Inflationary model in minimally modified gravity theories

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We have investigated inflationary model constructed from minimally modified gravity (MMG) theories. The MMG theory in the form of $f(\mathbf{H}) \propto \mathbf{H}^{1+p}$ gravity where, $\mathbf{H}$ is the Hamiltonian constraint in the Einstein gravity and p is constant, has been studied. An inflation is difficult to be achieved in this theory of gravity unless an additional scalar field playing a role of inflaton is introduced in the model. We have found that the inflaton with exponential potential can drive inflation with graceful exist different from the case of Einstein gravity. The slow-roll parameter for both the exponential and the power-law potentials is inversely proportional to number of e-folding similar to the case of the Einstein gravity. We also have found for the scalar perturbation that the curvature perturbation in the comoving gauge on super Hubble radius scales grows rapidly during inflation unless $p = 0$. For the tensor modes, the amplitude of the perturbations is constant on large scales, and sound speed of the perturbations can diviate from unity and can vary with time depending on the form of $f(\mathbf{H})$.

Keywords: minimally modified gravity theories, inflationary universe, inflationary predictions

I. INTRODUCTION

Cosmic inflation [1–3] is a standard framework addressing issues in the hot Big Bang model and providing mechanism for generation of primordial density perturbation. In the standard scenario, inflation can be achieved by introducing extra degrees of freedom in universe. In the case of Einstein gravity the extra degrees of freedom may be in the form of fields minimally couple to gravity called inflaton. Alternatively, the extra degrees of freedom can be parts of degrees of freedom of the gravitational interaction. The extra degrees of freedom of gravity can be obtained by assuming non-minimally coupling between extra field and curvature terms in the action. This class of theories

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is scalar-tensor theories of gravity [4]. Moreover, the extra degrees of freedom of the gravitational interaction can also be obtained due to non-linear curvature terms in the action. The simplest example of this class of gravity is $f(R)$ gravity [5].

However, in the cuscuton models [6–8], it has been shown that the acceleration of the universe can be achieved even though the minimally couple extra degree of freedom is non-dynamical field. This implies that theories which have two dynamical degrees of freedom can also drive acceleration of the universe. Theories of gravity beyond Einstein theory with have two degrees of freedom as Einstein theory have been studied in various contexts [9–14]. Such theories could be constructed by supposing that the temporal diffeomorphism is broken while the spatial diffeomorphism is still invariant. In general, if the diffeomorphism invariant is broken in this way, the theories can have an extra degree of freedom similar to scalar-tensor theories of gravity [15]. However, if the Lagrangian of theories is a linear function of the lapse function, the theories can have two degrees of freedom under suitable conditions. This class of theories is minimally modified gravity theories [10, 13]. Nevertheless, these conditions can not be satisfied if matter appears in the action. To ensure that this class of theories still has two degrees of freedom when matter appears in the theories, we have to impose the gauge fixing condition[11, 18, 19]. Cosmology with this class of theories has been investigated in [8, 19]. In [19], it has been shown that late time universe with this class of gravity theories is more preferred by observational data than ΛCDM model. In [16, 17, 19], matter coupling in this class of theories has been discussed.

Here, we investigate inflation due to this class of gravity theories. This work is organized as follow: firstly we review MMG theories in the next section. We investigate background inflation in Sec. (III). We study cosmological perturbation in Sec. (IV) and conclude in the last section.

II. MINIMALLY MODIFIED GRAVITY THEORIES

Minimally modified gravity theories are the modified theories propagating two degrees of freedom like Einstein theory of gravity. Generally, most of popular modified theories of gravity always generate extra degrees of freedom in the theories. The extra degrees of freedom are related to the broken diffeomorphism invariant in the construction of the theories. However, we can construct the theories that have two degrees of freedom even if the full diffeomorphism invariant is broken. We can construct MMG theories by writing the Hamiltonian of the theories to be linear in the lapse function and imposing a suitable constraint. Square root gravity and exponential gravity are the MMG theories that we obtain by using this method [10]. However, there is an interesting class of MMG theories, $f(\mathbf{H})$ theory. This class of MMG theories is constructed in another way by the Hamiltonian construction [13].

In order to construct the MMG theories, we break the temporal diffeomorphism invariant which is conveniently represented by the ADM decomposition. In the ADM formalism, one can write the line-element in the form

$$ds^2 = \left(-\mathcal{N}^2 + \mathcal{N}_i\mathcal{N}^i\right)dt^2 + h_{ij}\left(\mathcal{N}^i dt + \mathcal{N}dx^i\right)\left(\mathcal{N}^j dt + \mathcal{N}dx^j\right) , \tag{1}$$

where h_{ij}, $\mathcal{N}$ and $\mathcal{N}^i$ are the three-dimensional induce metric, the lapse function and the shift vector, respectively. We are interested in MMG theories in the form of $f(\mathbf{H})$ theory which the action can be written in the form

$$S[h_{ij},\mathcal{N},\mathcal{N}^i] = \frac{m_p^2}{2}\int d^4x\mathcal{N}\sqrt{h}\mathcal{L}_G = \frac{m_p^2}{2}\int d^4x\mathcal{N}\sqrt{h}\left[\frac{2}{f_{,c}(C)}(K_{ij}K^{ij} - K^2) - f(C)\right] , \tag{2}$$

where $m_p = 1/\sqrt{8\pi G}$ is the reduced Planck mass. Here, C can be computed from

$$C = \frac{K_{ij}K^{ij} - K^2}{[f_{,c}(C)]^2} - R\,. \tag{3}$$

From the above expressions, $f(C)$ is an arbitrary function of C, $f_{,c}$ denotes derivative of $f(C)$ with respect to C, and we see that C has the same dimension as R, i.e., its dimension is mass^2. Moreover, C is the Hamiltonian constraint in the Einstein gravity if $f_{,c} = 1$.

To study possible models of inflation from this theory of gravity, we add extra scalar field into the above action as

$$S[h_{ij},\mathcal{N},\mathcal{N}^i,\phi] = \int d^4x\mathcal{N}\sqrt{h}\left[\frac{m_p^2}{2}\mathcal{L}_G + X - V(\phi)\right] . \tag{4}$$

Here, we suppose that the field has standard kinetic term where $X = -\partial_\mu\phi\partial^\mu\phi/2$ is the kinetic term of the scalar field and V is the potential term. However, the degree of freedom in the theory increases when the scalar field is simply added in the action. To ensure that the theory still has two degrees of freedom, we have to fix the gauge degree of freedom in the theory. Using the choice of gauge presented in [19], the Hamiltonian of the gauge fixeing term is written in the form

$$H_{gf} = \int d^3x\sqrt{h}\tilde{\lambda}^i\partial_i\left(\frac{\pi}{\sqrt{h}}\right) , \tag{5}$$

where $\tilde{\lambda}^i$ is a Lagrange multiplier and π is the trace of momenta conjugate to the induce metric. Imposing this gauge fixing, the action for $f(\mathbf{H})$ becomes

$$S = \frac{1}{2}\int d^4x\mathcal{N}\sqrt{h}\Big\{m_p^2\Big[(C+R)\left[2 - \lambda_0 f_{,c}(C)\right]f_{,c}(C) - f(C) \tag{6}$$

$$+\lambda_0\left[K^{ij}K_{ij} - K^2 - \frac{2K}{\mathcal{N}}D_k\tilde{\lambda}^k - \frac{3}{2\mathcal{N}^2}\left(D_k\tilde{\lambda}^k\right)^2\right]\Big] + 2X - 2V(\phi)\Big\} , \tag{7}$$

where λ_0 is another Lagrange multiplier, and in this case C becomes

$$C = \frac{1}{[f_{,c}(C)]^2}\left[K^{ij}K_{ij} - K^2 - \frac{2K}{\mathcal{N}}D_k\tilde{\lambda}^k - \frac{3}{2\mathcal{N}^2}\left(D_k\tilde{\lambda}^k\right)^2\right] - R\,. \tag{8}$$

