LETTER

A Weighted Forward-Backward Spatial Smoothing DOA Estimation Algorithm Based on TLS-ESPRIT

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SUMMARY Based on TLS-ESPRIT algorithm, this paper proposes a weighted spatial smoothing DOA estimation algorithm to address the problem that the conventional TLS-ESPRIT algorithm will be disabled to estimate the direction of arrival (DOA) in the scenario of coherent sources. The proposed method divides the received signal array into several subarrays with special structural feature. Then, utilizing these subarrays, this paper constructs the new weighted covariance matrix to estimate the DOA based on TLS-ESPRIT. The auto-correlation and cross-correlation information of subarrays in the proposed algorithm is extracted sufficiently, improving the orthogonality between the signal subspace and the noise subspace so that the DOA of coherent sources could be estimated accurately. The simulations show that the proposed algorithm is superior to the conventional spatial smoothing algorithms under different signal to noise ratio (SNR) and snapshot numbers with coherent sources.

key words: *coherent signals, weighted spatial smoothing, DOA estimation, TLS-ESPRIT*

1. Introduction

The DOA estimation of sources is an important branch in the field of array signal processing. The ESPRIT algorithm was proposed by Roy, Paulraj and Kailath in [1], and it uses the rotation invariant property of covariance matrix of the received signal subspace to estimate signal angles. Moreover, based on total least square estimation of the signal subspace, the TLS-ESPRIT algorithm has become the research focus with its high-precision and super-resolution performance [2]. However, in coherent source environments, the covariance matrix of the received signals is singular, and its rank is smaller than the number of signal sources. As the results, the signal subspace is not orthogonal to the noise subspace, making the ESPRIT-based algorithm invalid. Shan et al. proposed a spatial smoothing method to preprocess the coherent sources in [3]. These kinds of methods are unable to extract the coherent signals with less data information of the covariance matrix. A weighted spatial smoothing algorithm is proposed in [4]. This method obtained the source orientation information roughly by the general estimation algorithm, and then constructed the weighted factor to realize the direction estimation of the coherent sources. The method proposed in [5] combined TLS-ESPRIT method with the spatial smoothing method to obtain the good performance. Similarly, the spatial smoothing-based method is also combined with multiple signal classification algorithm to estimate DOA, using to solve engineering application in [6]. For further improving the accuracy of TLS-ESPRIT algorithm in coherent sources, a weighted forward-backward spatial smoothing algorithm based on TLS-ESPRIT is proposed in this paper and is named as WFB-ESPRIT. In the proposed method, the received array will be twice divided into several subarrays. The first division aims to obtain auto- and cross- covariances among all the various subarrays. The weighting coefficient matrix is calculated by dividing secondly the original received array with special structural feature. Then, we make the DOA estimation by utilizing the rotational invariant property of signal subarray of the new weighted covariance matrix. The simulation results prove that the proposed method has better effectiveness than the traditional ESPRIT-based DOA estimation methods.

This paper is organized as follow. Section 2 formulates the problem and the signal model. Section 3 provides WFB-ESPRIT algorithm and analyzes its performance theoretically. Section 4 simulates the improved algorithm to verify its performance numerically. Section 5 concludes this paper.

2. Signal Model and Problem

Assuming the number of antenna elements is $M + 1$, there are K incident narrow-band signals with the incident angles $\theta_k (k = 1, 2, \ldots, K)$. The uniform line array (ULA) with array elements spaced d is shown in Fig. 1, where the first array element is used as the reference antenna.

The received signal of the mth array element is expressed as

$$x_m(t) = \sum_{k=1}^{K} a_m(\theta_k) s_k(t) + n_m(t), \tag{1}$$

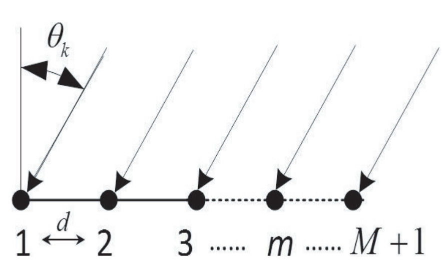

Fig. 1 Antenna array model

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where $m = 1, 2, \ldots, M+1$, $t = 1, 2, \ldots, N$, N is the number of snapshots, $a_m(\theta_k)$ is the transmit steering of the kth signal on the mth array element, $s_k(t)$ is the complex envelope of the kth source, and $n_m(t)$ is the noise on the mth array element at time t. Then, the received signal matrix of the ULA can be written as

