

# DETECTING DIVERSIFYING SELECTION FOR A TRAIT FROM WITHIN AND BETWEEN-SPECIES GENOTYPES AND PHENOTYPES

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**T. Latrille<sup>1</sup> , M. Bastian<sup>2</sup> , T. Gaboriau<sup>1</sup> , N. Salamin<sup>1</sup> **

<sup>1</sup>Department of Computational Biology, Université de Lausanne, Lausanne, Switzerland

<sup>2</sup>Laboratoire de Biométrie et Biologie Evolutive, UMR5558, Université Lyon 1, Villeurbanne, France

[thibault.latrille@ens-lyon.org](mailto:thibault.latrille@ens-lyon.org)

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## Abstract

2 To quantify selection acting on a trait, methods have been developed using either within or  
3 between-species variation. However, methods using within-species variation do not integrate  
4 the changes at the macro-evolutionary scale. Conversely, current methods using between-  
5 species variation usually discard within-species variation, thus not accounting for processes  
6 at the micro-evolutionary scale. The main goal of this study is to define a neutrality index  
7 for a quantitative trait, by combining within- and between-species variation. This neutrality  
8 index integrates nucleotide polymorphism and divergence for normalizing trait variation.  
9 As such, it does not require estimation of population size nor of time of speciation for  
10 normalization. Our index can be used to seek deviation from the null model of neutral  
11 evolution, and test for diversifying selection. Applied to brain mass and body mass at the  
12 mammalian scale, we show that brain mass is under diversifying selection. Finally, we show  
13 that our test is not sensitive to the assumption that population sizes, mutation rates and  
14 generation time are constant across the phylogeny, and automatically adjust for it.

15 **Keywords** Quantitative genetics · Trait evolution · Selection · Phylogenetics · Population genetics

## 16 Introduction

17 Determining whether a trait is under a particular regime of selection has been a long-standing goal in evolu-  
18 tionary biology. Fundamentally, distinguishing neutral evolution from selection requires determining which  
19 selective regime is supported by the observed variation of traits or sequences. The variation of phenotypes  
20 (traits) and genotypes (sequences) can be observed at different scales, across different development stages at  
21 the individual level, across different individuals and populations at the species level, and finally across differ-  
22 ent species at the phylogenetic level. All these systems require different assumptions and methodologies, and

23 the endeavor to determine the selective regime for a given trait has thus incorporated theories, methods, and  
24 developments across various fields of evolutionary biology such as quantitative genetics, population genetics,  
25 phylogenetics and comparative genomics (Lynch & Walsh, 1998; Walsh & Lynch, 2018).

26 Leveraging individual variations within the same species, Genome-Wide Association Studies (GWAS) in  
27 humans have shown that traits are mostly polygenic (many loci associated with a given trait) and under  
28 stabilizing selection, while the loci affecting those traits are mostly pleiotropic (many traits associated with  
29 a given locus) with additive effects (Sella & Barton, 2019; Simons et al., 2018). Across several populations,  
30 by contrasting both trait and genetic differentiation,  $Q_{ST}$ – $F_{ST}$  methods have been used to determine the  
31 selective regime and to quantify the strength of selection acting on a trait (Leinonen et al., 2008; Merilä  
32 & Crnokrak, 2001). A trait differentiation ( $Q_{ST}$ ) higher than genetic differentiation ( $F_{ST}$ ) is interpreted  
33 as a signature of diversifying selection due to adaptation in different optimum trait value in the different  
34 populations (Lamy et al., 2012). Contrarily,  $Q_{ST}$  lower than  $F_{ST}$  is interpreted as a signature of stabilizing  
35 selection. However,  $Q_{ST}$ – $F_{ST}$  methods have been found to require many populations (O’Hara & Merilä,  
36 2005), and that various factors can generate a spurious signal of selection (Edelaar et al., 2011; Pujol et al.,  
37 2008). Moreover, the test for diversifying selection is limited to recent local adaptation since the test is  
38 based on the variation observed within a single species. To disentangle selection from neutral evolution, trait  
39 variation can also be observed at a larger time scale. For example, change in mean trait value accumulates  
40 linearly with time of divergence from a sister species, and also proportionally to the trait variance (Lande,  
41 1980a; Turelli, 1984). Empirically, this effect can be observed for genes with larger within-species variation in  
42 gene expression level, which exhibits a faster accumulation of divergence in mean expression level (Khaitovich  
43 et al., 2004). Altogether, both the trait variance and the evolution in mean value can be used to test for trait  
44 selection in a pair of species (Walsh & Lynch, 2018).

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48 a similar accumulation as divergence in expression accumulates faster for genes with large within-species  
49 variation (Khaitovich et al., 2004). Altogether, both the trait variance and the evolution in mean value can  
50 be used to test for trait selection in a pair of species (Walsh & Lynch, 2018).

51 Alternatively, by accounting for the underlying relationships between several species, the selective regime  
52 for a quantitative trait can also be tested at the phylogenetic scale (Felsenstein, 1985). Under neutral evolution,  
53 the change in mean trait value along a given branch of the tree is normally distributed, with a variance  
54 proportional to divergence time (Hansen & Martins, 1996). As a result, the mean trait value can be modeled  
55 as a Brownian process branching at every node of the tree (Hansen & Martins, 1996; Harmon, 2018). Re-  
56 constructing the trait variation along the whole phylogeny as a Brownian process can thus constitute a null  
57 model of neutral trait evolution. Deviations from the assumptions of the Brownian process are however well  
58 known. When trait variation is constraint because of optimum mean trait values across or between species,  
59 the pattern of evolution can be modeled by the Ornstein-Uhlenbeck processes, which is often interpreted  
60 as a signature of stabilizing selection (Catalán et al., 2019). Alternatively, a trend in the Brownian process

61 (the tendency of a trait to evolve in a certain direction without fixed optimum) is interpreted as a signature  
62 of directional selection at the phylogenetic scale (Silvestro et al., 2019). However, studies have shown that  
63 such comparative approaches are subject to different biases (Harmon, 2018). First, a trait under stabilizing  
64 selection for which the optimal trait value is also evolving as a Brownian process will not deviate from a  
65 Brownian process, and thus be wrongly classified as neutral (Hansen & Martins, 1996). In other words, the  
66 better fit of a Brownian process does not necessarily constitute proof of the neutral model. Second, even for  
67 a trait evolving under a neutral regime, the Ornstein-Uhlenbeck process might sometimes be statistically  
68 preferred over a Brownian process due to sampling artifacts (Cooper et al., 2016; Price et al., 2022; Silvestro  
69 et al., 2015). Those limitations, altogether with the use of mean trait estimates leaving out the variance in  
70 traits between individuals, easily generate misclassification of selection from methods at the phylogenetic  
71 scale.

72 At the frontier between micro and macroevolution, comparative methods at the phylogenetic scale have  
73 acknowledged the importance of modeling within-species variation together with changes in mean trait  
74 value to either describe measurement errors (Hansen & Bartoszek, 2012; Lynch, 1991), incorporate values  
75 for individuals (Felsenstein, 2008) or to scale the rate of change in mean trait value (Gaboriau et al., 2020;  
76 Gaboriau et al., 2023; Kostikova et al., 2016). Within-species variation has also been used to infer diversifying  
77 selection by estimating the ratio of between to within variation of many traits and test for deviation from the  
78 average ratio across traits (Rohlf et al., 2014; Rohlf & Nielsen, 2015). Here, our goal was again to use both  
79 variances between and within species to determine the selective regime of a quantitative trait. We build a novel  
80 framework that integrates trait variation at the phylogenetic and population scales together with estimates of  
81 molecular divergence at both scales. It allowed us to define an expected ratio of normalized variance between  
82 and within species while setting the threshold of this ratio for neutral, stabilizing, and diversifying selection.  
83 The ratio that we propose can be considered as a neutrality index for a any quantitative trait articulating  
84 trait and nucleotide variation within and between species. Importantly, our neutrality index also leverages  
85 nucleotide divergence and polymorphism to normalize trait variation at both scales, such that it does not  
86 require estimating population size (within-species) or speciation time (between species). From the field of  
87 population genetics, our study can be seen as the macro-evolutionary generalization of  $Q_{ST}$ – $F_{ST}$  methods  
88 to account for phylogenetic relationships between species. From the field of phylogenetics, our study can be  
89 seen as an alternative to the EVE model (Rohlf et al., 2014; Rohlf & Nielsen, 2015) for a single trait, where  
90 we set a threshold for neutral evolution by leveraging species nucleotide polymorphism and divergence.

## 91 Materials and Methods

### 92 Neutrality index for a quantitative trait

93 Prior to developing our neutrality index, we review theoretical expectations for variations of quantitative  
94 traits and genomic sequences under neutral evolution for both within- and between-species variation.