The above expression for C can be obtained by varying the action Eq. (6) with respect to λ_0. Varying the action with respect to $C, \mathcal{N}$ and $\mathcal{N}^k$ yiels, respectively,

$$\lambda_0 = \frac{1}{f_{,c}}, \tag{9}$$

$$0 = f(C) - \frac{2}{m_p^2}\left[X - V - \frac{1}{\mathcal{N}^2}\left(\dot{\phi} - \mathcal{N}^i\partial_i\phi\right)^2\right], \tag{10}$$

$$0 = D_iK^{ik} - h^{ik}D_iK - h^{ik}D_iD_m\lambda_t^m - \frac{1}{m_p^2\mathcal{N}}\left(\dot{\phi} - \mathcal{N}^i\partial_i\phi\right)\partial_k\phi, \tag{11}$$

where we have used Eq. (8) and Eq. (9) to obtain Eq. (10) . Variation with respect to scalar field give us the evolution equation for scalar field as

$$\partial_0\left[\frac{\sqrt{h}}{\mathcal{N}}\left(\dot{\phi} - \mathcal{N}^j\partial_j\phi\right)\right] - \partial_i\left[\frac{\sqrt{h}}{\mathcal{N}}\mathcal{N}^i\dot{\phi} + \mathcal{N}\sqrt{h}\left(h^{ij} - \frac{\mathcal{N}^i\mathcal{N}^j}{\mathcal{N}^2}\right)\partial_j\phi\right] + \mathcal{N}\sqrt{h}V_\phi = 0, \tag{12}$$

where subscript $_\phi$ denotes derivative with respect to scalar field ϕ.

III. BACKGROUND EVOLUTION

We now consider the evolution of the spatially flat Friedmann universe for the theory described in the previous section. Due to the homogeneity and isotropy of the Friedmann universe, $\mathcal{N} = 1$, $\mathcal{N}^i = 0$ and therefore

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^idx^j, \tag{13}$$

where $a(t)$ is a cosmic scale factor and t is the cosmic time. For the Friedmann universe, the Hamiltonian constraint from Eq.(10) and the expression for C in Eq. (8) are given by

$$f = -\frac{2}{m_p^2}(X + V) = -\frac{1}{m_p^2}\left(\dot{\phi}^2 + 2V\right), \tag{14}$$

$$Cf_{,c}^2 = -6H^2, \tag{15}$$

where a dot denotes derivative with respect to time t, and $H \equiv \dot{a}/a$ is the Hubble parameter. The evolution equation for scalar field in the Friedmann universe is

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi = 0. \tag{16}$$

The slow-roll parameter $\epsilon \equiv -\dot{H}/H^2$ can be computed by differentiating Eq. (15) with respect to time to obtain $\dot{C}$, and substituting resulting $\dot{C}$ in to the time-derivative of Eq. (14). The result is

$$\epsilon = \frac{\eta f_{,c}}{2}\left(1 + 2\frac{Cf_{,cc}}{f_{,c}}\right), \tag{17}$$

where $\eta \equiv \dot{\phi}^2/(H^2m_p^2)$. The above relation reduces to the usual relation for ϵ for the Einstein gravity when $f_{,c} = 1$. It follows from Eq. (17) that $\epsilon \ll 1$, which is required during inflation, when $\eta \ll 1$ or

$|f_{,c} + 2Cf_{,cc}| \ll 1$. However, the latter condition is difficult to be achieved, so that slow-roll inflaton is need for inflation in this theory. The case $\eta \ll 1$ corresponds to the slow-roll evolution of the inflaton field ϕ. Under the slow-roll approximation, $|\ddot{\phi}| \ll |H\dot{\phi}|$, Eq. (16) becomes

$$\frac{d\phi}{dN} = -\frac{V_\phi}{3H^2}, \tag{18}$$

where $N \equiv \ln a$ is the number of e-folding.

In order to study the evolution of the background universe, we have to specify form of $f(C)$. Here, we suppose

$$f(C) = -\Lambda \left(-\frac{C}{\Lambda}\right)^{1+p}, \tag{19}$$

where Λ is a constant with dimension of mass^2 and p is a constant parameter. We then obtain from Eq. (15) that

$$f = -\Lambda \left[\frac{6H^2}{\Lambda(1+p)^2}\right]^{\frac{1+p}{2p+1}}, \tag{20}$$

$$f_{,c} = (1+p) \left[\frac{6H^2}{\Lambda(1+p)^2}\right]^{\frac{p}{2p+1}}. \tag{21}$$

Hence, we obtain the modified Friedmann equation by substituting the above expression in to Eq. (14) as

$$\left[\frac{6H^2}{\Lambda(1+p)^2}\right]^{\frac{1+p}{2p+1}} = \frac{1}{m_p^2\Lambda}\left(\dot{\phi}^2 + 2V(\phi)\right), \tag{22}$$

Using slow-roll condition, $V \gg \dot{\phi}$, we can write Eq. (22) as

$$H^2 = \frac{2^{\frac{2p+1}{1+p}}(1+p)^2\Lambda}{6}\left(\frac{V}{m_p^2\Lambda}\right)^{\frac{2p+1}{1+p}}. \tag{23}$$

Substituting Eq. (20) in to Eq. (18), and using Eq. (23), we get

$$\frac{d\phi}{dN} = -\frac{2^{-p/(1+p)}}{\Lambda(1+p)^2}\frac{V_\phi}{\tilde{V}^{\frac{2p+1}{1+p}}}, \tag{24}$$

where $\tilde{V} \equiv V/(m_p^2\Lambda)$. The above equation can be written in the integral form as

$$\int_0^{N_N} dN = 2^{p/(1+p)}\Lambda\,(1+p)^2 \int_{\phi_e}^{\phi_N} d\phi \frac{\tilde{V}^{\frac{2p+1}{1+p}}}{V_\phi}, \tag{25}$$

where subscript $_e$ denotes evaluation at the end of inflation, while subscript $_N$ denotes evaluation at the moment when particular modes of cosmological perturbations generated during inflation crosses the horizon. For the form of f given by Eq. (19), the slow-roll parameter ϵ in the slow-roll approximation is

$$\epsilon = \frac{2^{-\frac{2p+1}{1+p}}(2p+1)}{m_p^2\Lambda^2(1+p)^3}\frac{V_\phi^2}{\tilde{V}^{\frac{3p+2}{1+p}}}. \tag{26}$$

In the slow-roll approximation, we can write $f_{,c}$ in terms of the potential as

$$f_{,c} = (1+p)2^{\frac{p}{1+p}}\tilde{V}^{\frac{p}{1+p}}\,, \tag{27}$$

$$C = -\Lambda 2^{\frac{1}{1+p}}\tilde{V}^{\frac{1}{1+p}}\,. \tag{28}$$

To integrate Eq. (25), and compute ϵ in terms of the number of e-folding, we have to specify the potential V of scalar field. As the illustrative examples, we will consider two cases where V takes either exponential or power-law form.