$$\boldsymbol{X} = \boldsymbol{AS} + \boldsymbol{N}, \tag{2}$$

where $\boldsymbol{X}, \boldsymbol{N} \in \mathbb{C}^{(M+1)\times N}$, $\boldsymbol{A} \in \mathbb{C}^{(M+1)\times K}$ and $\boldsymbol{S} \in \mathbb{C}^{K\times N}$ are the matrices of the received signals, noise, transmit steering and signal sources, respectively. The covariance matrix of the received signals

$$\begin{aligned}\boldsymbol{R_X} =& E[\boldsymbol{XX}^H] = \boldsymbol{A}E[\boldsymbol{SS}^H]\boldsymbol{A}^H + E[\boldsymbol{NN}^H]\\ =& \boldsymbol{AR_S A}^H + \boldsymbol{R_N},\end{aligned} \tag{3}$$

where $(\cdot)^H$ is transposed conjugate, $\boldsymbol{R_S}$ and $\boldsymbol{R_N}$ are the covariance matrices of sources and noise, respectively. The relationship between two signals is represented by their correlation coefficient ρ, i.e.,

$$\begin{cases} s_i(t) \text{ and } s_k(t) \text{ are independent}, & \rho_{ik} = 0,\\ s_i(t) \text{ and } s_k(t) \text{ are correlation}, & 0 < |\rho_{ik}| < 1,\\ s_i(t) \text{ and } s_k(t) \text{ are coherent}, & |\rho_{ik}| = 1,\end{cases} \tag{4}$$

where

$$\rho_{ik} = \frac{E[s_i(t)s_k^*(t)]}{\sqrt{E[|s_i(t)|^2]E[|s_k(t)|^2]}}. \tag{5}$$

3. WFB-ESPRIT Algorithm and Analyses

In order to further improve the performance of the DOA estimation algorithm, this paper proposes a weighted forward-backward spatial smoothing algorithm based on TLS-ESPRIT. There are four steps to perform the proposed algorithm. According to the basic criterion of ESPRIT, we firstly divide the received array into two overlapped block arrays in step 1. The corresponding received signal matrix to the first M elements is denoted as $\boldsymbol{X}_\alpha$. The second received signal matrix from the 2nd element to the $(M+1)$th element is $\boldsymbol{X}_\beta$. These two matrices constitute a matrix pair, containing M element pairs with spacing d. The DOA estimation $\hat{\theta}_k$ based on ESPRIT [1] is expressed as

$$\hat{\theta}_k = \arcsin\{c \cdot \arg(\lambda_k)/(\omega_0 d)\}, \tag{6}$$

where λ_k is eigenvalue of the estimated signal subspace, and ω_0 is the carrier frequency. In the case of coherent sources, the second step of proposed method reconstitutes decoherence signal subspace by forward-back spatial smoothing. Taking the matrix $\boldsymbol{X}_\alpha$ as the reference, P subarrays with Q elements for each could be obtained by moving the window forward to divide the original matrix $\boldsymbol{X}_\alpha$ such that $P+Q = M+1, Q \geq K$. The received data of the pth subarray is $\boldsymbol{X}^f_{\alpha p} \in \mathbb{C}^{Q\times N}, p = 1, 2, \ldots, P$, where superscript f represents forward spatial smoothing. The correlation matrix of the pth subarray and qth subarray

$$\boldsymbol{R}^f_{\alpha pq} = E\left[\boldsymbol{X}^f_{\alpha p}\left(\boldsymbol{X}^f_{\alpha q}\right)^H\right] = \boldsymbol{F}_p \boldsymbol{R}_{X\alpha} \boldsymbol{F}^H_q \tag{7}$$

where $p, q = 1, 2, \ldots, P$, $\boldsymbol{F}_p = \left[\boldsymbol{0}_{Q\times(p-1)}|\boldsymbol{I}_Q|\boldsymbol{0}_{M-Q-p+1}\right]$. Furthermore, the correlation matrix of backward spatial smoothing subarray

