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4 **AlphaImpute2: Fast and accurate pedigree and population based imputation for hundreds**  
5 **of thousands of individuals in livestock populations**

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40 **Code availability**

41 The code used to simulate the data in this study is available from the authors upon a reasonable

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44

45 **Author contributions**

46 AW designed the imputation algorithm. AW and JH designed the simulations. AW ran  
47 the simulations and analyzed the results. All authors contributed to writing the manuscript and  
48 approved the final manuscript

49

50

## Abstract

51        In this paper we present a new imputation algorithm, AlphaImpute2, which performs fast  
52        and accurate pedigree and population based imputation for livestock populations of hundreds of  
53        thousands of individuals. Genetic imputation is a tool used in genetics to decrease the cost of  
54        genotyping a population, by genotyping a small number of individuals at high-density and the  
55        remaining individuals at low-density. Shared haplotype segments between the high-density and  
56        low-density individuals can then be used to fill in the missing genotypes of the low-density  
57        individuals. As the size of genetics datasets have grown, the computational cost of performing  
58        imputation has increased, particularly in agricultural breeding programs where there might be  
59        hundreds of thousands of genotyped individuals. To address this issue, we present a new  
60        imputation algorithm, AlphaImpute2, which performs population imputation by using a particle  
61        based approximation to the Li and Stephens which exploits the Positional Burrows Wheeler  
62        Transform, and performs pedigree imputation using an approximate version of multi-locus  
63        iterative peeling. We tested AlphaImpute2 on four simulated datasets designed to mimic the  
64        pedigrees found in a real pig breeding program. We compared AlphaImpute2 to AlphaImpute,  
65        AlphaPeel, findhap version 4, and Beagle 5.1. We found that AlphaImpute2 had the highest  
66        accuracy, with an accuracy of 0.993 for low-density individuals on the pedigree with 107,000  
67        individuals, compared to an accuracy of 0.942 for Beagle 5.1, 0.940 for AlphaImpute, and 0.801  
68        for findhap. AlphaImpute2 was also the fastest software tested, with a runtime of 105 minutes a  
69        pedigree of 107,000 individuals and 5,000 markers was 105 minutes, compared to 190 minutes  
70        for Beagle 5.1, 395 minutes for findhap, and 7,859 minutes AlphaImpute. We believe that  
71        AlphaImpute2 will enable fast and accurate large scale imputation for agricultural populations as  
72        they scale to hundreds of thousands or millions of genotyped individuals.

73 **Introduction**

74 In this paper we present a new imputation algorithm, AlphaImpute2, which performs fast  
75 and accurate pedigree and population based imputation for livestock populations of hundreds of  
76 thousands of individuals. Genetic imputation is a commonly used tool in agricultural and human  
77 genetics. It can be used to decrease the cost of genotyping individuals by allowing only a small  
78 number of individuals to be genotyped on a high-cost high-density genotyping platform, and the  
79 remaining individuals to be genotyped on a lower-cost lower-density platform. Shared haplotype  
80 segments between the low-density and the high-density individuals are then used to fill in  
81 missing genotypes for the low-density individuals [1,2]. Low cost genotypes are important for  
82 increasing the rate of genetic gain in animal and plant breeding programs [3–5]. As genotyping  
83 animals has become a routine part of breeding operations, many agricultural datasets contain  
84 hundreds of thousands, or even millions of genotyped individuals [6,7] which means that  
85 imputation algorithms must be to scale to ever expanding datasets.

86 Genetic imputation algorithms use either (1) pedigree or family information to perform  
87 imputation, which rely on long shared haplotype segments between an individual and their  
88 parents, (2) population information to perform imputation, which rely on haplotype sharing  
89 between an individual and distant relatives, or (3) both sources of information in a combined  
90 algorithm. Pedigree based imputation tends to be fast and accurate, but requires the pedigree of  
91 the population to be known, and many of the founders to be genotyped at high density [8–11].  
92 Population based imputation tend to be slower and less accurate, particularly at low marker  
93 densities, but can perform imputation on individuals with unknown parents and no known  
94 genotyped relatives [2,12]. Population and pedigree based imputation can be effectively  
95 combined for livestock populations: pedigree information is used to impute the genotypes of

96 most individuals, and population information is used to impute the remaining genotypes,  
97 particularly those of founders or individuals with ungenotyped parents [9,13,14]. When aiming  
98 to improve the scaling of a combined imputation algorithm, most of the runtime tends to occur in  
99 the population imputation steps [13].

100 There have been a large number improvements in the runtime of population based  
101 imputation algorithms, particularly those based on the “Li and Stephens” hidden Markov model  
102 framework [2]. In this framework, an individual’s genotypes are modelled as a mosaic of pairs of  
103 haplotypes from the reference library. The reference library represents possible ancestral  
104 haplotypes in the population, and generally consists of all of the phased haplotypes of the high-  
105 density individuals. By itself, this algorithm scales poorly, with a runtime that is quadratic with  
106 the number of haplotypes in the reference library. Runtime can be improved by either using a  
107 fixed subset of haplotypes from the reference library [15,16], or by using a phasing algorithm to  
108 pre-phase the data and running haploid hidden Markov model separately on each phased  
109 chromosome [17,18]. Because the haploid hidden Markov model only needs to consider one  
110 chromosome at a time, it scales linearly with the number of haplotypes in the reference panel,  
111 allowing it to scale to reference panels of tens of thousands of haplotypes.

112 For reference haplotypes with hundreds of thousands of haplotypes, scaling can be  
113 improved by employing the Positional Burrows Wheeler Transform (PBWT; [19]). The PBWT  
114 is an opportunistic data structure which lexicographically sorts the haplotypes at each loci. By  
115 sorting the library in this way, it is possible to search through the haplotype reference library for  
116 a given haplotype segment in constant time (independent of the size of the library). The creation  
117 of the PBWT is linear in both the number of markers and number of individuals, but once  
118 created, it can be re-used for all of the individuals genotyped with the same set of markers. There

119 are a growing number of approaches for using the PBWT to speed up the runtime of imputation,  
120 e.g., by using it to find a fixed-number of reference haplotypes to use for haploid imputation  
121 [20], find “maximally matching” haplotype segments [19], or implement a Viterbi algorithm by  
122 using a branch and bound search [21].

123 In this paper we first present a new population imputation algorithm which uses the  
124 PBWT to perform a guided stochastic search through the haplotype reference library. The idea  
125 behind this algorithm is to focus on combinations of haplotypes that have high posterior  
126 probability. We do this by creating a series of particles and having them explore the high  
127 probability paths through the haplotype reference library. Normally the number of particles we  
128 would need to use would scale based on the size of the haplotype reference library. We solve this  
129 issue by having the particles represent all of the haplotypes in a region with the same genotype  
130 state. We then use the PBWT to update each of these particles in constant time, which allows this  
131 approach to scale to large reference haplotype libraries.

132 We also present a refined version of multi-locus iterative peeling which has greatly  
133 reduced runtime and memory requirements compared to previous versions [22]. Multi-locus  
134 iterative peeling is a probabilistic method for performing pedigree based imputation, that has  
135 high accuracy particularly in the presence of genotyping errors [9,22,23]. However, multi-locus  
136 iterative peeling has traditionally been too computationally intensive to use for routine  
137 imputation, and most pedigree based imputation algorithms use heuristic methods to perform  
138 population imputation [8,11]. We found that it was possible to greatly increase the speed of  
139 multi-locus iterative peeling by approximating the joint genotype probabilities of an individual’s  
140 parents, and by calling the segregation and genotype states when estimating an offspring’s

141 contribution to their parent's genotypes. These approximations appear to have limited impact on  
142 imputation accuracy.