A. Exponential potential

We first consider the potential in the form

$$V(\phi) = V_0\Lambda m_p^2 e^{\lambda\tilde{\phi}}\,, \tag{29}$$

where $\tilde{\phi} \equiv \phi/m_p$, while V_0 and λ are the dimensionless constants. Substituting the above potential in Eq. (25), and performing an integration, we get

$$N_N = \frac{2^{p/(1+p)}\,(1+p)^3}{\lambda^2 p V_0^{-p/(1+p)}}\left[e^{\lambda\tilde{\phi}_N p/(1+p)} - e^{\lambda\tilde{\phi}_e p/(1+p)}\right]\,. \tag{30}$$

We can calculate ϕ_e by using the slow-roll parameter. Since $\epsilon = 1$ at the end of inflation, we get from Eq. (26) that

$$e^{\lambda\tilde{\phi}_e p/(1+p)} = \frac{\lambda^2(2p+1)}{2^{\frac{2p+1}{p+1}}(1+p)^3 V_0^{p/(1+p)}}\,. \tag{31}$$

Substituting the above equation into Eq. (30), we get

$$N_N + N_* = \frac{2^{p/(1+p)}(1+p)^3}{\lambda^2 p V_0^{-p/(1+p)}}e^{\lambda\tilde{\phi}_N p/(1+p)}\,, \tag{32}$$

where

$$N_* \equiv \frac{2p+1}{2p}\,. \tag{33}$$

Inserting Eq. (32) in to Eqs. (26) and (18), we can write ϵ and η in terms of the number of efolding as

$$\epsilon_N = \frac{N_*}{N_N + N_*}\,. \quad \eta_N = \frac{(p+1)^2}{\lambda^2 p^2 (N_N + N_*)^2}\,. \tag{34}$$

Using Eqs. (27) and (32), we have

$$f_{,c}(N) = f_{,c*}(N_N + N_*) = \frac{\lambda^2(2p+1)}{2(1+p)^2}\frac{1}{\epsilon}\,, \tag{35}$$

where $f_{,c*}$ is defined as

$$f_{,c*} \equiv \frac{\lambda^2 p}{(1+p)^2}\,, \tag{36}$$

It follows from the above calculations that the inflaton with exponential potential has graceful exit in this theory of gravity. This result is different from that in Einstein theory of gravity. The moment at graceful exit is described by Eq. (31).

B. Power-law potential

In this section, we apply the potential of the form,

$$V(\phi) = V_0 m_p^2 \Lambda \tilde{\phi}^q \,, \tag{37}$$

to Eq. (25). After integrating, we obtain

$$N_N = \frac{2^{p/(1+p)}(1+p)^3 V_0^{p/(1+p)}}{q(pq+2p+2)} \left[\tilde{\phi}_N^{\frac{pq+2p+2}{1+p}} - \tilde{\phi}_e^{\frac{pq+2p+2}{1+p}} \right] . \tag{38}$$

Using the condition $\epsilon = 1$ at the end of inflation, we can calculate $\tilde{\phi}_e$ as

$$\tilde{\phi}_e^{\frac{pq+2p+2}{1+p}} = \frac{2^{-\frac{2p+1}{1+p}} q^2 (2p+1)}{(1+p)^3 V_0^{p/(1+p)}} . \tag{39}$$

Substituting the above expression into Eq. (38), we get

$$N_N + N_* = \frac{2^{p/(1+p)}(1+p)^3 V_0^{p/(1+p)}}{q(pq+2p+2)} \tilde{\phi}_N^{\frac{pq+2p+2}{1+p}} \,, \tag{40}$$

where

$$N_* \equiv \frac{q\,(2p+1)}{2\,(pq+2p+2)} . \tag{41}$$

Then we can calculate

$$\epsilon_N = \frac{N_*}{N_N + N_*} \,, \quad \text{and} \quad \eta_N = \left[\frac{q^{2p+2}(1+p)^{2(pq-p-1)}}{2^{2p} V_0^{2p} (pq+2p+2)^{2(pq+p+1)}} (N_N + N_*)^{-2(pq+1+p)} \right]^{1/(pq+2p+2)} . \tag{42}$$

Using Eqs. (27) and (40), we have

$$f_{,c}(N) = f_{,c*}(N+N_*)^{\frac{pq}{pq+2p+2}} = f_{,c*} \left(\frac{q(2p+1)}{2(pq+2p+2)\epsilon} \right)^{\frac{pq}{pq+2p+2}} \,, \tag{43}$$

where

$$f_{,c*} \equiv \left(\frac{4^p V_0^{2p} q^{qp} \, (pq+2p+2)^{qp}}{(1+p)^{2qp-2p-2}} \right)^{1/(pq+2p+2)} . \tag{44}$$

C. Numerical results

In this subsection, we solve the evolution equations for the background universe numerically and plot the results in Figs. (1)-(3). The models in our plots are shown in Table (I). In Fig. (1), we plot evolution of ϵ for both exponential and power-law potentials cases. From this figure, we see that for both forms of potential, inflationary epoch can be taken place such that slow-roll parameter ϵ increases from small value during early stage towards one at the end of inflation. The main different

No. Model	1	2	3	4	5	6
potential	e^{ϕ}	ϕ^2	ϕ^4	$0.7\phi^{1/2}$	$0.085\phi^{1/2}$	$0.002\phi^2$
p	1	1	1	1	1/5	1/21
No. Model	7	8	9	10	11	12
potential	$0.05\phi^1$	$0.02\phi^1$	$0.0005\phi^2$	$0.0001\phi^2$	$0.02\phi^2$	$0.5\phi^2$
p	1/10	1/5	1/21	1/21	1/21	1/21

TABLE I: Model used in the numerical calculation

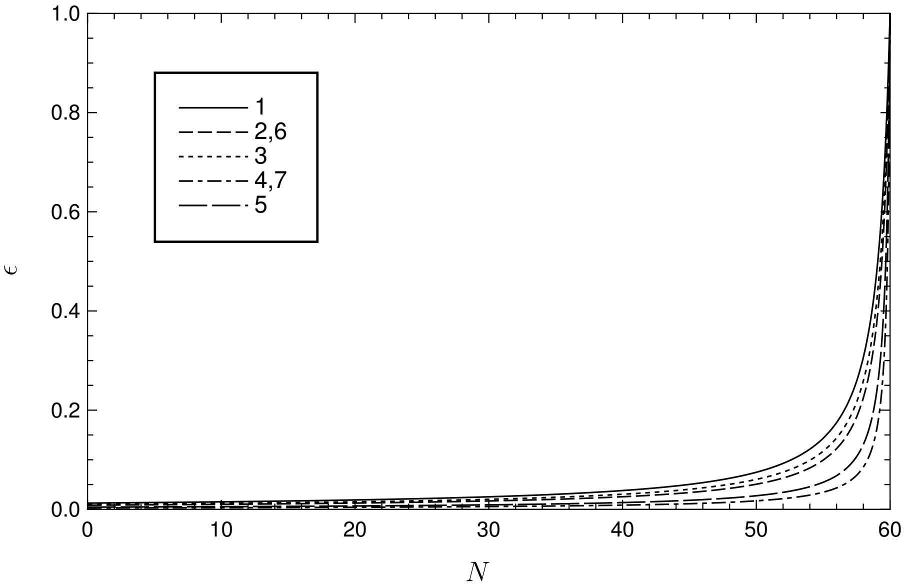

FIG. 1: Plots of slow-roll parameter ϵ as a function of number of e-folding for the models 1 - 7. In the plots, models 1 - 7 correspond to lines 1 - 7, respectively.

feature of the models comes from different evolution of $f_{,c}$. As will be seen in the next section, $f_{,c}$ controls evolution of the curvature perturbation during inflation. Evolutions of $f_{,c}$ are plotted in Figs. (2) and (3). According to Eq. (35), $f_{,c}$ is proportional to $1/\epsilon$ for the exponential potential, so that for this form of potential $f_{,c}$ can increase several order of magnitude through out inflationary epoch. This conclusion agrees with the plot in Fig. (2). However, for the power-law potential, Eq. (43) shows that the rate of change of $f_{,c}$ decreases when q and p decrease. When $p \to 0$, the model for power-law case reduces to Einstein gravity such that $f_{,c} = 1$. Nevertheless, it follows from Eq. (34) that there is no Einstein limit for the case of the exponential potential. Dependence of $f_{,c}$ on parameters p and q for the case of power-law potential is shown in Figs. (2 and 3). From the figures, we see that the variation of $f_{,c}$ reduces when p and q decrease.