95 **Within-species trait variations**

For a given trait, the genetic architecture is mainly defined by the number of loci encoding the trait ( $L$ ) and the random additive effect of a mutation on the trait ( $a$ ). New mutations are generating trait variance and the average effect of a mutation on the trait is  $\sigma_M^2 = L \cdot \mathbb{E}[a^2]$ . At the individual level, the mutational variance ( $V_M$ ) is the rate at which new mutations contribute to the trait variance per generation. As shown in Lande (1979, 1980b),  $V_M$  is a function of the mutation rate per loci per generation ( $\mu$ ) and  $\sigma_M^2$ :

$$V_M = 2\mu \cdot \sigma_M^2. \quad (1)$$

While in an infinitesimal model mutations supply new genetic variants, random genetic drift depletes standing variation (Barton et al., 2017; Sella & Barton, 2019; Turelli, 2017). For a neutral trait at equilibrium between mutation and drift (Lynch et al., 1998), the additive genetic variance in a species ( $V_A$ ) is a function of the mutational variance ( $V_M$ ) and the effective number of individuals in the population ( $N_e$ ):

$$V_A = 2N_e \cdot V_M, \quad (2)$$

$$= 4N_e \cdot \mu \cdot \sigma_M^2 \text{ from eq. 1.} \quad (3)$$

For any neutral genomic region of interest, the nucleotide diversity,  $\pi$ , is measured as the number of mutations segregating in the population divided by the length of the region. Any segregating mutations will eventually reach fixation or extinction due to random genetic drift and  $\pi$  is also at a balance between mutations and drift. As shown in Tajima (1989),  $\pi$  is a function of the mutation rate per loci per generation ( $\mu$ ) and the effective population size ( $N_e$ ):

$$\pi = 4N_e \cdot \mu. \quad (4)$$

We define  $\sigma_W^2$  as the ratio of additive genetic variance of the trait ( $V_A$ ) over  $\pi$  of any neutral genomic region of interest. This ratio allows removing the effect of  $N_e$  and  $\mu$ , which are parameters not related to the genetic architecture of the trait, giving  $\sigma_W^2$  as a proxy of  $\sigma_M^2$ :

$$\sigma_W^2 \stackrel{\text{def}}{=} \frac{V_A}{\pi}, \quad (5)$$

$$= \frac{4N_e \cdot \mu \cdot \sigma_M^2}{4N_e \cdot \mu} \text{ from eq. 1 and 4,} \quad (6)$$

$$= \sigma_M^2. \quad (7)$$

The additive genetic variance is also equal to the observed phenotypic variance ( $V_P$ ) multiplied by narrow-sense heritability ( $h^2$ ; (Hill et al., 2008)), which leads to  $\sigma_W^2$  being a function of  $V_P$  and  $h^2$ :

$$\sigma_W^2 = \frac{h^2 \cdot V_P}{\pi}. \quad (8)$$

96 **Between-species trait variations**

For a given species, we denote by  $\bar{P}_t$  the mean value of the trait across the individuals in the species at generation  $t$ . If the trait is neutral and encoded by many loci as assumed by the infinitesimal model,  $\bar{P}_t$

evolves as a Brownian process (Felsenstein, 1985; Hansen & Martins, 1996). The variance of  $\bar{P}_t$  after  $t$  generations,  $\text{Var}[\bar{P}_t]$  is given (Hansen & Martins, 1996) by:

$$\text{Var}[\bar{P}_t] = \frac{V_A}{N_e} \cdot t \quad (9)$$

$$= 4t \cdot \mu \cdot \sigma_M^2, \text{ from eq. 3,} \quad (10)$$

Moreover, for any genomic region under neutral evolution, some mutations will eventually reach fixation due to random genetic drift, resulting in a substitution of a nucleotide at the species level. The probability of fixation ( $P_{\text{fix}}$ ) of a neutral mutation is  $1/2N_e$  (Kimura, 1962). We can derive the substitution rate per generation  $q$  as the number of mutations per generation ( $2N_e \cdot \mu$ ) multiplied by the probability of fixation for each newly arisen mutations  $P_{\text{fix}}$  (McCandlish & Stoltzfus, 2014), giving:

$$q = 2N_e \cdot \mu \cdot P_{\text{fix}}, \quad (11)$$

$$= 2N_e \cdot \mu \cdot \frac{1}{2N_e}, \quad (12)$$

$$= \mu. \quad (13)$$

97 That is, if mutations are neutral, the rate of substitution within a genomic region equals the rate at which  
98 new mutations arise per generation for the same genomic region (Kimura, 1968).

After  $t$  generations and assuming that no multiple substitutions occurred at the same site, the nucleotide divergence  $d$ , which is the fraction of the genomic region that generated a substitution, will be  $t$  multiplied by the nucleotide substitution rate per generation ( $q$ ):

$$d = t \cdot q \quad (14)$$

$$= t \cdot \mu \text{ from eq. 13.} \quad (15)$$

We define  $\sigma_B^2$  as the variance in the mean trait value ( $\text{Var}[\bar{P}_t]$ ) normalized by the nucleotide divergence of any neutral genomic region ( $d$ ). This ratio allows removing the effect of  $t$  and  $\mu$ , which are parameters not related to the genetic architecture of the trade, giving  $\sigma_B^2$  as another proxy of  $\sigma_M^2$ :

$$\sigma_B^2 \stackrel{\text{def}}{=} \frac{\text{Var}[\bar{P}_t]}{4d}, \quad (16)$$

$$= \frac{4t \cdot \mu \cdot \sigma_M^2}{4t \cdot \mu} \text{ from eq. 10 and 15,} \quad (17)$$

$$= \sigma_M^2. \quad (18)$$

## 99 Neutrality index

The variability between either individuals or species can be obtained for both quantitative traits and genomic sequences. At the population level, the variability of the trait between individuals can be combined with the nucleotide diversity of any neutrally evolving genomic region to obtain  $\sigma_W^2$ , which equals  $\sigma_M^2$  if the trait is neutrally evolving (see above). At the phylogenetic level, the variability of the mean trait value between species can be combined with the nucleotide divergence of any neutrally evolving genomic region to obtain  $\sigma_B^2$ . Similarly,  $\sigma_B^2 = \sigma_M^2$  if the trait is neutrally evolving and the genetic architecture of the trait has not

