

Genome Analysis

Pangenome graph layout by Path-Guided Stochastic Gradient Descent

Simon Heumos ^{1,2,†}, Andrea Guarracino ^{3,4,†}, Jan-Niklas Manuel Schmelzle ^{5,6}, Jiajie Li ⁶, Zhiru Zhang ⁶, Jörg Hagmann ⁷, Sven Nahnsen ^{1,2}, Pjotr Prins ³, Erik Garrison ^{3,*}

¹Quantitative Biology Center (QBiC), University of Tübingen, Tübingen 72076, Germany

²Biomedical Data Science, Department of Computer Science, University of Tübingen, Tübingen 72076, Germany

³Department of Genetics, Genomics and Informatics, University of Tennessee Health Science Center, Memphis, TN 38163, USA

⁴Genomics Research Centre, Human Technopole, Milan 20157, Italy

⁵Department of Computer Engineering, School of Computation, Information and Technology (CIT), Technical University of Munich, Munich 80333, Germany

⁶School of Electrical and Computer Engineering, Cornell University, Ithaca, NY 14853, USA

⁷Computomics GmbH, Eisenbahnstr. 1, 72072 Tübingen, Germany

*To whom correspondence should be addressed.

†The authors wish it to be known that, in their opinion, the first two authors should be regarded as Joint First Authors.

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Abstract

Motivation: The increasing availability of complete genomes demands for models to study genomic variability within entire populations. Pangenome graphs capture the full genetic diversity between multiple genomes, but their layouts may exhibit complex structures due to common, nonlinear patterns of genome variation and evolution. These structures hamper downstream analyses, visualization, and interpretation.

Results: In response, we introduce a novel graph layout algorithm: the Path-Guided Stochastic Gradient Descent (PG-SGD). PG-SGD uses the genomes, represented in the pangenome graph as paths, to move pairs of nodes in parallel applying a modified HOGWILD! strategy. We show that our implementation efficiently computes the layout of gigabase-scale pangenome graphs, unveiling their biological features.

Availability: We integrated PG-SGD in *ODGI* which is released as free software under the MIT open source license. Source code is available at <https://github.com/pangenome/odgi>.

Contact: egarris5@uthsc.edu

1 Introduction

Reference genomes are widely used in genetics, serving as a foundation for a variety of analyses, including gene annotation, read mapping, and variant detection (Singh *et al.*, 2022). However, this linear model is becoming obsolete given the accessibility to hundreds or even thousands of high-quality genomes. A single genome can not fully represent the genetic diversity of any species, resulting in reference bias (Ballouz *et al.*, 2019). In contrast, a pangenome models the entire set of genomic elements of a given population (Tettelin *et al.*, 2008; Computational Pan-Genomics Consortium, 2018; Eizenga *et al.*, 2020; Sherman and Salzberg, 2020). Pangenomes can be represented as a sequence graph incorporating sequences as nodes and their relationships as edges (Hein, 1989). In the

variation graph model (Garrison *et al.*, 2018), genomes are encoded as paths traversing the nodes in the graph.

A pangenome graph layout is the arrangement of nodes and edges in an N -dimensional space to produce a human-readable visualization of genetic variation between multiple genomes. Layout algorithms aim to find optimal node coordinates in order to minimize overlapping nodes or edges, reduce edge crossings, and promote an intuitive graph understanding. One popular approach is force-directed graph drawing (Cheong and Si, 2022) which produces aesthetic layouts. This is prone to get stuck in local minima, but stochastic gradient descent (SGD) implementations alleviate such a problem (Zheng *et al.*, 2019). SGD uses the gradient of its individual terms to approximate the gradient of a sum of functions.

Typically, force-directed layouts are hard to compute (Wang *et al.*, 2014), but the lock-free HOGWILD! method offers a highly parallelizable

and thus scalable SGD approach that can be applied when the optimization problem is sparse (Recht *et al.*, 2011).

In practice, multidimensional scaling (MDS) is applied to minimize the difference between the visual distance and theoretical graph distance. This can be accomplished by using pairwise node distances to minimize an energy function. Since pangenome graphs represent genomes as paths in the graph, a reasonable distance metric would be the nucleotide distance between a pair of nodes traversed by the same path.