143 Finally, we present a combined algorithm which integrates the population and pedigree  
144 imputation algorithm.

145 We have implemented the population, pedigree, and combined imputation algorithms in a  
146 new software package, AlphaImpute2. We compared the performance of AlphaImpute2 to  
147 AlphaImpute [8], AlphaPeel [22], findhap version 4 [24] , Beagle 4.1 [25], and Beagle 5.1 [26]  
148 on a series of simulated datasets designed to mimic four real pig pedigrees. We find that  
149 AlphaImpute2 has high accuracy and low runtimes, achieving an average accuracy of .99 for  
150 low-density individuals across all four pedigrees. The runtime for imputing a single chromosome  
151 of a pedigree of 107,000 individuals with 5,000 markers was just over two hours. Compared to  
152 the other software, AlphaImpute2 had higher accuracy and lower run-times in most situations.

153

154 **Materials and Methods**

155 **Population imputation using particles**

156 AlphaImpute2 performs population based imputation using an approximate version of the Li and  
157 Stephens algorithm [2]. In the Li and Stephens algorithm models an individual's genotype is  
158 constructed as a mosaic of haplotypes from a haplotype reference panel. This can be  
159 implemented in a hidden Markov model where the state space consists of pairs of haplotype  
160 identifiers from the reference panel at each loci, and inference is done to find a high-likelihood  
161 path, or sequence of haplotype identifiers, through this space. The path can then be used to  
162 impute and phase missing genotypes by looking at that state of each haplotype along the path.

163 Because the state space grows quadratically with the size of the reference panel, approximations  
164 are needed to perform inference.

165 Lunter [21] published an algorithm to produce an exact maximum likelihood (Viterbi)  
166 path for a diploid Li and Stephens algorithm in constant time. This approaches uses the  
167 Positional Burrows Wheeler Transform [19] as an opportunistic data structure that allows  
168 searching across many similar haplotypes at the same time. The approach we use here is loosely  
169 based on the framework of Lunter [21]. Instead of taking the maximum likelihood path, we  
170 instead generate samples from an approximate posterior distribution over all possible paths  
171 through the haplotype reference library. The logic behind this choice is that there may be many  
172 paths with similarly high likelihoods, and by combining information from multiple samples it  
173 may be possible to obtain more accurate genotypes than from any single sample.

174 To run the algorithm we construct a series of series of particles. Each particle consists  
175 of a pair of ranges of haplotypes at each loci,  $(\phi_x^i, \psi_y^i)$  where  $\phi_x^i$  and  $\psi_y^i$  give the set of  
176 haplotypes at loci  $i$ , whose values at preceding loci are given by the sequences  
177  $x = \{x_{i-n}, x_{i-n+1}, \dots, x_i\}$  or  $y = \{y_{i-n}, y_{i-n+1}, \dots, y_i\}$ .  $\phi_x^i$  gives the haplotypes for the paternal  
178 chromosome, and  $\psi_y^i$  gives the haplotypes for the maternal chromosome. At each loci we  
179 probabilistically update the each particle to a new state  $(\phi_{x*}^{i+1}, \psi_{y*}^{i+1})$  using a guided stochastic  
180 search algorithm.

181 To create the final imputed genotypes and haplotypes, we use this approach to generate  
182 between 40-100 particles, and then merge the particles to form a set of consensus haplotypes.  
183 These haplotypes can be used to either phase high-density individuals, or impute low-density  
184 individuals.

185

186 *Updating a particle*

187 At each loci, particles are updated by probabilistically selecting a new genotype state, and  
188 recombination state. We consider a 4x4 grid of possible steps that the particle can take. In the  
189 first dimension the particle can move to one of the four phased genotype states. In the second  
190 dimension the particle can make the move by having a recombination on either none of their  
191 haplotypes, on either their maternal, or paternal haplotypes, or on both haplotypes. An example  
192 of this update is given in Figure 1.

193 The probability of selecting to move to genotype  $g_{i+1}$  with recombination  $r_i$  is:

$$p(g_{i+1}, r_i | \phi_x^i, \psi_y^i) \propto f(g_{i+1}^p | r_{i+1}^p, \phi_x^i) f(g_{i+1}^m | r_{i+1}^m, \psi_x^i) p(r_i^p) p(r_i^m) p(g_i | d_i) \#(1)$$

194 Where  $f(g_i^p | r_i^p, \phi_x^i)$  is a function which gives the probability of the paternal part of the genotype  
195 state given the recombination state (for either the paternal or maternal haplotypes) and the  
196 current set of haplotypes considered;  $p(r_i^p)$  is the probability of the paternal part of the  
197 recombination state; and  $p(g_i | d_i)$  is the probability of the full genotype state conditional on the  
198 data observed;  $p(g_i | d_i)$  is the same as in multi-locus iterative peeling and is given later, in  
199 Equation 8.

200 We calculate  $f(g_i^p | r_i^p, \phi_x^i)$  as

$$f(g_{i+1}^p = 0 | r_{i+1}^p, \phi_x^i) = \begin{cases} \frac{|\phi_{(x,0)}^{i+1}|}{|\phi_{(x,0)}^{i+1}| + |\phi_{(x,1)}^{i+1}|} & \text{if } r_{i+1}^p \text{ does not indicate a recombination} \\ \frac{|\phi_0^{i+1}|}{|\phi_0^{i+1}| + |\phi_1^{i+1}|} & \text{if } r_{i+1}^p \text{ indicates a recombination} \end{cases} \#(2)$$

201 where  $|\phi_{(x,0)}^{i+1}|$  gives the number of haplotypes in  $\phi_x^i$  that have a 0 at locus  $i + 1$ . Similarly  
202  $|\phi_{(x,1)}^{i+1}|$  gives the number of haplotypes in  $\phi_x^i$  that have a 1 at locus  $i + 1$ .  $\phi_0^{i+1}$  represents the set  
203 of all haplotypes that have a 0 at locus  $i + 1$ . Using the Positional Burrows Wheeler Transform

204 we can calculate  $|\phi_{x,0}^{i+1}|$  in constant time, independent of the size of the haplotype reference  
205 library or the number of haplotypes in  $\phi_x^i$  [21].

206 We calculate the probability of the maternal and paternal recombination  $p(r_i^p)$  as either:

$$p(r_i^p) = \begin{cases} 1 - \gamma & \text{if } r_{i+1}^p \text{ does not indicate a recombination} \\ \gamma & \text{if } r_{i+1}^p \text{ indicates a recombination} \end{cases} \#(3)$$

207 Where  $\gamma$  is a recombination rate which, we estimate as  $\frac{1}{n_{loci}}$ . This value underestimates of the  
208 effective recombination rate since individuals are inheriting haplotypes from distant relatives  
209 [16], however the accuracy of this algorithm seems to largely insensitive to the recombination  
210 rate given, and this serves as a good approximation.

211 In order to determine how well a particle matches the data at a particular locus, we assign  
212 a score to each particle at each locus:

$$score_i = \log(p(r_i^p)p(r_i^m)p(d_i|g_i)) \#(4)$$

213 , which gives a higher score to particles that do not have a recombination and fit the observed  
214 genotype data. We use the score to combine particles into a single consensus haplotype.

215

216 *Combining particles*

217 We create a consensus haplotype and genotype state out of multiple particles by using a two-  
218 stage approach. In the first stage we call consensus genotypes for each locus. In the second stage  
219 we phase the loci that are called as heterozygous.