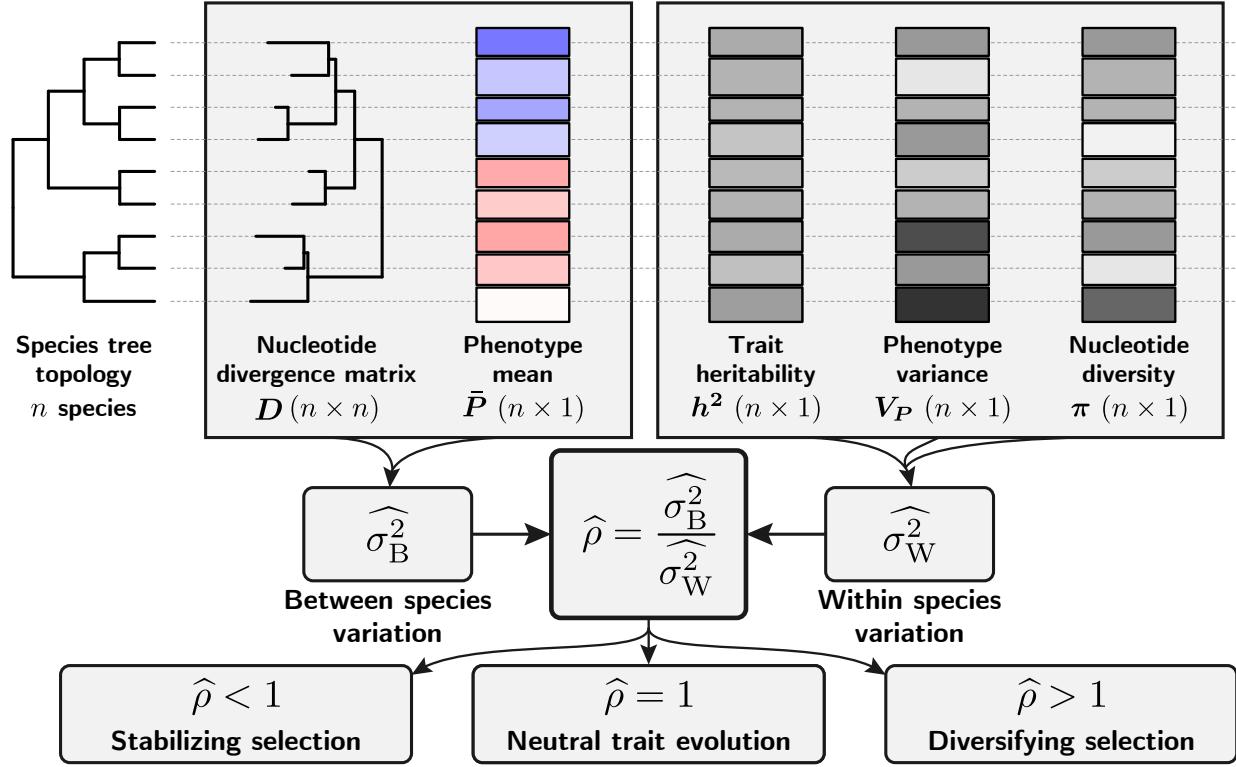


Figure 1: Between species, the change along the phylogeny of the mean phenotypic trait allows the estimation of between-species trait variation,  $\widehat{\sigma}_B^2$ , which is normalized by nucleotide divergence. Within species, the genetic variance allows the estimation of within-species trait variation,  $\widehat{\sigma}_W^2$ , which is normalized by nucleotide diversity.  $\widehat{\rho}$  is the ratio of  $\widehat{\sigma}_B^2$  over  $\widehat{\sigma}_W^2$ . Under neutral evolution,  $\widehat{\rho}$  is expected to be equal to one. Under diversifying selection, the trait is heterogeneous between species, but homogeneous within species, leading to  $\widehat{\rho}$  greater than one. Under stabilizing selection, the trait is homogeneous between species, leading to  $\widehat{\rho}$  smaller than one. Importantly, the sequence from which nucleotide diversity and divergence are estimated should be neutrally evolving, but they are not necessarily linked to the quantitative trait under study, they allow for discarding the confounding effect on mutation rate diversity, population size and divergence time.

changed along the phylogenetic tree. We thus have, for a neutrally evolving trait:

$$\sigma_W^2 = \sigma_B^2 \text{ from eq. 7 and 18,} \quad (19)$$

$$\Rightarrow \rho \stackrel{\text{def}}{=} \frac{\sigma_B^2}{\sigma_W^2} = 1. \quad (20)$$

100 We define a neutrality index  $\rho = \sigma_B^2 / \sigma_W^2$  that will equal 1 for a trait evolving neutrally. Both  $\sigma_B^2$  and  $\sigma_W^2$   
 101 can be estimated using quantitative trait and genomic sequences within and between species, while neither  
 102 the mutation rate ( $\mu$ ), nor the effective population size ( $N_e$ ) or time of divergence ( $t$ ) need to be estimated.  
 103 Moreover, the sequence from which  $\pi$  and  $d$  are estimated should be neutrally evolving, but they are not  
 104 necessarily linked to the quantitative trait under study.

105 **Estimate**

106 Based on the comparative framework that can account for phylogenetic inertia (Felsenstein, 1985; O'Meara  
 107 et al., 2006), we provide a maximum likelihood estimate for  $\rho$  as well as a Bayesian estimate to derive  
 108 posterior probabilities that the null model of neutrality (i.e.  $\rho = 1$ ) is rejected.

109 **Maximum likelihood estimate**

At the phylogenetic scale, for  $n$  taxa in the tree,  $\mathbf{D}$  ( $n \times n$ ) is the distance matrix computed from the branch lengths ( $d$  as nucleotide divergence in units of substitutions per site) and the topology of the phylogenetic tree. The diagonal  $\mathbf{D}_{i,i}$  represents the total distances from the root of the tree to each taxon ( $i$ ). The off-diagonal elements ( $\mathbf{D}_{i,j} = \mathbf{D}_{j,i}$ ) are the distances between the root and the most recent common ancestor of taxa  $i$  and  $j$ . The state  $P_0$  at the root of the tree for the trait can be estimated from the  $n \times 1$  vector of mean trait values  $\bar{\mathbf{P}}$  at the tips of the tree using maximum likelihood (O'Meara et al., 2006):

$$P_0 = (\mathbf{1}^\top \times \mathbf{D}^{-1} \times \mathbf{1})^{-1} \cdot (\mathbf{1}^\top \times \mathbf{D}^{-1} \times \bar{\mathbf{P}}), \quad (21)$$

110 where  $\mathbf{1}$  is an  $n \times 1$  column vector of ones.

Finally, between-species variation  $\widehat{\sigma}_B^2$  is estimated as (O'Meara et al., 2006):

$$\widehat{\sigma}_B^2 = \frac{1}{4} \frac{(\bar{\mathbf{P}} - P_0 \cdot \mathbf{1})^\top \times \mathbf{D}^{-1} \times (\bar{\mathbf{P}} - P_0 \cdot \mathbf{1})}{n-1}. \quad (22)$$

For a given species  $i$  with inter-individual data available, additive genetic variance of a trait ( $V_{A,i}$ ) is the product of heritability ( $h_i^2$ ) and phenotypic variance ( $V_{P,i}$ ). The ratio of  $V_{A,i}$  over nucleotide diversity of neutrally evolving sequences ( $\pi_i$ ) is a sample estimate of  $\sigma_W^2$ . Averaged across all species, we obtain the estimate  $\widehat{\sigma}_W^2$  as:

$$\widehat{\sigma}_W^2 = \frac{1}{n} \sum_{i=1}^n \frac{V_{A,i}}{\pi_i} = \frac{1}{n} \sum_{i=1}^n \frac{V_{P,i} \cdot h_i^2}{\pi_i}. \quad (23)$$

As depicted in fig. 1, the neutrality index is estimated as:

$$\widehat{\rho} = \frac{\widehat{\sigma}_B^2}{\widehat{\sigma}_W^2}. \quad (24)$$

111 **Bayesian estimate**

112 The Bayesian framework allows obtaining the posterior distribution of the neutrality index ( $\widehat{\rho}$ ) for a given  
 113 trait. Even though  $\widehat{\rho}$  is estimated independently for each trait of interest in the maximum likelihood frame-  
 114 work (previous section), here we generalize to  $K$  traits co-varying along the phylogenetic tree using the  
 115 *BayesCode* software (Latrille et al., 2021). Trait variation along the phylogenetic tree is modeled as a  $K$ -  
 116 dimensional Brownian process  $\mathbf{B}$  ( $1 \times K$ ) starting at the root and branching along the tree topology (Huelsen-  
 117 beck & Rannala, 2003; Lartillot & Poujol, 2011; Lartillot & Delsuc, 2012; Latrille et al., 2021). The rate  
 118 of change of the Brownian process is determined by the positive semi-definite and symmetric covariance  
 119 matrix between traits  $\Sigma$  ( $K \times K$ ). The off-diagonal elements of  $\Sigma$  are the covariance between traits, and  
 120 the diagonal elements are the variance of each trait, thus corresponding to  $\widehat{\sigma}_B^2$  (see section S2.1). With an

121 inverse Wishart distribution as the prior on the covariance matrix, the posterior on  $\Sigma$ , conditional on  $\mathcal{B}$   
122 is also an invert Wishart distribution (see section S2.2). We used Metropolis-Hastings algorithm to sample  
123  $\mathcal{B}$ , while the posterior distribution of  $\Sigma$  is sampled using Gibbs sampling. For each trait and each species,  
124 the prior on heritability ( $h^2$ ) for each species is set as a uniform distribution with user-defined boundaries.  
125 Heritability and phenotypic variance for each trait are combined with nucleotide diversity to compute  $\widehat{\sigma_W^2}$  for  
126 each species before being averaged across species (as in eq. 23). From  $\widehat{\sigma_W^2}$  and  $\Sigma$ , the posterior distribution  
127 of  $\widehat{\rho}$  (as in eq. 24) is obtained for each trait. The posterior distribution of  $\widehat{\rho}$  thus allows testing for deviation  
128 from neutrality (Fig. 1), for example, by computing  $\mathbb{P}[\widehat{\rho} > 1]$  to test for evidence of diversifying selection  
129 and  $\mathbb{P}[\widehat{\rho} < 1]$  to test for evidence of stabilizing selection.

### 130 **Applicability to empirical data**

131 Our method assumes that the narrow-sense heritability ( $h^2$ ) of a trait is known such as to estimate additive  
132 genetic variance ( $V_A$ ) from phenotypic variance ( $V_P$ ) as  $V_A = h^2 \cdot V_P$ . Fortunately, if heritability is not  
133 known, the test for diversifying selection can still be performed, although it is underpowered. Indeed, if  
134 the additive genetic variance is substituted by phenotypic variance, it is equivalent to assuming complete  
135 heritability ( $h^2 = 1$ ). Because  $h^2 \leq 1$  by definition, we overestimate the within-species variation and thus  
136 underestimate  $\widehat{\rho}$ . It is, however, possible to test for diversifying selection because testing for  $\widehat{\rho} > 1$  while using  
137 phenotypic variance instead of additive genetic variance means that knowing the additive genetic variance  
138 would have only increased the evidence for diversifying selection. Similarly, using the broad-sense heritability  
139 ( $H^2$ ) instead of narrow-sense heritability ( $h^2$ ) results in an underestimation of  $\widehat{\rho}$  since  $h^2 \leq H^2$ . In contrast,  
140 the test for stabilizing selection is invalid if  $\widehat{\rho}$  is underestimated. Several assumptions made by our test might  
141 not hold on empirical data and their consequences on the neutrality index and the test that can be performed  
142 are shown in Table 2.

