is an anaerobic bioreactor, ecologically distinct to each individual,
34 that transforms dietary and host substrates into bioactive molecules important to host health [1–
35 3]. Disruptions to the ecological composition of the gut have been shown to mediate the
36 progression of various complex diseases [4–8]. Furthermore, the ecological dynamics of the gut
37 appear to be relevant to both health and disease states [9, 10]. However, the biological
38 interpretation of densely-sampled adult human fecal microbiome time series is fraught.

39 Various dynamical models have been applied to gut microbial abundance data collected
40 from adult human donors [11–15]. These models often assume, either explicitly or implicitly, that
41 day-to-day changes in abundance are proportional to population growth and/or death [16].
42 However, the underlying data often do not match this assumption [11, 16–20]. The gut is a flow-
43 through ecosystem and commensal gut bacteria must grow fast enough to avoid dilution-to-
44 extinction. As such, gut bacterial doubling times tend to be fast, ranging from minutes-to-hours
45 [21–23]. However, stool sampling frequency is usually limited to, at most, about once per day.
46 Consequently, rapid internal population dynamics likely cannot be directly estimated from the
47 day-to-day measurements obtained from stool [16].

48 In the absence of major perturbations, is it possible to extract meaningful information
49 about commensal population dynamics from adult human gut microbiome time series, despite
50 the fundamental limitations in sampling timescales mentioned above? One work around to
51 inferring growth rates of bacterial populations *in situ* is to leverage metagenome-inferred
52 replication rates [22]. Briefly, instantaneous replication rates can be estimated for abundant
53 bacterial populations in metagenomic samples by taking advantage of the fact that fast-growing
54 taxa show an asymmetry in reads mapping to different genomic loci, with higher read depth
55 near the origin of replication and a lower depth near the terminus due to the initiation of multiple

56 replication forks [21–23]. However, even when replication rates and population abundances can
57 both be estimated from the same metagenomic samples, it is unclear how these measurements
58 are related to the *in situ* growth phase of a population (e.g. lag, log, or stationary phases). As
59 such, biological interpretations regarding population size and replication rate fluctuations in flow-
60 through ecosystems like the human gut, where internal dynamics are much faster than sampling
61 rates, remain challenging.

62 Early experiments by Jaques Monod [24] identified distinct growth phases for bacterial
63 populations in culture, which can be captured by the stochastic logistic growth equation (sLGE)
64 [25]. The sLGE has been shown to be a good fit for bacterial population growth *in vitro* and in
65 real-world, steady-state ecosystems [26–31]. We used the sLGE to study statistical
66 relationships between population sizes and growth rates across the various phases of growth
67 (i.e., lag, acceleration, log, deceleration, and stationary phases) to see if we could extract *in situ*
68 growth phase information from longitudinal data from a steady-state, flow-through ecosystem
69 sampled at a consistent frequency. Overall, the sLGE model yields statistical relationships that
70 can be leveraged to identify the *in situ* growth phase of a bacterial population periodically
71 sampled from a continuous-flow ecosystem, like the human gut.

72 To assess our model predictions, we calculated population abundance and
73 growth/replication rate trajectories from more than a dozen organisms in four densely sampled
74 human gut metagenomic time series [32]. On average, gut commensal growth rates and
75 population sizes were positively correlated within each of the stool donor time series, which
76 suggests that most abundant taxa in the gut are growing exponentially. We were able to identify
77 signatures of specific growth phases in 20–40% of the abundant bacterial populations in the guts
78 of these four individuals. We describe how our growth phase inference approach can serve to
79 inform more accurate mechanistic modeling of flow-through ecosystems (e.g., community-scale
80 metabolic models, which usually assume exponential growth), which could have broad

81 implications for the gut microbiome and host health [8, 33, 34], flow-through agricultural systems
82 [35, 36], climate change [35, 37, 38], and industrial bioreactor production processes [39, 40].

83

84 **RESULTS**

85 ***Framing the gut as an anaerobic flow-through bioreactor***

86 The mammalian gut can be understood as an anaerobic bioreactor with a continuous input (i.e.,
87 dietary and host substrates) and output (i.e., stool) [41], and microbial taxa must grow fast
88 enough to avoid dilution to extinction (Fig. 1A). Stool sampling captures the endpoint of internal
89 gut bacterial population dynamics. For example, in our cartoon figure we see that Taxon 1 starts
90 growing higher up in the colon and is in stationary phase by the time a stool sample is collected,
91 while Taxon 3 starts growing lower in the colon and is still growing exponentially at the point of
92 stool sampling (Fig. 1A). Overall, the daily abundances of Taxa 1-3 represent the average (μ)
93 steady-state population size, plus or minus some amount of biological and technical noise, at
94 the time of stool sampling (Fig. 1A). To investigate improved methods for interpreting the
95 dynamics of human gut microbial time series, we downloaded shotgun metagenomic time series
96 data from the BIO-ML cohort (i.e., health-screened stool donors who provided fecal-transplant
97 material to the stool bank OpenBiome) [32]. The BIO-ML cohort contained 74 donors, 70 of
98 which had 1-3 time points collected [32]. To filter for dense longitudinal data, we selected a
99 subset of donors with more than 50 time points. Four donors (i.e. donors ae, am, an, and ao)
100 met this criterion, with 3-5 fecal samples per week for >50 days (Fig. 1B).

101

102 ***Characterizing the relationships between gut commensal population size and growth rate***
103 ***using metagenomic time series data***

104 We first investigated the statistical properties of day-to-day fluctuations in gut bacterial
105 population sizes, estimated from metagenomic time series. Specifically, we looked at the

106 associations between population abundance estimates (t_n) and the changes in abundance
107 estimates (i.e., deltas) between time points ($t_{n+1} - t_n$). Naïvely, if most bacterial populations in
108 the stool were growing exponentially, we would expect that population abundances and growth
109 rates would be positively correlated. However, prior work has indicated an overall negative
110 correlation between abundances and changes in abundances in stool 16S rRNA gene amplicon
111 sequencing data generated from densely sampled human stool time series [15]. Indeed, we
112 found that abundant bacterial populations in the stool of the four BIO-ML donors maintained
113 stable average abundances over time (μ), with day-to-day fluctuations above and below this
114 average, as pictured in the example of *Bacteroides cellulosilyticus* in donor am (Fig. 2A-B). This
115 kind of pattern fits a regression-to-the-mean model, which one would expect when randomly
116 sampling from a stationary distribution (Fig. 2B). This kind of regression-to-the-mean process
117 will give rise to a negative correlation between population abundances and changes in
118 abundance between time points, consistent with what has been observed previously [15]. We
119 observed that the deltas ($t_{n+1} - t_n$) for the same gut taxon (*Bacteroides uniformis*) measured
120 across each donor time series, when plotted against their respective normalized abundances
121 (t_n), showed the expected negative association (Fig. 2C). Furthermore, similar negative
122 associations were observed across all taxa analyzed, across all donors (Fig. 2D). Overall, these
123 results support our assertion that stool samples provide steady-state population abundance
124 estimates for gut commensal bacteria, which are representative of the endpoint of internal
125 dynamics.

126 Next, we looked at the statistical associations between calculated peak-to-trough ratios
127 (i.e., PTRs; a proxy for growth-rate) [22] of abundant bacterial populations from each
128 metagenomic sample and their respective metagenomic population abundance estimates. If the
129 deltas, presented above, were truly proportional to growth and/or death rates, we would expect
130 that the statistical relationships between deltas and population size would be similar to those

131 between PTRs and population size. However, unlike the regression-to-the-mean signature
132 identified for the deltas, we found variable statistical relationships between PTR and centered
133 log-ratio (CLR) transformed population abundances for the same taxon (*Bacteroides ovatus*)
134 across the four donors (Fig. 3A). Similarly, we saw a wide range of positive, negative, and null
135 associations between PTRs and CLR abundances across all measured taxa within each donor
136 (Fig. 3B). These results are inconsistent with a regression-to-the-mean signal, and suggest a
137 more complex relationship between growth rate and population size [42–44]. Finally, we
138 calculated temporally-averaged PTRs and population sizes for each abundant taxon within each
139 of the donors. Overall, there was a significantly positive (linear regression, *p*-values = 0.0318,
140 0.125, 0.155, 0.031 for donors ae, am, an, and ao, respectively; combined *p*-value using
141 Fisher's method = 0.005), albeit noisy, association between average PTR and average CLR
142 abundance across all four donors (Fig. 3C), indicating that taxa with higher average population
143 sizes tend to have higher average growth rates. This result is consistent with what we would
144 expect to observe in exponentially-growing populations.