Here, we present a new pangenome graph layout algorithm which applies a path-guided stochastic gradient descent (PG-SGD) to move pairs of nodes in parallel with a modified HOGWILD! strategy. The algorithm computes the pangenome graph layout that best reflects the nucleotide sequences in the graph. To our knowledge, no algorithm takes into account such biological information to compute the graph layouts. PG-SGD can be extended in any number of dimensions. In the ODGI toolkit (Guarracino *et al.*, 2022), we provide implementations for 1-dimensional (1D) and 2-dimensional (2D) layouts. These algorithms have already been successfully applied to construct and visualize large-scale pangenome graphs of the Human Pangenome Reference Consortium (HPRC) (Liao *et al.*, 2023; Guarracino *et al.*, 2023).

2 Algorithm

While PG-SGD is inspired by Zheng *et al.* (2019), we designed the algorithm to work on the variation graph model (Definition 2.1).

Definition 2.1. Variation graphs are a mathematical formalism to represent pangenome graphs (Garrison, 2019). In the variation graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{P})$, nodes (or vertices) $\mathcal{V} = v_1 \dots v_{|\mathcal{V}|}$ contain nucleotide sequences. Each node v_i has a unique identifier i and an implicit reverse complement $\bar{v}_i$. The node strand o represents the node orientation. Edges $\mathcal{E} = e_1 \dots e_{|\mathcal{E}|}$ connect ordered pairs of node strands ($e_i = (o_a, o_b)$), defining the graph topology. Paths $\mathcal{P} = p_1 \dots p_{|\mathcal{P}|}$ are series of connected steps s_i that refer to node strands in the graph ($p_i = s_1 \dots s_{|p_i|}$); the paths represent the genomes embedded in the graph.

We report PG-SGD’s pseudocode in Algorithm 1 and its schematic in Figure 1. In brief, the algorithm moves one pair of nodes (v_i, v_j) at a time, minimizing the difference between the layout distance ld_{ij} of the two nodes and the nucleotide distance nd_{ij} of the same nodes as calculated along a path that traverses them. In the 2D layouts, nodes have two ends. When moving a pair of nodes, we actually move one end of each node. For clarification, an example is given in Figure 1. v_i is the node associated with the step s_i sampled uniformly from all the steps in $\mathcal{P}$. v_j is the node associated with the step s_j sampled from the same path of s_i by drawing a uniform or a Zipfian distribution (Zipf, 1932). The difference between nd_{ij} and ld_{ij} guides the update of the node coordinates in the layout. The magnitude r of the update depends on the learning rate μ . The number of iterations steers the annealing step size η which determines the learning rate μ . A large η in the first iterations leads to a globally linear (in 1D) or planar (in 2D) layout. By decreasing η , the layout adjustments become more localized, ensuring that the nodes are positioned to best reflect the nucleotide distances in the paths (i.e., in the genomes).

Originating from empirical inspection of word frequency tables, Zipf’s law states that a word with rank n occurs $1/n$ times as the most frequent one. This law is modeled by the Zipf distribution. Sampling s_j from a Zipf distribution fixed in the s_i ’s path position space increases the possibility to draw a nucleotide position close to s_i . So there is a high chance to use small nucleotide distances nd_{ij} to refine the layout of nodes comprising a few base pairs. The Zipf distribution is also long-tailed, with many occurrences of low frequency events. However, extremely long-range correlations might not be captured sufficiently, resulting in