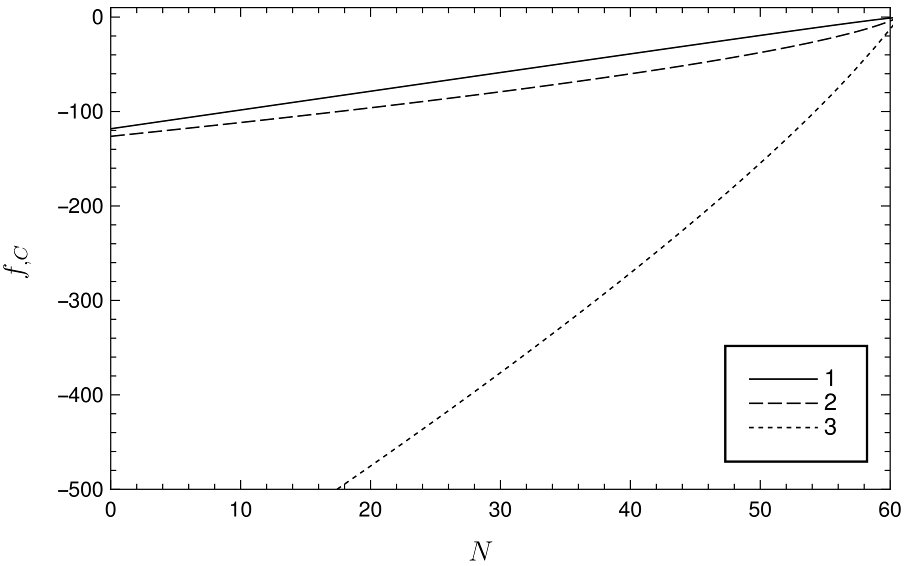

FIG. 2: Plots of f_C as a function of number of e-folding. In the plots, lines 1 - 3 represent models 1 - 3, respectively.

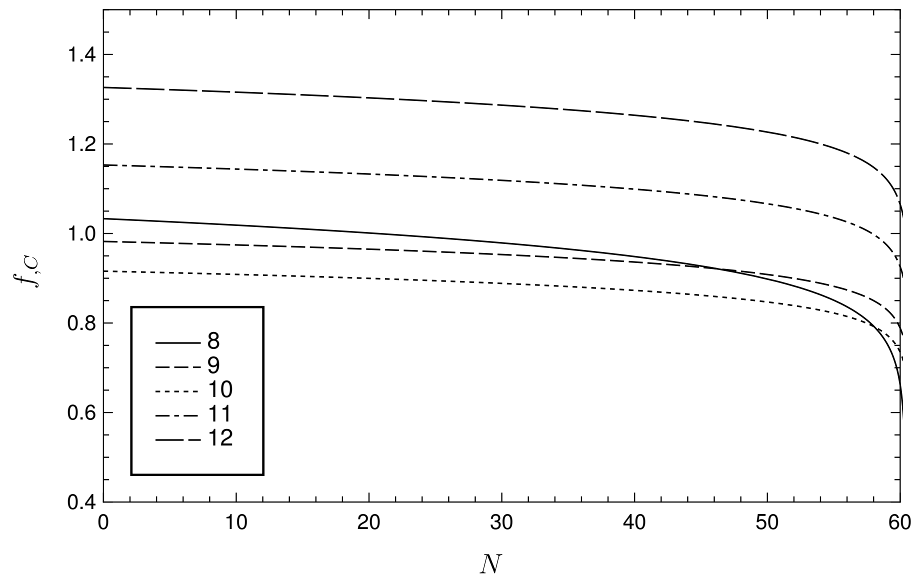

FIG. 3: Plots of f_C as a function of number of e-folding. In the plots, lines 8 - 12 represent models 8 - 12, respectively.

IV. EVOLUTION OF PRIMORDIAL DENSITY PERTURBATIONS

In this section we consider evolution of primordial perturbations generated in inflationary model introduced in Sec. (II). In the following consideration, we concentrate on scalar and tensor perturbation which usually provide predictions of the model.

A. Scalar perturbations

In principle, to investigate the primordial density perturbations generated during inflation, we should construct the action for second order perturbation in which the primordial perturbation are described by canonical variables. However, the action for perturbation for this theory is rather complicated due to the scale-dependence of gauge fixing term in the action. Thus instead of constructing this action, we start from the evolution equations for perturbation in Newtonian gauge presented in [19]. Transforming these equations to comoving gauge, we obtain the evolution equations for curvature perturbation in comoving gauge as

$$\zeta'' + \alpha\zeta' + \beta\zeta = 0 \tag{45}$$

where a prime denotes derivative with respect to the conformal time $\tau \equiv \int a dt$, while coefficients α and β are function of number of e-folding, wavenumber k and Hubble parameter $\mathcal{H} \equiv a'/a$. The explicit expression of these coefficients are presented in the appendix. In the region where $k^2/\mathcal{H}^2 > \mathcal{O}(\epsilon)$, Eq. (45) can be written up to the lowest order in slow-roll parameters as

$$v'' - \frac{z''}{z}v + c_s^2 k^2 v = 0\,. \tag{46}$$

where $v = z\zeta$ and in this case

$$\frac{z''}{z} = \frac{1}{4}(8 + 18f_C - 9f_C^2 - 18f_C^3 + 9f_C^4)\mathcal{H}^2\,, \quad \text{and} \quad c_s^2 = 1 + \mathcal{O}(\epsilon). \tag{47}$$

The expression for z is computed from

$$z = a exp\left\{\int d\tau \left(\frac{3}{2}f_{,c}\mathcal{H}\left(1 - f_{,c}\right)\right)\right\}\,, \tag{48}$$

where z reduces to $z = a$ in the Einstein limit. For the subhorizon modes, $k \gg \mathcal{H}$, Eq. (46) is satisfied by the solution [20]

$$v = \frac{e^{-ikc_s\tau}}{\sqrt{2c_s k}}\,. \tag{49}$$

For the superhorizon modes, where $k \ll \mathcal{H}$ but $k^2/\mathcal{H}^2$ still larger than $\mathcal{O}(\epsilon)$, Eq. (46) is solved by the solution $v \propto z$, where the proportional constant could be computed by matching the solution for the subhorizon limit with that for the superhorizon limit. However, we are not interested in such calculation here because the condition $k/\mathcal{H} > \mathcal{O}(\epsilon)$ is violated just a few numbers of e-folding after the horizon crossing. When this condition is violated, the evolution of ζ is time dependent as we will see below. For the case where $k^2/\mathcal{H}^2 < \mathcal{O}(\epsilon)$, the evolution equation for the curvature perturbation up to the dominant contribution from $k/\mathcal{H}$ can be written in the form

$$\frac{d^2\zeta_k}{dN^2} + (3 + A)\frac{d\zeta_k}{dN} + (\Xi + B)\,\zeta_k = 0\,. \tag{50}$$