145

146 ***Stochastic logistic growth equation provides insights into growth phases***

147 In order to better understand and interpret the varying relationships we observe between PTRs
148 and CLR abundances, we turn to modeling. The basic properties of growth curves of microbial
149 taxa can be captured using the logistic growth equation (Fig. 4). This model is defined such that
150 the change in abundance for each taxon *i* (dx_i/dt) is captured by the current abundance at time *t*,
151 $x_i(t)$, multiplied by the maximal growth rate, *r*, and the carrying capacity (*k*) term ($1-x_i(t)/k$) [45]. In
152 this model, population size over time shows a sigmoidal curve, with the abundance
153 asymptotically approaching *k* (Fig. 4A, top panel). The derivative of this curve with respect to
154 time yields the change in growth rate over time, which peaks during log-phase growth (Fig. 4A,
155 middle panel). The second derivative of abundance with respect to time, which is the

156 instantaneous change in growth with respect to time and is often referred to as the acceleration
157 rate, shows a peak during the acceleration phase and a trough during the deceleration phase
158 (Fig. 4A, bottom panel). Based on this second-derivative curve, we show the expected
159 relationships between growth rate and abundance as you move across the logistic growth
160 curve, along the time axis (Fig. 4B). These expected relationships provide a potential path
161 forward for inferring the *in situ* growth phase of a bacterial population sampled at a consistent
162 frequency from a flow-through ecosystem.

163 The logistic growth model is a deterministic equation. However, the abundances of
164 commensal bacterial populations in the gut fluctuate due to myriad factors including interspecies
165 competition, resource fluctuations, and stool residence time [46]. In order to approximate these
166 fluctuations in our modeling, we introduced a stochastic term to the logistic growth model (Fig.
167 5A). Herein, σ denotes the noise magnitude and $\omega(t)$ represents a white noise term [27]. Five
168 growth phases (i.e., lag, acceleration, log, deceleration, and stationary phases) were defined
169 using the half-maximum and half-minimum, respectively, of the second derivative LGE curve
170 (Fig. S1A). We grouped these phases into three major categories: lag-acceleration phase, log
171 phase, and deceleration-stationary phase. We simulated 100 iterations of the stochastic logistic
172 growth equation (sLGE) for each of a range of parameterizations (see Methods), which
173 recapitulated the expected statistical relationships between growth rates and abundances for
174 populations consistently sampled within our three major growth phase categories (Fig. 5A-C).
175 For example, the Pearson's R values between growth rates and abundances were significantly
176 positive in lag-acceleration phase and significantly negative in deceleration-stationary phase
177 (Fig. 5B). Log phase growth was more variable, but showed little-to-no significant association
178 between growth rates and abundances. These results were reproduced across a wide range of
179 parameter space and were robust to varying the noise term (Fig. S1B). Overall, the relationships

180 between growth rate and abundance across growth phases were highly consistent with our
181 expectations (Fig. 5C).

182

183 ***Inferring in situ growth phases for abundant gut commensal populations sampled in***
184 ***metagenomic time series***

185 Based on these sLGE results, we assigned putative *in situ* growth phases to abundant gut
186 bacterial populations from the four BIO-ML gut metagenomic time series. Specifically, we
187 suggest that significantly positive associations (linear regression, adjusted *p*-value < 0.05, with a
188 positive beta-coefficient) between PTRs and CLR abundances indicate early-phase exponential
189 growth (i.e. acceleration phase; we can likely exclude lag phase due to the fact that we would
190 be unlikely to detect taxa with very low levels of biomass), significantly negative associations
191 (linear regression, adjusted *p*-value < 0.05, with negative beta-coefficient) indicate deceleration
192 or stationary phases, and the absence of a significant association could indicate either log-
193 phase growth or a false negative (i.e., not powered enough to detect a positive or negative
194 association with the number of time points sampled). *Bacteroides cellulosilyticus*, *Bacteroides*
195 *ovatus* 1, and *Megasphaera eldenii* showed significantly positive PTR-abundance associations
196 within donor ae (Figs. 6A and S2). *Bacteroides ovatus* 1 and *Parabacteroides distasonis*
197 showed positive PTR-abundance associations, while *Alistipes finegoldii*, *Bacteroides uniformis*,
198 and *Bacteroides xylanisolvans* showed negative associations in donor am (Figs. 6A and S3).
199 *Alistipes shahii*, *Bacteroides intestinalis*, *Bacteroides thetaiotaomicron*, *Bacteroides uniformis*,
200 *Bacteroides xylanisolvans*, and *Odoribacter splanchnicus* showed significantly negative PTR-
201 abundance associations in donor an (Fig. 6A and S4). Finally, *Favonifractor plautii* showed a
202 positive PTR-abundance association and *Bacteroides fragilis*, *Bacteroides ovatus* 1,
203 *Bacteroides uniformis*, and *Bacteroides xylanisolvans* showed negative associations in donor ao
204 (Fig. 6A and S5).

205 We observed a slight difference in the number of significantly positive and negative PTR-
206 abundance associations between donors ae/am, and an/ao, with donors an and ao tending to
207 have a larger proportion of negative associations when compared to donors ae and am.
208 Interestingly, donors an and ao had a lower average defecation frequency (≤ 1 per day) than
209 donors ae and am (> 1 per day). Concordantly, based on our flow-through model of the gut
210 ecosystem (Fig. 1A), we would expect that bacterial populations would be pushed towards
211 earlier growth phases at faster flow rates (Fig. 6B). Over half of the taxa with PTR and
212 abundance time series data did not show significant associations (Fig. 6A and Fig. S2-5). This
213 suggests that either these taxa are in the log growth phase or we were not powered enough to
214 detect significant positive or negative associations for these taxa given the effect sizes and the
215 number of samples. We suggest that many of these taxa may well be in log phase, due to the
216 significant association observed between average PTRs and average CLR abundances across
217 donors (Fig. 3C). Overall, our approach provides a new path toward identifying the *in situ* growth
218 phase of microbial populations in flow-through ecosystems.

219

220 **DISCUSSION**

221 Many prior studies assumed, either implicitly or explicitly, that the growth and death rates of gut
222 bacterial populations were proportional to day-to-day changes in abundances, as measured
223 from human stool samples. However, we outline how this assumption is likely invalid due to the

224 fact that human gut bacterial population growth/death processes inside the intestinal tract are
225 known to be faster (minutes-to-hours) than our sampling timescales (days). In support of this
226 assertion, we show how the statistical relationships between changes in abundance ($t_{n+1} - t_n$)
227 and abundances (t_n), estimated from stool metagenomic time series, indicate a regression-to-
228 the-mean effect that one would expect when sampling from a stationary distribution (Figs. 1-2).
229 Thus, as prior work has indicated [15], bacterial taxa in the gut have stable average population
230 sizes, which likely represent steady-state endpoints of internal dynamics (Figs. 1-2). Despite the
231 fundamental mismatch between gut bacterial population dynamics and sampling timescales, we
232 attempt to identify statistical signatures within these daily-sampled human gut time series that
233 might provide accurate insights into *in situ* population dynamics.

234 While changes in abundance between time points (i.e., deltas) do not appear to be
235 related to population growth, PTRs enable direct estimates of *in situ* growth rates from
236 metagenomic samples [22, 23, 47–49]. Unlike the relationships between deltas and
237 abundances, which were always negative (Fig. 2C-D), the relationships between PTRs and
238 abundances were quite variable (Fig. 3A-3B). While regression-to-the-mean is a plausible
239 mechanism for the consistent negative delta-abundance relationships (Fig. 2), the underlying
240 processes driving variable PTR-abundance relationships appear to be more nuanced (Fig. 3).

241 We turned to the sLGE to explore relationships between growth rate and abundance
242 across different phases of growth (Fig. 4). The sLGE recapitulated the key relationships
243 observed in the metagenomic time series between PTRs and abundances and provided
244 predictions for *in situ* growth phases (Figs. 5-6). Consistent with our sLGE predictions, we found
245 that individuals with higher defecation rates tended to be enriched for taxa in earlier growth
246 phases (Fig. 6). Thus, our results reveal a promising approach to inferring *in situ* growth phases
247 for abundant organisms detected in human gut metagenomic time series. A major limitation of
248 this approach is our lack of knowledge about the effect size distributions for these relationships

249 in real-world data and an understanding of the statistical power needed for detecting these
250 associations from metagenomic time series. The absence of a PTR-abundance relationship
251 could represent a false negative or it could indicate log-phase growth. Future *in vitro*
252 experimental work (e.g., in chemostats) should focus on better quantifying these PTR-
253 abundance relationships across parameter space to build a more quantitative understanding of
254 these phenomena (e.g., through varying maximal growth rates, the carrying capacities, the flow
255 rates, and volumes).