220 To create a consensus genotype, we score each particle at each locus by taking the sum of  
221 the scores of each particle (given by Equation 4) within a 50 marker window. We then select the  
222 genotype state of the particle with the best score as the called genotype. If multiple particles have  
223 the highest score, we use the most frequent genotype state of those particles the called genotype.

224 In the second stage we phase heterozygous loci by looking at transitions between  
225 neighbouring heterozygous states. We track whether the alternative alleles are on the same or  
226 different haplotypes, e.g., whether the phased genotype state transitions from  $aA$  to  $aA$  where the  
227 alternative allele is on the maternal haplotype, or from  $aA$  to  $Aa$  where the alternative allele  
228 transitions between the maternal to the paternal haplotypes. We pick the transition that is most  
229 frequent in all of the particles that are heterozygous at both loci.

230

231 *Backward information*

232 In a traditional hidden Markov model, inference is done by combining information from a  
233 forward pass (information from the first locus to the current locus) with a backward pass  
234 (information from the last locus to the current locus). The search algorithm we present only takes  
235 into account information in the forward pass, and does not make selections based on genotype  
236 data from loci after the current locus. Information from backward pass can be useful for  
237 imputation by filling in spontaneous missing markers, and phasing genotype states. To  
238 incorporate backwards information, we first run a series of particles in reverse, i.e., from the end  
239 of the chromosome to the beginning of the chromosome, and then run a forward pass of particles  
240 where we replace  $p(g_i|d_i)$  in Equation 1 with:

$$p(g_i|d_{i:n}, \phi, \psi) = p(g_i|d_i) \sum_{r_i} f(g_i^p|r_i^p, \phi_x^{i-1}) f(g_i^m|r_i^m, \psi_x^{i-1}) p(r_i^p) p(r_i^m) \#(5)$$

241 In this equation we project each of the reverse particles forward by one locus to see what  
242 genotype state they are likely to carry at the next locus. When multiple particles are run on the  
243 backward pass,  $p(g_i|d_{i:n}, \phi, \psi)$  is averaged across all particles in the backward pass.

244

245 *Imputation*

246 This algorithm can also be used to impute missing markers. To perform imputation, we evaluate  
247 particles on the non-missing markers. We track the loci where recombination occurs, and what  
248 the set of haplotypes are for each particle at those loci. This creates an ordered pair of haplotype  
249 regions  $\{(start, stop), (\phi_x^{stop}, \psi_y^{stop})\}$ . For each locus, we select the interval that contains the  
250 locus, and fill in missing markers on the corresponding haplotypes from the middle haplotype in  
251  $\phi_x^{stop}$  on the paternal side, and  $\psi_y^{stop}$  on the maternal side.

252

253 *Creating the haplotype library*

254 In many animal populations pre-phased haplotype libraries are not available and so the haplotype  
255 used for population imputation needs to be constructed. This is done by iteratively (1)  
256 constructing a haplotype library from the high-density individuals, (2) phasing those individuals  
257 with that haplotype library, and (3) re-building the library using the phased haplotypes from the  
258 previous iteration. To initialize the haplotype library, we randomly phase heterozygous loci and  
259 fill in spontaneous missing genotypes. We then run 5 rounds of phasing and imputation,  
260 rebuilding the haplotype library at the end of each round. Running more rounds of phasing and  
261 imputation can increase the quality of the haplotype library, but we found that 5 rounds was  
262 sufficient in pilot simulations for accurate imputation.

263 During this process we keep track of location of which haplotypes the individual  
264 contributes to the haplotype library, and remove those haplotypes as options for the particle  
265 steps. This is done by modifying  $|\phi_{x,0}^{i+1}|$  to be either

$$|\phi_{x,0}^{i+1}| = \begin{cases} |\phi_{x,0}^{i+1}| & \text{if } \phi_{x,0}^{i+1} \text{ contains none of the individual's haplotypes} \\ |\phi_{x,0}^{i+1}| - 1 & \text{if } \phi_{x,0}^{i+1} \text{ contains one of the individual's haplotypes} \\ |\phi_{x,0}^{i+1}| - 2 & \text{if } \phi_{x,0}^{i+1} \text{ contains both of the individual's haplotypes} \end{cases} \quad \#(6)$$

266

267 *Array Clustering*

268 In order to run this algorithm, we want  $p(g_i|d_i)$  to be as informative as possible for each locus.

269 Having an informative value for  $p(g_i|d_i)$  allows us to have more confidence in taking each step.

270 If the individual does not have genotype data at a locus, then  $p(g_i|d_i)$  will be relatively  
271 uninformative at that locus. One solution would be to only evaluate the individual's genotypes at  
272 non-missing loci. This would require re-calculating the Positional Burrows Wheeler Transform  
273 on an individual-by-individual basis to take into account each individual's pattern of missing  
274 genotypes, which would be prohibitively computationally expensive. Instead we cluster  
275 individuals based on the SNP array they are genotyped on, and build the Positional Burrows  
276 Wheeler Transform on an array-by-array basis. This greatly reduces the number of times the  
277 Positional Burrows Wheeler Transform needs to be calculated, and backwards information is  
278 used to guide decisions on any remaining spontaneous missing markers.

279

280 **Pedigree based imputation using approximate multi-locus iterative peeling**

281 AlphaImpute2 performs pedigree based imputation using an approximate version of multi-locus  
282 iterative peeling [9,22]. In an iterative peeling framework the probability of an individual's  
283 genotypes is based on three sources of information [23]:

$$p(g_i) = \text{anterior}(g_i)\text{posterior}(g_i)\text{penetrance}(g_i) \#(7)$$

284 , where the anterior term represents information about an individual's genotypes based on  
285 information from the ancestors of the individual filtered through their parents, the posterior  
286 represents information about an individual's genotypes based on the decedents of the individual

287 filtered through their offspring, and the penetrance term represents information about an  
288 individual's genotypes based on their own genetic data (i.e., SNP array or sequence data).

289 In order to take linkage information into account, multi-locus iterative peeling builds on  
290 the normal iterative peeling framework by having the anterior terms and the posterior terms  
291 depend on the segregation state of the individual, i.e., which pair of parental haplotypes that  
292 individual inherited at each loci [22].

293 Performing exact inference in a peeling framework is challenging because the anterior  
294 and posterior terms for an individual, depend on the genotypes of their parents and offspring,  
295 which themselves need to be estimated. Because of this, AlphaImpute2 takes an iterative  
296 approach to update the anterior and posterior terms in a series of passes up and down the  
297 pedigree. This is summarized in the following algorithm:

298 1. Downward pass: Starting from individuals in the first generation to the last generation  
299 a. Re-estimate the segregation probabilities for each individual.  
300 b. Re-estimate the anterior terms for each individual based on their parent's  
301 genotypes.

302 2. Upward pass: starting from the last generation to the first generation  
303 a. Re-estimate the segregation probabilities for each individual.  
304 b. Re-estimate the posterior terms for each individual based on their offspring's  
305 genotypes.

306 In order to enable these passes, we sort the pedigree according to an individual's generation. The  
307 generation of each individual is the minimum of the generation of their sire and dam plus one.

308 The peeling algorithm needs to be run in a series of passes. We have found that 5 passes  
309 of peeling is often enough to obtain high-quality genotype probabilities. The purpose of running

310 multiple passes is primarily to transmit information horizontally across the pedigree, i.e.,  
311 between children of a shared parent. The amount of information that is passed horizontal quickly  
312 decays as individuals become more genetically distant.

313 In order to perform peeling we need to specify how we calculate the penetrance term, and  
314 how we update the anterior, posterior, and segregation probabilities.