Here,

$$
\begin{aligned}
A \;\equiv\;& \frac{1}{18\eta\,(f_{,c}-1)^2}\Big[\eta^2 f_{,c}^2\{9+f_{,c}\,(\Xi-9)\,\}+4\Big\{\epsilon^2\,(4f_{,c}-3)-\eta_1\big(3f_{,c}^5-6f_{,c}^4+6f_{,c}^2+\epsilon f_{,c}\\
&-6f_{,c}-\epsilon+3\big)-6\left(3f_{,c}^5-6f_{,c}^4+6f_{,c}^2-4f_{,c}+1\right)+\epsilon\big(6f_{,c}^5-12f_{,c}^4+12f_{,c}^2-17f_{,c}\\
&+\epsilon_1\,(f_{,c}-1)+8\big)\Big\}\Xi+2\eta\Big\{5f_{,c}^2\Xi-2f_{,c}\Xi-\epsilon\big(18f_{,c}^4-36f_{,c}^3+f_{,c}^2(5\Xi-27)\\
&-3f_{,c}(\Xi-27)-36\big)+27f_{,c}^4-27f_{,c}^3+9\eta_1(f_{,c}-1)^3(f_{,c}+1)-81f_{,c}^2+135f_{,c}-54\Big\}\Big] \qquad (51)\\
B \;\equiv\;& -\frac{1}{54\eta^2(f_{,c}-1)^2}f_{,c}\Big[4\eta\Xi\Big\{\epsilon\big(-\epsilon_1(f_{,c}-1)(f_{,c}(\Xi-9)+9)-6f_{,c}^6\Xi+12f_{,c}^5\Xi+108f_{,c}^4\\
&-12f_{,c}^3(\Xi+18)+f_{,c}^2(37\Xi-99)+f_{,c}(387-19\Xi)-180\big)+\eta_1(f_{,c}-1)\big(\epsilon\big(9f_{,c}^3-9f_{,c}^2\\
&+f_{,c}(\Xi-18)+18\big)+3\left(f_{,c}^5\Xi-f_{,c}^4\Xi-f_{,c}^3(\Xi+9)+f_{,c}^2(\Xi+9)-f_{,c}(\Xi-18)-18\right)\big)\\
&+\epsilon^2\left(-18f_{,c}^4+36f_{,c}^3-9f_{,c}^2(\Xi-4)+f_{,c}(6\Xi-99)+45\right)+3\big(6f_{,c}^6\Xi-12f_{,c}^5\Xi\\
&-f_{,c}^4(\Xi+54)+2f_{,c}^3(7\Xi+54)+f_{,c}^2(18-13\Xi)+2f_{,c}(\Xi-72)+\Xi+72\big)\Big\}\\
&+8(\epsilon-3)\Xi^2\Big\{\epsilon\left(\epsilon_1(f_{,c}-1)+6f_{,c}^5-12f_{,c}^4+12f_{,c}^2-17f_{,c}+8\right)+\eta_1\big(-\epsilon f_{,c}+\epsilon-3f_{,c}^5\\
&+6f_{,c}^4-6f_{,c}^2+6f_{,c}-3\big)+\epsilon^2(4f_{,c}-3)-6\left(3f_{,c}^5-6f_{,c}^4+6f_{,c}^2-4f_{,c}+1\right)\Big\}\\
&-2\eta^2\Big\{9f_{,c}^5\Xi(\eta_1-2\epsilon+6)-18f_{,c}^4\Xi(\eta_1-2\epsilon+6)+f_{,c}^2\big(9\Xi(2\eta_1-10\epsilon+13)\\
&+(3\epsilon-2)\Xi^2+243\big)-9f_{,c}(\Xi(\eta_1-4\epsilon+5)+27)+f_{,c}^3\left((8-6\epsilon)\Xi^2+18(2\epsilon-1)\Xi-81\right)\\
&+81\Big\}+\eta^3\left(-f_{,c}^2\right)\Xi\left(f_{,c}^2\Xi-9f_{,c}+9\right)\Big]-\Xi \qquad (52)
\end{aligned}
$$

where $\Xi \equiv k^2/\mathcal{H}^2 < \mathcal{O}(\epsilon) \ll 1$, $\epsilon_1 \equiv \dot{\epsilon}/(H\epsilon)$ and $\eta_1 \equiv \dot{\eta}/(H\eta)$. Since the analytic solution for the above equation is difficult to be computed due to time dependence of $f_{,c}$, which is not necessary slowly varying with time, we will study the important features of the solution for this equation numerically in the next section. However, from the structure of this equation, we expect that the dominant solution for Eq. (50) should be time dependent unless $f_{,c} = 1$. One can check that for $f_{,c} = 1$, coefficients A and B vanish, which corresponds to Einstein gravity.

B. Numerical result

To confirm rough analytic estimation in the previous section, we solve the evolution equation for curvature perturbation numerically. We start the numerical integration at the time when physical wavelength of perturbation is well inside the Hubble radius. The initial conditions are chosen according to Eq. (49) by spitting ζ to the real and the imaginary parts. We integrate Eq. (45) for the real $\zeta_{\rm real}$ and the imaginary $\zeta_{\rm imaginary}$ parts of ζ separately, and plot the absolute value $\zeta = \sqrt{\zeta_{\rm real}^2 + \zeta_{\rm imaginary}^2}$ in the following figures. According to discussion in the previous section, the main features of ζ-evolution depend on $f_{,c}$. Hence, we consider evolution of ζ for models 8, 10, 11 and 12 in which $f_{,c}$ varies by a few multiplication factor, $f_{,c}$ is nearly constant with $f_{,c} \lesssim 1$, $f_{,c} \gtrsim 1$ and $f_{,c} \sim 1$. From the