$$\boldsymbol{R}^b_{\alpha pq} = \boldsymbol{F}_p \boldsymbol{J} \boldsymbol{R}^*_{X\alpha} \boldsymbol{J} \boldsymbol{F}^H_q \tag{8}$$

where $\boldsymbol{J}$ is an exchange matrix whose anti-diagonal elements are 1, and $(\cdot)^*$ is conjugate. Therefore, the forward covariance matrix $\boldsymbol{R}^f_\alpha$ and backward covariance matrix $\boldsymbol{R}^b_\alpha$ are composed of their elements $\boldsymbol{R}^f_{\alpha pq}$ and $\boldsymbol{R}^b_{\alpha pq}$ respectively. The forward-backward covariance matrix is expressed as

$$\boldsymbol{R}^{fb}_\alpha = \frac{1}{2}\left(\boldsymbol{R}^f_\alpha + \boldsymbol{R}^b_\alpha\right). \tag{9}$$

The conventional spatial smoothing algorithm and the weighted spatial smoothing algorithm only utilized the diagonal block matrices $\boldsymbol{R}^{fb}_{\alpha pp}, p = 1, 2, \ldots, P$. It is just able to estimate DOA in the scenario of large spaced sources and high SNR by only utilizing auto-correlation information.

To improve the resolution of DOA estimation in coherent sources, we extract the covariance information from the received original data $\boldsymbol{X}$ to produce a special weighted matrix in the step 3. To avoid confusion, we rename $\boldsymbol{X}$ as $\boldsymbol{W}$ and the corresponding block matrix is $\boldsymbol{W}_\alpha$. The matrix $\boldsymbol{W}_\alpha$ is divided by forward windows with dimension of P, where P equals to the number of subarrays of the first division. Therefore, the qth sub-array is denoted as $\boldsymbol{W}^f_{\alpha q} \in \mathbb{C}^{P\times N}, q = 1, 2, \ldots, Q$. Similarly, the new auto-covariance matrices of the forward and backward spatial smoothing subarray are $\boldsymbol{R}^f_{\boldsymbol{W}\alpha qq}$ and $\boldsymbol{R}^b_{\boldsymbol{W}\alpha qq} \in \mathbb{C}^{P\times P}$. The weighted matrix is expressed as

$$\begin{aligned}\bar{\boldsymbol{W}}_\alpha =& \frac{1}{2Q}\sum_{q=1}^{Q}\left(\boldsymbol{R}^f_{\boldsymbol{W}\alpha qq} + \boldsymbol{R}^b_{\boldsymbol{W}\alpha qq}\right)\\ =& \frac{1}{2Q}\sum_{q=1}^{Q}\left(\boldsymbol{F}_q\left(\boldsymbol{R}_{\boldsymbol{W}\alpha} + \boldsymbol{J}\boldsymbol{R}^*_{\boldsymbol{W}\alpha}\boldsymbol{J}\right)\boldsymbol{F}^H_q\right),\end{aligned} \tag{10}$$

whose elements are $\bar{w}_{\alpha pq}, p, q = 1, 2, \ldots, P$. Obviously, using each block matrices of $\boldsymbol{R}^{fb}_\alpha$ in (9) and $\bar{\boldsymbol{W}}_\alpha$, the modified covariance matrix is rewritten as

$$\bar{\boldsymbol{R}}_{X\alpha} = \frac{1}{P\times P}\sum_{q=1}^{P}\sum_{p=1}^{P}\bar{w}_{\alpha pq}\boldsymbol{R}^{fb}_{\alpha pq}. \tag{11}$$

The same processing applies to another block $\boldsymbol{X}_\beta$ of matrix pair, which has been divided in step 1. And then the corresponding modified covariance matrix is $\bar{\boldsymbol{R}}_{X\beta}$.