256 We observed that the average PTR and average abundance of a given taxon over time
257 were positively correlated, which is consistent with exponentially-growing populations (Fig. 3C).
258 This result is especially relevant to the metabolic modeling community. Ecological interactions
259 within free-living and host-associated microbial communities are largely governed by exchanges
260 of small-molecule metabolites [50, 51]. Genome-scale metabolic modeling and flux-balance
261 analysis (FBA) has been effective mechanistic tools for simulating these metabolic exchanges,
262 especially in controlled bioreactor systems [52]. The objective function used to find a unique
263 solution to bacterial FBA models is often biomass maximization, which assumes that these
264 organisms are growing exponentially at steady state. Exponential growth is a valid assumption
265 for organisms in acceleration or log growth phases, but this assumption is violated for
266 organisms in deceleration or stationary phases. Prior work has demonstrated that biomass
267 composition can change depending on the growth phase of a population, which ideally would be
268 taken into account to more accurately model metabolic fluxes within the system [53–55].
269 Overall, our work suggests that most organisms in the human gut are amenable to FBA, and our
270 growth phase estimation approach allows for the identification of populations that may not fit
271 classical FBA assumptions.

272 In conclusion, we provide a new path forward for the biological interpretation of
273 metagenomic time series data generated from adult human stool samples. We hope that *in situ*

274 growth phase estimation will be applied more broadly to other kinds of flow-through
275 environments to improve our understanding of internal dynamics in these systems and provide
276 improved constraints for mechanistic modeling of microbial communities.

277

278 **METHODS**

279 **Shotgun metagenomics data processing and analysis**

280 Longitudinal shotgun metagenomics sequencing data from healthy human stool samples was
281 downloaded from NCBI BioProject accession PRJNA544527, and the associated metadata was
282 downloaded from the associated article [32]. Raw FASTQ files were filtered and trimmed using
283 FASTP [56], removing the first 5 nucleotides of the read 5' end to avoid leftover primer and
284 adapter sequencing not removed during demultiplexing and an adaptive sliding window filter on
285 the 3' end of the read with a required minimum quality score of 20. Reads containing ambiguous
286 base calls, having a mean quality score less than 20, or with a length smaller than 50nt after
287 trimming were removed from the analysis. Taxonomic assignment on the read level was
288 performed with Kraken2 using the Kraken2 default database [57]. Abundances on the kingdom,
289 phylum, genus, and species ranks were then obtained using Bracken [58]. Trimmed and filtered
290 reads were then aligned to 2,935 representative bacterial reference genomes taken from the
291 IGG database (version 1.01) using Bowtie2 [59, 60]. Coverage profiles and log2 estimates of
292 peak-to-trough ratios were estimated using COPTR v1.1.2 on the species level within each
293 sample [61]. PTR estimates were then merged with Bracken abundance estimates, retaining
294 only those species identified by both methods (Kraken2 and Bowtie2 alignment to IGGdb).

295 The processed data containing the raw reads and \log_2 peak-to-trough ratios (\log_2 PTRs)
296 were read into R version 4.1.3 for analysis ([62]). All plots were generated using ggplot2 [63],
297 unless indicated otherwise. Donor time series were selected by only retaining individuals with
298 over 50 metagenomic time points, resulting in four time series (i.e., donors ae, am, an, and ao).

299 Distinct *Bacteroides ovatus* strains across all four donors contained duplicated taxon names
300 with unique taxonomic identifiers, and were renamed to “*Bacteroides ovatus_1*” and
301 “*Bacteroides ovatus_2*.” Raw read counts for a given taxon within a sample were centered log-
302 ratio (CLR) transformed [64]. Taxa that had matched log₂PTR information available across more
303 than 5 time points within an individual, with time differences between samples less than three
304 days, were used in subsequent analyses. Changes in normalized abundance were calculated
305 as *Abundance changes(delta)* = $x(t + 1) - x(t)$, where $\Delta t < 3$ days. To assess the regression-
306 to-the-mean effect, CLR-normalized abundances were plotted against deltas for each taxon,
307 and the regression coefficients, aggregating all microbial taxa, were plotted as boxplots
308 (showing median and interquartile range), summarized by donor.

309 For each donor, to estimate the growth phase of each individual taxon, we used linear
310 regression of CLR-normalized abundances vs. log₂PTRs, followed by a Benjamini-Hochberg p-
311 value correction to control for the false discovery rate (FDR) in base R. FDR-adjusted p-values
312 < 0.05 were considered significant. Taxa with significantly positive or negative associations
313 were considered to be in lag-acceleration or deceleration-stationary phase, respectively. Those
314 with no correlation were not assigned a growth phase, as this result could either be a false
315 negative or indicative of log-phase growth. Linear regression was also used to test whether or
316 not average CLR-normalized abundances and average log₂PTRs were significantly associated
317 within each donor, and p-values from individual tests were combined using Fisher’s method
318 [65].

319

320 **Stochastic logistic growth model simulation**

321 The stochastic logistic growth equation (SLE) was implemented as: $\frac{dx_i}{dt} = rx_i(t) \left(1 - \frac{x_i(t)}{K}\right) +$
322 $\sigma x_i(t)\omega(t)$, where t is time, r is the growth rate, x_i is the abundance of taxon i , K is the carrying
323 capacity, σ is the noise magnitude term, and $\omega(t)$ is the noise distribution term. Using the R

324 package sde [66], taxonomic growth was simulated with $x_{i,0} = 1$, $t_0 = 1$ to $t_{final} = 100$, for 100
325 iterations. The other parameters were varied as described in the results and below. To
326 investigate the impact of noise on sLGE trajectories, noise levels were set from 0.001 to 1, with
327 r and K ranging from 1 to 3 and 10 to 1000, respectively. To investigate the statistical
328 relationships between deltas and abundances across growth phases and across model
329 parameterizations, Pearson's R coefficients and p-values were calculated for each of the three
330 growth phase categories. The growth phases for each model parameterization were defined
331 using the non-stochastic logistic growth equation (LGE): $\frac{dx_i}{dt} = rx_i(t) \left(1 - \frac{x_i(t)}{K}\right)$, the solution for
332 which can be written as $x_i = \frac{x_{i,0}Ke^{rt}}{(K-x_{i,0})+x_{i,0}e^{rt}}$.

333 The x_i values for each simulated time point from solving the LGE were used to calculate
334 the first derivative (i.e., the growth rate), which is exactly equal to the LGE. The second
335 derivative (i.e., growth acceleration), $\frac{d^2x_i}{dt^2} = K^2x_i \left(1 - \frac{x_i}{K}\right) \left(1 - \left(\frac{2x_i}{K}\right)\right)$, was calculated using
336 solved x_i values. Growth phases from the SLM were defined using the second derivative curves.
337 First, the intersections of the acceleration curve and the half-max, a_1 and a_2 , and the half-
338 min, a_3 and a_4 , were calculated (Fig. S1). The corresponding simulated time points of a_j ,
339 denoted as s_j , where $j = 1 - 4$, were then used to define growth phases as follows: lag
340 phase: $t < s_1$; acceleration phase: $s_1 < t < s_2$; log phase: $s_2 < t < s_3$; deceleration phase: $s_3 <$
341 $t < s_4$; and stationary phase: $t > s_4$. Here, lag and acceleration, and deceleration and stationary
342 phases were combined, as these phases display similar delta-abundance relationships along
343 the logistic growth curve. Conceptual diagrams were created using BioRender.

344

345 **Data and code availability**

346 Nextflow pipelines implementing the processing of metagenomic shotgun sequencing data from
347 raw reads to taxonomic abundance matrices and PTR estimates can be found at

348 <https://github.com/Gibbons-Lab/pipelines/> (metagenomics pipelines). R scripts and code used to
349 analyze the data, run the sLGE simulations, and produce the figures in the manuscript have
350 been deposited at [https://github.com/Gibbons-Lab/human-microbiome-time-series-growth-](https://github.com/Gibbons-Lab/human-microbiome-time-series-growth-phase-estimation)
351 [phase-estimation](https://github.com/Gibbons-Lab/human-microbiome-time-series-growth-phase-estimation).

352

353 **Acknowledgements**

354 We would like to thank Shijie Zhao for suggesting that we investigate PTR-abundance
355 relationships in the BIO-ML data set. We would also like to thank Nitin Baliga, Amy Willis, Julia
356 Cui, and the members of the Gibbons Lab for helpful discussions of this work. SMG and CD
357 were supported by a Washington Research Foundation Distinguished Investigator Award and
358 by startup funds from the Institute for Systems Biology. JL was supported by the Environmental
359 Pathology/Toxicology training grant (ES007032).