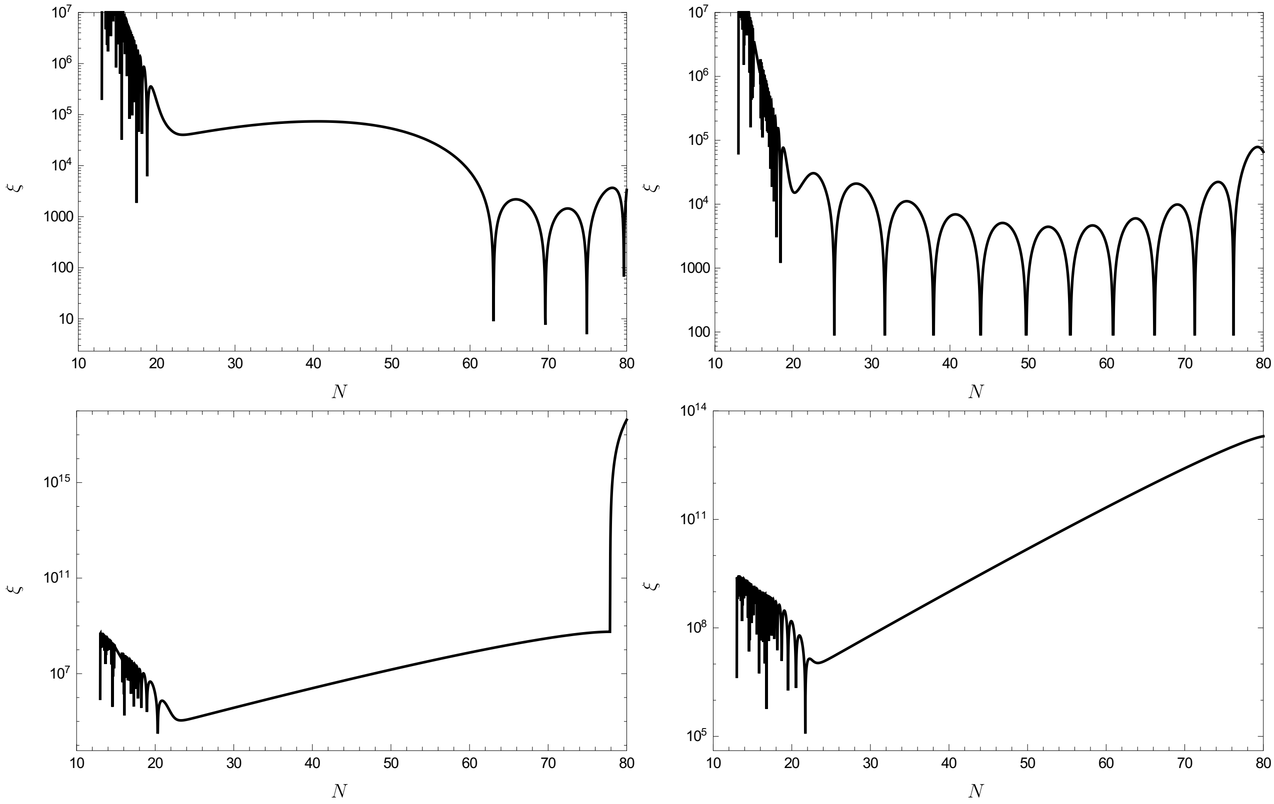

FIG. 4: Plots of ζ as a function of the number of e-folding. The left top, right top, left bottom and right bottom panels represent evolutions of ζ for models 8, 10, 11 and 12, respectively. In all plots, the perturbation crosses Hubble radius at $N = 20$.

plots in Fig. (4), we see that ζ can rapidly grow on super Hubble radius scales although $f_{,c}$ changes only fews percents around one through out inflation

C. Tensor perturbations

To study the tensor modes of perturbation, we write the metric tensor in the form of the background metric and tensor perturbations as

$$h_{ij} = a^2 \left(\delta_{ij} + \gamma_{ij}\right) , \qquad h^{ij} = a^{-2} \left(\delta^{ij} - \gamma^{ij}\right) , \tag{53}$$

where $\gamma^i_i = 0$ and $\partial_i \gamma^{ij} = 0$. Since the gauge-fixing term does not depend on tensor quantity, the tensor perturbation does not depend on the gauge and therefore the tensor perturbation computed from Eq. (6) and Eq. (4) are equivalent. Hence for convenience, we insert the metric from Eq. (53) into Eq. (4) and expand the action up to second order in perturbation. We obtain the second order action for the tensor perturbation as

$$S^{(2)}_T = \int dt dx^3 a^3 \left(\frac{1}{8f'}\dot{\gamma}_{ij}\dot{\gamma}^{ij} - \frac{f'}{8}\partial_i \gamma^{kl} \partial^i \gamma_{kl}\right) , \tag{54}$$

where the divergent term is omitted. The tensor perturbation γ_{ij} can be expanded in terms of the polarization tensors as

$$\gamma_{ij} = \int \frac{d^3k}{(2\pi)^3} \sum_{s=\pm} \epsilon^s_{ij}(k)\gamma^s_k(\tau)e^{i\vec{k}\cdot\vec{x}}\,, \tag{55}$$

where $\epsilon_{ii} = k^i\epsilon_{ij} = 0$ and $\epsilon^s_{ij}(k)\epsilon^{s'}_{ij}(k) = 2\delta_{ss'}$. According to the action Eq. (54), each of the mode functions $\gamma^s_k(\tau)$ obeys

$$\gamma^{s}_k{}'' + \frac{(a^2/f_{,c})'}{a^2/f_{,c}}\gamma^{s}_k{}' + k^2c_T^2\gamma^s_k = 0\,, \tag{56}$$

where $c_T^2 = f_{,c}^2$ is the sound speed squared of the tensor perturbations. As in the usual calculation, we define [20]

$$v^s_T \equiv z_T\gamma^s_k\,, \quad \text{where} \quad z_T^2 \equiv \frac{a^2}{4f_{,c}}\,, \tag{57}$$

so that Eq. (56) becomes

$$v^s_T{}'' + k^2c_T^2v^s_T - \frac{z_T''}{z_T}v^s_T = 0\,. \tag{58}$$

Applying the standard calculation, we have

$$|v^s_T|^2_c = \left.\frac{1}{2c_Tk}\right|_c\,, \tag{59}$$

which implies that the amplitude of tensor perturbation is constant on large scale, and we can compute the power spectrum for the tensor perturbations as

$$P^T_k \equiv \frac{k^3}{2\pi^2}\left(|\gamma^+_k|^2_c + |\gamma^-_k|^2_c\right) = \frac{2}{\pi^2}\frac{H^2}{f_{,c}^2}\,, \tag{60}$$

where the tensor perturbations cross the sound horizon at $aH = c_Tk$. The spectral index for the tensor perturbations can be computed as

$$n_T \equiv \frac{d\ln P^T_k}{d\ln k} = -2\epsilon - 2\frac{\dot{f}_{,c}}{Hf_{,c}}\,. \tag{61}$$

where $c_T^2 \equiv f_{,c}^2$. Using $\dot{c}_s/(Hc_s) \simeq \dot{f}_{,c}/(Hf_{,c})$, and Eqs. (22), (30) and (38), the tensor spectral index for the exponential and the power law potentials are respectively given by

$$n_T = -2\frac{n_* - 1}{N_N + N_*}\,, \tag{62}$$

$$n_T = -\frac{2}{(2n-1)}\frac{N_*}{N_N + N_*}\,. \tag{63}$$

V. CONCLUSIONS

We have studied models of inflation in the MMG theory. We have concentrated on $f(\mathbf{H})$ gravity in the form $f(C) = -\Lambda(-C/\Lambda)^{1+p}$, where C is the Hamiltonian constraint in the Einstein gravity, and p as well as Λ are constant. It can be checked that this theory reduces to Einstein gravity when $p = 0$. It is difficult for this theory to drive inflation without introducing an inflaton field We have examined inflationary models in which potential of the inflaton takes the exponential and the power-law forms. We have found that the slow-roll parameter ϵ is inversely proportional to the number of e-folding similar to case of Einstein gravity. The expression for ϵ in the case of power-law potential takes the form as in the Einstein gravity when $p = 0$. Nevertheless, there is no Einstein limit for the case of exponential potential. According to the evolution equation for the perturbations, it can be seen that evolution equation of perturbations depends on $f_{,c}$. For the case of exponential potential, $f_{,c}$ is inversely proportional to ϵ, so that $f_{,c}$ can vary a few order of magnitudes through out inflation. However, for the case of power-law potential, $f_{,c}$ becomes nearly constant when p is close to zero. From the numerical integration, we have found that the curvature perturbation on large scales can grow extremely large if $f_{,c}$ is significantly vary in time. The curvature perturbation becomes constant on large scales when $f_{,c} = 1$. For tensor perturbation, the sound speed of tensor mode can significantly deviate from unity and vary with time if $p \neq 0$.