The step 4 of the proposed method calculates the direction of signals as (6) based on TLS-ESPRIT, using the modified covariance matrix pair $\bar{\boldsymbol{R}}_{X\beta}$ and $\bar{\boldsymbol{R}}_{X\alpha}$ shown in (11). Taking the eign decomposition of $\bar{\boldsymbol{R}}_{\boldsymbol{X}}$ shown as in (12), we extract the first K signal eigenvectors from $\boldsymbol{U_X}$ and denote it as $\boldsymbol{U}_{\hat{\boldsymbol{S}}}$.

$$\bar{\boldsymbol{R}}_X = \begin{bmatrix} \bar{\boldsymbol{R}}_{X\alpha} & \boldsymbol{0} \\ \boldsymbol{0} & \bar{\boldsymbol{R}}_{X\beta} \end{bmatrix} = \boldsymbol{U}_X \boldsymbol{V}_X \boldsymbol{U}_X^H = \left[\boldsymbol{U}_{\hat{S}}\ \boldsymbol{U}_{\hat{N}}\right] \boldsymbol{V}_X \begin{bmatrix} \boldsymbol{U}_{\hat{S}}^H \\ \boldsymbol{U}_{\hat{N}}^H \end{bmatrix}. \tag{12}$$

Then, dividing $\boldsymbol{U}_{\hat{S}}$ into two parts, we have

$$\boldsymbol{U}_{\hat{S}} = \begin{bmatrix} \boldsymbol{U}_A \\ \boldsymbol{U}_B \end{bmatrix} \in \mathbb{C}^{2Q\times K}, \tag{13}$$

where $\boldsymbol{U}_A$ and $\boldsymbol{U}_B$ satisfy rotational invariance. By taking eign decomposition,

$$\begin{bmatrix} \boldsymbol{U}_A^H \\ \boldsymbol{U}_B^H \end{bmatrix} [\boldsymbol{U}_A\ \boldsymbol{U}_B] = \boldsymbol{U}\Lambda\boldsymbol{U}^H. \tag{14}$$

Then, we take part $\boldsymbol{U}$ into four submatrices with dimension of $K \times K$, i.e.,

$$\boldsymbol{U} = \begin{bmatrix} \boldsymbol{U}_{11} & \boldsymbol{U}_{12} \\ \boldsymbol{U}_{21} & \boldsymbol{U}_{22} \end{bmatrix}. \tag{15}$$

And,

$$\hat{\boldsymbol{U}} = -\boldsymbol{U}_{12}\boldsymbol{U}_{22}. \tag{16}$$

The total least square estimation could be calculated as in (6), where λ_k is the eignvalue of $\hat{\boldsymbol{U}}$.

From these four steps mentioned before, the essence of the WFB-ESPRIT algorithm is dividing the original matrix with different construction in each time of spatial smoothing processing. By the first division in step 2, we can obtain auto- and cross- covariances of all submatices. In the second division of step 3, we change the size of window to ensure that the number of elements of the weighted matrix equals to the number of covariances in step 1. Obviously, if the weighted matrix equals to identity matrix, the WFB-ESPRIT algorithm degrades as the normal forward, backward and forward-backward spatial smoothing methods with utilizing (7), (8) and (9), respectively. The proposed method makes full use of the auto- and cross-correlation information of subarrays, improving the ability of decoherence in coherent sources condition, especially in scenarios of the low SNR and small spaced sources.

4. Numerical Simulations

In simulations, a uniform linear array of 33 array elements with half-wavelength space d is considered. The signal frequency f_c is 300MHz. The size of spatial smoothing window Q in step 2 is 24 and the size of spatial smoothing window P in step 3 is 9. Therefore, we can weighted sum of 81 auto- and cross-covariance matrices whose size is 24×24. The TLS-ESPRIT algorithm, the FB-ESPRIT algorithm and the WFB-ESPRIT algorithm are compared for their accuracies in the following simulations.

Simulation 1 shows the DOA estimation with the SNR of 5dB and 500 snapshots. The incident angles of the coherent signal sources are 10°, 20°, 30°, respectively. The correlation coefficient of signal 1 and signal 2 are 0.6906, and that of signal 1 and signal 3 are 0.4014.