360

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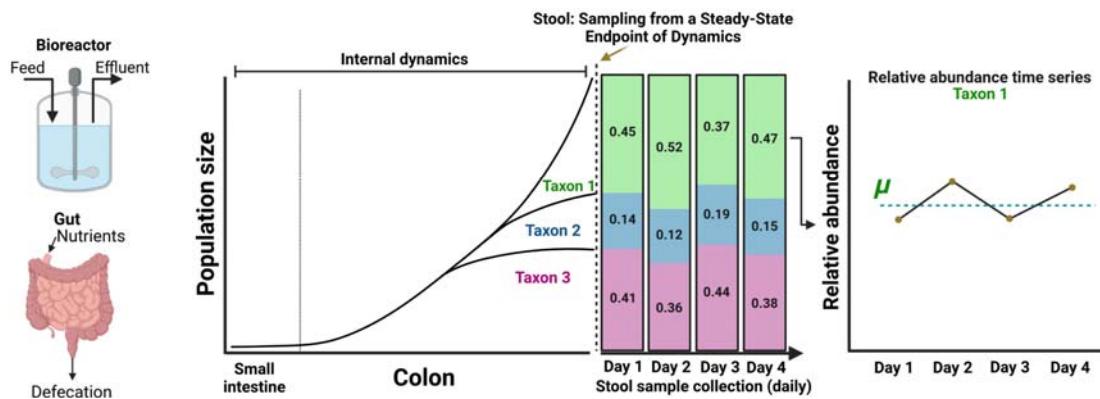
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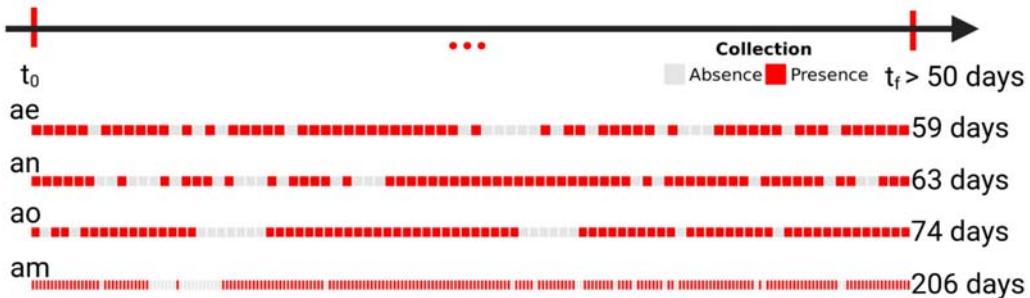
527 **FIGURES**

A



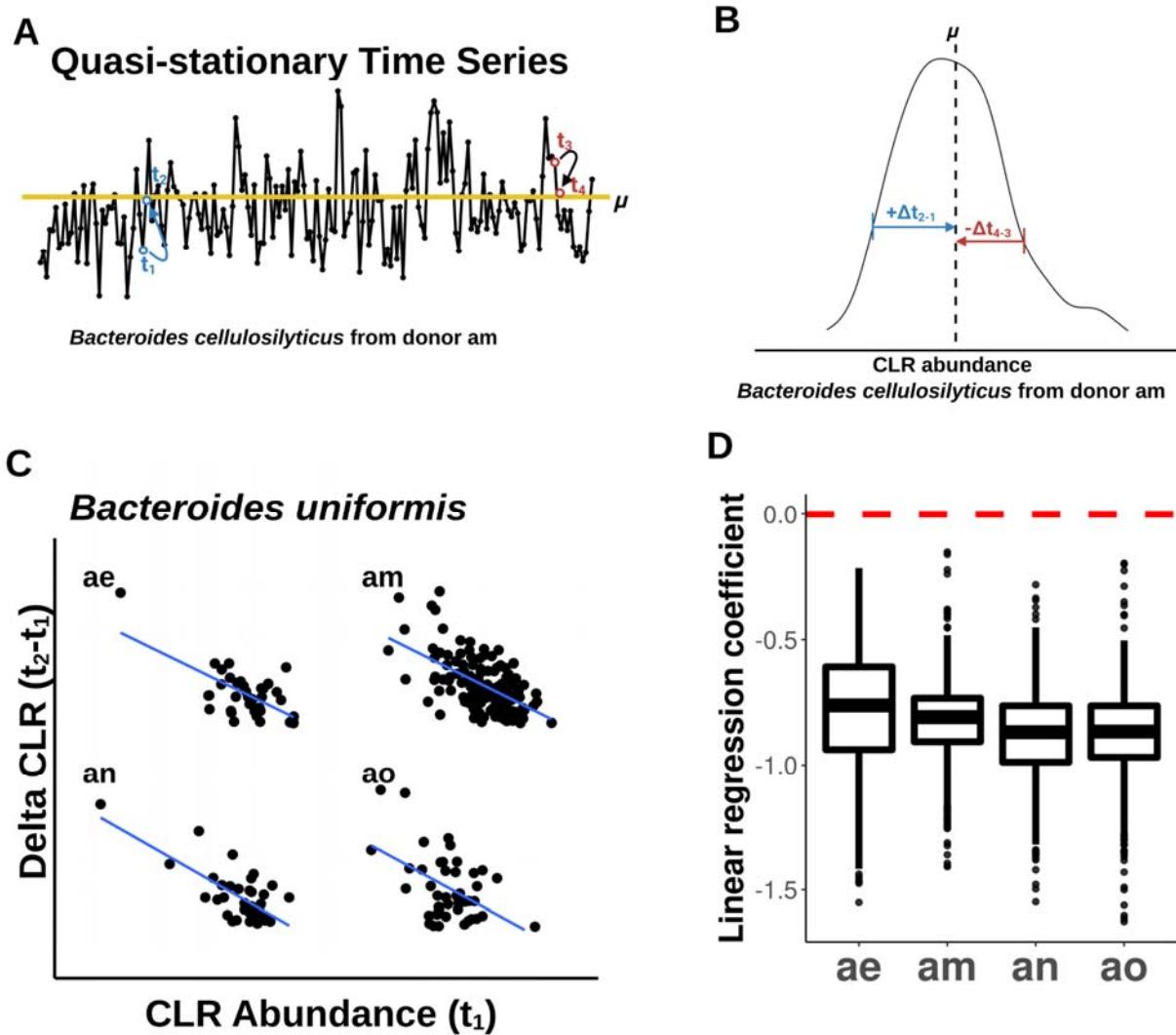
B

4 healthy human donors longitudinal fecal microbiome collection



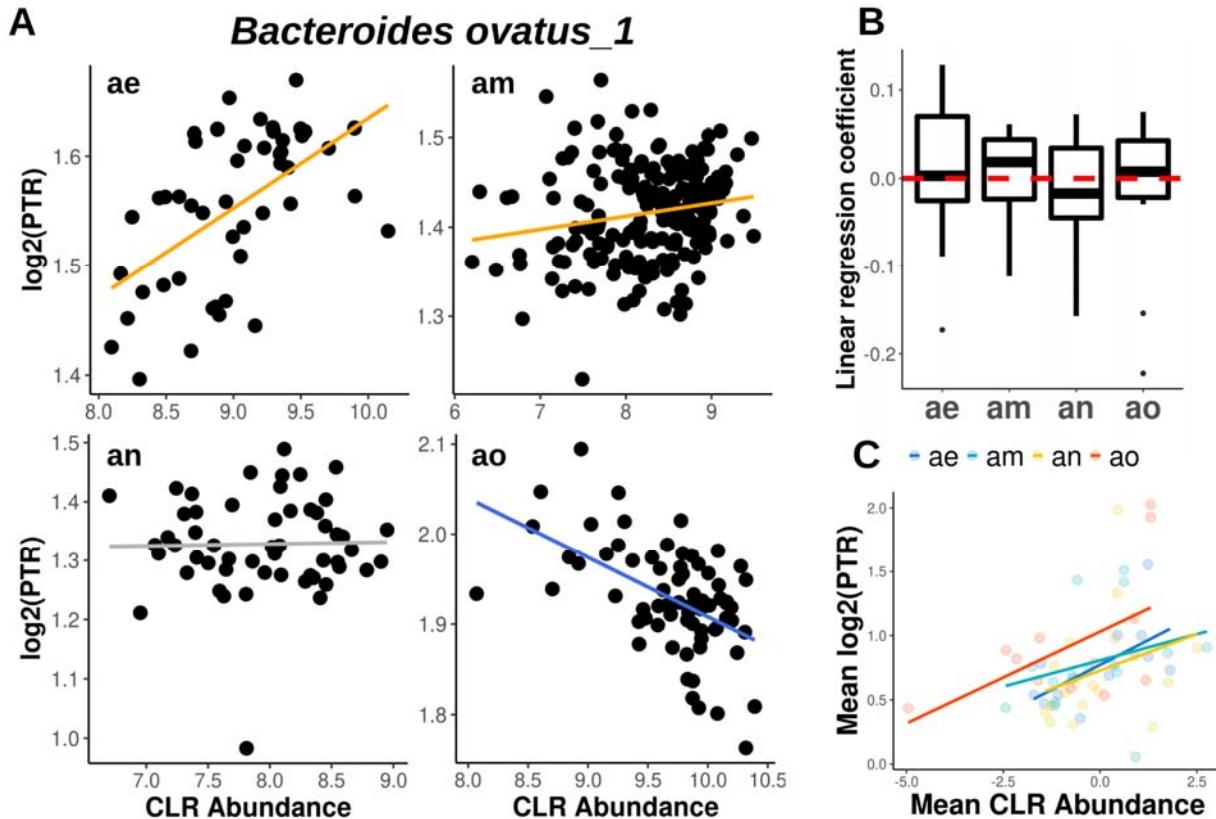
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529 **Figure 1. Conceptual figure showing two flow-through microbial ecosystems: a**
530 **bioreactor and a human gut. A.** Both bioreactors and guts are continuous flow-through
531 systems. Prior to reaching the measured abundances in stool, taxa grow in the large intestine
532 with varying growth rates, carrying capacities, and steady-state population sizes, which may be
533 in different growth phases at the time of measurement. For example, see dynamics for Taxa 1-
534 3. Daily stool collections show variation in abundances, but this variation likely does not reflect
535 internal growth dynamics in the gut. **B.** Healthy BIO-ML stool donors (subject IDs: ae, am, an,
536 and ao) with samples collected 3-5 days per week for a total of >50 time points. Red indicates
537 presence of shotgun metagenomic sequencing data and gray represents absence of
538 metagenomic data from consecutive daily time points.