Appendix: The expressions for the coefficients α and β

In this appendix, we present the explicit form of the coefficients α and β in Eq. (45). Firsly, we decompose them as

$$\alpha = \frac{n_1}{d_1}, \quad \beta = \frac{n_2}{d_2}, \tag{64}$$

where the expressions of n_1, n_2, d_1 and d_2 are given by

$$\begin{aligned}
n_1 &= a_1 + a_2 k_H^2 + a_3 k_H^4 + a_4 k_H^6 + a_5 k_H^8 + a_6 k_H^8 \,, \\
d_1 &= 4(3\eta + 2 f_C k_H^2)\Big(-8(-3+\epsilon) f_C^2 k_H^6 + 9\eta^3\big(-9 + f_C(9 + k_H^2)\big) \\
&\quad + 4\eta f_C k_H^4\big(18 - 6\epsilon - 9 f_C + f_C^2(9 + k_H^2)\big) + 6\eta^2 k_H^2\big(9 - 3\epsilon - 18 f_C + k_H^2 + f_C^2(18 + k_H^2)\big)\Big) \,, \\
n_2 &= b_1 + b_2 k_H^2 + b_3 k_H^4 + b_4 k_H^6 + b_5 k_H^8 + b_6 k_H^{10} + b_7 k_H^{12} \,, \\
d_2 &= 6(3\eta + 2 f_C k_H^2)^2\Big(-8(-3+\epsilon) f_C^2 k_H^6 + 9\eta^3\big(-9 + f_C(9 + k_H^2)\big) \\
&\quad + 4\eta f_C k_H^4\big(18 - 6\epsilon - 9 f_C + f_C^2(9 + k_H^2)\big) + 6\eta^2 k_H^2\big(9 - 3\epsilon - 18 f_C + k_H^2 + f_C^2(18 + k_H^2)\big)\Big) \,.
\end{aligned}$$

Here, a_i are

$$\begin{aligned}
a_1 &= -96 f_C^3(-2 - 3 f_C + 3 f_C^2)\mathcal{H} \,, \\
a_2 &= -432\eta f_C^3(-3 + 2 f_C - 2 f_C^2 + f_C^3)\mathcal{H} + 16 f_C^3\big(6\eta_1 + (2\epsilon - \eta f_C)(-2 - 3 f_C + 3 f_C^2)\big)\mathcal{H} \,, \\
a_3 &= 16\epsilon f_C^3(2\epsilon_1 - 2\eta_1 + 2\epsilon - \eta f_C)\mathcal{H} - 648\eta^2 f_C(2 - 5 f_C + 4 f_C^2 - 4 f_C^3 + f_C^4)\mathcal{H} \\
&\quad + 72\eta f_C^2\big(2\eta_1(2 + f_C^2) + 2\epsilon(2 - 5 f_C + 2 f_C^2) - \eta(-1 + f_C - 2 f_C^2 + f_C^3)\big)\mathcal{H} \,, \\
a_4 &= 324\eta^3(-4 + 15 f_C - 18 f_C^2 + 6 f_C^3 + 3 f_C^4)\mathcal{H} + 24\eta f_C\Big(f_C\big(6\epsilon_1\epsilon + \eta^2 - \epsilon\eta f_C(3 + 2 f_C^2) \\
&\quad + \epsilon^2(2 + 4 f_C^2)\big) + \eta_1\big(-2\epsilon f_C(2 + f_C^2) + \eta(-1 + f_C^4)\big)\Big)\mathcal{H} - 36\eta^2\Big(\eta(2 - 3 f_C + f_C^2 \\
&\quad - 6 f_C^3 + 9 f_C^4) - 6 f_C\big(\epsilon(6 - 11 f_C + 6 f_C^2 + 2 f_C^3 - 2 f_C^4) + \eta_1(1 + f_C + f_C^2 - f_C^3 + f_C^4)\big)\Big)\mathcal{H} \,, \\
a_5 &= 36\eta^2 f_C\Big(6\epsilon_1\epsilon - 2\epsilon^2 - 3\epsilon\eta f_C + 8\epsilon^2 f_C^2 + \eta^2 f_C^2 - 2\epsilon\eta f_C^3 + \eta_1\big(\eta f_C(-1 + f_C^2) \\
&\quad - 2\epsilon(1 + 2 f_C^2)\big)\Big)\mathcal{H} - 54\eta^3\Big(\epsilon(-20 + 66 f_C - 60 f_C^2 - 24 f_C^3 + 24 f_C^4) + f_C\big(\eta(4 - 3 f_C + 6 f_C^2) \\
&\quad - 6\eta_1(2 - f_C - 2 f_C^2 + 2 f_C^3)\big)\Big)\mathcal{H} \,, \\
a_6 &= 54\eta^3\big(2\epsilon_1\epsilon + \eta_1\eta f_C(-1 + f_C^2) + \epsilon^2(-2 + 4 f_C^2) + \epsilon f_C(\eta - 2\eta_1 f_C - 2\eta f_C^2)\big)\mathcal{H} \,,
\end{aligned} \tag{65}$$