Table 1 The DOA estimation results of simulation 1

Angle	10°	20°	30°
TLS-ESPRIT	9.9002	20.1538	30.0390
FB-ESPRIT	9.9821	19.99630	29.9943
WFB-ESPRIT	9.9925	19.9654	30.0008

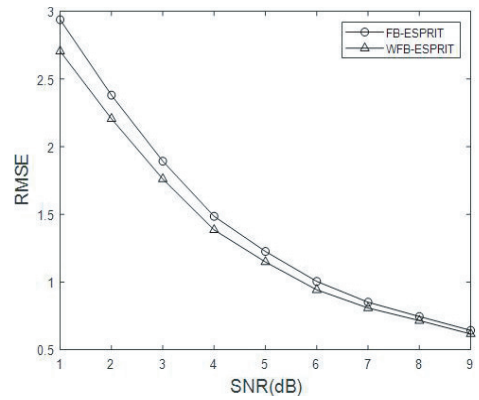

Fig. 2 RMSE vs SNR when the snapshot is 500

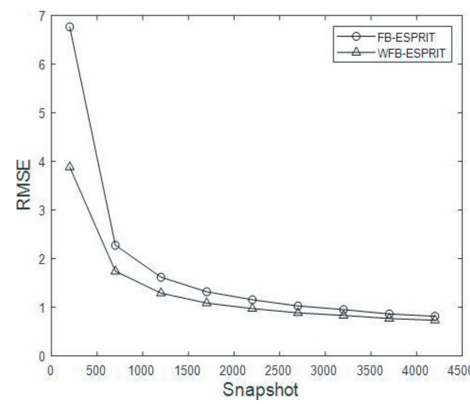

Fig. 3 RMSE vs snapshot when SNR is 5dB

Table 1 shows the estimation results of these three algorithms. As shown in results, the TLS-ESPRIT algorithm and the FB-ESPRIT algorithm can estimate the source directions, but the WFB-ESPRIT algorithm is significantly more accurate than these two algorithms, indicating that the WFB-ESPRIT algorithm has a better performance in the scenario of coherent sources.

Simulation 2 shows the performance of the WFB- ESPRIT algorithm and the FB- ESPRIT algorithm in the case of different SNR and snapshots with 10000 Monte Carlo trials. The incident angles of the coherent signal sources are 20°, 30°, 40°, 50°, 60°, respectively.

Figure 2 and Fig. 3 show the root mean square errors (RMSE) of the algorithms under different SNR and snapshots. The figures illustrate that these two algorithms are able to accurately estimate the direction of coherent sources, but the error of the WFB-ESPRIT algorithm is obviously smaller than that of the FB-ESPRIT algorithm under the

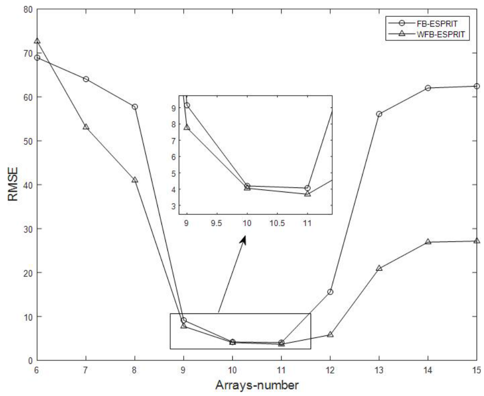

Fig. 4 RMSE for the different element number of subarrays divided in step 2

same condition, indicating that the WFB-ESPRIT algorithm is superior to the FB-ESPRIT algorithm in the case of coherent sources DOA estimation.

Simulation 3 compares the performances of two algorithms accompanying with the size of subarray changing. The minimum size of subarray is 6 because there are 5 coherent sources with the same incident angles shown in simulation 2. The snapshots are 100, SNR is 5dB, and Monte Carlo trials are 10000 in this simulation.

Figure 4 shows that two algorithms are unable to effectively estimate the direction of coherent sources until the element number of subarray divided in step 2 is increased up to 9. Furthermore, the performance of estimation continues to decline when the element number of subarray is greater than 12. When the element number of subarray equals to 11, both algorithms get the best results. And the WFB-ESPRIT algorithm performs a slightly better than the FB-ESPRIT algorithm.

5. Conclusions

In this paper, we proposed a weighted special smoothing DOA estimation algorithm based on TLS-ESPRIT to solve the problem that the performances of conventional algorithms deteriorate drastically in case of coherent signals. The proposed algorithm makes full use of the auto- and cross- correlation information of the received signals, improving the orthogonality of signal subspace and noise subspace. Two times subarray division and weighted summation of covariances are crucial to improve the performance of resolution and precision. The experimental results show that the WFB-ESPRIT algorithm is superior to the FB-ESPRIT algorithm, indicating the effectiveness of the proposed algorithm.

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