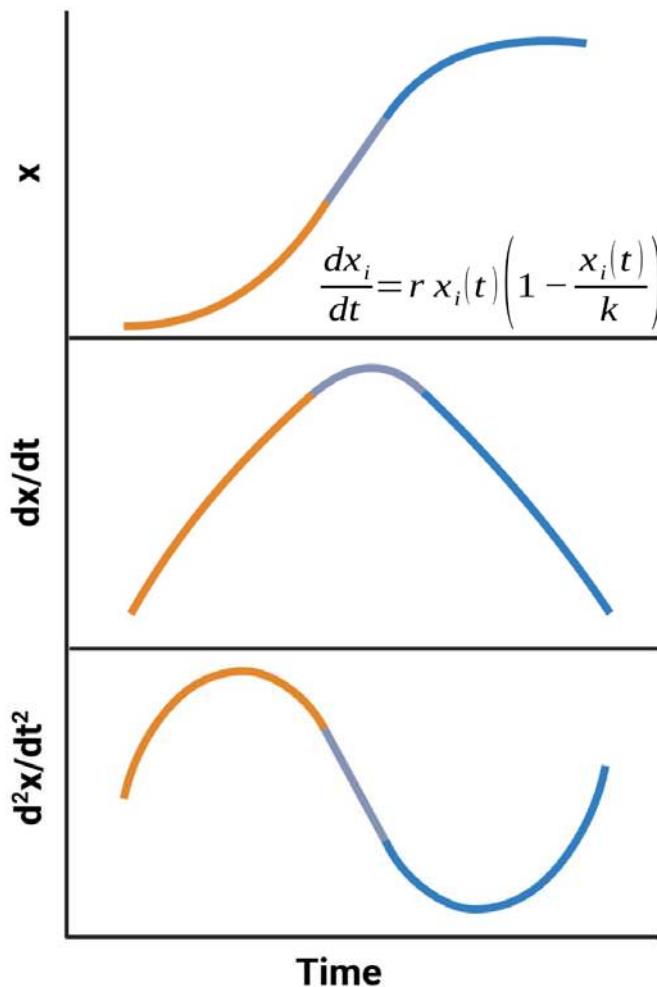
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540 **Figure 2. Regression-to-the-mean effect in human microbial time series data.** A. Yellow
541 line represents the mean abundance (μ) of *Bacteroides cellulosilyticus* over time in donor am.
542 Time points t_1 and t_3 indicate fluctuations below and above the mean abundance, and t_2 and t_4
543 show the return to the mean abundance. B. Distribution of time series delta values (e.g., t_2-t_1)
544 for *Bacteroides cellulosilyticus* in donor am, which is approximately normally distributed. C.
545 Deltas vs. abundances for *Bacteroides uniformis* time series from donors ae, am, an, and ao.
546 D. Boxplots (showing median and interquartile range) of linear regression coefficients for deltas
547 vs. abundances across all taxa time series in all four donors. Red line indicates a regression
548 coefficient of 0.

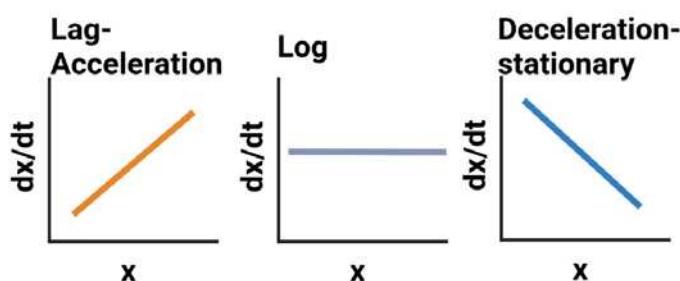


550 **Figure 3. Variable relationships between PTRs and CLR-normalized abundances across**
551 **human gut microbial time series. A.** Log₂(PTR) and CLR-normalized abundance relationships
552 for donors ae, am, an, and ao. Orange and blue lines show significantly positive and negative
553 linear regression coefficients (linear regression, FDR adjusted *p*-value < 0.05), respectively.
554 Gray lines indicate no statistically significant association. **B.** Boxplots (showing median and
555 interquartile range) of linear regression coefficient combined for all filtered taxa for each donor.
556 **C.** Mean log₂(PTR) and mean CLR-normalized abundance for all abundant taxa in each donor
557 (*p*-values for regressions run within each donor were combined using Fisher's method;
558 combined *p*-value = 0.005).

A Logistic growth of microbiota

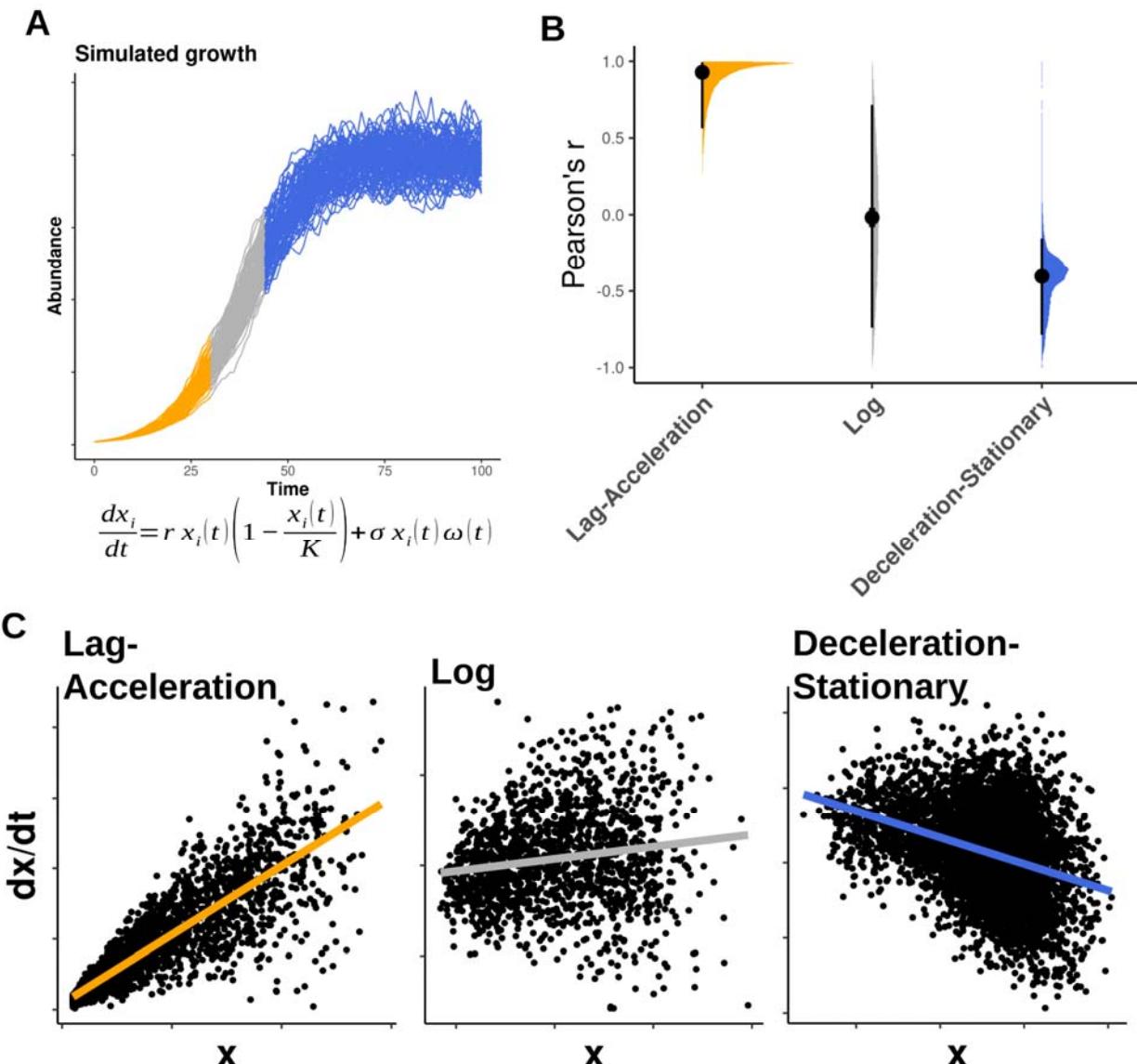


B



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560 **Figure 4. Logistic growth model. A.** The logistic growth curve models abundance (x) with
561 respect to time (top panel). The first derivative of the logistic growth curve models the growth
562 rate with respect to time (middle panel). The second derivative of the logistic growth curve
563 models growth rate acceleration with respect to time (bottom panel). **B.** Expected relationships
564 between abundance and growth rate at different locations along the logistic growth curve.