and b_i are

$$
\begin{aligned}
b_1 &= 576f_C^4\mathcal{H}^2+5184f_C^4(-2+f_C+f_C^2)\mathcal{H}^2\,,\\
b_2 &= 192f_C^4(-2\epsilon+\eta f_C)\mathcal{H}^2+2592\eta f_C^3(-18+18f_C-f_C^2-4f_C^3+5f_C^4)\mathcal{H}^2\\
&\quad+864f_C^3\Big(\eta(1-f_C+f_C^3+2f_C^4)-2f_C\big(\eta_1-\eta_1 f_C+\epsilon(-6+3f_C+2f_C^2)\big)\Big)\mathcal{H}^2\,,\\
b_3 &= 16f_C^4(-2\epsilon+\eta f_C)^2\mathcal{H}^2+7776\eta^2f_C^2(-9+18f_C-11f_C^2-5f_C^3+7f_C^4)\mathcal{H}^2\\
&\quad+144f_C^2\Big(\eta^2(1+4f_C^2-f_C^3+4f_C^4+f_C^6)-4\epsilon f_C^2\big(\epsilon_1(-1+f_C)f_C+\epsilon(6-3f_C-2f_C^2)\\
&\quad+\eta_1(-2+f_C+f_C^2)\big)+2\eta f_C\big(\epsilon(-3+f_C-3f_C^3-3f_C^4)+2\eta_1(-1+f_C^4)\big)\Big)\mathcal{H}^2\\
&\quad+432\eta f_C^2\Big(\eta(-5-4f_C-3f_C^2+10f_C^3+8f_C^4-6f_C^5+6f_C^6)+6f_C\big(2\epsilon(9-8f_C+f_C^2\\
&\quad+2f_C^3-2f_C^4)+\eta_1(-3+5f_C-2f_C^2-f_C^3+f_C^4)\big)\Big)\mathcal{H}^2\,,\\
b_4 &= 34992\eta^3(-1+f_C)^2f_C(-1+3f_C+2f_C^2)\mathcal{H}^2+648\eta^2f_C\Big(\eta(-5+f_C-14f_C^2+17f_C^3-5f_C^4\\
&\quad-18f_C^5+18f_C^6)+18f_C\big(\eta_1(-1+f_C)^2(-1+f_C+f_C^2)+2\epsilon(3-5f_C+3f_C^2+2f_C^3-2f_C^4)\big)\Big)\mathcal{H}^2\\
&\quad+72\eta f_C\Bigg(\eta^2(-1+12f_C^2-6f_C^3+31f_C^4-6f_C^5+6f_C^6)+2\eta f_C\big(2\epsilon(1+3f_C+2f_C^2-18f_C^3+3f_C^5\\
&\quad-3f_C^6)+3\eta_1(-3-2f_C^2+f_C^3+4f_C^4-f_C^5+f_C^6)\big)-12\epsilon f_C^2\Big(\eta_1(-6+5f_C+f_C^2-f_C^3+f_C^4)\\
&\quad+2\big(2\epsilon_1(-1+f_C)f_C+\epsilon(9-7f_C+f_C^3-f_C^4)\big)\Big)\Bigg)\mathcal{H}^2-24f_C^2\Bigg(16\epsilon^3f_C^2(-1+f_C^2)\\
&\quad-\eta^2f_C(1+f_C^2)\big(\eta+2\eta f_C^2+2\eta_1 f_C(-1+f_C^2)\big)-4\epsilon^2f_C\big(2\eta_1 f_C(-1+f_C^2)+\eta(1+f_C^2+4f_C^4)\big)\\
&\quad+2\epsilon\eta\Big(\eta(1+f_C^2)^2(1+2f_C^2)+2f_C\big(\epsilon_1-\epsilon_1 f_C^2+2\eta_1(-1+f_C^4)\big)\Big)\Bigg)\mathcal{H}^2\,,\\
b_5 &= 8748\eta^4f_C(9-13f_C+2f_C^2+2f_C^3)\mathcal{H}^2+972\eta^3f_C\big(6\eta_1(-1+7f_C-6f_C^2-3f_C^3+3f_C^4)\\
&\quad-12\epsilon(-3+12f_C-11f_C^2-6f_C^3+6f_C^4)+\eta(2-11f_C+16f_C^2-16f_C^3-18f_C^4+18f_C^5)\big)\mathcal{H}^2\\
&\quad+36\eta f_C^2\Big(-4\epsilon^2\eta+48\epsilon^3f_C-8\epsilon\eta^2f_C+4\epsilon^2\eta f_C^2+5\eta^3f_C^2-48\epsilon^3f_C^3-24\epsilon\eta^2f_C^3+40\epsilon^2\eta f_C^4\\
&\quad+5\eta^3f_C^4-8\epsilon\eta^2f_C^5+8\epsilon_1\epsilon\eta(-1+f_C^2)+2\eta_1(-1+f_C^2)\big(12\epsilon^2f_C+\eta^2f_C(1+2f_C^2)\\
&\quad-2\epsilon\eta(3+5f_C^2)\big)\Big)\mathcal{H}^2+108\eta^2f_C\Big(72\epsilon^2f_C(-3+4f_C-2f_C^2-f_C^3+f_C^4)+\eta^2f_C(-2-3f_C\\
&\quad+23f_C^2-15f_C^3+9f_C^4)+6\eta_1(-1+f_C)\big(-6\epsilon f_C(2-f_C+f_C^3)+\eta(1+f_C+5f_C^2+2f_C^3+3f_C^5)\big)\\
&\quad+\epsilon\big(-72\epsilon_1(-1+f_C)f_C^2+\eta(10+6f_C+44f_C^2-114f_C^3+66f_C^4+36f_C^5-36f_C^6)\big)\Big)\mathcal{H}^2\,,\\
b_6 &= 4374\eta^4f_C\big(2\eta_1(2-2f_C-f_C^2+f_C^3)+\eta f_C(2-3f_C-2f_C^2+2f_C^3)-2\epsilon(7-8f_C-4f_C^2+4f_C^3)\big)\mathcal{H}^2\\
&\quad+54\eta^2f_C\Big(-4\epsilon^2\eta+48\epsilon^3f_C-12\epsilon^2\eta f_C^2+\eta^3f_C^2-48\epsilon^3f_C^3-16\epsilon\eta^2f_C^3+40\epsilon^2\eta f_C^4+5\eta^3f_C^4-8\epsilon\eta^2f_C^5\\
&\quad+4\epsilon_1\epsilon\eta(-1+f_C^2)+4\eta_1(-1+f_C^2)\big(6\epsilon^2f_C+\eta^2f_C^3-\epsilon(\eta+5\eta f_C^2)\big)\Big)\mathcal{H}^2-972\eta^3f_C\Big(-\eta^2(-2+f_C)f_C^3\\
&\quad-4\epsilon^2(-3+9f_C-8f_C^2-3f_C^3+3f_C^4)+\eta_1(-1+f_C)\big(\eta f_C(-2+f_C-3f_C^3)+2\epsilon(2-5f_C+3f_C^3)\big)\\
&\quad+2\epsilon f_C\big(4\epsilon_1(-1+f_C)+\eta(-3+7f_C-7f_C^2-3f_C^3+3f_C^4)\big)\Big)\mathcal{H}^2\,,\\
b_7 &= 81\eta^3f_C(-2\epsilon+\eta f_C)^2\big(\eta f_C+2\eta_1(-1+f_C^2)-4\epsilon(-1+f_C^2)\big)\mathcal{H}^2-729\eta^4f_C\Big(-20\epsilon^2+4\epsilon_1\epsilon(-1+f_C)\\
&\quad+24\epsilon^2f_C+8\epsilon\eta f_C+8\epsilon^2f_C^2-12\epsilon\eta f_C^2+\eta^2f_C^2-8\epsilon^2f_C^3-4\epsilon\eta f_C^3+4\epsilon\eta f_C^4+2\eta_1(-1+f_C)\big(\eta f_C-\eta f_C^3\\
&\quad+2\epsilon(-2+f_C^2)\big)\Big)\mathcal{H}^2\,.
\end{aligned}
$$

In the above expressions, $k_H = k/\mathcal{H}$, and we use Eqs. (14), (15) and (17) to write $f_{,cc}$ as

$$f_{,cc} = -\frac{1}{12X} f_{,c}^2 \left(\frac{\epsilon - f_{,c}\eta}{2} \right) . \tag{66}$$

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[1] A. H. Guth, Phys. Rev. D **23** (1981) 347.

[2] A. D. Linde, Phys. Lett. B **108** (1982) 389.

[3] A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48** (1982) 1220.

[4] Y. Fujii and K.-i Maeda, (Cambridge: Cambridge University Press), 2007

[5] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Physics Reports **513**, (2012) 1, [arXiv:1106.2476 [astro-ph.CO]].

[6] N. Afshordi, D. J.H. Chung, M. Doran, and G. Geshnizjani, Phys. Rev. D **75** (2007) 123509, [astro-ph/0702002].

[7] A. Iyonaga, K. Takahashi, and T. Kobayashi, JCAP **12** (2018) 002, [arXiv:1809.10935 [gr-qc]].

[8] A. Iyonaga, K. Takahashi, and T. Kobayashi, JCAP **07** (2020) 004, [arXiv:2003.01934 [gr-qc]].

[9] J. Khoury, G. E.J. Miller, and A. J. Tolley, Phys. Rev. D **85** (2012) 084002, [arXiv:1108.1397 [hep-th]].

[10] C. Lin and S. Mukohyama, JCAP **10** (2017) 033, [arXiv:1708.03757 [gr-qc]].

[11] K. Aoki, C. Lin, and S. Mukohyama, Phys. Rev. D **98** (2018) 044022, [arXiv:1804.03902 [gr-qc]].

[12] K. Aoki, A. De Felice, C. Lin, S. Mukohyama, and M. Oliosi, JCAP **01** (2019) 017, [arXiv:1810.01047 [gr-qc]].

[13] S. Mukohyama and K. Noui, JCAP **07** (2019) 049, [arXiv:1905.02000 [gr-qc]].

[14] An. De Felice, A. Doll, and S. Mukohyama, JCAP **09** (2020) 034, [arXiv:2004.12549 [gr-qc]].

[15] X. Gao, Phys. Rev. D **90** (2014) 081501, [arXiv:1406.0822 [gr-qc]].

[16] C. Lin, JCAP **05** (2019) 037, [arXiv:1811.02467 [gr-qc]].

[17] C. Lin and Z. Lalak, [arXiv:1911.12026 [gr-qc]].

[18] R. Carballo-Rubio, F. Di Filippo, and S. Liberati, JCAP **06** (2018) 026, [arXiv:1802.02537 [gr-qc]].

[19] K. Aoki, A. De Felice, S. Mukohyama, K. Noui, M. Oliosi, and M. C. Pookkillath, Eur.Phys.J.C **80** (2020) 708, [arXiv:2005.13972 [astro-ph.CO]].

[20] J. Garriga and V. F. Mukhanov, Phys. Rev. Lett.B **485** (1982) 219.