PG-SGD ($\mathcal{G}$):

```

input: variation graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{P})$ 
output:  $N$ -dimensional layout  $\mathcal{L}$  with  $|\mathcal{V}|$  nodes
 $\mathcal{XP} \leftarrow \text{PathIndex}(\mathcal{G})$  // for path position
                                lookup
 $\mathcal{L} \leftarrow \text{LayoutInitialization}(\mathcal{V}, N)$ 
 $\mathcal{Z} \leftarrow \text{InitZipf}(\mathcal{G}, \mathcal{XP})$  // Zipfian distribution
for  $\eta$  in annealing schedule:
    for each planned term update:
         $s_i \leftarrow \text{Unif}(\mathcal{XP})$  // uniform sampling of a
                                step from  $\mathcal{P}$ 
         $p \leftarrow \text{Path}(s_i, \mathcal{XP})$  // path of  $s_i$ 
        if (cooling || flip) then
             $s_j \leftarrow \text{Unif}(\text{StepCount}(p, \mathcal{XP}))$  // uniform
                                sampling of a step from  $p$ 
        else
             $s_j \leftarrow \text{Zipf}(p)$  // Zipfian sampling of a
                                step from  $p$ 
        end
         $p_i \leftarrow \text{StepPos}(s_i)$  // nuc. position
         $p_j \leftarrow \text{StepPos}(s_j)$  // nuc. position
         $nd_{ij} \leftarrow \|p_i - p_j\|$  // nuc. distance
         $ld_{ij} \leftarrow \|l_i - l_j\|$  // layout distance
         $w_{ij} \leftarrow \frac{1.0}{nd_{ij}}$  // term weight
         $\mu \leftarrow w_{ij}\eta$  // learning rate
        if  $\mu > 1$ :
             $\mu \leftarrow 1$ 
        end
         $\delta \leftarrow \mu \cdot \frac{ld_{ij} - nd_{ij}}{2}$  // the actual delta
        if  $\text{abs}(\delta) \leq 0$  then
             $\text{STOP}$  // we can’t optimize more
         $r \leftarrow \delta - ld_{ij}$  // size of the update
         $l_i \leftarrow l_i + r \cdot ld_{ij}$  // update  $v_i$  coordinates
         $l_j \leftarrow l_j - r \cdot ld_{ij}$  // update  $v_j$  coordinates
    end
end

```

Algorithm 1: Pseudocode of PG-SGD in 1D.

collapsed layouts for structures that are otherwise linear. To provide balance between global and local layout updates, in half of the updates (flip flag in Algorithm 1), the s_j is sampled uniformly instead from a Zipf distribution, with uniform sampling being more favorable for global updates. Furthermore, to enhance local linearity (in 1D) or planarity (in 2D) of the graph layout, a cooling phase skews the Zipfian distribution after half of iterations have been completed. This increases the likelihood of sampling smaller nucleotide distances for the layout updates.

3 Implementation

We implemented PG-SGD in ODGI (Guarracino *et al.*, 2022): the 1D version can be found in *odgi sort* and the 2D version in *odgi layout*. To efficiently retrieve path nucleotide positions, we implemented a path index. This index is a strict subset of the XG index (Garrison *et al.*, 2018) where we avoid to use succinct SDSL data structures (Gog *et al.*, 2014). Instead, we rely on bit-compressed integer vectors, enabling efficient retrieval of path nucleotide positions to quickly compute nucleotide distances without having to store all pairwise distances between nodes in memory.