315

316 *Calculating the penetrance term*

317 The penetrance term give the probability of the observed genetic data, conditional on the  
318 individual's genotype state. We consider four phased genotype states, aa, aA, Aa, AA, where the  
319 first allele is the paternal allele, and the second allele is the maternal allele. We assume that the  
320 observed genotype data is the number of observed alternative alleles for SNP data.

$$\text{penetrance}(g_i) = p(d_i|g_i) = \begin{cases} 1 - e & \text{if } g_i \text{ is consistent with } d_i \\ e & \text{otherwise} \end{cases} \# \quad (8)$$

321 Where  $e$  represents the genotyping error rate with a default value of 0.01.

322

323 *Updating the anterior term*

324 In each downward pass the anterior term is updated for each individual. To perform this update  
325 we re-estimate which genotypes the individual inherited from their parents using the current  
326 estimate of their parents' genotypes. We calculate the anterior term for individual  $o$  at locus  $i$  as:

$$\text{anterior}(g_{o,i}) = \sum_{seg_{o,i}} \sum_{g_{s,i}, g_{d,i}} p(g_{o,i}|g_{s,i}, g_{d,i}, seg_i) p^*(g_{s,i}) p^*(g_{d,i}) p^*(seg_{o,i}) \# \quad (9)$$

327 where  $p(g_{o,i}|g_{s,i}, g_{d,i}, seg_i)$  is a transmission function which gives the probability that the  
328 offspring inherits a genotype conditional on the offspring's segregation states and the genotypes  
329 of their sire and dam. This value will be either 0 or 1 depending on if an inherited genotype is

330 consistent with the individual's segregation state and the genotypes of their parents. The term  
331  $p^*(g_{s,i})$  gives the probability of the genotype state of the sire, based on the information from the  
332 last pass of peeling.

333 This formulation of the anterior term is an approximation to the term used in previous  
334 papers [22,23]. In the traditional peeling framework the parents genotypes are given by  
335  $p_{-o}(g_{s,i}, g_{d,i})$  which gives the joint genotype probabilities of the parents ignoring information  
336 from the offspring,  $o$ . By not excluding the offspring's genotypes we are effectively double  
337 counting the offspring's genotypes: first information on the offspring's genotypes is used in the  
338 penetrance function, and second information from the offspring's genotypes will be used to  
339 calculate the posterior term of the parent, which will then be used to estimate the anterior term of  
340 the offspring. In practice, we find that the posterior term from a single offspring only provides a  
341 small amount of information to their parent, and that the double counting of information here  
342 does not lead to a substantial loss in accuracy. We also assume that the genotype probabilities of  
343 the parents are independent, i.e.,  $p(g_{s,i}, g_{d,i}) = p(g_{s,i})p(g_{d,i})$ .

344

345 *Updating the posterior term*

346 In each upward pass the posterior term is updated based on the genotypes of the offspring. This  
347 update is performed on a family-by-family basis and the result is combined across families. For a  
348 sire,  $s$ , with mates,  $M = \{m_1, m_2, \dots\}$  the posterior term is:

$$posterior(g_{s,i}) \propto \prod_{m \in M} posterior_m(g_{s,i}) \#(10)$$

349 where posterior term for each mate is given by

$$posterior_m(g_{s,i}) \propto \sum_{g_{m,i}} \prod_o \sum_{seg_{o,i}} \sum_{g_{o,i}} p(g_{o,i} | g_{s,i}, g_{d,i}, seg_i) p_{-parents}^*(g_{o,i}) p^*(seg_{o,i}) p^*(g_{o,i}) \#(11)$$

350

351 Where the product is on all of the shared offspring between  $m$  and  $s$ . Similar terms are used to  
352 generate the posterior estimate for each dam.

353 In order to avoid double counting the genotypes of the parents, we exclude the anterior  
354 term from the calculation of the genotypes of the offspring in  $p_{-parents}(g_{o,i})$ . As when  
355 calculating the anterior term, we do not exclude the contribution of the offspring when  
356 calculating the genotypes of the mate  $p(g_{m,i})$ .

357

358 *Calculating the probability of each segregation state*

359 The probability of each segregation state are calculated by using a hidden Markov model  
360 to determine which segregation state the individual carries at different loci across the genome.  
361 We consider four segregation states ( $mm$ ,  $pm$ ,  $mp$ ,  $pp$ ) where the first letter gives whether the  
362 individual inherits their sire's maternal or paternal haplotype, and the second letter gives whether  
363 the individual inherits their dam's maternal or paternal haplotype.

364 Hidden Markov models are defined by a series of emission probabilities which give the  
365 likelihood of the observations given a hidden state, and transmission probabilities, which give  
366 the probability of transitioning between hidden states. The emission probabilities of this model  
367 are:

$$p(seg_{o,i}) \propto \sum_{g_{o,i}} \sum_{g_{d,i}, g_{o,i}} p(g_{o,i} | g_{s,i}, g_{d,i}, seg_i) p^*(g_{s,i}) p^*(g_{d,i}) p_{-parents}^*(g_o) \#(12)$$

368 Which uses Bayes' rule to estimate the probability of each segregation state conditional on the  
369 estimated genotypes of the individual and their parents. The transmission function is given by  
370 (Whalen 2018):

$$p(seg_i = s | seg_i = s') = (1 - \gamma)^{2-d} \gamma^d \#(13)$$

371 where  $d$  is the number of recombinations required to move between  $s$  and  $s'$ , i.e.,

372  $p(seg_{i,j} = pp | seg_{i,j-1} = pm) = (1 - \gamma)\gamma$ , and  $\gamma$  is the recombination rate. We found that

373 accuracy was largely insensitive to the recombination rate and so set it a default value of  $\frac{1}{n_{loci}}$

374 where  $n_{loci}$  is the number of loci on the chromosome. This assumes markers are evenly spaced

375 (in genetic map distance) across an 100 cM chromosome. The assumed total chromosome

376 genetic map length can be changed using a command line option.

377 We use the forward-backward algorithm [27] to calculate segregation probabilities across

378 each loci. To simplify the amount of information stored at each loci, we assume the segregation

379 probabilities for the maternal and paternal haplotypes are independent and set e.g.,

$$380 p(seg_{o,pat,i} = m) = p(seg_{o,i} = mp) + p(seg_{o,i} = mm).$$

381

382 *Calling genotype probabilities for the posterior term*

383 In order to reduce runtime we call the offspring segregation and genotype probability values

384 when calculating the posterior terms for their parents. We use calling threshold of 0.99 for

385 calling the segregation values, and a calling threshold of 0.99 for the genotypes on the first round

386 of peeling, and a threshold of 0.95 for subsequent rounds of peeling. For genotype or segregation

387 probabilities that do not reach the threshold, the genotypes or segregation values are set with

388 each state being equally likely.

389 By calling the segregation values and genotype values, we are able to store part of the

390 posterior update,

$$\sum_{seg_{o,i}} \sum_{g_{o,i}} p(g_{o,i} | g_{s,i}, g_{d,i}, seg_i) p_{-parents}(g_{o,i}) p(seg_{o,i}) \#(14)$$

391 , in a look-up table, which substantially reduces runtime. In addition, we do not consider the  
392 dependency between the uncertainty of segregation states at nearby loci. The un-modelled linked  
393 uncertainty between segregation states can lead to errors in imputing the parental genotypes. By  
394 calling the segregation probabilities we mitigate the impact of this simplification by only  
395 considering non-equal segregation probabilities where there is minimal uncertainty in the  
396 segregation state.

397 After running the final round of peeling we also call the genotype probabilities of all  
398 individuals in the population to get best-guess genotypes for each individual.