566 **Figure 5. Distinguishing growth phases using the stochastic logistic growth model. A.**
567 Stochastic logistic growth curves with growth rate (r) = 1.2, carrying capacity (K) = 100, and
568 noise level (σ) = 0.1 across 100 iterations. Major growth phase groups in orange (lag-
569 acceleration), gray (log), and blue (deceleration-stationary). **B.** Pearson's R values between
570 abundances and growth rates in each of our three growth phase windows across variable model
571 parameterizations (r = 1-3, K = 10-1000) and a fixed noise level (σ = 0.1). **C.** Scatter plots
572 showing relationships between abundances and deltas across the three growth phase regions
573 defined in panel A.

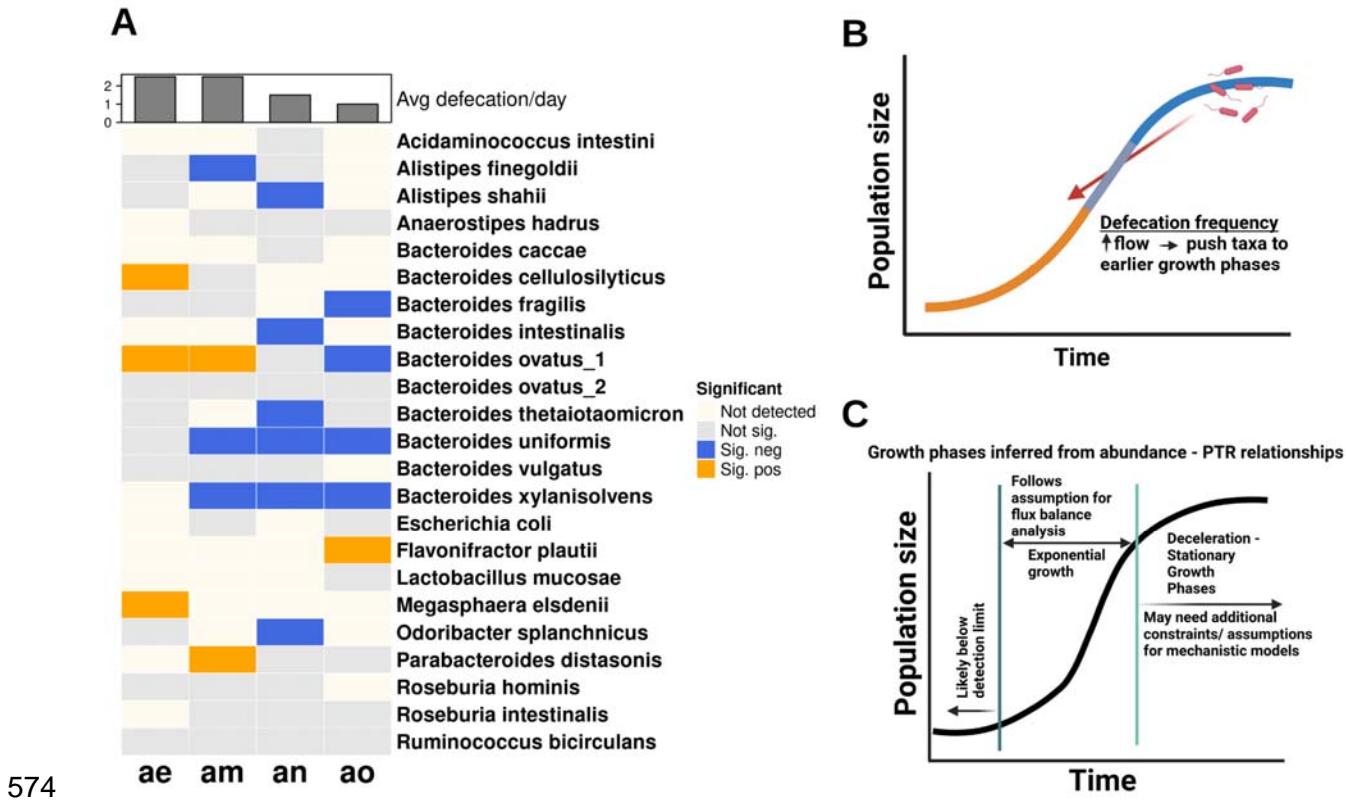
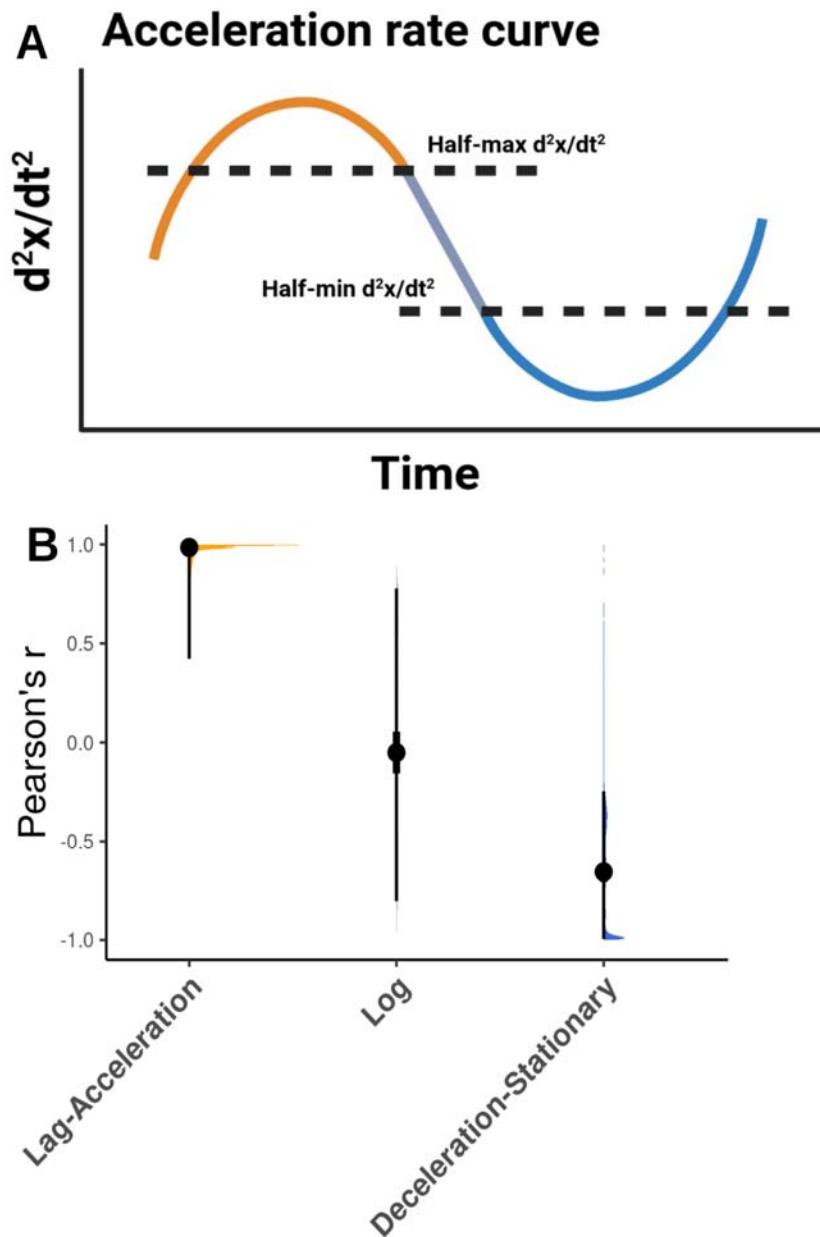