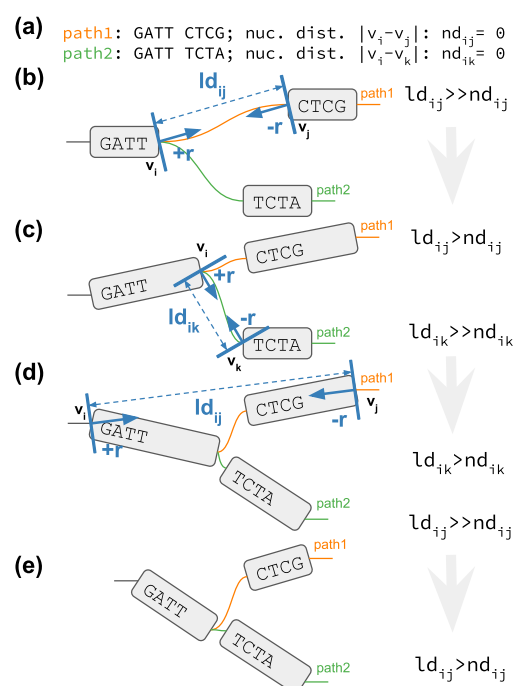


Fig. 1: 2D PG-SGD update operation sketches. (a) The path information of the graph. *path1* and *path2* both visit the same first node. Then their sequence diverges and they visit distinct nodes. (b-e) v_i/v_j or v_i/v_k is the current pair of nodes to update. ld_{ij}/ld_{ik} is the current layout distance. $r, -r$ is the current size of the update. (b) Initial graph layout highlighting the future update of the two nodes of *path1*. (c) The graph layout after the first update. The nodes appear longer now, because we updated at the end of the nodes. Highlighted is the future update of the two nodes of *path2*. (d) The graph layout after the second update. Highlighted is the future update of the two nodes of *path1*. (e) Final graph layout after three updates using the 2D PG-SGD.

This approach ensures to scale on large pangenome graphs representing thousands of whole genomes.

Graph layout initialization can significantly influence the quality of the final layout. In the 1D implementation, by default, nodes are placed in the same order as they appear in the input graph, although we also provide support for random layout initialization. In 2D, we offer several layout initialization techniques. One approach places nodes in the first layout dimension according to their order in the input graph, adding either uniform or Gaussian noise in the second dimension. Another strategy arranges nodes along a Hilbert curve, an approach that often favors the creation of planar final layouts. We also support fixing node positions to keep nodes in the same order as they are in a selected path, such as a reference genome. This feature allows us to build reference-focused graph layouts (Figure S1d).

Our implementation is multithreaded and uses shared memory for storing the layout in a vector, according to the HOGWILD! strategy (Recht *et al.*, 2011). Threads perform layout updates without any locking for additional speed up. This approach is feasible since pangenome graphs are typically sparse (Guarracino *et al.*, 2022), with low average node degree. As a result, the updates only modify small parts of the entire layout. While the HOGWILD! SGD algorithm writes the layout updates to a shared non-atomic double vector, PG-SGD stores node coordinates in a vector of atomic doubles. This vector prevents any potential memory overwrites. Our tests revealed basically no performance loss with respect to the non-atomic counterpart.

4 Results

4.1 Performance

We apply the 2D PG-SGD to the human pangenome (Liao *et al.*, 2023) from the Human Pangenome Reference Consortium (HPRC) to show the scalability of the algorithm. Experiments were conducted on a cluster with 24 Regular nodes (32 cores / 64 threads with two AMD EPYC 7343 processors with 512 GB RAM) and 4 HighMem nodes (64 cores / 128 threads with two AMD EPYC 7513 processors with 2048 GB RAM). We downloaded pangenome graphs for each autosome (24 in total) and for the mitochondrial DNA. Each graph represents 90 whole human haplotypes: 44 diploid individuals plus the GRCh38 (Schneider *et al.*, 2017) and CHM13 (Nurk *et al.*, 2021) haploid human references (see Supplementary Table S1 for graph statistics). When applied to these pangenome graphs using one Regular node for each calculation, 2D PG-SGD obtains the graph layouts in 50 minutes on average, with the highest run time observed being chromosome 16 (Supplementary Table S1). This is expected since chromosome 16 has one of the highest levels of segmentally duplicated sequence among the human autosomes (Martin *et al.*, 2004). Repetitive sequences lead to graph nodes with a very high number of path traversals, which are computationally expensive to work with (Guarracino *et al.*, 2022). Memory consumption is 29.66 GB of RAM on average, with the memory peak again occurring with chromosome 16, due to the path index building phase. Given its scalability, we even applied PG-SGD to the full graph with all chromosomes together using a HighMem node (Supplementary Table S1). *BandageNG* (<https://github.com/as1/BandageNG>, last accessed Jul 2023), the current state-of-the-art for graph visualization, was not able to produce a layout within 7 days, hitting the wall clock time limit of the cluster. On average, PG-SGD is ~8X faster than *BandageNG* while using ~2X less memory.