### 143 **Simulation**

144 We tested the performance of our neutrality index ( $\rho$ ) to detect selection on a quantitative trait using  
145 simulations. We performed simulations under different selective regimes (neutral, stabilizing, diversifying),  
146 different demographic histories (constant or fluctuating population size) and different evolution of the mu-  
147 tation rate (constant or fluctuating). Simulations were individual-based and followed a Wright-Fisher model  
148 with mutation, selection and drift for a diploid population including speciation along a predefined ultrametric  
149 phylogenetic tree (Fig. 2A&B). Each individual phenotypic value was the sum of genotypic value and an  
150 environmental effect. The environmental effect was normally distributed with variance  $V_E$ . We assumed that  
151 the genotypic value was encoded by  $L = 5,000$  loci, with each locus contributing an additive effect that was  
152 normally distributed with standard deviation  $a = 1$  (Fig. 2A and section S1.1 for theoretical formulation).  
153 We assumed a trait with a narrow-sense heritability of  $h^2 = 0.2$  and computed the theoretical  $V_E$  accordingly  
154 (see section S1.1). Assuming a diploid panmictic population of size  $N_e = 50$  at the root of the tree, and with  
155 non-overlapping generations, we simulated explicitly each generation along an ultrametric phylogenetic tree.  
156 For each offspring, the number of mutations was drawn from a Poisson distribution with mean  $2 \cdot \mu \cdot L$ , with  
157 the mutation rate per generation  $\mu$ . From the empirical mammalian dataset (see next section), we computed

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158 an average nucleotide divergence from the root to leaves of 0.18 and average genetic diversity of 0.00276.  
159 We scaled parameters in our simulations to fit plausible values for mammals. We thus used a mutation rate  
160 of  $\mu = 0.00276/4N_e = 1.38 \times 10^{-5}$  per generation per locus and a total of  $t = 0.18/1.38 \times 10^{-5} = 13,500$   
161 generations from root to leaves, and the number of generations along each branch was proportional to the  
162 branch length.

163 The changes in  $\log-\mu$  and  $\log-N_e$  along the lineages were both modeled by a geometric Brownian process  
164 ( $\mathcal{B}(0, \sigma_\mu = 0.0086)$  and  $\mathcal{B}(0, \sigma_{N_e} = 0.0086)$ ), which led to a standard deviation of  $0.0086 \cdot \sqrt{13,500} = 1.0$   
165 in log-space from root to leaves. An Ornstein-Uhlenbeck process was overlaid to the instant value of  $\log-$   
166  $N_e$  provided by the geometric Brownian process to account for short-term changes between generations  
167 ( $\text{OU}(0, \sigma_{N_e} = 0.1, \theta_{N_e} = 0.9)$ ). The geometric Brownian motion accounted for long-term fluctuations (low  
168 rate of changes  $\sigma_{N_e}$  but unbounded), while the Ornstein-Uhlenbeck introduced short-term fluctuations (high  
169 rate of changes  $\sigma_{N_e}$  but bounded and mean-reverting). The simulation started from an initial sequence at  
170 equilibrium at the root of the tree and, at each node, the process was split until it finally reached the leaves  
171 of the tree. From a speciation process perspective, this was equivalent to an allopatric speciation over one  
172 generation.

173 A random genetic drift was introduced by resampling individuals at each generation, with each parent  
174 having a probability of being sampled that was proportional to its fitness ( $W$ ). Selection was modeled as a one-  
175 dimensional Fisher's geometric landscape, with the fitness of an individual being a monotonously decreasing  
176 function of the distance between the individual and the optimal phenotype (Blanquart & Bataillon, 2016;  
177 Tenaillon, 2014). More specifically, the fitness of an individual was given by  $W = e^{(P-\lambda)^2/\alpha}$ , where  $P$  was  
178 the trait value of the individual,  $\lambda = 0.0$  was the optimal trait value, and  $\alpha = 0.02$  was the strength of  
179 selection. Mutations were considered as a displacement of the phenotype in the multidimensional space.  
180 Beneficial mutations moved the phenotype closer to the optimum, while deleterious mutations moved it  
181 further away. Stabilizing selection was implemented by fixing the optimum phenotype to a single value  
182 ( $\lambda = 0.0$ ). Diversifying selection was implemented by allowing the optimum phenotype to move along the  
183 phylogenetic tree as a geometric Brownian process (Hansen, 1997) ( $\lambda \sim \mathcal{B}(0, \sigma_\lambda = 1.0)$ ). Neutral evolution  
184 was implemented by fixing the fitness landscape ( $W = 1$ ), which meant that each individual had the same  
185 probability of being sampled at each generation.

186 Nucleotide diversity ( $\pi$ ) was measured as the heterozygosity of neutral markers that were simulated  
187 along the phylogenetic tree but not linked to the trait simulated. Nucleotide divergence ( $d$ ) was measured  
188 as the number of substitutions per site of neutral markers along the branches of the phylogenetic tree. The  
189 additive genetic variance was measured as phenotypic variance multiplied by heritability. Heritability was  
190 estimated from the slopes of the regression of offspring's phenotypic trait values on parental phenotypic trait  
191 values (Lynch & Walsh, 1998) averaged over the last 10 simulated generations. Heritability was thus not a  
192 given parameter of the simulations, but rather measured as it would be in empirical data.

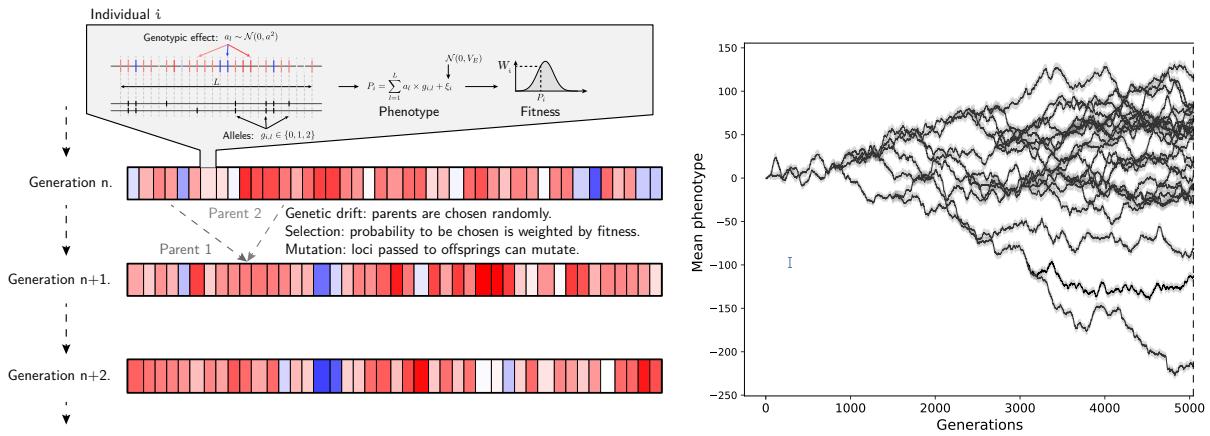


Figure 2: Wright-Fisher simulations with mutation, selection and drift. Left panel: For a given individual, the trait phenotypic value is the sum of genotypic value and a environmental effect (standard deviation  $V_E$ ). The trait's genotypic value is encoded by  $L$  loci, with each locus contributing additively to the genotypic value. Parents are selected for reproduction to the next generation according to their phenotypic value, with a probability proportional to their fitness. Mutations are drawn from a Poisson distribution, with each locus having a probability  $\mu$  to mutate. Drift is modeled by the resampling of parents. Right panel: examples of a trait evolving along a phylogenetic tree, with the mean phenotype (black line) and the variance of the trait genotypic value (gray area).

193 **Empirical dataset**

194 We analyzed a dataset of body and brain masses from mammals. The log-transformed values of body and  
 195 brain masses were taken from Tsuboi et al. (2018). We removed individuals not marked as adults and split  
 196 the data into males and females due to sexual dimorphism in body and brain masses. We discarded species  
 197 with only one representative sample. The mammalian nucleotide diversity was obtained from the Zoonomia  
 198 project (Genereux et al., 2020), with nucleotide divergence obtained on a set of neutral markers in Foley  
 199 et al. (2023), and with nucleotide diversity measured as heterozygosity in Wilder et al. (2023).