399

#### 400 **Integrating population and pedigree based imputation**

401 Past work has found that combining pedigree and population imputation algorithms can increase  
402 accuracy in populations where pedigree information is available [13,14]. The goal of this  
403 combination is to use the population-based imputation algorithms to phase and impute the  
404 individuals at the top of the pedigree. These genotypes can then be dropped through the rest of  
405 the pedigree using the pedigree based imputation algorithm. To combine the pedigree and  
406 population algorithms in AlphaImpute2, we perform imputation using a three step approach  
407 where we first perform an initial run of pedigree imputation, we then perform population  
408 imputation on a limited set of “pseudo-founders”, and we finish with a final run of pedigree  
409 imputation to fill in the remaining missing genotypes.

410

##### 411 *Step 1: Initial pedigree imputation*

412 In Step 1, 5 rounds of multi-locus iterative peeling are run on the population. After the final  
413 round all of the genotypes in the population are called with a genotype calling threshold of 0.9.

414 We then split the population to three parts: (1) high-density individuals that have fewer than 10%  
415 missing markers, (2) low-density individuals who are “pseudo-founders” (see below) and (3)  
416 low-density individuals who are not “pseudo founders”. After splitting the population, the  
417 genotypes of individuals in group (3) are reset to their original genotype values before pedigree  
418 based imputation.

419 Pseudo-founders are individuals who genotyped at a higher density than their parents  
420 (accounting for the fact that their parents may be imputed to a higher density using pedigree  
421 based imputation). To detect pseudo-founders we go through the pedigree from the start to the  
422 end and calculate the effective genotyping density of an individual:

$$score_{ind} = \begin{cases} \min(score_{sire}, score_{dam}) & (a) \text{ if } missing_{ind} * 0.9 < \min(score_{sire}, score_{dam}) \\ missing_{ind} & (b) \text{ otherwise} \end{cases}$$

423 where  $missing_{ind}$  is the percentage of non-missing markers the individual has. The value 0.9 is  
424 used to give a slight preference to using the genotype of the parents if the individuals are at a  
425 similar marker density. Individuals in group (b) are the “pseudo-founders” of the population.

426

427 *Step 2: Population imputation*

428 In Step 2, we use the population imputation algorithm to phase the high-density individuals  
429 detected in Step 1, and use the haplotype constructed from their phased haplotypes as the  
430 reference library to impute the low-density “pseudo-founders”. We perform an initial 5 rounds of  
431 phasing to iteratively build the reference haplotype library using 40 particles to phase each  
432 individual. For imputation we use 100 particles to impute each individual. At the end of this step,  
433 we reset the genotypes and haplotypes of high-density individuals that are not “pseudo-founders”  
434 to their original genotype states at the start of Step 1. The number of particles selected for

435 phasing and imputation were chosen based on pilot simulations. Larger numbers of particles may  
436 yield more accurate results, but the improvement in accuracy will likely be small.

437

438 *Step 3: Final pedigree imputation*

439 In Step 3, we re-run 5 rounds of multi-locus iterative peeling, using the new phased genotypes  
440 for the “pseudo-founder” generated in Step 2. In order to reduce the negative impact of switch  
441 errors, we perform peeling on a lesioned pedigree where the link between a “pseudo-founder”  
442 and both of their parents is removed. After running multi-locus iterative peeling, the genotypes  
443 are set to the best-guess genotypes.

444

445 **Testing the algorithm**

446 We tested the performance of AlphaImpute2 on four simulated datasets based on pedigrees taken  
447 from a commercial pig breeding program. The pedigrees had either 18,349 (18k), 34,425 (34k),  
448 63,872 (63k), or 107,815 (107k) individuals and were genotyped on four SNP arrays which  
449 ranged from 350 (*very low density*), 10,000 (*low density*), 33000 (*medium density*), and 46,000  
450 (*high density*) markers. Although these marker densities are lower than highest density SNP  
451 arrays available for humans and livestock, they represent commonly used marker densities for  
452 performing genomic selection in many animal breeding programs [28–30].

453 We compared the performance of AlphaImpute2 to that of Beagle 4.1, Beagle 5.1,  
454 AlphaImpute, AlphaPeel, and findhap. We evaluated each software on their accuracy, runtime,  
455 and memory requirements.

456

457

458 **Simulated genetic data**

459 We simulated the four pedigrees by generating a set of founder haplotypes using a Markovian  
460 Coalescent Simulator [31], and then dropped them through each pedigrees.

461 The founder haplotypes were generated by assuming there were 18 100-cM long  
462 chromosomes that were simulated using a per site mutation rate of  $2.5 \times 10^{-8}$ , and an  
463 effective population size (Ne) that changed over time based on estimates for the Holstein cattle  
464 population [32]. Ne was set to 100 in the final generation of simulation and to 1256, 4350, and  
465 43,500 at 1000, 10,000, and 100000 generations ago, with linear changes in between. The  
466 number of markers per chromosome varied between 1,231 and 4690 based on the marker  
467 densities on each chromosome in the real genotype data.

468 The founder haplotypes were then dropped through the pedigree using AlphaSimR [33].  
469 The genotypes of each individual were then masked to reflect the pattern of missingness for that  
470 individual in the real genotype data.

471

472 **Comparison with other software**

473 We evaluated the performance of AlphaImpute2 when using either the population only  
474 algorithm, the pedigree only algorithm, or the combined algorithm.

475 We compared the performance of AlphaImpute2 with the performance Beagle 4.1,  
476 Beagle 5.1, AlphaPeel, findhap, and AlphaImpute. Beagle 4.1 and Beagle 5.1 were run using  
477 default parameters except the effective population size which was set to 200. AlphaImpute and  
478 AlphaPeel were run with default parameters. For AlphaImpute we rounded the genotype  
479 probabilities that it outputs before calculating accuracy to make it consistent with the other

480 software packages. findhap was run with the recommend parameters of maxlen = 600, minlen =  
481 75, and errate = .004.

482 Our goal in running a large number of other software packages was to evaluate the  
483 performance of both the population only, and pedigree only algorithms separately, and to  
484 evaluate the performance of the combined algorithm.

485 Beagle 4.1 and Beagle 5.1 were chosen to serve as a benchmark for the population only  
486 algorithm. Both software packages are commonly used in the human and animal imputation  
487 literature, and Beagle 5.1 has incorporated a number of (as of yet unpublished) improvements for  
488 phasing.

489 AlphaPeel was chosen to serve as a benchmark for the pedigree only algorithm.  
490 AlphaPeel implements a version of multi-locus iterative peeling, which is approximated by the  
491 pedigree only algorithm in AlphaImpute2. Our goal in making this comparison was to see how  
492 much accuracy was sacrifice to increase runtime in AlphaImpute2.

493 findhap and AlphaImpute were chosen to serve as a benchmark for a combined pedigree  
494 and population imputation algorithm. Both programs are currently in use in commercial breeding  
495 programs, and AlphaImpute2 could serve as a possible candidate to replace them.

496

## 497 **Performance measurements**

498 Imputation accuracy was measured as the correlation between an individual's imputed  
499 genotype and their true genotype, corrected for the population minor allele frequency [34]:

$$accuracy = \text{cor}(G_{impute} - 2 \text{maf}, G_{true} - 2 \text{maf})$$

500 Accuracy was averaged across all of the 18 simulated chromosomes.

501 We also measured the runtime and memory usage of each program. All programs were  
502 run on the Edinburgh Compute and Data Facility cluster using 4 cores. Programs were given at  
503 most eight days to impute each chromosome. The results are given only for programs that  
504 successfully finished on all of the chromosomes.