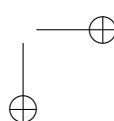
4.2 Pangenome graph layouts reveal biology features

Graph visualization is essential for understanding pangenome graphs and the genome variation they represent. We show how 2D PG-SGD allows us gaining insight into biological data by looking at the graph layout structure. In Figure 2a, the chromosomes of the HPRC graph show the large scale structural variations in the centromeres. Focusing on the major histocompatibility complex (MHC) of chromosome 6 (Figure 2b), the 2D layout reveals the positions and diversity of all MHC genes (Figure 2c). In Figure 2d the C4A and C4B genes are highlighted. Complementary, we provide various 1D visualizations in Supplementary Figure S1.

5 Discussion

We presented Path-Guided Stochastic Gradient Descent (PG-SGD), the first layout algorithm for pangenome graphs that leverages the biological information available within the genomes represented in the graph. Our implementation efficiently computes the layout of pangenome graphs representing thousands of whole genomes.

Graph visualization is key for understanding genome variations and the layouts produced by PG-SGD offer an unprecedented high-level perspective on pangenome variation. We implemented PG-SGD to generate layouts in 1D and 2D. These graph projections have already been employed in constructing and analyzing the first draft human pangenome reference (Liao *et al.*, 2023), as well as in the discovery of heterologous recombination of human acrocentric chromosomes (Guarracino *et al.*, 2023). Furthermore, they are applied in the creation and analysis of pangenome graphs for any species (Guarracino *et al.*, 2022; Garrison *et al.*, 2023). Of note, there still remains a gap in interactive and scalable solutions that merge layouts of large pangenome graphs with annotation. Our algorithm will underpin new pangenome graph browsers for studying



graph layouts and the genome variation they represent (<https://github.com/chfi/waragraph>) last accessed Jul 2023).

The performance analysis shows that our 2D implementation outperforms *BandageNG* when handling large, complex pangenome graphs. While *BandageNG* was not able to deliver a layout of the whole HPRC graph within 1 week, our 2D PG-SGD calculated one within one day. There are some possible optimization approaches for future work to further improve the performance of PG-SGD, making it possible for interactive use. The data structure could be optimized to improve cache performance. Moreover, the high-degree of parallelism could be further exploited by using a GPU.

PG-SGD can be extended to any number of dimensions. It can be seen as a graph embedding algorithm that converts high-dimensional, sparse pangenome graphs into low-dimensional, dense, and continuous vector spaces, while preserving its biologically relevant information. This enables the application of machine learning algorithms that use the graph layout for variant detection and classification. Our future research involves leveraging these graph projections to detect structural variants and to identify and correct assembly errors. Moreover, we are considering extending the algorithm to RNA and protein sequences to support pantranscriptome graphs (Sibbesen *et al.*, 2023) and panproteome graphs (Dabbaghie *et al.*, 2023), respectively.

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Competing interests

Author J.H. is employed by Computomics GmbH.

Software and data availability

Software versions, code, and links to data resources used to prepare this manuscript can be found at <https://github.com/pangenome/sorting-paper>. Animations of the algorithm are deposited at <https://doi.org/10.5281/zenodo.8288999>.

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Table S1: Performance evaluation of computing a 2D layout of all chromosomal HPRC pangenome graphs. From GFA to the actual layout. **BandageNG* did not finish within the job wall clock time limit of 7 days. Therefore, no layout was produced. **32T**: Number of threads: 32. **64T**: Number of threads: 64.