200 We also analyzed a dataset of primate species, with the nucleotide variation obtained from Kuderna  
 201 et al. (2023) and the quantitative trait variation also from Tsuboi et al. (2018), using the same filtering as  
 202 for the mammalian dataset. However, the primate nucleotide divergence was not obtained on a set of neutral  
 203 markers as for the mammalian dataset, but across the whole genome.

204 **Results**

205 **Neutrality index**

206 For a neutral trait, the genetic architecture, meaning the number of loci encoding the trait and the average  
 207 effect of a mutation on the trait, is formally related to both within and between-species variation of the  
 208 trait. We defined the neutrality index as  $\rho = \sigma_B^2 / \sigma_W^2$ , which equals 1 for a neutral trait (see Materials  
 209 and Methods), suggesting that traits for which this relationship was not verified were putatively under

selection. Under stabilizing selection, the variation between species is depleted because the mean trait value is maintained similar between different species, which leads to  $\rho < 1$ . In contrast, under diversifying selection, the variation between species is inflated because species will have potentially different trait values (Hansen, 1997), which leads to  $\rho > 1$ . Our neutrality index for a quantitative trait leveraged the data for any number of species, and took advantage of the signal over the whole phylogenetic tree, while at the same time taking into account phylogenetic inertia and addressing the non-independence between species (Fig. 1). This statistic was obtained as a maximum likelihood estimate ( $\hat{\rho}$ ), from eq. 23 and 22. We also devised a Bayesian estimate to obtain the posterior distribution of the neutrality index, and test for diversifying selection as  $\mathbb{P}[\hat{\rho} > 1]$ , and stabilizing selection as  $\mathbb{P}[\hat{\rho} < 1]$ .

Our neutrality index made a series of assumptions that we described in details in the Material and Methods section. Table 2 summarized these assumptions and outlined possible consequences for the neutrality test that we proposed.

## Results against simulations

The inference framework was first tested on independently simulated datasets matching an empirically relevant mammalian empirical regime (see Materials and Methods). Under constant population size ( $N_e$ ) and constant mutation rate ( $\mu$ ) across the phylogenetic tree (fig. 3, top row), we found no false negative for simulations of stabilizing ( $\mathbb{P}[\hat{\rho} < 1] > 0.975$ ; blue in fig. 3) or diversifying ( $\mathbb{P}[\hat{\rho} > 1] > 0.975$ ; red in fig. 3) selection. For simulations under neutral evolution, 77% of those were correctly identified ( $0.025 \leq \mathbb{P}[\hat{\rho} > 1] \leq 0.975$ ; yellow in fig. 3), while 21% and 2% were wrongly detected as stabilizing or diversifying selection, respectively. Once we introduced fluctuating  $N_e$  and  $\mu$  (Fig. 3, bottom row), our ability to identify simulations under either diversifying or stabilizing selection remained the same with all cases detected correctly. For simulations under neutral evolution, 51% of the simulations were correctly detected ( $0.025 \leq \mathbb{P}[\hat{\rho} > 1] \leq 0.975$ ), while 49% were detected as stabilizing selection ( $\mathbb{P}[\hat{\rho} < 1] > 0.975$ ) and none as diversifying selection.

## Results on empirical data

For mammalian body and brain mass, we obtained male ( $\sigma$ ) and female ( $\varphi$ ) trait variations. Combined with nucleotide diversity and divergence, we estimated  $\hat{\rho}$  and posterior probabilities of diversifying selection under different assumptions for trait heritability as shown in the Table 1. Assuming complete heritability, brain mass was found to be under diversifying selection with posterior probabilities of 0.0 for both males and females. If we assumed that heritability ( $h^2$ ) of body mass was uniformly distributed between 20% and 40% (Hu et al., 2022), posterior probabilities of diversifying selection became 0.635 for males and 0.324 for females. Mammalian brain mass was found to be under diversifying selection with posterior probabilities of 0.877 for males and 0.972 for females when complete heritability was assumed. Assuming a uniform distribution between 20% and 40% for heritability led to posterior probabilities of diversifying selection of 1.0 for both males and females.

We also analyzed a similar dataset for body mass focusing this time only at Primates (Table 1). For primates body mass, we found posterior probabilities of diversifying selection of 1.0 for males and 0.914

## TRAIT SELECTION FROM WITHIN AND BETWEEN-SPECIES VARIATION

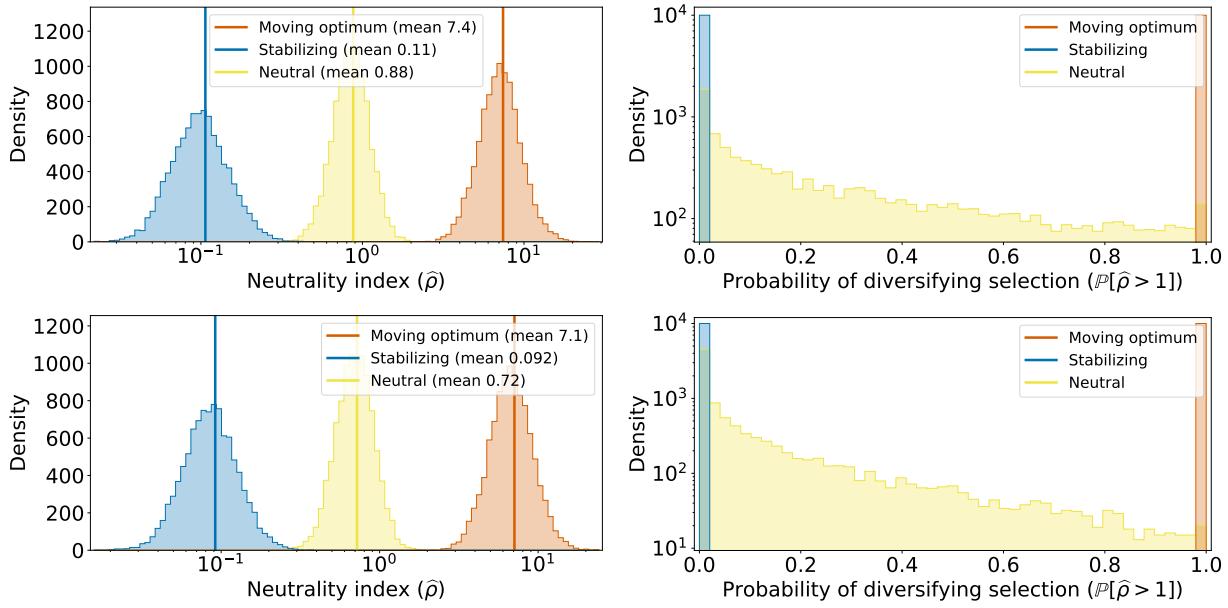


Figure 3: 10,000 simulations of trait evolution along a phylogenetic tree under different selection regimes. Traits simulated under stabilizing selection (blue), under a neutral evolution (yellow), and under a moving optimum (red). Histogram of ratio of between-species trait variation ( $\widehat{\sigma}_B^2$ ) over within-species trait variation  $\widehat{\sigma}_W^2$  with  $\widehat{\rho} = \widehat{\sigma}_B^2 / \widehat{\sigma}_W^2$  estimated from each simulated data (left) and probabilities of  $\widehat{\rho}$  being greater than 1 (right). Effective population size ( $N_e$ ) and mutation rate ( $\mu$ ) were either constant (top row), or fluctuating as a Brownian process along the phylogenetic tree (bottom row).

246 for females when assuming a uniform distribution for the heritability of body mass between 20% and 40%.  
 247 Assuming complete heritability of body mass did not change the posterior probability for males, but increased  
 248 the one for female to 1.0. Evidence for diversifying selection on body mass was therefore more pronounced in  
 249 Primates than in mammals. However, the genetic markers used to normalize trait variance with nucleotide  
 250 divergence were not necessarily neutral, which could create spurious false positives by artificially inflating  $\widehat{\rho}$   
 251 (Table 2 and methods).