505

506 **Results**

507 We found that AlphaImpute2 had high accuracy and low run-times across all four pedigrees.  
508 Imputation accuracy for the 107k pedigree was .988 for high-density individuals, .988 for  
509 medium density individuals, .993 for low-density individuals, and .81 for very-low-density  
510 individuals. Imputation took 105 minutes and 14.4 GB of memory for Chromosome 1 (4,600  
511 Markers and 107,000 individuals). AlphaImpute2 had higher accuracy than the alternative  
512 algorithms and comparable run times to findhap and Beagle 5.1, both of which are significantly  
513 faster than Beagle 4.1. The accuracy, runtime, and memory usage of each algorithm is given in  
514 Table 1.

515

516 **Accuracy of the full AlphaImpute2 algorithm**

517 The accuracy of AlphaImpute2 depended primarily on the genotyping density of the  
518 individuals and their relative position in the pedigree. We found similar accuracies across all four  
519 pedigrees and so focus on the 18k pedigree to enable comparisons to Beagle 4.1 which did not  
520 finish on all pedigrees.

521 On the 18k pedigree the accuracy of the full AlphaImpute2 algorithm was .998 for high-  
522 density individuals, .944 for medium-density individuals, .990 for low-density individuals, and  
523 .827 for very-low-density individuals. The lower accuracy for medium-density individuals

524 compared to low-density individuals was likely driven by their relative position in the pedigree.  
525 All of the medium-density individuals appeared in the first quarter of the pedigree compared to  
526 only 3% of the low-density individuals and 0.2% of the high-density individuals.

527 The accuracy of the pedigree only algorithm was .998 for high-density individuals, .661  
528 for medium-density individuals, .987 for the low-density individuals, and .862 for the very-low-  
529 density individuals. Compared to the full algorithm, the pedigree only algorithm had much lower  
530 accuracy on the medium-density individuals (.661 compared to .944), and similar accuracies on  
531 the high-density, low-density, and very-low-density individuals.

532 The accuracy of the population only algorithm was .987 for the high-density individuals,  
533 .929 for the medium-density individuals, .973 for the low-density individuals, and .257 for the  
534 very-low-density individuals. Compared to the full algorithm, the population only algorithm had  
535 much lower accuracy on the very-low-density individuals (0.257 compared to 0.827), and  
536 between 1-2% lower accuracies on the high-density, medium-density, and low-density  
537 individuals.

538 The full AlphaImpute2 algorithm had higher accuracy than both the population only or  
539 pedigree only algorithms except in the case of very-low-density individuals where the pedigree  
540 only algorithm had a slightly higher accuracy (.862 compared to .827).

541

#### 542 **Pedigree only imputation accuracy compared to AlphaPeel**

543 The pedigree only algorithm in AlphaImpute2 uses an approximate version of multi-locus  
544 iterative peeling that is implemented in AlphaPeel. The accuracy of the two algorithms are  
545 similar, with the accuracy of AlphaPeel on the 18k pedigree being .997 for high-density

546 individuals, .733 for medium-density individuals, .984 for low-density individuals, .855 for very-  
547 low-density individuals.

548 The correlation between the genotypes imputed by the pedigree-only imputation  
549 algorithm and AlphaPeel was high. On Chromosome 1 for the 18k pedigree, the correlation  
550 between the genotypes imputed between the two algorithms was on average .973, with a  
551 correlation of .999 for high-density individuals, .960 for medium-density individuals, .994 for  
552 low-density individuals, and .804 for very-low-density individuals. The lower correlation for the  
553 medium-density and very-low-density individuals is due to the lack of high-density parents for  
554 these individuals. In AlphaPeel, the observed minor allele frequency is used as a prior for the  
555 genotypes of the founder individuals, whereas a minor allele frequency of 0.5 is used as a prior  
556 for founder individuals in AlphaImpute2. We return to this difference in the Discussion.

557

### 558 **AlphaImpute2 accuracy compared to Beagle 4.1 and Beagle 5.1**

559 For the 18k pedigree, the accuracy of Beagle 4.1 was .995 for the high-density  
560 individuals, .944 for the medium-density individuals, .969 for the low-density individuals, and  
561 .327 for the very-low-density individuals. The accuracy of Beagle 5.1 was .626 for the high-  
562 density individuals, .909 for the medium-density individuals, .939 for the low-density  
563 individuals, and .219 for the very-low-density individuals.

564 The accuracy of Beagle 4.1 was slightly higher than that of Beagle 5.1 in all cases, with  
565 the largest difference being on filling spontaneous missing markers in the high-density  
566 individuals, where the accuracy of Beagle 4.1 was .995 but the accuracy of Beagle 5.1 was .626.

567        The accuracy of the population only algorithm in AlphaImpute2 was similar to Beagle  
568        4.1 with lower accuracies on the medium-density individuals (.929 compared to .944), and very-  
569        low-density individuals (.257 compared to .327).

570

571        **Combined algorithms: findhap and AlphaImpute2**

572        For the 18k pedigree, the accuracy of findhap was .719 for the high-density individuals,  
573        .627 for the medium-density individuals, .774 for the low-density individuals, and .445 for the  
574        very-low-density individuals. The accuracy of findhap was between 20-40% lower than  
575        combined algorithm in AlphaImpute2 in all cases (Table 1).

576        The accuracy of AlphaImpute2 was .940 for the high-density individuals, .875 for the  
577        medium-density individuals, .931 for the low-density individuals, and .641 for the very-low-  
578        density individuals. The accuracy of the combined algorithm in AlphaImpute2 was higher than  
579        AlphaImpute in all cases, with the largest differences for medium density individuals (.944  
580        compared to .857) and very-low-density individuals (.827 compared with .641).

581

582        **Runtime**

583        AlphaImpute2 was faster than all of the other software packages tested. For the pedigree  
584        of 18k individuals, AlphaImpute2 took 15 minutes, followed by findhap which took 17 minutes,  
585        Beagle 5.1 which took 28.1 minutes, AlphaImpute which took 348 minutes, and Beagle 4.1  
586        which took 2,250 minutes.

587        For the pedigree of 107k individuals, AlphaImpute2 took 105 minutes, Beagle 5.1 took  
588        190 minutes, findhap took 395 minutes, and AlphaImpute took 7,859. Beagle 4.1 did not finish  
589        on the 107k pedigree within eight days of run time.

590

591 **Discussion**

592 In this paper we present a new population and pedigree based imputation algorithm,  
593 AlphaImpute2, and demonstrate its performance on four simulated datasets based on real  
594 livestock pedigrees. We find that it is able to perform fast and accurate imputation in a range of  
595 scenarios, and performs competitively with other existing imputation software. In the remainder  
596 of the discussion we discuss the advantages of combining pedigree and population based  
597 imputation information for imputation, ways to further decrease the runtime of AlphaImpute2,  
598 the performance of the approximate iterative peeling framework, compare the population  
599 imputation algorithm to already existing population imputation algorithms, and the particle based  
600 approach for approximating the Li and Stephens algorithm.

601

602 **Combining pedigree and population imputation increases accuracy**

603 In line with previous research, we find that combining population and pedigree  
604 imputation can increase accuracy compared to running either the population or pedigree  
605 imputation algorithms alone [13].

606 Compared to the pedigree only algorithm, the combined algorithm delivers high-  
607 accuracy phasing and imputation for the individuals at the top of the pedigree. These phased  
608 genotypes can then be used to impute individuals further down in the pedigree. This improves  
609 the imputation accuracy for both the “pseudo-founders” of the pedigree, but also other  
610 descendants who may be genotyped at lower densities. A similar effect was seen in LDMIP  
611 which used a population based imputation algorithm to impute and phase the founders of the  
612 pedigree before running multi-locus iterative peeling [9].