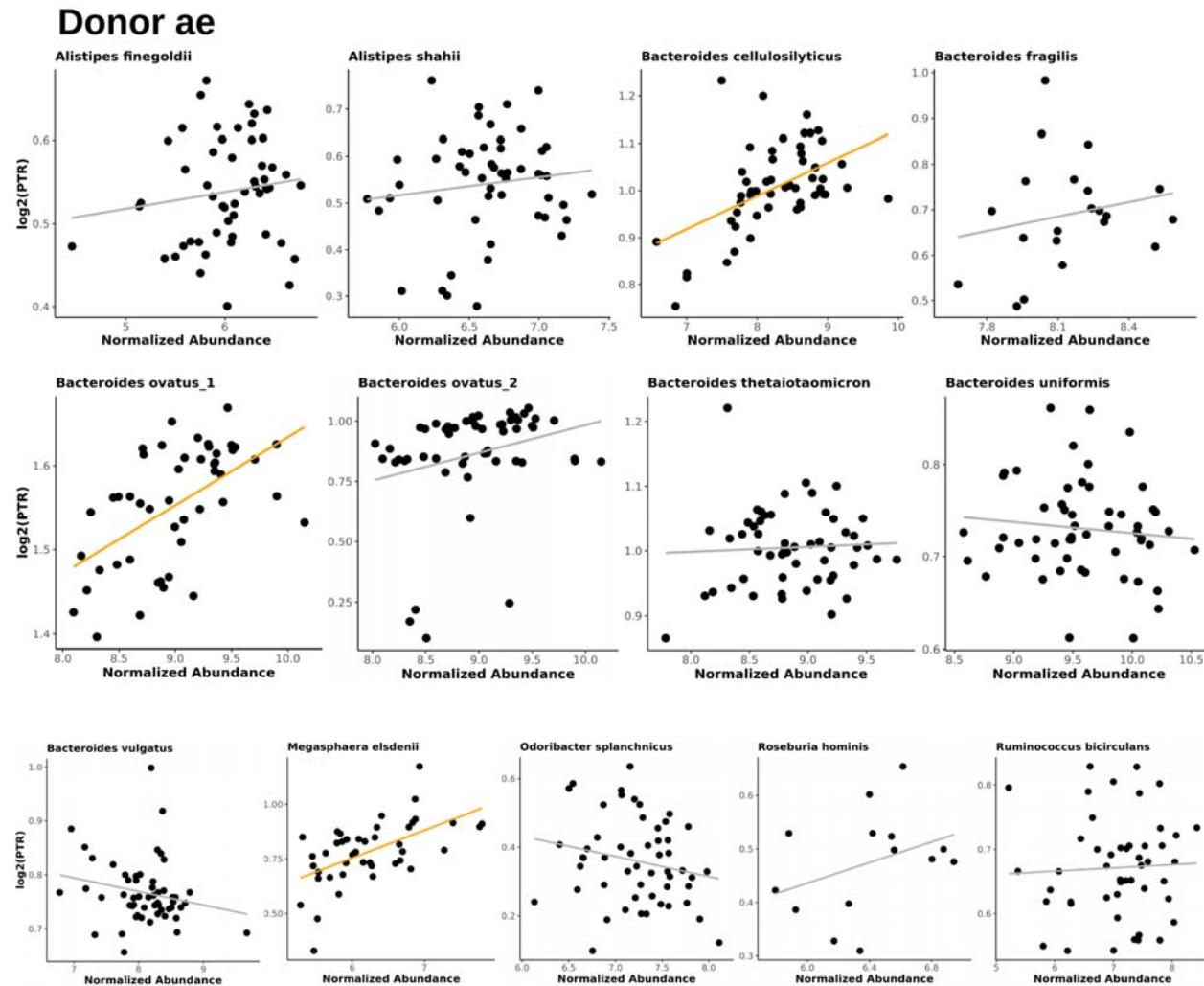
Figure 6. *in vivo* growth phase estimation. **A.** We find variable relationships between PTRs and population abundances across taxa in each of the four donors, consistent with the growth phase patterns observed in sLGE simulations. Donors with higher defecation rates tended to have a larger fraction of taxa with positive PTR-abundance associations and fewer with negative associations, indicating lag-acceleration and deceleration-stationary phases, respectively. **B.** We suggest that higher defecation rates (i.e., higher dilution rates) push bacterial populations towards earlier growth phases, which is consistent with our results in panel A. **C.** Growth phase estimates can be leveraged to identify taxa that are more-or-less amenable to metabolic modeling techniques, like Flux Balance Analysis, which assumes exponential growth.

591 **SUPPLEMENTAL FIGURES**



592

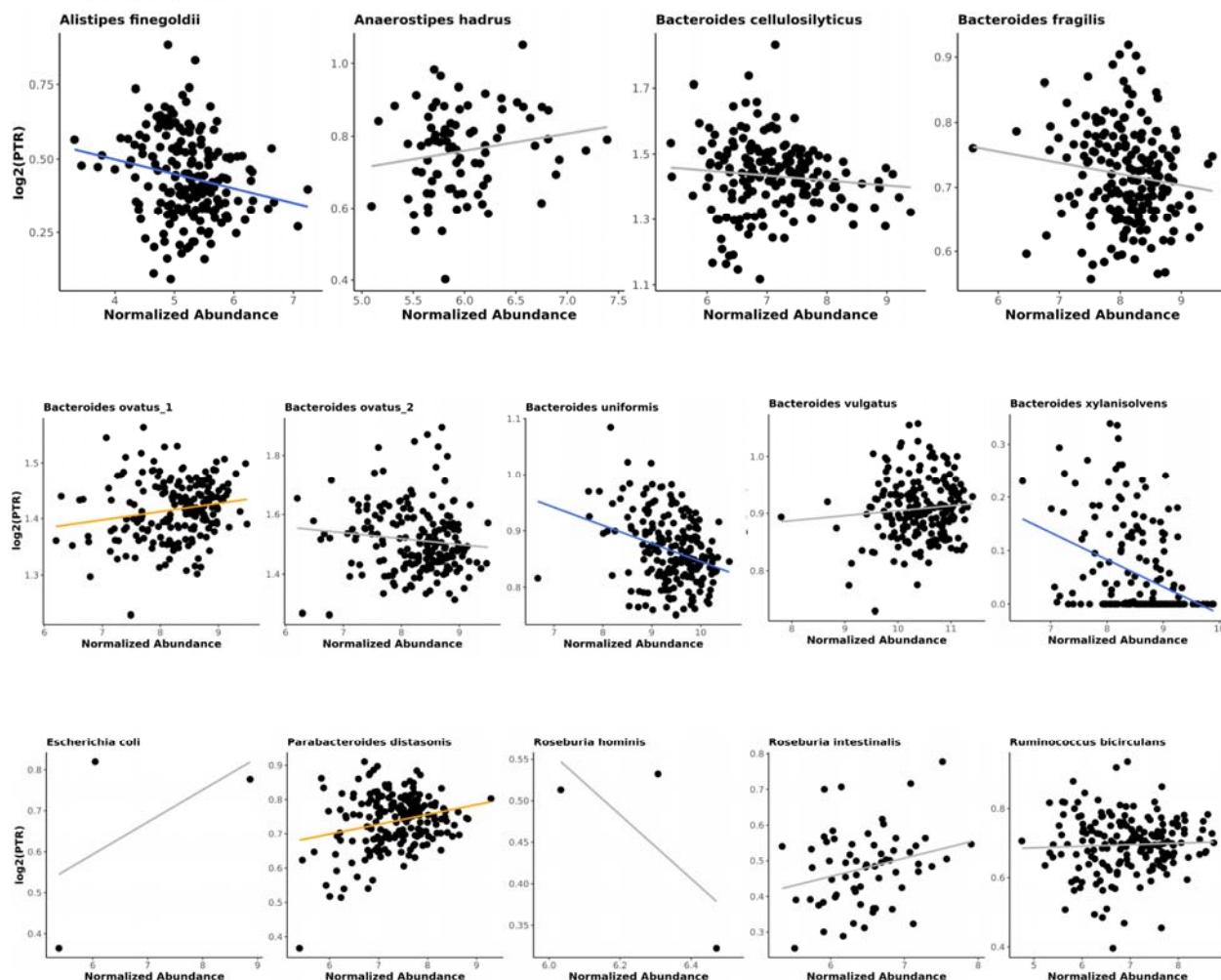
593 **Figure S1. Definition of major growth phases using the stochastic logistic growth model.**
594 **A.** The half-maximum of the peak and half-minimum of the trough of the second derivative of
595 abundance were used to define growth phases across model parameterizations. **B.** Pearson's R
596 values between abundances and growth rates in the three growth phase categories obtained
597 from combined sLGE simulation results across a range of growth rates ($r = 1-3$), carrying
598 capacities ($k = 10-1000$), and noise levels ($n = 0.001-1$).