## 252 Discussion

253 In this study, we proposed a neutrality index for a quantitative trait that can be used within a statistical  
 254 framework to test for selection. Our neutrality index for a trait,  $\rho$ , is calculated as the ratio of the normalized  
 255 within- to between-species variation and it allowed the identification of the evolutionary regime of a quanti-  
 256 tative trait. At the phylogenetic scale, trait variation between species was normalized by sequence divergence  
 257 obtained from a neutral set of markers. Similarly, trait variation within species was normalized by sequence  
 258 polymorphism obtained also from a neutral set of markers. Our estimate of  $\widehat{\rho}$  could be tested for deviation  
 259 from the value of 1.0 expected under the null hypothesis of neutrality. Technically, the neutrality index can  
 260 be estimated either as a maximum likelihood point estimate, or as a mean posterior estimate from a Bayesian  
 261 implementation (see section S3). The latter also enabled the estimation of the posterior credible interval to

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Dataset	Trait	$h^2$	Sex	$n$	$\hat{\rho}$	95% CI for $\hat{\rho}$	$\mathbb{P}[\hat{\rho} > 1]$
Mammals	Body mass	1.0	$\sigma$	36	0.340	0.217-0.523	0.000
Mammals	Body mass	1.0	$\varphi$	26	0.277	0.160-0.490	0.000
Mammals	Body mass	$\mathcal{U}(0.2, 0.4)$	$\sigma$	36	1.124	0.721-1.754	0.635
Mammals	Body mass	$\mathcal{U}(0.2, 0.4)$	$\varphi$	26	0.936	0.523-1.715	0.324
Mammals	Brain mass	1.0	$\sigma$	36	1.351	0.851-2.173	0.877
Mammals	Brain mass	1.0	$\varphi$	26	1.727	0.991-2.938	0.972
Mammals	Brain mass	$\mathcal{U}(0.2, 0.4)$	$\sigma$	36	4.527	2.831-7.091	1.000
Mammals	Brain mass	$\mathcal{U}(0.2, 0.4)$	$\varphi$	26	6.001	3.288-10.941	1.000
Primates	Body mass	1.0	$\sigma$	71	0.558	0.401-0.784	0.000
Primates	Body mass	1.0	$\varphi$	65	0.389	0.278-0.547	0.000
Primates	Body mass	$\mathcal{U}(0.2, 0.4)$	$\sigma$	71	1.875	1.288-2.695	1.000
Primates	Body mass	$\mathcal{U}(0.2, 0.4)$	$\varphi$	65	1.296	0.899-1.821	0.914
Primates	Brain mass	1.0	$\sigma$	71	1.929	1.395-2.616	1.000
Primates	Brain mass	1.0	$\varphi$	65	1.950	1.399-2.790	1.000
Primates	Brain mass	$\mathcal{U}(0.2, 0.4)$	$\sigma$	71	6.479	4.658-8.944	1.000
Primates	Brain mass	$\mathcal{U}(0.2, 0.4)$	$\varphi$	65	6.522	4.664-9.294	1.000

Table 1: Test of diversifying selection on a mammal and a primate dataset, by splitting males ( $\sigma$ ) and females ( $\varphi$ ). Traits considered were body mass or brain mass (log-transformed). Heritability ( $h^2$ ) was either assumed complete ( $h^2 = 1.0$ ) or uniformly distributed between 20% and 40% ( $h^2 \sim \mathcal{U}(0.2, 0.4)$ ).  $n$  was the number of species in the dataset.  $\hat{\rho}$  was the posterior estimate of our neutrality index, with the 95% credible interval (CI) for  $\hat{\rho}$  also computed.  $\mathbb{P}[\hat{\rho} > 1]$  was the estimated posterior probability of diversifying selection.

262 test for departure from a neutrally evolving trait (e.g.  $\mathbb{P}[\hat{\rho} > 1]$ ). We tested our statistical procedure against  
 263 simulated data and showed that our test was able to correctly detect simulations under diversifying selection  
 264 (test of  $\hat{\rho} > 1$ ) or under stabilizing selection (test of  $\hat{\rho} < 1$ ). However, our test detected a spurious signal  
 265 of stabilizing selection ( $\hat{\rho} < 1$ ) when we simulated the evolution of a neutral trait. We thus argue that our  
 266 method should be used to detect diversifying selection, but that it had low accuracy to detect stabilizing  
 267 selection due to false positives.

268 Our results showed that our method significantly improved over currently available methods to detect  
 269 selection acting on a trait at the phylogenetic scale. Current methods relying on evolution of the mean trait  
 270 value between species also tend to statistically prefer a model of stabilizing selection over a Brownian process  
 271 when the trait is neutral (Cooper et al., 2016; Price et al., 2022; Silvestro et al., 2015). Our approach could  
 272 in theory be applied to detect stabilizing selection at the phylogenetic scale, but we showed that it did  
 273 not have the statistical power to identify those cases. In contrast, we showed that our method was able to

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Broken assumption	Consequences	$\widehat{\sigma}_W^2$	$\widehat{\sigma}_B^2$	Test $\rho > 1$	Test $\rho < 1$
Trait encoded by few loci	Between-species trait variation is underestimated	–	Underestimated	Conservative	Invalid
Sexual dimorphism	Within-species trait variation is overestimated	Overestimated	–	Conservative	Invalid
Inbreeding	Nucleotide diversity ( $\pi$ ) is underestimated	Overestimated	–	Conservative	Invalid
Markers for polymorphism are negatively selected	Nucleotide diversity ( $\pi$ ) is underestimated	Overestimated	–	Conservative	Invalid
Markers for polymorphism are positively selected	Nucleotide diversity ( $\pi$ ) is underestimated	Overestimated	–	Conservative	Invalid
Markers for divergence are positively selected	Nucleotide divergence ( $d$ ) is overestimated	–	Underestimated	Conservative	Invalid
Markers for polymorphism under balanced selection	Nucleotide diversity ( $\pi$ ) is overestimated	Underestimated	–	Invalid	Conservative
Markers for divergence are negatively selected	Nucleotide divergence ( $d$ ) is underestimated	–	Overestimated	Invalid	Conservative
Multiple nucleotide substitutions at the same locus	Nucleotide divergence ( $d$ ) is underestimated	–	Overestimated	Invalid	Conservative

Table 2: Assumptions breaks and their consequences on the estimation of within-species variation ( $\widehat{\sigma}_W^2$ ), between-species variation ( $\widehat{\sigma}_B^2$ ), and on the neutrality index  $\rho = \widehat{\sigma}_B^2/\widehat{\sigma}_W^2$ . The last two columns indicate whether the test for diversifying selection ( $\rho > 1$ ) and for stabilizing selection  $\rho < 1$  are conservative or invalid due to violated assumptions.

274 identify correctly cases of diversifying selection, which is a clear an improvement over current methods that  
 275 model only mean trait value. Indeed, under diversifying selection, mean trait value will not deviate from  
 276 a Brownian process, and thus cannot be distinguished from neutral evolution (Hansen & Martins, 1996;  
 277 Harmon, 2018). For example, testing the selective regime in the expression level of the majority of genes  
 278 led to the selection of a Brownian process as the prefered model and the interpretation that the expression  
 279 was evolving neutrally (Catalán et al., 2019). Our diversity index has the advantage to discriminate the  
 280 alternative model of diversifying selection from the neutral case by comparing within- and between-species  
 281 variation correctly normalized to remove confounding factors. Our approach is not the first one to normalize  
 282 between-species variation to detect selection, but this was done by using within-species variations (Rohlf  
 283 et al., 2014; Rohlf & Nielsen, 2015) and not estimates of neutral molecular divergence as done in our study.  
 284 These studies have further compared their statistic across a pool of traits, which allowed them to identify  
 285 outlier traits putatively under diversifying selection but without testing for selection on a single trait at a  
 286 time (Gillard et al., 2021; Rohlf & Nielsen, 2015). Instead, our procedure can be applied to a single trait,  
 287 estimating the neutrality index and giving a statistical test for departures from the null model of neutral  
 288 evolution for a single test. Our diversity index opens new avenues to revisit these studies and better test  
 289 for the selective regime affecting the quantitative traits, assuming we have access to genomic datasets to  
 290 estimate nucleotide divergence and polymorphism.

291 The main novelty of our study was to use the nucleotide divergence and polymorphism to normalize trait  
 292 variation between and within species. In the context of within species variation,  $Q_{ST}-F_{ST}$  tests have been  
 293 developed to compare trait and sequence across several populations to test for selection (Leinonen et al., 2013;  
 294 Martin et al., 2008). Our neutrality index also used the genetic sequences from which nucleotide divergence  
 295 and polymorphism are estimated. Although the sequences should be neutrally evolving, they do not have to be  
 296 necessarily linked to the quantitative trait under study. Nucleotide variation allows normalizing for diversity  
 297 driven by confounding factors such as population sizes ( $N_e$ ), mutation rates ( $\mu$ ) and generation time (Hansen  
 298 & Martins, 1996; Harmon, 2018). Thus our test avoids the estimation of the parameters, which are complex  
 299 to correctly infer, and it also bypasses the estimation of divergence time, which was necessary in previous

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300 approaches (Walsh & Lynch, 2018). But importantly, by normalizing with sequence variation, we also showed  
301 using simulated data that our test was not sensitive to the assumption that  $N_e$ ,  $\mu$  and generation time were  
302 constant across the phylogenetic tree, an unmet assumption empirically (Bergeron et al., 2023; Wilder et al.,  
303 2023). Indeed, under the neutral case of evolution, changes in  $N_e$ ,  $\mu$  and generation time impacted similarly  
304 trait and sequence variation. The normalization by nucleotide divergence and polymorphism automatically  
305 absorbed long-term and short-term changes in  $N_e$ ,  $\mu$  and generation time, which canceled out in the ratio  
306 of trait variation  $\hat{\rho}$ .