613       Compared to the population only algorithm, the combined algorithm delivers higher-  
614   accuracy imputation across the board, particularly for very-low-density individuals. For these  
615   individuals imputation accuracy is improved by using pedigree information to decrease the  
616   number of haplotypes that need to be considered – the four parental haplotypes in the case of  
617   pedigree based imputation, compared to tens of thousands for population imputation – which  
618   makes it easier to find the correct haplotypes with a limited number of low-density markers.

619       The only place where accuracy of the combined algorithm was lower than that of the  
620   pedigree only algorithm, was for very-low-density individuals, particularly those at the  
621   beginning of the pedigree. The lower accuracy on very-low-density individuals is likely due to a  
622   lack of high-density or medium-density ancestors for these individuals. In AlphaPeel, the minor  
623   allele frequency is used as a prior for missing genotypes of founders in the population. This  
624   allows AlphaPeel to take the uncertainty in the genotypes of these individuals into account. In  
625   contrast, in the combined algorithm the founders and “pseudo founders” are imputed with the  
626   population imputation algorithm, and the resulting genotypes are treated as observed genotypes  
627   (with a default 1% error rate). The population imputation algorithm tends to have low error rates  
628   for high, medium, and low-density individuals, but high error rates for very-low-density  
629   individuals. Treating these imputed genotypes as observed in the final round of population  
630   imputation may be the cause of the lower imputation accuracy. A solution to this problem may  
631   be to include a minimum genotyping density required for population imputation (e.g., 10-50 non-  
632   missing markers per chromosome), and using the minor allele frequency as a prior for the  
633   genotypes of “pseudo founders” who do not reach this density.

634

635   **Decreasing the runtime of AlphaImpute2**

636 We found that the combined imputation algorithm had lower runtime than the population  
637 only algorithm, but a higher runtime than the pedigree-only algorithm. The lower runtime  
638 compared to the population only algorithm is likely due to the fact that in the combined  
639 algorithm imputation is only run on a small set of “pseudo founders”. This does not lead to a  
640 large reduction in runtime since all the of the high-density individuals are still need to be phased  
641 to build the haplotype reference library.

642 One option to decrease runtime would be to bypass phasing completely by using the  
643 high-density individuals who have been fully phased via pedigree imputation to construct the  
644 haplotype reference library. We tested this in a small number of pilot simulations and found that  
645 this approach reduced run time by 50%, but also decreased accuracy by 1-2%. The lower  
646 accuracy is likely driven by having a less relevant set of haplotypes included in the reference  
647 library, particularly from those individuals at the top of the pedigree.

648 Another option to decrease runtime of the population imputation algorithm would be to  
649 decrease the number of particles that are run. We chose 40 particles for phasing the haplotype  
650 reference panel, and 100 for imputing low-density individuals since those values seemed to give  
651 good accuracies in pilot simulations. The number of particles used for phasing the haplotype  
652 reference panel was lower than that for imputation, since errors in the haplotype reference panel  
653 can be corrected in imputation, and the cost of each additional particle is higher for phasing since  
654 phasing is run five times on the high-density individuals to refine the haplotype reference library.

655

### 656 **Approximate iterative peeling**

657 One of the goals in this paper was to improve the scaling of multi-locus iterative peeling. We  
658 have previously found that multi-locus iterative peeling is a robust imputation algorithm for

659 performing imputation in large livestock pedigrees [22], but has suffered from long run-times  
660 that make it impractical for regular use. We found that by approximating the full multi-locus  
661 iterative peeling algorithm we were able to reduce both runtime and memory by 80% by a factor  
662 while maintaining the similar accuracies. The speed improvements in AlphaImpute2, exploit the  
663 fact that offspring provide relatively little information on their parent's genotype, and in many  
664 cases it is possible to call the segregation values at most loci. This allows us to re-use the  
665 parent's genotype probabilities in the peel-down steps for all of their offspring instead of re-  
666 calculating these probabilities on an offspring-by-offspring basis, and to use lookup tables to  
667 calculate the summations in the peeling-up step (particularly Equation 11).

668 In terms of accuracy the AlphaPeel and the pedigree only algorithm in AlphaImpute2 had  
669 similar accuracies in both datasets. This suggests that the use of the approximations lead minimal  
670 decreases in accuracy on these datasets. The primary difference between algorithms was on how  
671 the founders of the population were imputed. For missing genotypes in the founders, AlphaPeel  
672 imputes the individuals based on the minor allele frequency in the population. AlphaImpute2  
673 imputes these individuals assuming a minor allele frequency of 0.5. We used a neutral minor  
674 allele frequency in AlphaImpute2, to prevent the algorithm from incorrectly calling genotypes  
675 with a low minor allele frequency for the combined algorithm, and assume that the genotypes of  
676 these individuals will eventually be imputed using the population imputation algorithm. This  
677 means that when run alone, the pedigree only algorithm in AlphaImpute2 may give lower  
678 accuracies than AlphaPeel, but we find that the combined algorithm in AlphaImpute2 gives  
679 higher accuracies than AlphaPeel in most cases.

680

681 **Comparison of population imputation algorithms**

682       Compared to the other imputation algorithms tested, AlphaImpute2 obtained generally  
683       higher imputation accuracies at lower runtimes on all four simulated datasets.

684       In terms of speed, we found that AlphaImpute2 and Beagle 5.1 scaled the best out of the  
685       software packages tested. findhap had initially low-runtimes on the 18k pedigree, but the  
686       performance substantially decreased as the number of reference haplotypes grew larger. The  
687       runtime of findhap increased from 15 minutes to 395 minutes between the 18k pedigree and the  
688       107k pedigree, where the runtime of AlphaImpute2 only increased from 15 minutes to 105  
689       minutes. The poorer scaling of findhap is likely due to it searching through the haplotype  
690       reference library for each individual, a task that gets harder as more high-density individuals are  
691       genotyped. AlphaImpute2 addresses this issue by applying the positional Burrows Wheeler  
692       transform to enable constant-time searches through large haplotype reference libraries.

693       In terms of accuracy, we found that AlphaImpute2 had a higher accuracy than most of the  
694       other software tested. For the pedigree only algorithm, AlphaImpute2 had a similar accuracy to  
695       AlphaPeel. For the population only algorithm, AlphaImpute2 had a similar accuracy to Beagle  
696       5.1 and Beagle 4.1. For the combined algorithm AlphaImpute2 had a higher accuracy than all of  
697       the other algorithms including findhap and AlphaImpute. These results suggest that the  
698       approximations used in multilocus peeling had limited impact on imputation accuracy for  
699       pedigree based imputation, that the approximate Li and Stephens algorithm used performs as  
700       well as other techniques used to fit the Li and Stephens model, and that the way that population  
701       and pedigree based imputation are integrated leads to better performance compared to existing  
702       software packages.

703       We were surprised by the large speed improvement between Beagle 4.1 and Beagle 5.1.  
704       Beagle 4.1 had the longest runtime of any of the software packages analysed with a runtime of

705 36 hours on the 18k pedigree, whereas Beagle 5.1 had a runtime similar to AlphaImpute2 with a  
706 runtime of just 28 minutes on the 18k pedigree. The improvement in speed between Beagle 4.1  
707 and Beagle 5.1 are impressive, but the changes to the phasing algorithm are (to our knowledge)  
708 as yet unpublished. The paper on Beagle 5 [26] only described the improvements to the haploid  
709 imputation algorithm which primarily improve speed on imputing whole genome sequence data.