name	len	nodes	edges	paths	steps	time in minutes				memory in gigabytes			
						32T		64T		32T		64T	
						pg – sgd	bng	pg – sgd	bng	pg – sgd	bng	pg – sgd	bng
chr1	1.12e+09	1.11e+07	1.54e+07	2.26e+03	6.01e+08	110	1439	68	1427	55.73	149.91	56.00	195.33
chr2	3.47e+08	6.68e+06	9.27e+06	1.65e+03	3.89e+08	67	576	47	521	37.31	81.97	37.29	81.97
chr3	4.06e+08	6.20e+06	8.62e+06	1.56e+03	4.55e+08	81	473	52	481	41.34	81.41	41.71	93.83
chr4	2.73e+08	5.91e+06	8.23e+06	1.35e+03	4.97e+08	88	422	56	423	44.90	79.40	45.02	79.48
chr5	3.35e+08	5.39e+06	7.51e+06	1.20e+03	4.04e+08	73	349	46	375	35.83	75.13	36.48	75.10
chr6	2.29e+08	4.70e+06	6.56e+06	1.41e+03	4.03e+08	70	270	46	271	36.74	71.25	37.22	71.26
chr7	2.71e+08	5.17e+06	7.25e+06	1.22e+03	4.10e+08	70	328	46	346	37.39	73.70	37.88	73.81
chr8	1.93e+08	4.26e+06	5.95e+06	8.55e+02	4.29e+08	71	224	47	233	37.73	54.72	38.07	54.70
chr9	1.01e+09	8.80e+06	1.23e+07	8.67e+02	3.31e+08	44	931	38	957	31.76	131.93	31.79	131.96
chr10	2.56e+08	4.50e+06	6.26e+06	8.79e+02	2.72e+08	36	256	32	260	25.32	67.85	25.25	67.87
chr11	2.83e+08	4.73e+06	6.54e+06	6.53e+02	2.38e+08	31	277	28	286	21.81	68.49	21.77	68.54
chr12	2.44e+08	4.10e+06	5.71e+06	7.68e+02	2.54e+08	44	210	27	206	23.55	51.19	23.99	51.22
chr13	3.47e+08	4.34e+06	6.08e+06	2.58e+03	3.12e+08	52	242	34	237	27.98	54.02	28.64	85.85
chr14	2.73e+08	4.15e+06	5.79e+06	1.82e+03	2.62e+08	45	222	28	222	23.56	51.67	24.17	78.13
chr15	5.64e+08	5.20e+06	7.26e+06	2.06e+03	4.02e+08	64	347	35	334	35.20	74.27	35.69	102.97
chr16	3.39e+08	3.91e+06	5.53e+06	1.52e+03	6.91e+08	152	216	512	244	58.88	53.00	61.02	53.00
chr17	1.73e+08	2.76e+06	3.93e+06	1.42e+03	3.25e+08	50	102	33	102	27.83	40.68	28.69	49.50
chr18	2.44e+08	2.83e+06	3.98e+06	1.27e+03	3.00e+08	44	108	31	106	26.61	40.80	26.78	45.01
chr19	2.91e+08	3.02e+06	4.21e+06	1.07e+03	2.03e+08	31	123	21	117	18.12	40.14	18.43	40.18
chr20	1.87e+08	2.82e+06	3.97e+06	8.24e+02	2.35e+08	35	114	25	108	20.79	39.02	21.04	39.05
chr21	2.74e+08	2.76e+06	3.88e+06	3.03e+03	2.21e+08	33	110	23	103	18.79	38.07	19.12	46.47
chr22	4.64e+08	3.76e+06	5.22e+06	1.76e+03	2.05e+08	32	181	22	183	18.30	44.73	18.65	45.13
chrX	2.07e+08	3.46e+06	4.89e+06	2.42e+03	2.70e+08	41	156	28	155	24.66	43.05	24.84	43.05
chrY	8.80e+07	3.18e+05	4.41e+05	3.07e+02	1.34e+07	2	5	1	5	1.47	4.65	1.57	4.65
chrM	1.76e+04	1.40e+03	1.89e+03	4.40e+01	4.06e+04	1	1	1	1	0.21	0.04	0.49	0.04
all chrs	8.42e+09	1.11e+08	1.55e+08	3.48e+04	8.12e+09	1630	-*	1020	-*	737.15	-*	738.76	-*

6 Supplement

6.1 Supplementary data

6.1.1 Performance evaluation

The results of the performance evaluation are given in Table S1.

6.1.2 1D visualizations

The 1D PG-SGD algorithm creates a 1D layout of the nodes of the graph. Theoretically, it is possible that 2 nodes have the same 1D coordinate or

overlap. But, in our 1D visualizations, we arrange the nodes from left to right. Therefore, we project the 1D coordinates into a 1D node order: We sort the final layout by graph component, graph position, and node rank.