307 Even though our test was developed for a quantitative trait, analogies with other tests of selection  
308 developed for molecular sequences also provided insight into its behavior. First, we acknowledge that our  
309 test took inspiration from the McDonald and Kreitman (1991) test devised for protein-coding DNA sequences,  
310 where synonymous mutations were used to determine the neutral expectation, and the inflation of divergence  
311 was compared to polymorphism within species. Second, because  $\rho$  was compared to 1, our test ultimately bear  
312 analogy to the codon-based test of selection, where the ratio of non-synonymous to synonymous substitutions  
313 ( $\omega$ ) is compared to 1 (Goldman & Yang, 1994; Muse & Gaut, 1994). As  $\omega < 1$  is interpreted as purifying  
314 selection acting on the protein,  $\rho < 1$  is interpreted as stabilizing selection acting on the trait. Similarly, the  
315 interpretation of adaptation for  $\omega > 1$  is analogous to diversifying selection for  $\rho > 1$ . With this analogy  
316 in mind, we could leverage the vast literature discussing and interpreting the results of these tests and  
317 their pitfalls (Anisimova & Kosiol, 2009; Jensen et al., 2019; Nielsen, 2005). First, not rejecting the neutral  
318 null model of  $\rho = 1$  did not necessarily imply that the trait was effectively neutral, since diversifying and  
319 stabilizing selection could compensate each other resulting in  $\rho = 1$ , analogously to  $\omega = 1$  under a mix  
320 of adaptation and purifying selection (Nielsen, 2005). Second, empirical evidence for  $\rho < 1$  did not rule  
321 out diversifying selection, but rather that this diversifying selection was not strong enough to overcome the  
322 stabilizing selection, similarly to strong purifying selection resulting  $\omega < 1$  even though those genes and  
323 sites are under adaptation (Latrille et al., 2023). By explicitly modeling stabilizing selection as a moving  
324 optimum, it would theoretically be possible to tease apart the effect of diversifying and stabilizing selection  
325 in the context of quantitative traits to obtain a statistically more powerful test.

326 In the context of detecting diversifying selection on a trait, we argue that the main drawback of our  
327 method is that the additive genetic variance of the trait is required instead of the phenotypic variance. If  
328 phenotypic variance was used instead of additive genetic variance to estimate  $\hat{\rho}$ , meaning that we assumed  
329 complete heritability, the neutrality index  $\hat{\rho}$  was ultimately underestimated. Similarly, using broad-sense  
330 heritability instead of narrow-sense heritability would result in underestimated  $\hat{\rho}$ . In such context, the test  
331 of stabilizing selection ( $\hat{\rho} < 1$ ) would be statistically invalid. However, the test of diversifying selection  
332 ( $\hat{\rho} > 1$ ) was underpowered although not invalidated, meaning that absence of evidence would not be evidence  
333 of absence. As an example, even though we assumed complete heritability for brain mass, we uncovered  
334 diversifying selection in mammals since  $\hat{\rho} > 1$ .

335 The development of our neutrality index was also based on several assumptions that could be relaxed  
336 in future studies. First, we cannot predict the behavior of our test in the context of population structures,  
337 gene flow and introgression. These factors should be thoroughly investigated using simulations. Second, loci

338 were assumed to contribute additively to the phenotype. Although the effects of dominance and epistasis  
339 is typically weak compared to the additive effects on the quantitative traits, their influence should be as-  
340 sessed (Crow, 2010; Hill et al., 2008). Third, the genetic architecture of the trait was assumed to be constant  
341 across the phylogenetic tree, whereas it might actually be variable among individuals and species (Huber  
342 et al., 2015; Tung et al., 2015). Such an assumption can theoretically be relaxed and changes in genetic  
343 architecture along the phylogenetic tree could jointly be estimated (Arnold et al., 2008; Gaboriau et al.,  
344 2020; Hohenlohe & Arnold, 2008; Kostikova et al., 2016). Finally, our Bayesian estimation could integrate  
345 uncertainty from the estimation of genetic variation, using sequences as input instead of estimated values of  
346 nucleotide diversity and divergence.

347 From an empirical point of view, our method required integrating genomic and trait variation, which  
348 could reduce the possible datasets to be used. However, such datasets will become more and more accessible  
349 and we showed the applicability of our method by applying it to the illustrative example of mammals brain  
350 and body mass. Because our test was also based on several assumptions that might not hold on empirical  
351 data, we also provided a table containing the main assumptions and their consequences on the neutrality  
352 index and the test that can be performed (Table 2). For example, at the primate scale, the evidence for  $\hat{\rho} > 1$   
353 does not necessarily imply that the brain mass was evolving under diversifying selection since the markers  
354 used for nucleotide divergences were not neutral, which can lead to a spurious  $\hat{\rho} > 1$ . In conclusion, our  
355 study provided a statistical framework to test for diversifying selection acting on a quantitative trait while  
356 integrating the trove of genomic data available both within and between species, and we believe that our  
357 new approach is a promising tool to investigate the evolution of quantitative traits.

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### 362 Competing interests:

363 The authors declare no conflicts of interest.

### 364 Data and materials availability:

365 The data that support the findings of this study are openly available in GitHub at  
366 [github.com/ThibaultLatrille/MicMac](https://github.com/ThibaultLatrille/MicMac). Snakemake pipeline, analysis scripts and documentation are  
367 available in the repository to replicate the study.

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# DETECTING DIVERSIFYING SELECTION FOR A TRAIT FROM WITHIN AND BETWEEN-SPECIES GENOTYPES AND PHENOTYPES

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**T. Latrille<sup>1</sup> , M. Bastian<sup>2</sup> , T. Gaboriau<sup>1</sup> , N. Salamin<sup>1</sup> **

<sup>1</sup>Department of Computational Biology, Université de Lausanne, Lausanne, Switzerland

<sup>2</sup>Laboratoire de Biométrie et Biologie Evolutive, UMR5558, Université Lyon 1, Villeurbanne, France

[thibault.latrille@ens-lyon.org](mailto:thibault.latrille@ens-lyon.org)

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## 562 **Supplementary materials**

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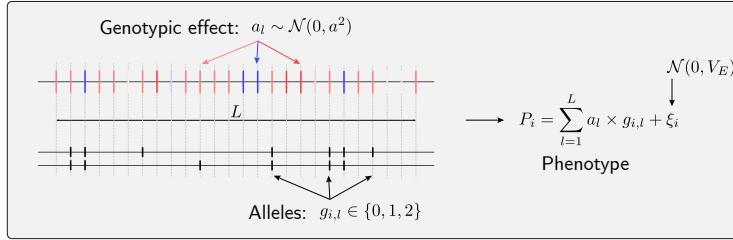
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578 **1 Genetic architecture of the trait**

579 **1.1 Genotype-phenotype map**

- 580 •  $L$  is the number of loci encoding the trait.
- 581 •  $a_l \sim \mathcal{N}(0, a^2)$  is the effect of a mutation on the trait at locus  $l \in \{1, \dots, L\}$ .
- 582 •  $N_e$  is the effective number of individuals.
- 583 •  $g_{i,l} \in \{0, 1, 2\}$  is the genotypic value at locus  $l$  for individual  $i \in \{1, \dots, N_e\}$ .
- 584 •  $G_i = \sum_{l=1}^L a_l \times g_{i,l}$  is the genotypic value for individual  $i$ .
- 585 •  $\xi_i \sim \mathcal{N}(0, V_E)$  is the effect of environment on the trait for individual  $i$ .
- 586 •  $P_i = G_i + \xi_i$  is the phenotype for individual  $i$ .