710

## 711 **Particle based approximation to the Li and Stephens model**

712 The particle based implementation of the Li and Stephens model in AlphaImpute2 takes a  
713 different approach for increasing the speed of the Li and Stephens algorithm. Previous work has  
714 increased the speed of diploid imputation by first pre-phasing the data, and then running a  
715 haploid imputation algorithm on the phased haplotypes [18]. The split between phasing and  
716 imputation is important, because for many phasing algorithms increased speed by running the  
717 algorithm to directly infer the phased genotype (typically groups of heterozygous loci) instead of  
718 inferring the underlying haplotypes of origin. This allowed the algorithms to scale better for  
719 known loci, but means that that a haploid imputation algorithm needed to be run after pre-  
720 phasing the data to fill in missing loci [35,36]. These phasing and imputation algorithms have  
721 then been extended to utilize the Positional Burrows Wheeler transform to increase the speed of  
722 both the phasing step [37,38] and imputation step [20].

723 In contrast, the population imputation algorithm in AlphaImpute2 runs phasing and  
724 imputation together in a full diploid Li and Stephens model. In order to make this  
725 computationally tractable, we approximate the Li and Stephens model using a small number of  
726 particles to search for paths with high posterior probability, and use the Positional Burrows  
727 Wheeler Transform to update entire sets of paths at once. Our approach is most similar to that of

728 FastLS [21], with the difference that we generate multiple (approximate) samples from the  
729 posterior distribution instead of calculating a single maximum-likelihood path. This has the  
730 advantage of guaranteeing a constant-time runtime for each particle, and may increase accuracy  
731 if multiple paths have similar high posterior probability. We believe that techniques like  
732 AlphaImpute2 and FastLS may offer an alternative avenue for performing imputation using a Li  
733 and Stephens style model.

734

### 735 **Conclusion**

736 In this paper we present a new imputation algorithm AlphaImpute2, which combines  
737 high-accuracy pedigree imputation with high-accuracy population imputation. The pedigree  
738 imputation was performed by an approximate form of multi-locus iterative peeling, and the  
739 population imputation was performed using a new algorithm which uses particles to approximate  
740 a Li and Stephens style hidden Markov models. We find that in four simulated datasets that the  
741 algorithm has higher accuracies and lower runtimes compared to other existing imputation  
742 software packages, and that it scales well enough to run imputation on hundreds of thousands of  
743 pedigree individuals in a matter of hours. We believe that as the size of agricultural populations  
744 increase, this software will provided a much needed tool for performing imputation while scaling  
745 to the datasets available.

746

### 747 **Declarations**

### 748 **Ethics approval and consent to participate**

749 Not applicable.

### 750 **Consent for publication**

751 Not applicable.

752 **Availability of data and material**

753 The base dataset used in this study cannot be made available due to commercial considerations.

754 However, a copy of the simulation pipeline using a purely simulated dataset is available from the  
755 authors upon a request.

756 **Competing interests**

757 The authors declare no competing interests.

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763 **Authors' contributions**

764 AW and JMH designed the simulation study. AW carried out the simulations and analysed the  
765 results. All authors contributed to writing the manuscript.

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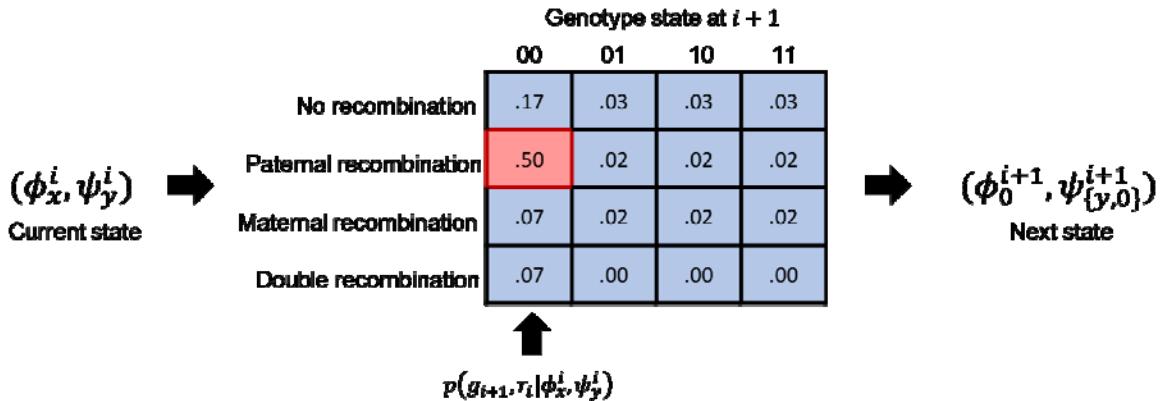
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863  
864 Figure 1: A pictorial representation of a particle update. For each particle at locus  $i$  we  
865 consider a 4x4 grid of possible updates depending on the genotype state at the next locus, and the  
866 recombination state between loci. The probability of selecting of each option is given by  
867 Equation 1. In the example, the selected state, in red, is homozygous for the reference allele (00),  
868 and has a recombination on the paternal haplotype. This means that the paternal path is reset to  
869 , while the maternal path is extended to .  
870

871           Table 1: Imputation accuracy, memory, and runtime for each algorithm. Beagle 4.1 did  
 872           not complete within 8 days on the 63k or 107k pedigrees. Imputation accuracies were averaged  
 873           across all 18 chromosomes. Time and memory are given for Chromosome 1.

	High Density	Medium Density	Low Density	Very Low Density	Time (m)	Memory (GB)
<b>18k Pedigree</b>						
Alphalimpute2	.998	.944	.990	.827	15	2.8
<i>Pedigree Only</i>	.998	.661	.987	.862	3	2.5
<i>Population Only</i>	.987	.929	.963	.257	13	2.1
AlphaPeel	.997	.733	.984	.855	15	9.1
Beagle 4.1	.995	.944	.969	.327	2,250	12.4
Beagle 5.1	.626	.909	.939	.219	28	17.1
Alphalimpute	.940	.875	.931	.641	348	10.5
findhap	.719	.627	.774	.445	15	4.6
<b>34k Pedigree</b>						
Alphalimpute2	.998	.961	.989	.699	24	4.2
<i>Pedigree Only</i>	.998	.653	.987	.843	5	4.1
<i>Population Only</i>	.992	.952	.962	.199	25	3.6
AlphaPeel	.997	.748	.986	.838	21	16.5
Beagle 4.1	.996	.959	.972	.269	4,353	17.5
Beagle 5.1	.601	.937	.936	.148	46	26.0
Alphalimpute	.955	.883	.946	.611	707	15.4
findhap	.731	.678	.787	.438	36	4.6
<b>63k Pedigree</b>						
Alphalimpute2	.998	.984	.991	.858	53	8.6
<i>Pedigree Only</i>	.976	.541	.974	.864	9	7.7
<i>Population Only</i>	.988	.978	.962	.188	57	6.3
AlphaPeel	.980	.596	.979	.855	74	29.8
Beagle 4.1						
Beagle 5.1	.578	.910	.937	.140	88	26.5
Alphalimpute	.898	.869	.921	.625	2,556	27.0
findhap	.757	.732	.796	.451	156	4.8
<b>107k Pedigree</b>						
Alphalimpute2	.998	.987	.993	.821	105	14.4
<i>Pedigree Only</i>	.991	.719	.991	.869	15	12.6
<i>Population Only</i>	.990	.981	.963	.148	106	11.6
AlphaPeel	.992	.786	.990	.858	84	50.4
Beagle 4.1						
Beagle 5.1	.682	.948	.942	.114	190	43.7
Alphalimpute	.942	.929	.940	.639	7,859	41.2
findhap	.778	.761	.801	.455	395	5.0