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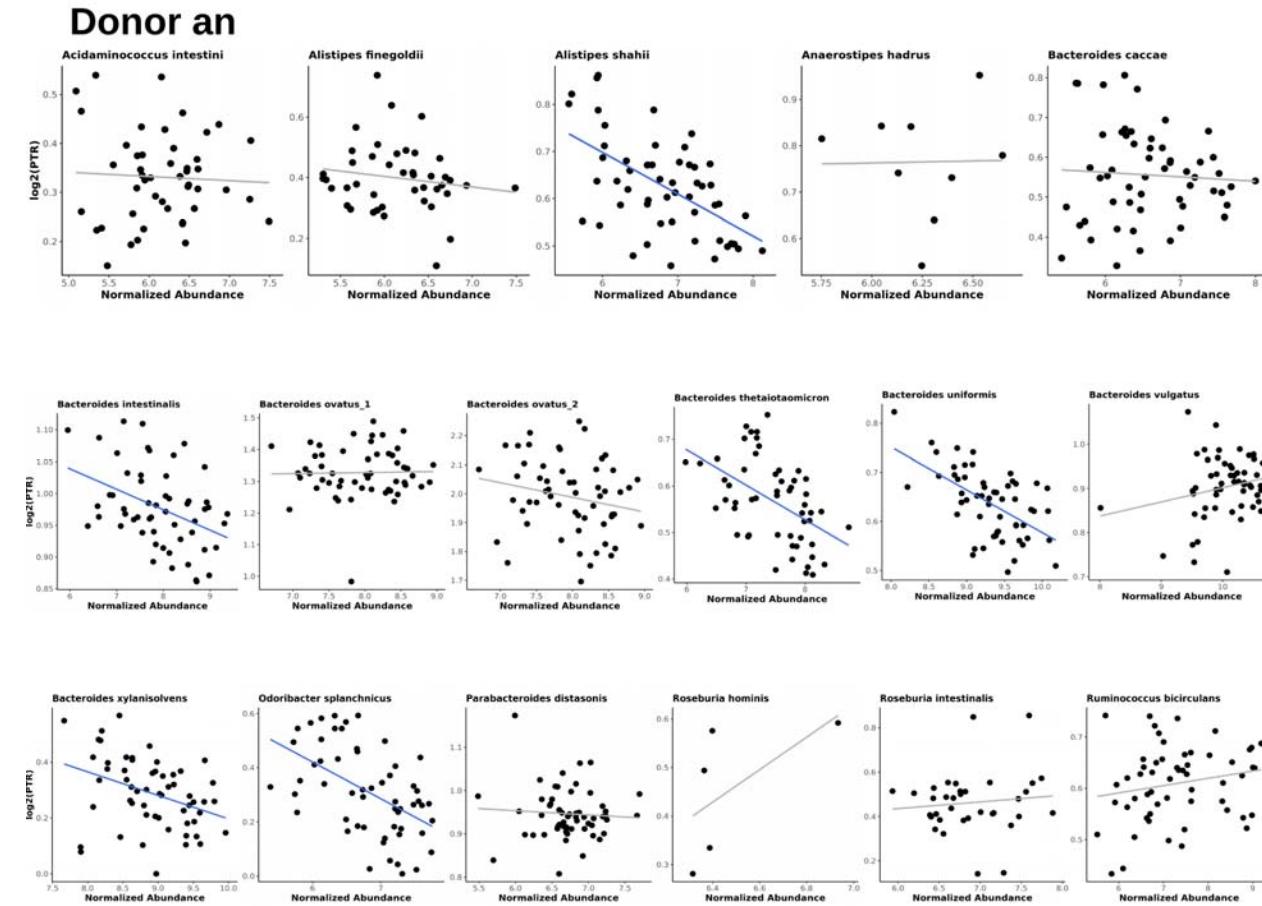
Figure S2. Relationships between abundance and $\log_2(\text{PTR})$ for abundant taxa in donor ae. Abundant taxa with relatively dense longitudinal PTR and abundance data (at least 5 matched data points; time differences between adjacent samples less than three days) were selected for analysis. Gray trend lines show no significant correlations, orange trend lines indicate significant positive correlations, and blue trend lines represent significant negative correlations (linear regression, BH-FDR < 0.05).

Donor am

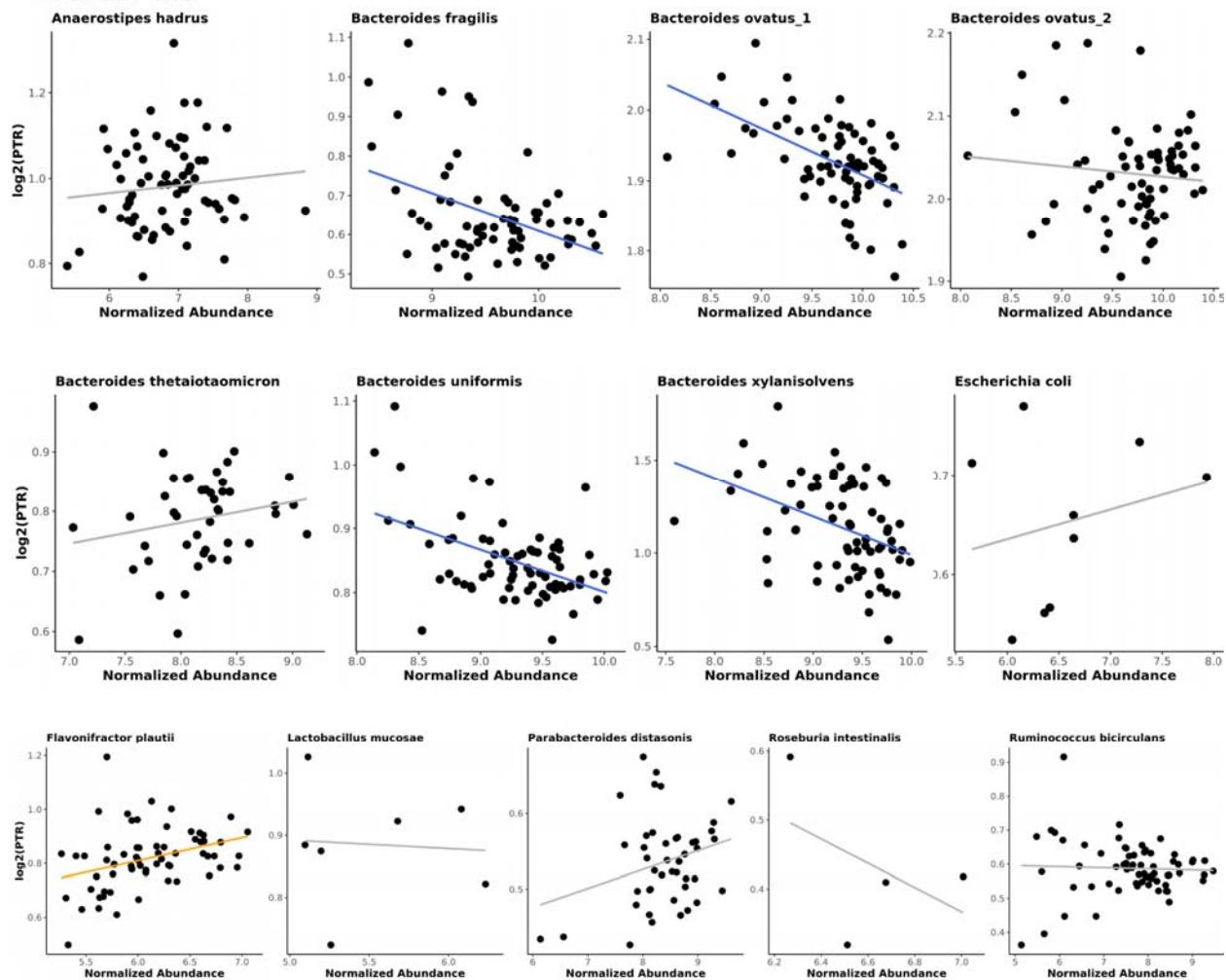


606

607 **Figure S3. Relationships between abundance and $\log_2(\text{PTR})$ for abundant taxa in donor**
608 **am.** Abundant taxa with relatively dense longitudinal PTR and abundance data (at least 5
609 matched data points; time differences between adjacent samples less than three days) were
610 selected for analysis. Gray trend lines show no significant correlations, orange trend lines
611 indicate significant positive correlations, and blue trend lines represent significant negative
612 correlations (linear regression, BH-FDR < 0.05).



Donor ao



620

621 **Figure S5. Relationships between abundance and log₂(PTR) for individual taxon in donor**
622 **ao.** Abundant taxa with relatively dense longitudinal PTR and abundance data (at least 5
623 matched data points; time differences between adjacent samples less than three days) were
624 selected for analysis. Gray trend lines show no significant correlations, orange trend lines
625 indicate significant positive correlations, and blue trend lines represent significant negative
626 correlations (linear regression, BH-FDR < 0.05).

627