587 Figure S1: summary of trait's genetic architecture.



588

589 within-species, the mean ( $\bar{G}$ ) and variance ( $V_A$ ) of the genotype are:

$$590 \quad \bar{G} = \frac{1}{N_e} \sum_{i=1}^{N_e} G_i \quad \text{and} \quad V_A = \frac{1}{N_e} \sum_{i=1}^{N_e} (G_i - \bar{G})^2 \quad (25)$$

590 The theoretical additive genetic variance ( $V_A$ ) is a function of the number of loci ( $L$ ) and the effect of a  
591 mutation ( $a$ ) as:

$$592 \quad V_A = 4N_e \cdot \mu \cdot L \cdot a^2 \quad (26)$$

592 The mean ( $\bar{P}$ ) and variance ( $V_P$ ) of the phenotype are:

$$593 \quad \bar{P} = \frac{1}{N_e} \sum_{i=1}^{N_e} P_i \quad \text{and} \quad V_P = \frac{1}{N_e} \sum_{i=1}^{N_e} (P_i - \bar{P})^2 \quad (27)$$

593 Heritability ( $h^2$ ) is defined as:

$$594 \quad h^2 = \frac{V_A}{V_P} = \frac{V_A}{V_A + V_E} \quad (28)$$

594 Altogether, effective population size ( $N_e$ ), the number of loci ( $L$ ) and the effect of a mutation ( $a$ ), we can  
595 compute the variance of the environment ( $V_E$ ) that is required to reach a given heritability ( $h^2$ ) as:

$$595 \quad V_E = V_A \cdot \left( \frac{1}{h^2} - 1 \right) = 4N_e \cdot \mu \cdot L \cdot a^2 \cdot \left( \frac{1}{h^2} - 1 \right) \quad (29)$$

596 **2 Bayesian estimate**

597 **2.1 Multivariate Brownian process**

598 Here we generalize to  $K$  traits evolving along the phylogeny and are correlated between them. Their variation  
 599 along the phylogeny is modeled as a  $K$ -dimensional Brownian process  $\mathcal{B}$  ( $1 \times K$ ) starting at the root and  
 600 branching along the tree topology. The rate of change of the Brownian process is determined by the positive  
 601 semi-definite and symmetric covariance matrix between traits  $\Sigma$  ( $K \times K$ ). Along branch  $j$  with length  $d_j$ ,  
 602 the Brownian process start at the ancestral node  $\mathcal{A}(j)$  with value  $\mathcal{B}(\mathcal{A}(j))$ , and ends at node  $\mathcal{R}(j)$  with value  
 603  $\mathcal{B}(\mathcal{R}(j))$ . The independent contrast  $\mathbf{C}_j$  defined as change in trait along the branch normalized by  $\sqrt{d_j}$  is a  
 604 multivariate Gaussian:

$$C_j = \frac{\mathcal{B}(\mathcal{R}(j)) - \mathcal{B}(\mathcal{A}(j))}{\sqrt{d_j}} \sim \mathcal{N}(\mathbf{0}, \Sigma). \quad (30)$$

605 **2.2 Sampling the covariance matrix**

606 From the independent contrast at each branch of the tree ( $\mathbf{C}_j$ ), we can define the  $K \times K$  scatter matrix,  $\mathbf{A}$ ,  
 607 as:

$$\mathbf{A} = \sum_{j=1}^{2n-2} \mathbf{C}_j \times [\mathbf{C}_j]^\top, \quad (31)$$

608 where  $2n - 2$  is the number of branches in the tree and  $n$  the number of taxa.

609 The prior on the covariance matrix is an inverse Wishart distribution, with  $K + 1$  degrees of freedom:

$$\Sigma \sim \text{Wishart}^{-1}(\mathbf{I}, K + 1). \quad (32)$$

610 By Bayes theorem, the posterior on  $\Sigma$ , conditional on a particular realization of  $\mathcal{B}$  (and thus of  $\mathbf{C}$ ) is an  
 611 invert Wishart distribution, of parameter  $\mathbf{I} + \mathbf{A}$  and with  $2n + 1$  degrees of freedom.

$$\Sigma \sim \text{Wishart}^{-1}(\mathbf{I} + \mathbf{A}, 2n + 1) \quad (33)$$

612 This invert Wishart distribution can be obtained by sampling  $2n + 1$  independent and identically distributed  
 613 multivariate normal random variables  $\mathbf{Z}_k$  defined by

$$\mathbf{Z}_k \sim \mathcal{N}(\mathbf{0}, [\mathbf{I} + \mathbf{A}]^{-1}). \quad (34)$$

614 And from these multivariate samples,  $\Sigma$  is Gibbs sampled as:

$$\Sigma = \left( \sum_{k=1}^{2n+1} \mathbf{Z}_k \times [\mathbf{Z}_k]^\top \right)^{-1} \quad (35)$$

### 615 3 Bayesian and Maximum-likelihood implementation

616 Implementation is included within the *BayesCode* software, available at <https://github.com/ThibaultLatrille/bayescode>.

#### 618 3.1 Data formatting

619 Running the analysis on your dataset and compute posterior probabilities requires three files:

620 1. A phylogenetic tree in newick format, with branch lengths in number of substitutions per site (neutral  
621 markers).

622 2. A file containing the mean trait values for each species.

623 3. A file containing the variation within-species for each trait and the genetic variation within-species  
624 (neutral markers).

##### 625 3.1.1 Phylogenetic tree

626 The phylogenetic tree must be in newick format, with branch lengths in substitutions per site (neutral  
627 markers).

##### 628 3.1.2 Mean trait for each species

629 The file containing mean trait values for each species must be in a tab-delimited file with the following  
630 format:

TaxonName	Body_mass	Brain_mass
Panthera_tigris	12.26	5.676
Pithecia_pithecia	7.256	3.436
Colobus_angolensis	9.176	4.284
Saimiri_boliviensis	6.845	3.279
:	:	:

632 The columns are:

633 • *TaxonName*: the name of the taxon matching the name in the alignment and the tree.

634 • As many columns as traits, without spaces or special characters in the trait.

635 • The values can be NaN to indicate that the trait is not available for that taxon.

636 **3.1.3 Trait variation for each species**

637 The file containing trait variation for each species must be in a tab-delimited file with the following format:

638

TaxonName	Nucleotide_diversity	Body_mass_variance	Body_mass_heritability	Brain_mass_variance	Brain_mass_heritability
Pithecia_pithecina	0.0016	0.22871	0.2	0.00737	0.2
Colobus_angolensis	0.0017	0.00393	0.2	0.00416	0.2
Saimiri_boliviensis	0.0013	0.00022	0.2	0.00045	0.2
Pygathrix_nemaeus	0.0016	0.00347	0.2	0.00097	0.2
:	:	:	:	:	:

639

- 640 • *TaxonName*: the name of the taxon matching the name in the alignment and the tree.
- 641 • *Nucleotide\_diversity*: the nucleotide diversity within-species (neutral markers), cannot be `NaN`.
- 642 • As many columns as traits, without spaces or special characters in the trait.
- 643 • *TraitName\_variance*: the phenotypic variance of the trait within-species, can be `NaN` to indicate that
- 644 the trait variance is not available for that taxon.
- 645 • *TraitName\_heritability* (optional): the heritability of the trait within-species, between 0 and 1, cannot
- 646 be `NaN`.
- 647 • The columns with the suffix `_variance` and `_heritability` are repeated for each trait.
- 648 • *TraitName\_heritability\_lower* (optional): the lower bound of the heritability of the trait within-
- 649 species, between 0 and 1, cannot be `NaN`.
- 650 • *TraitName\_heritability\_upper* (optional): the upper bound of the heritability of the trait within-
- 651 species, between 0 and 1, cannot be `NaN`.
- 652 • If the columns with the suffix `_heritability_lower` and `_heritability_upper` are present, the
- 653 heritability is randomly drawn from a uniform distribution between the lower and upper bounds.
- 654 • If the columns with the suffix `_heritability` is present, it is taken as is.
- 655 • If the additive genetic variance (instead of phenotypic variance) is available for a trait, the heritability
- 656 can be omitted and will automatically be set to 1.0.

657 **3.2 Bayesian estimation**

658 The executable `nodetraits` from *BayesCode* is used to run the Bayesian estimation of the model, and the  
659 executable `readnodetraits` is used to read the results.

660 Assuming that the file `data/body_size/mammals.male.tsv` contains the mean trait values for  
661 each species, the file `data/body_size/mammals.male.var_trait.tsv` contains the variation within-  
662 species for each trait and the genetic variation within-species (neutral markers), and the file  
663 `data/body_size/mammals.male.tree` contains the phylogenetic tree, the following commands are used to  
664 run the model and read the results.

665 **3.2.1 Running the model**

666 `nodetraits` is run with the following command:

```
667 nodetraits --until 2000
668     --tree data/body_size/mammals.male.tree
669     --traitsfile data/body_size/mammals.male.tsv
670     run_mammals_male
```

671 **3.2.2 Reading the results**

672 Once the model has run, the chain `run_mammals_male` is used to compute the posterior distribution of the  
673 ratio of between-species variation over within-species variation with `readnodetraits`:

```
674 readnodetraits --burnin 1000
675     --var_within data/body_size/mammals.male.var_trait.tsv
676     --output results_mammals_male.tsv
677     run_mammals_male
```

678 The file `data.empirical/chain_name.ratio.tsv` then contains the posterior mean of the ratio of between-  
679 species variation over within-species variation, the 95% and 99% credible interval, and the posterior proba-  
680 bility that the ratio is greater than 1.

681 **3.3 Maximum likelihood estimation**

682 To obtain the ratio (without the posterior credible interval and probability) using maximum likelihood  
683 computation, the following python script can be used:

```
684 python3 utils/neutral_index.py --tree data/body_size/mammals.male.tree
685     --traitsfile data/body_size/mammals.male.tsv
686     --var_within data/body_size/mammals.male.var_trait.tsv
687     --output results_ML_mammals_male.tsv
```